Networks and systems
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Lecture-8
Unit Impulse or Delta Function

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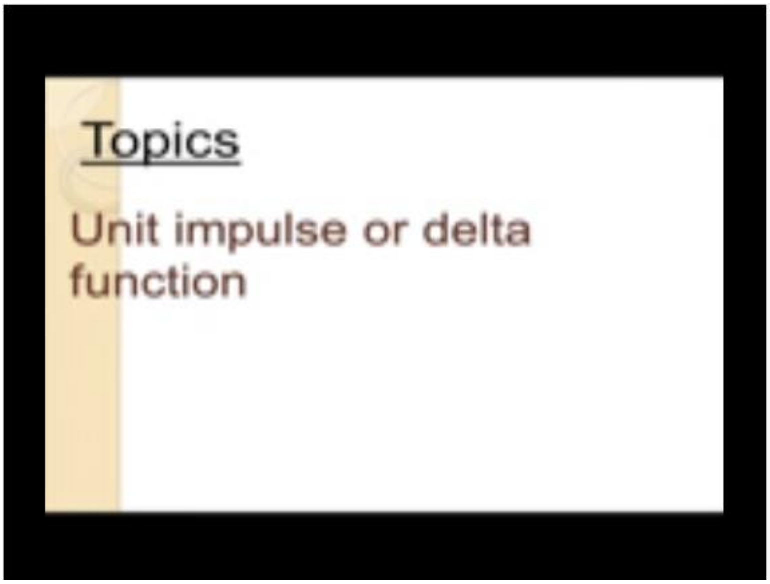

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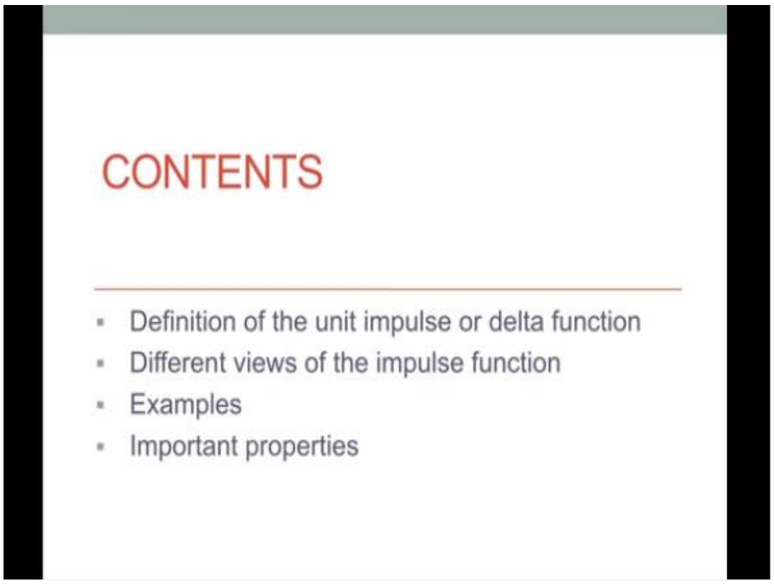

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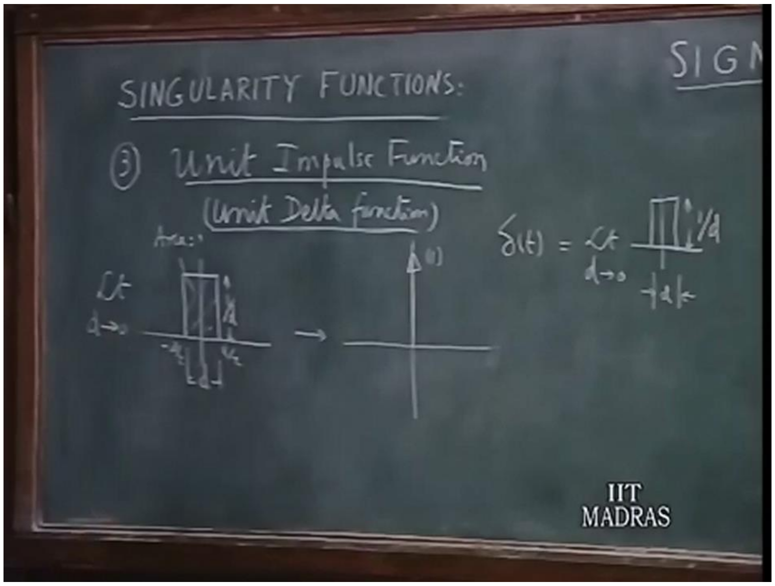

The last singularity function which we will talk about is the unit delta function or unit impulse function. It is also called unit delta function also referred as dirac delta function because it was introduced by dirac. Now, 1 can introduce this in this fashion suppose i have, a symmetrically situated pulse symmetrically situated around the origin a width d and height 1 over d.

So, this pulse has a width d and a height 1 over d. Now let us imagine, what happens when d is reduced to by a factor 2 then the pulse width diminishes, but the height increases. So, that the area under the curve is 1. So, the area is equal to 1. So, maintaining that area as you make d smaller and smaller then the pulse width becomes smaller, but the height grows.

So, that the area under the curve is maintained equal to 1 and this is d by 2 this is minus d by 2 and it is still centered around the origin. So, imagine what happens when d goes to 0, when d goes to 0 you should have identically ideally a pulse which has got 0 width but a height which is infinitely large. So, that is what is called the impulse function or delta function.

Which is indicated because, we can't mark the point at infinity in a black board or in your note book. We will indicate an arrow like this and say this is 1 indicating that the area under this curve is equal to 1 and this is called delta t this function is called delta t. So, delta t is we can define it in this manner limit as d goes to 0 of this symmetrically situated pulse width d and height 1 over d that is 1 way of defining this delta function. You can also think of sort with other wave forms like triangle wave form and take the limit and it goes 1 that is also possible, but we will confine our discussion to this.

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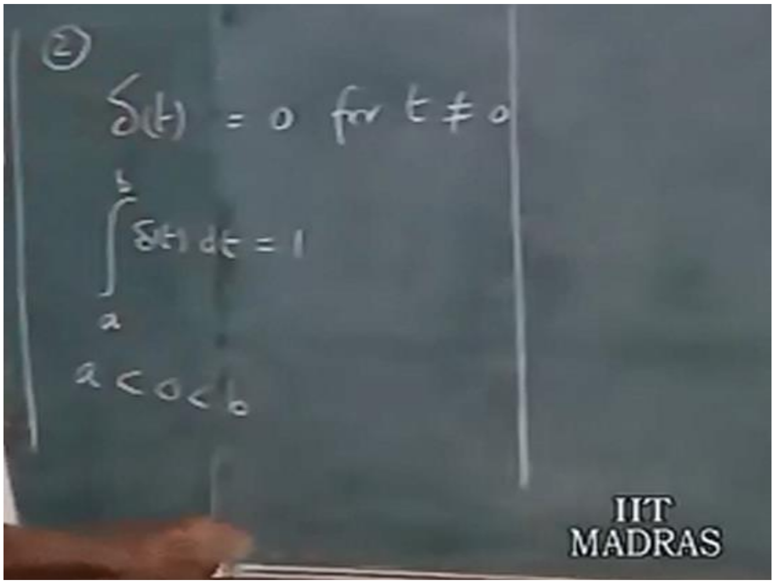

A second way of defining delta t mathematically would be say delta t equals 0 for t not equal to 0. So, this is 0 for every value of t except t equals 0. For t equals 0 it has infinite type. We do not know how to describe this. So, all we can say is if you integrate delta t from an interval a b, such that 0 is in between a and b that means, the range of integration includes that delta.

This is equal to 1. That means, the area under this curve this infinitely large amplitude 0 width curve is said to be equal to 1 and therefore, as you integrate through the origin integral from a to b of delta t d t equal to 1. So, that is 1 way of defining this.

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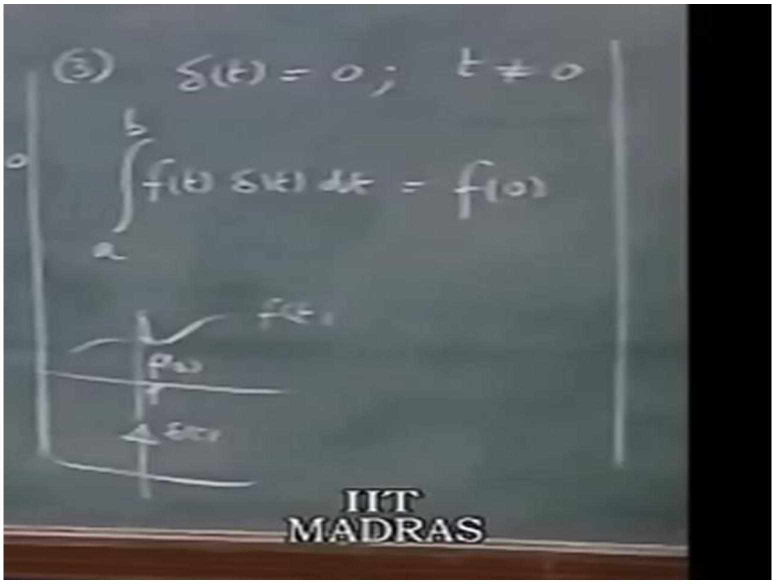

A third way of looking at this delta function is this. Delta t equals 0 for t not equal to 0 as before, but suppose you take a function f of t multiply by delta t and integrate from a to b. Then, what do you have, you have this delta function multiplying a function f of t. So, you have a continuous function. Let us say, f of t is a continuous function and then you are having this delta function.

When you multiply these it is 0 everywhere except at this point. At this point f of t is multiplied by delta t. So, f0 this value f0 is this value is multiplying this delta t. And the integral delta t d t is equal to 1 all we have now is instead of delta t dt you have f0 delta t dt therefore this is equal to f 0. So, this is another way of looking at a delta function any continuous function f of t if you multiply by delta t and integrate it samples that value of f of t at the point where the delta is situated delta is situated t equals 0.

So, you want to find out the value of the function f of t at t equals 0 you multiply f of t delta t dt integrate to get f of 0. So, these are all equivalent ways of looking at this delta function. We can adopt this as a definition sometimes we can even use this as a definition or you can take this as a definition all are equivalent ways of looking at this. Important thing is that this delta function is something again as I mentioned which may not be which is not coming under the fold of the classical mathematics.

So, originally when it is introduced it was introduced by applied mathematicians and engineers. And later on, mathematicians established a theory called theory of distributions which makes the manipulations of delta functions puts it on a more sound basis of rigorous mathematics and therefore, we don't go into the theory of distributions, but there is a branch of mathematics called theory of distributions which gives a theoretical basis to the use on manipulation of delta functions okay.

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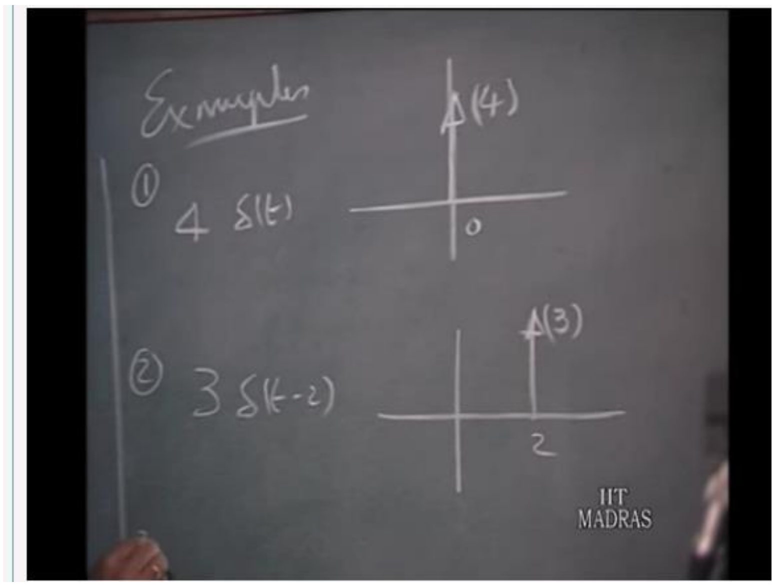

So, let us now let me now plot some examples of, how different types of delta functions can be taught can be conceived. Suppose I have 4 delta t, it is an impulse function standing at t equals 0 and its magnitude is 4. I mean, we indicate its magnitude 4 in the sense that it is no doubt infinity but the area under the curve is 4 units.

So, the magnitude of a delta is indicated within brackets in this fashion. Suppose, I have 3 delta t minus 2 it means the magnitude of the delta function is 3, but it is now sitting at t equals 2 because t wherever the argument of the delta is 0 then only delta exists otherwise it is 0 that means, this will be t equals 2 and its magnitude is 3 thats how 3 delta t minus 2 looks like.

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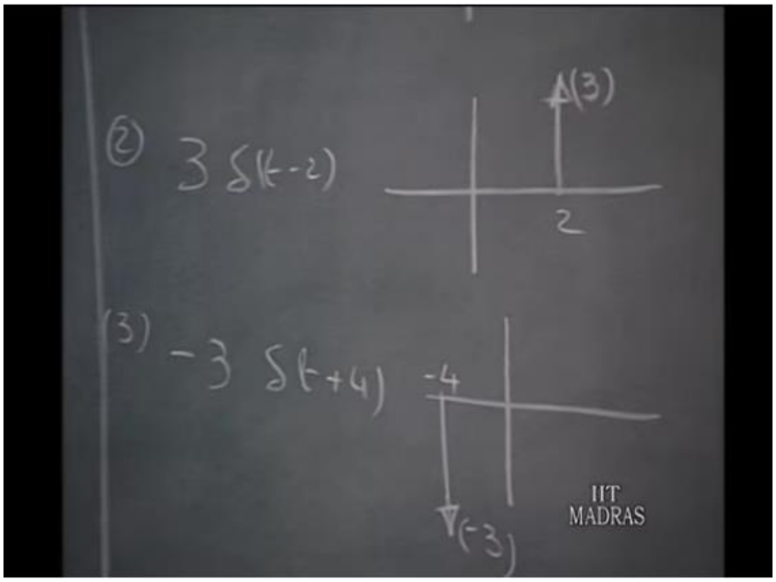

Suppose you have minus 3 delta 3 plus 4 it means that at equals minus 4 that is the position where delta is fixed and it is a value equal to minus 3 therefore, it is a negative going impulse with a magnitude of minus 3 thats how it goes.

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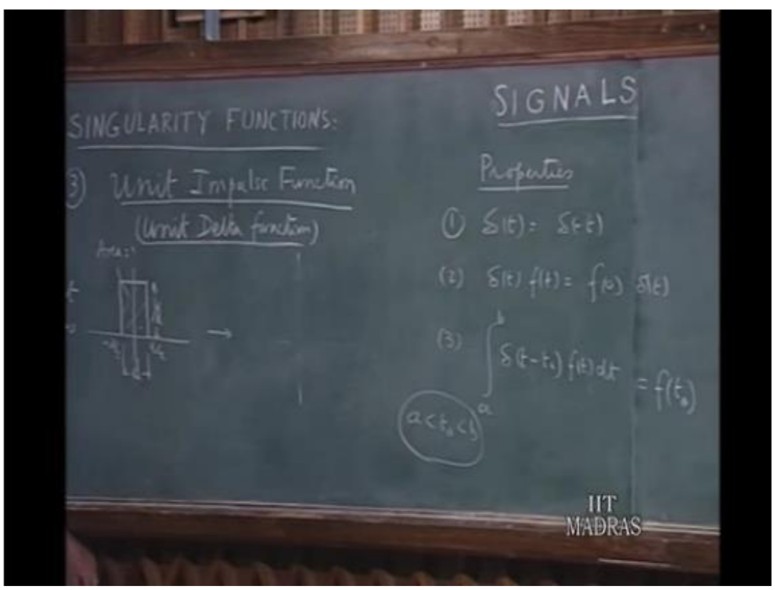

A few further properties, Delta t equals delta minus t, it is an even function of time. So, delta t exists at t equals 0 Delta minus t also exists at t equals 0 and nothing else.

Therefore, whether it is plus 0 or minus 0 it makes no difference therefore, this is an even function of time. Secondly, delta t multiplied by f of t equals f 0 multiplied by delta t because f of t multiplied by delta t is 0 for all values of t except t equals 0. So, the value if at all exists it exists only at t equals 0 at the time f 0 time delta t is the value that we have.

Thirdly if I integrate delta t minus t 0 times f of t dt over a range which includes t 0 over a range which includes t 0 then obviously, this will be everywhere this integrand is 0 except when t equals t 0 because at that time delta t minus t 0 exists. So, at that time f t 0, which is a constant delta t minus t 0 is delta function is being integrated therefore, this will yield f t 0.

So, this will yield f t 0 that is how it goes. In other words, if you multiply f of t delta t 0 this is after all f of t 0 delta t minus t 0 and you integrate f of t 0 delta t minus t 0 dt. Whenever you integrate through a delta the value increases by 1 unit and that 1 unit multiplied by f of t 0 leads this result.

So, this can be put in this fashion. Whenever you integrate through a delta at a particular point whenever whatever, suppose you have some kind of curve like this which includes a delta function. If you integrate that at this point because of delta it jumps like 1 because of the delta function.

Close to summarize we had in this course of this lecture reviewed the concept of complex frequency signal and identified the complex frequency components present in some example signals which we have taken up and then later on we introduced ourselves to the 3 singularity functions which play a very important role in describing functions which are either continuous or discontinuous which are either discontinuous by themselves or are discontinuous derivatives.

The 3 functions that we have familiarized ourselves with are the unit step function, the unit ramp function and the unit delta function. All these play a very important role in describing either response variables or excitation variables which are piecewise continuous, but have discontinuities at some points either in themselves or in the derivatives we will take up some examples involving the use of these functions in our next lecture.