

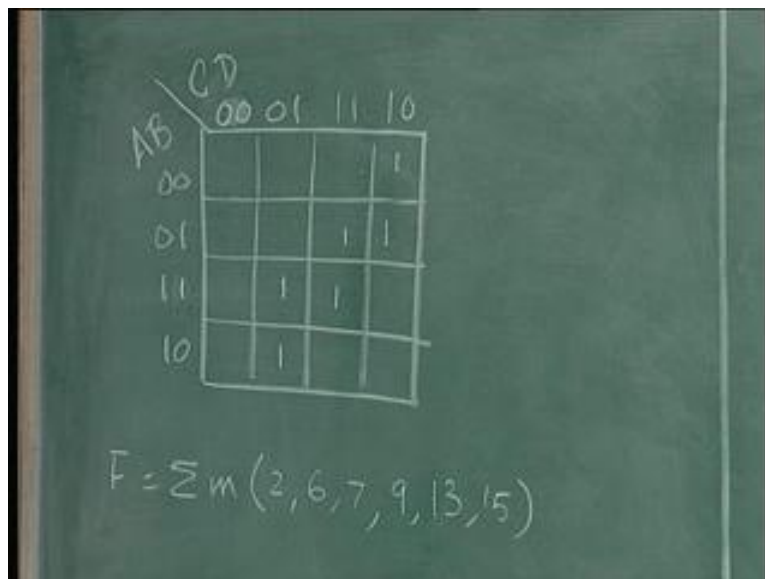
Digital Circuits and Systems
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Lecture - 7
Logic Minimization using Karnaugh Maps

In the last lecture we introduced you to the map method of Boolean simplification. we drew a Karnaugh Map which is a graphical representation of a truth table filled this graph with 1s corresponding to the cells for which whose min terms had a output true and 0s of course we didn't mark 0s to avoid cluttering of the map. Whereas we did not mark 1 in the map the entry was a 0 which was purposely left out so that the map doesn't get cluttered.

The objective is to identify groups of 1s as large as possible of course satisfying the adjacency rule and remove or knock off as many variables as possible so that by repeatedly doing this we can get a simpler representation of a given Boolean function. So we look at a few more examples today some special cases and all that.

We will take an example of a map like this, four variables I will call these variables ABCD, want to say this map as 1s in the following cells, this is the pattern of 1s. That means this is a function this expresses the sum of the following min terms, this is min term number 2 so you will have to sort out from here 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15, so we take (2, 6, 7, 9, 13, 15) this is the map.

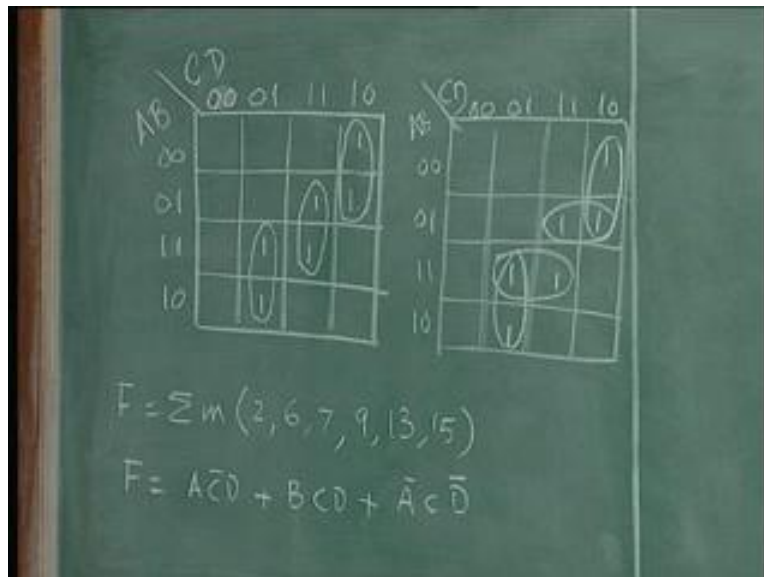
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Now the idea is to group the adjacent ones with as large groups as possible keeping in mind the group should be as large as possible. If you have a smaller group which can be totally submerged into a larger group we should not consider the smaller group and we have to make sure that all 1s are covered and we do not mind one being covered more than once these are the rules. Make as large groups as possible keeping in the mind the adjacency rule, make sure all 1s are covered, do not worry about combining a given 1 more than once. So in this map groups of two 1s are only possible 1 2 3 4 so I can say this because this one has to be covered any way, this is standing out separately.

The most efficient way would probably be not covering anything more than once. You can cover 1 more than once if it can result in a smaller group. If something is not going to result in a smaller group then there is no point of covering 1 more than once. So in this case there are only two ones this is going to be a group, this is going to be a group, this is going to be a group all groups are having two 1s so there is no point in unnecessarily doing this and doing this. So this is how I will write this three groups of two 1s each and what would this be in terms of the product expression? F would be sum of these products 1 2 3 this would be $AC\bar{D}$ plus then this will be BCD plus, this would be $A\bar{C}D$ bar

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Now instead of doing this way suppose I marked it slightly differently. That means suppose I did it like this it is obviously not the most efficient way you can see that because there is a small map with few 1s but in general when you have a large map with large possible groups you may miss out and may do things like this. I brought this up to explain some of the definitions we gave in the last lecture. I talked about implicant, the prime implicant and an essential prime implicant. An implicant is any group of 1s and a prime implicant is the largest possible group of 1s in the given group of course that cannot be an essential prime implicant, a prime implicant is a group which cannot become part of another implicant. An essential prime implicant is one which I said if you

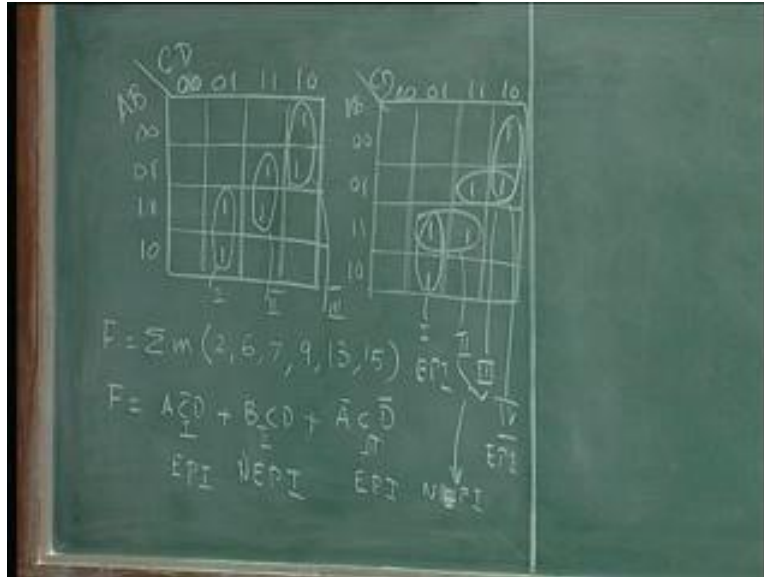
remember has at least an entry of one not covered in any other prime implicant. So this way here (refer Slide Time: 9:30) this is the essential prime implicant I will call this 1 2 3, this is 1, 1 is an essential prime implicant because this one cannot be covered otherwise, 3 is an essential prime implicant because this cannot be covered otherwise.

Now is 2 an essential prime implicant or not is the question? Do you think 2 is an essential prime implicant or not?

The way I draw it is an essential prime implicant because these two 1s cannot be covered anyway. But when I draw this map there are two ways of combining these two 1s. This is an essential prime implicant, this is an essential prime implicant (Refer Slide Time: 10:28), is this a prime implicant? What is a prime implicant? The largest group of 1s possible within that, it's a largest group of 1s possible in that group. So all the four here I call this 1 2 3 4, here all the four are prime implicants, here all the three are prime implicants but one and three are clearly essential prime implicant, 2 is not an essential prime implicant it is a non essential prime implicant for the simple reason I can cover these two 1s in two other ways. So in this case again 1 is an essential prime implicant, 4 is an essential prime implicant so 2 is a non EPI.

But the reason that this one could have been combined with this I could have it small it is an essential prime implicant. I could have covered this to make another prime implicant. So this is the only way in which I can make prime implicant otherwise that one gets left out only then it is an essential prime implicant. We are not talking about the most efficient way here remember that I am only talking about definitions. Just because it is an essential prime implicant it does not mean that it is the most efficient way of doing it. so these are non essential prime implicant, it is a non essential prime implicant because this one could have been combined with this to get another term, this is two in this case, this term could have been combined with this term to get another term 3 in this case so this is a non essential prime implicant (Refer Slide Time: 12:14) but the same argument 1 and 4 are EPI's and 2 and 3 are non essential prime implicants. Having drawn in this case F is equal to 1 or 2 or 3. In this case F is 1 or 2 or 3 or 4 and all are two terms, two 1s all are two variables, a three variable term there are only three terms in this case, four terms in this case and it is very obvious that this is a less efficient simplification compared to this.

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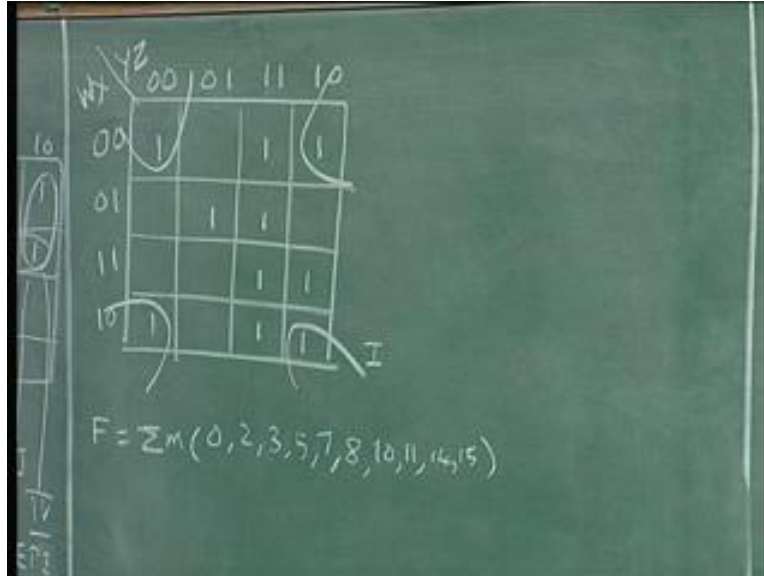
But the fact remains that there are some non essential prime implicants that has to come to the final expression in order to complete the expression. This is a very clear case of a non essential prime implicant inefficiently used. In this case a non essential prime implicant has been used efficiently because it was very clear and obvious apparent just looking into this map.

Sometimes there may be two non essential prime implicants, each of them can lead to a final solution which is correct and both of them may have the same simplicity or complexity in which case you have to choose between 1 and 2 or one or the other of the essential prime implicant in which case the choice is not unique but in this case the choice is almost unique. I would not choose this against this if my aim is to reduce the logic which is again you have been proclaiming I will not go for this. But I will give you another example where I could have combined in two ways both of them resulting in a same simplified expression in terms of number of variables and number of terms involving a non essential prime implicant that has been an interesting case. So we will take another example.

For a change I will call it WXYZ and this is my map (Refer Slide Time: 14:59). So what is the map now? F is sigma M (0 2 3 5 7 8 10 11 14 15) these are the min terms for which the output is true for all the min term outputs. I just wrote the map and then wrote the min term. Normally it is the other way. Normally you are given the min terms for which the output is 1 from the truth table or from the word description of the problem. Either a word description of the problem or a truth table is given to you which will tell you what are the min terms for which the output is 1 and automatically the other outputs are 0. But in this case just to illustrate a point in terms of implicants, prime implicant, essential prime implicants, non essential prime implicants and so on I am just taking maps to give you examples so I have to first draw the map and then tell you what are the min terms which are true in this map.

Now what are the different ways of combining them? We have to again efficiently do it.

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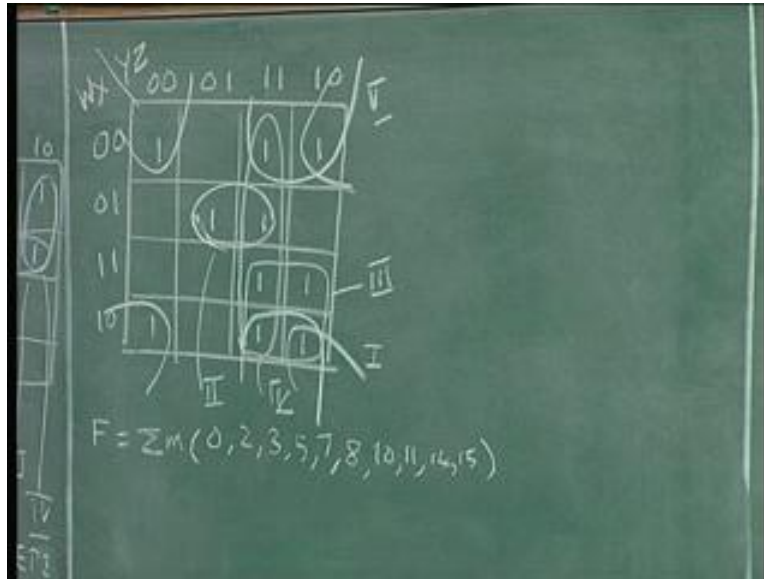
You have to make smallest possible number of prime implicants and make a combination which is the minimum possible combination. So now this one, this one, this one, this one together form a map (Refer Slide Time: 16:50) group a prime implicant because they roll over top to bottom and left to right so I will call this first prime implicant.

I said the other day the adjacency, this cell is adjacent to this cell as well as to this cell, this cell is adjacent to this cell as well as to this cell, this is adjacent to this as well as this and so forth. Hence all the four cells are adjacent to each other, it is very similar to something like this or something like this. and this is in essential prime implicant because this one would have been left out otherwise or this one for that matter because I could have combined this some other way but not this, I could have combine this but not this (refer Slide Time: 17:50). So because of these two 1s **we present** this becomes an essential prime implicant and essential prime implicant has to necessarily find its place in the final expression.

The debate is only about how many non essential prime implicants to be included to keep the function minimum and at the same time not omitting any one in the table that is the only thing will have to be concerned with. now this one again has to be combined there is no other way I could combine this one except this way otherwise this one gets left out completely so I will call this 2, one this is 2 now I have two options, I can combine these four into an essential prime implicant. In fact I should combine this way because this one would be left out otherwise of course I can always combine these two, these two are combining is not a solution because a prime implicant is one which absorbs smaller implicants so you put this one is not a solution because when I have four larger prime implicant I cannot put this one as an implicant I have to cover this. So this is an essential

prime implicant for the simple fact this one would not be covered otherwise so this will be number 3. Thus the only one that is left out is one. In order to complete my simplification I have an uncovered one and that one has to be included so this I can do it in two ways I can do it in this way I will call this 4 and these four 1s are adjacent to each other because this is adjacent to this and so forth or I could have combine these two 1s and these two 1s so either this way or this way these two 1s.

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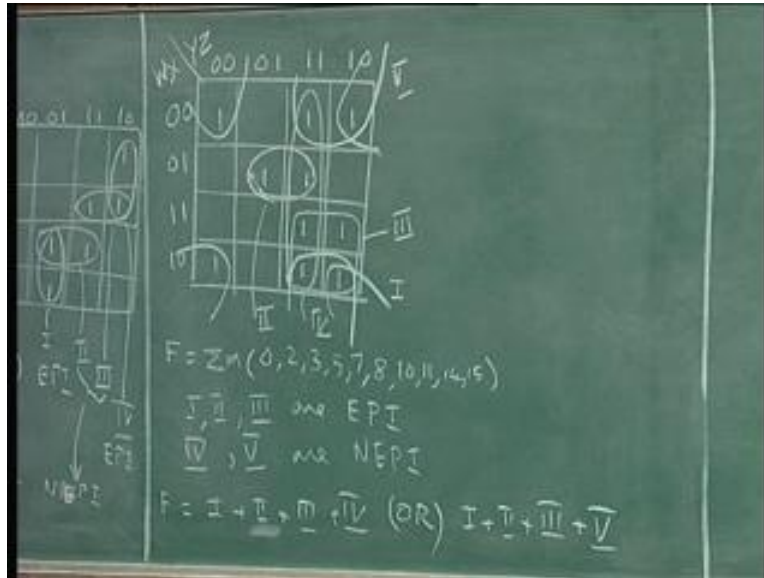


Now this one was what was left out remember, this min term number 3 was left out I had to include it in order to include it in a largest possible group of 1s I can do it this way or I can do it this way and this one I will call 5. Unlike in this case where we had an option which is obviously less efficient. We now an option in which one is essential prime implicant, II is essential prime implicant, III is essential prime implicant because of these individual things, IV and V are non essential prime implicants because essential prime implicant is 1 in which at least one unique 1 is covered in that group no other way it should be able to cover that one that is not the case with IV and V because this one can cover this way or this way so everyone of this 1 can be covered and all these other three 1s can be already taken care off. In this case all these have been taken care off except this one this has been taken care of, this has been taken care off, everything other than this has been taken care off so this one can be covered either this way or this way so when IV and V are two options of covering this min term represented by this III then it is not an essential prime implicant, IV and V are non essential prime implicants.

Therefore I II III are EPI's and IV and V are non EPIs. But it is enough one of IV or V is included because four and five both cover this one and this is the only one which has uncovered. After considering I II and III the only entry in the truth table which was not covered in our simplification was this entry which will be covered either by this or this. Both IV and V can cover this one so where is the need for both of them to be present and the choice is not easy because both are of same complexity, this is four ones this also is

four ones. If I is 4 1s the other is 8 1s so the complexity is different so you know what to choose. If both have equal complexity where is anything to choose from?

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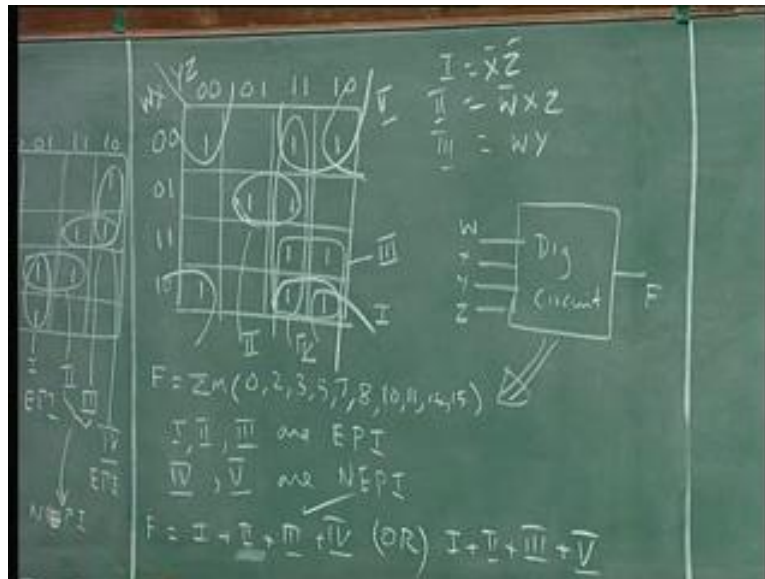
So now I can write the final expression as I OR II OR III OR IV this is one option or this OR is English OR not the operator OR I OR II OR III OR IV, this explains to you a non essential prime implicants and its role in the final simplification. So I wanted to bring about the definitions of implicant, prime implicant, essential prime implicant and non essential prime implicant and the role of non essential prime implicant in simplification. It is not that they are trivial they can be ignored they are essential sometimes so you may have to include. But how many of them you will include and what are the non essential prime implicants you will include depends on the overall goal of minimum sum of products.

I don't want to re write this now, you know each of these. We can complete this, I will have I II III IV but I II III IV are variables of or product terms with variables ABCD or in this case what WXYZ. So I II III IV and V are each product terms of variables WXYZ you write it and finally get the simplified minimum sum of product expression.

Now we can have a four variable map. What is meant by this? Let us do this for example. we started with simple gates and we drew the simple structure using gates and wrote down the expression and then said that write a truth table for that expression and we wrote the truth table found out the truth table had many more entries than the original circuit had so we tried to see why and we saw that the simplification of the truth table resulted in a smaller circuit that we started and we said simplification is possible using either Boolean algebra which you saw or by mapping method which you have seen so let us revisit that and see now. We will do the reverse now. This is the sum of product expression you have given. a system of four variables WXYZ having an output F circuit of system has the following Boolean functionality as a functional relationship between

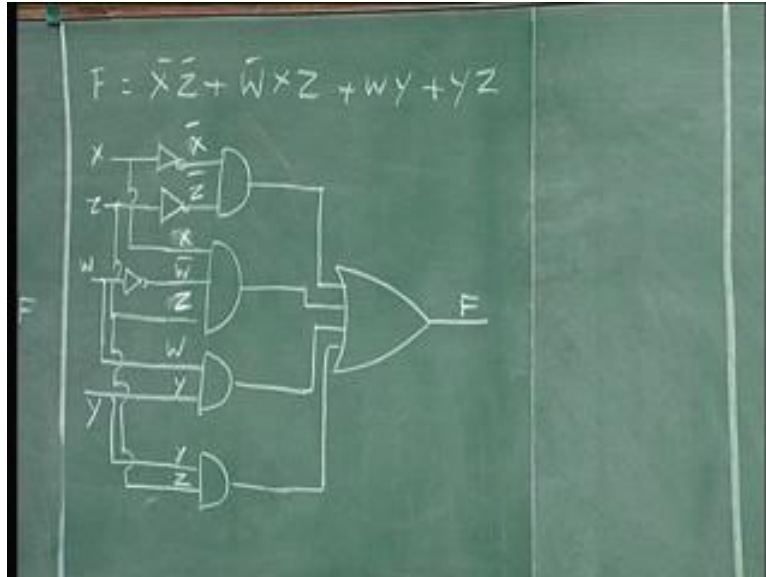
input and output given by this Boolean equation not identity Boolean expression this Boolean expression gives the relationship between F on one hand of the output and WXYZ on the other hand as inputs.

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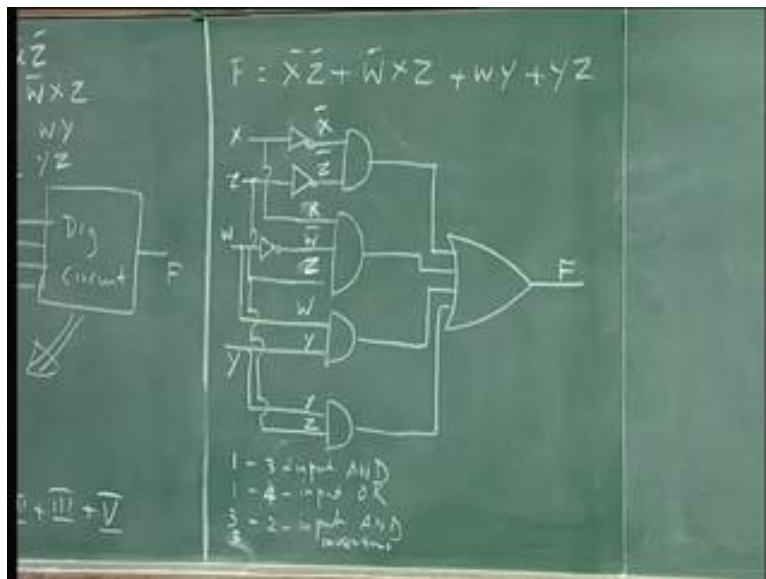
What will it look like we will see because we have simplified this? For that I need to explain this 1 2 3 4 5. I have no reason to choose this or this so we will go with this. So let us quickly write what is 1. 1 is these four 1s, what are the expression terms of WXYZ? This is X bar Z bar so I is X bar Z bar, II is these two W bar XZ, III is this which is WY and IV I am going to take instead of V so IV would be YZ. So my expression is done X bar Z bar W bar XZ WY YZ these are gates with inputs X and Z of course inverted WXZ where W is inverted W and Y, Y and Z these are four AND gates, outputs of these four AND gates tied together by an OR gate that is, this is X again, W, Z second term, third term is WY and fourth term is YZ, Z has to come from here which is here.

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So if you don't want to lose track of these inputs you can write it one more time here, YZ, WY, this is ZW bar X, XW bar Z, this is W bar X bar Z bar all of them combined in an OR gate with four inputs. This is the function represented by this circuit whose Karnaugh Map is this and whose functional description is this.

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Of course I can always write the truth table. The truth table is only an expansion of this, I will have to put WXYZ all the four combinations 0 0 0 0 to 1 1 1 1 and then put here finally 1 0. So it requires one 3 input AND gate, one 4 input OR gate, three 2 input AND gates and three inverters, this is inventory. You have to ask for these components in order

to build a circuit and you can now associate with WXYZ some signals as we did in the example previously about some system which is to represent a particular condition WXYZ may be four parameters we are monitoring and on these incidence of parameters in this particular way will trigger an event at the output of this system that could be an example of the use of this.

So you go for a definition of the word description of the truth table and the Boolean relationship mapping and then simplification and then drawing the circuit and then of course you have to get the components, build it and test it. Now the variables XYZ we have used three or four WXYZ and IV or V you may have used, VI we may use, seven we may use and digital systems today are very complex. A system like a micro processor, a system like a controller, a system like a automotive system or a fuel injection or even very complex systems like missiles I already told you missiles and aircrafts and things like that so I don't you can work with three variables four variables and three variables and four variables classroom exercises.

What will happen if variables are more than four? We have already represented a two dimensional now it has to go for three dimensional. For a five variables what do we do? So let us say we have a five variables let us call these variables ABCDE.

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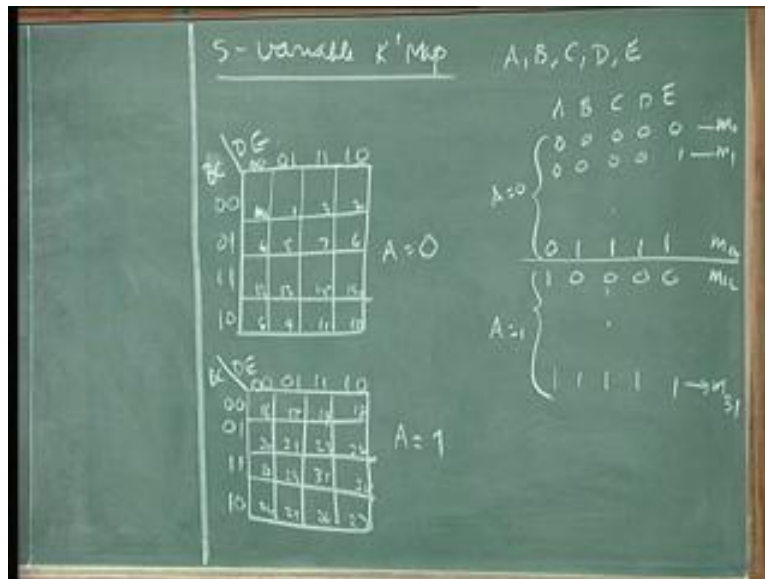


I can only represent sixteen cells in a four variable map two dimensional and five variables will have how many min terms? It will have 32, 2 power 5 so we need to have one variable. The variables we start listing them in a truth table will become starting with 0 0 0 0 0 0 0 0 1 and finally it will become (Refer Slide Time: 34:38) this is min term 0, min term 1, min term 31 or I told you that you will have to start always with 0 min term and go on till the last min term which is 2 power n minus 1, 5 is 2 power 5 is 32 minus 1 is equal to 31 so 0 to 31 will be min terms. So one way to do this would be draw two maps considering only four of these five variables each. You leave the A out because first

sixteen entries of the truth table m_{15} would be 0 1 1 1 1, m_{16} would be 1 0 0 0 0 so these entries will have A is equal to 1, these entries will have A is equal to 0. First sixteen entries of the truth table will have A is equal to 0 and second sixteen will have A is equal to 1. So I will draw this map with variables BCDE always remembering A is equal to 0 in this map. Likewise I will do BCDE one more time always remembering A is equal to 1 in this map.

Now the min terms would be m_0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 so whichever min term is one output you map it map in the corresponding cell either here or here if it is the min term corresponding to terms between 16 and 31 the 1 will up here so it will go in this part of the map. So you can map it the way you would map the four variables map graphically but while reading you have to be careful. The adjacency works between these two maps. The one here and the one here are considered adjacent (Refer Slide Time: 37:54) because the only variable different from these two maps this is B is equal to 0 C is equal to 0 D is equal to 0 E is equal to 0 and this is also B is equal to 0 C is equal to 0 D is equal to 0 E is equal to 0. If I have 1 here and a 1 here the variable A is 0 here, variable A is 1 here so these two cells are adjacent, the cell number 0 and 16 are adjacent.

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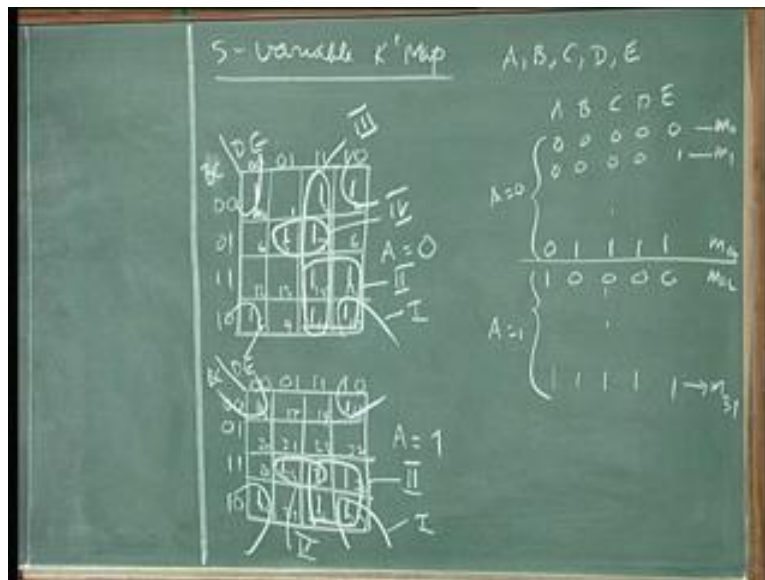
Therefore the corresponding cells in this map and this map are adjacent that property has to be built in while reading the map that's all. So I map the entire variable so we will use the same arbitrary thing, you will do this 0 2 and this for a change I will put here. Now these four 1s which you called prime implicant one essential prime implicant one here will appear here also in both the maps.

So this I will call 1 including this, these two together are 1 (Refer Slide Time: 39:37). Now these four again I don't know what we called it here we called it III but don't worry

about it, this and this are again are common between these two maps the same cells so I will call it II this and this. Now what is left out?

These four are left out I could have combined this way or this way doesn't matter I will combine this way but this is not common to this map 1 1 1 1 here unique to A is equal to 0 OR not to A is equal to 1. So I am going to call this III in which I will have to make A is equal to 0 in that. This one is unique to this map but not to this map. So this is III and these two 1s I will call it IV because this is not here, this one is unique not to the other map so V.

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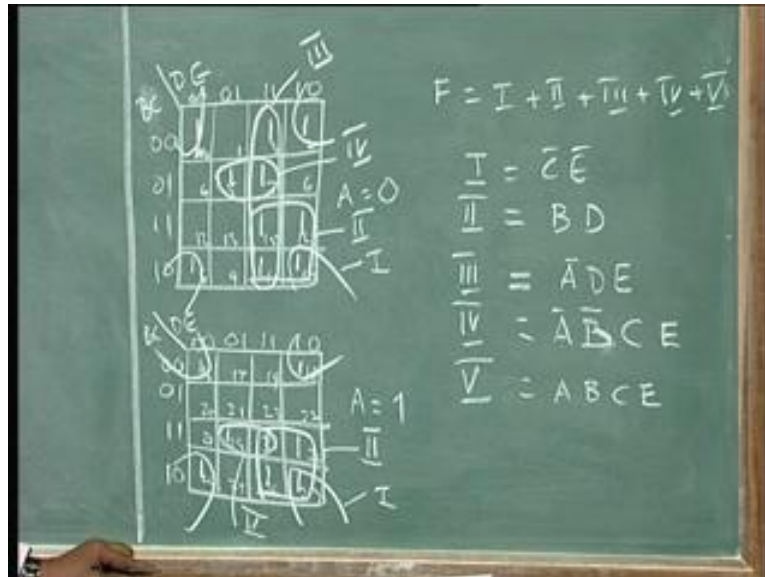


So there are five prime implicants all are essential I made sure that is simple because it is an example, with five variables I wanted to drive home the point of how to simplify a five variable map so I made them essential prime implicants so you need not worry about non essential prime implicants or proper choice of that so we have five EPI's 1 2 3 4 and 5, actually writing the minimum sum of product expression based on this is called reading the map. Somehow they call this reading the map so we should be able to read the map now, they use the term reading the map.

After putting all these 1s and grouping them identifying them in groups you are writing the final expression and that operation is called reading the map. So now I have to read the map to get 1 2 3 4 5 and then put it together as one expression of F in terms of ABCD and E. already we have said that I was here this is C prime E prime and since this is common between A is equal to 0 A is equal to 1 so 1 is C prime E prime or C bar E bar want to call it C prime E prime C bar E bar or NOT C AND NOT E whatever, II is again common between these two maps that would be BD then comes the third map third EPI and the third EPI is this between these four cells which is DE. But this is not available here so I will have to include the information and it is only available in the A bar part of the map. I will make sure that you know it is in A bar part of the map and not A part of

the map so I have to say A bar and B so III is A bar DE. And IV is again only this essential prime implicant which is B bar C B bar CE but you have to add A bar to that so it is A bar B bar CE but it is found in the A bar part of the map so it is A bar. And finally this fifth prime implicant this part A is equal to 1 part of the map which is BCE with A.

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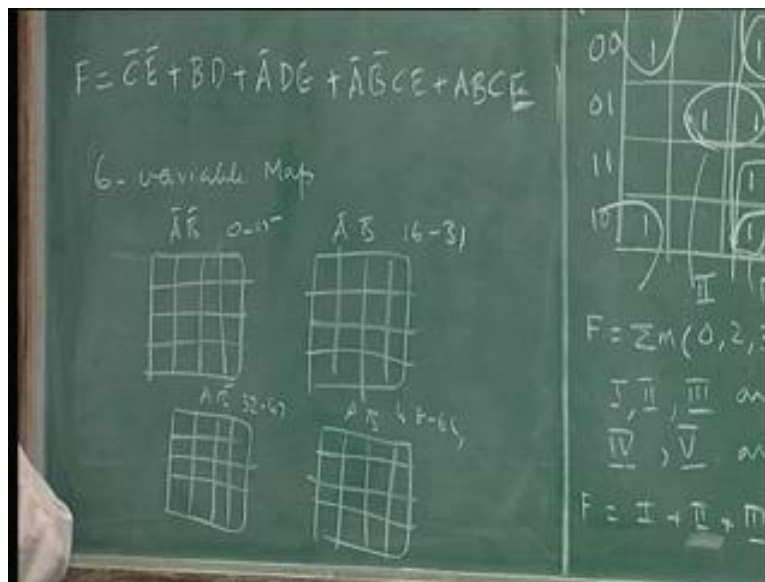
So now my final expression if you want we can write it as F is equal to C bar E bar BD A bar DE A bar B bar CE and you can draw your gate structure or whatever it is. This is how you read a five variable map identify the common cells. Of course some people draw it side by side, some people draw it together left and right and so on. It is not a mathematical procedure or anything it is only for convenience whichever way you feel comfortable with you can do it.

Now I can extend this to six variables. if I want six variables I want four maps each of sixteen cells I will have to say six variable map A bar B bar, A bar B, A B bar, and AB. So min terms 0 to 16, 16 to 31, 30 to 47, 48 to 64 and then you have to see adjacency see the adjacency between these adjacency between these, adjacency between all the four and all that and you can go on writing the expression and have unlimited fun all these checker games you play. But there is a limitation, why I am saying all these is because there is a limit beyond which after six how will you do? Of course you can always find a method of doing it and then it becomes inefficient.

We started with the map method because of which simplicity an efficiency then we are reaching a point where the simplicity is lost it would become an ordeal so beyond six variable, five variables is most comfortable, four is good we would like to have a problem with four in the exam may be five manage, six is okay but seven and above it becomes cumbersome.

After all this is a procedure what we have done is adjacency. Find 1s which are adjacent to each other when you represent in a graphical way. So it is always possible to write a computer program to find out the adjacent 1s in a truth table and keep on repeatedly doing it. The advantage of the problem whichever way you see it in the computers what you have to do is step by step. Combine two 1s at a time not more than two 1s at a time then it is too much to handle for computers. Put two 1s at a time and then two 1s at a time will become 1 1 then you say put two more so keep on giving it in steps you write a program algorithm they call it. So, algorithm is a step by step program. You write an algorithm for a program to systematically find out all the 1s in a truth table merge them to the best possible way and repeatedly do this.

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Sometimes there may be several terms in which some redundancy may be there. The essential prime implicant and non essential prime implicant that the computer cannot resolve you know. After all the computer is what you program computer, it cannot become more intelligent than you are because you are the one who is feeding in the program in the computer.

In a computer the program is limited by your intelligence cannot be more than your intelligence. So finally we may have intervened in some place and say we will list all essential prime implicants and non essential prime implicants and make a choice of the right combination that we have to do. Such a method is called a tabulation method as against the map method. We will not do that and **I thought I should tell you.** So the number of variables becomes extremely large so you go for what is known as the tabulation method. These are computer based methods. Of course you can also do it by hand to understand the algorithm. Like the same example of 31 cells or even 61 cells you can take for fun, 64 cells 0 to 63 or even 31, 0 to 31. Do it the way the computer will do it. Combine two two at a time and do it like that just to get an understanding of this method.

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I am not going to teach this in this class but it is available in many books, many books talk about this method called tabulation method. Some people call it prime implicant method. Some books call it prime implicant method; some books call it graphical method a tabulation method. You can do that to understand the concept behind this but these are all computer programs very easy to understand once you know how it works. So you can do a simple example of the same example we do in the class try to do it using this method so you will have an understanding of this program. We will stop here for today.