

Semiconductor Device Modeling
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Lecture - 05
Semi-classical Bulk Transport: Qualitative Model

So in the previous lecture, we have discussed about the equilibrium condition. We said that equilibrium can be defined as the state in which for every process there is an inverse process going on at the same rate. So that is the statement of a balance. Then, we talked about the balance between generation and recombination processes. We talked about scattering of the electron and holes.

We mentioned that the entire semiconductor under equilibrium at any temperature $> 0K$ can be visualised as consisting of randomly moving particles namely electrons, holes, phonons and protons and randomly located impurities all fixed in their place. Now this is the so-called particle approximation where we said that between 2 collisions or between 2 scattering events, an electron or a hole can be regarded as a particle.

However, the scattering event itself and the mass of the particle between the 2 collisions, these 2 entities have to be derived from quantum mechanics. We explain why quantum mechanical basis is necessary for explaining scattering and why the nature of randomness of motion or particles is very essential for scattering to happen. Then we said that at the microscopic level equilibrium state can be visualised as intense motion but no net motion.

So while the carriers are moving about rapidly they are not contributing to any current in any particular direction. Then, we began our discussion on charge transport, we said that if you superimpose on this random thermal motion a directed motion, then we can get current out of the semiconductor. Now how do we impose a directed motion, so we said there are 3 driving forces, the electric field, magnetic field and the heat flux.

So we can either talk in terms of the fluxes namely the electric, magnetic and heat fluxes, or we can talk in terms of the electric field, the magnetic flux and the heat flux. So these 3 fluxes give rise to a hole flux, an electron flux and a displacement current flux. Now the

electron, hole flux and displacement current flux in turn give rise to their own electric, magnetic and heat fluxes.

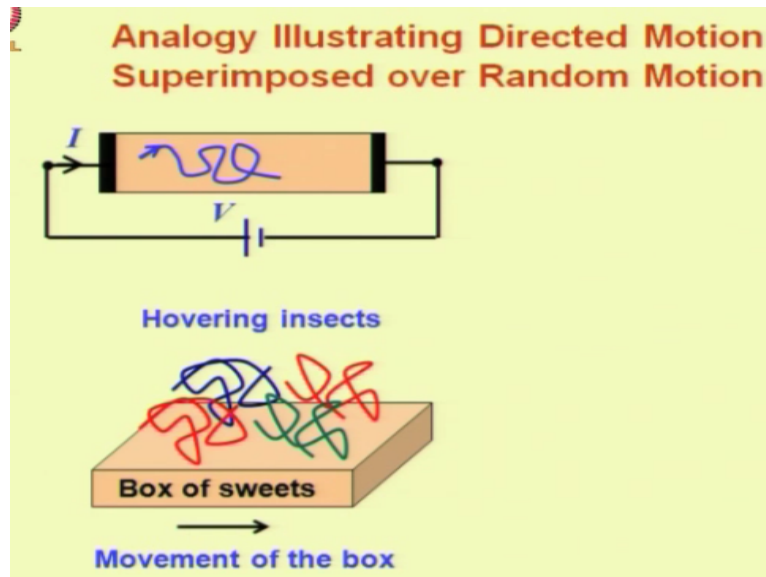
Now how does the electron current give rise to or a hole current give rise to electric field, well during flow the electrons and holes can get redistributed and therefore positive and negative charges can arise in the device. And this positive and negative charges can contribute to the electric field. Now this electric field is different from the driving electric field. Now similarly when the electrons and holes are moving, we know that around the current there is a magnetic field.

Also when there is voltage drop across a device in some regions, there can be a power dissipation and then the temperature in that region of the device will go up. For example, we gave an example of a power device near the junction, there is lot of power dissipation and so junction is at higher temperature than let us say the base or contact of the device. So this is how the current of electrons and holes and displacement current they give rise to their own electric, magnetic and heat fluxes.

And these fluxes then interact with the driving fluxes, driving, electric, magnetic and heat fluxes, and then that is how the whole current can be visualised. Now one important point is that when we make the particle approximation of the carrier for visualising the equilibrium state or the carrier transport, we must know the validity of this particle approximation. When is it valid? Now that is the question that we will defer for the moment, we will take it up later.

Right now, let us visualise how the directed motion can be super imposed over the random motion. We will do so with the help of an analogy.

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Now here is the analogy, this part of the diagram shows a semiconductor in which the random motion of a carrier has been traced, a voltage has been applied and there is a current flow. Now this current can be visualised as directed motion superimposed over random motion. Now that superimposition principle can be understood with the help of this analogy, where we have shown a box of sweets on which some insects are hovering.

Now when the box of sweet is stationary we can the system is under equilibrium all the insects are hovering around over the box of sweets and there is no net movement of the insects in any direction. Now when we start moving the box to the right, what will happen is the insects will keep hovering randomly and at the same time try to follow the box of sweets, because they would like to remain over the sweets.

So the motion of the insects is actually replicating the motion of electrons or holes in the semiconductor. So in this analogy, the carrier motion is analogous to the insect motion the force and the carriers, due to the electric field is longing of insects for sweets. So this is how a directed motion superimposed over a random motion can be visualised. Now let us return to the case of 6 couple flows, so visualisation of this directed motion, superimposed or random motion and the directed motion due to the 6 coupled flows, visualisation of this is fairly complex.

Therefore, we need to make some approximations regarding the 6 flows, let us see what approximations can we make. So what we will do is, we will tabulate the flows, their approximations and what is the implication of the approximations that we are making.

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Flow	Approximation	Implication
E	Quasi-Static (Q-S)	• J_D negligible * • B is Q-S *
B	Negligible	E is non-circulating and so expressible in terms of a scalar potential ψ
Q	Negligible	∇T_L assumed as small
J_D	Negligible	E assumed as Q-S *

* As $\partial E / \partial t$ rises, J_D comes into play followed by B

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Let us take up the electric field, the magnetic flux B , heat flux Q and displacement current density. Related to E we make the approximation that the rate of change of E with time is small, this is called the quasistatic approximation, so we assume E to be quasistatic. The consequence of this would be that J_D displacement current is negligible, because you know that J_D is given by $\epsilon_0 \partial E / \partial t$.

So if $\partial E / \partial t$ is small that is the quasistatic assumption J_D would be small. Next consequence is that B is quasistatic, you know that changing electric field give rise to a circulating change in magnetic field around the electric field. Now if the rate of change electric field is small then rate of change of magnetic field also is small with time and therefore B is quasistatic.

Now let us discuss some further approximation of B , in fact what we will do is that we neglect B altogether. As a result, the E will be non-circulating and so expressible in terms of a scalar potential ψ . So you know that when there is no magnetic field the electric field can be expressed as negative gradient of potential and that is ψ . Similarly, we shall neglect the heat flux altogether, this amounts to assuming that the gradient of the lattice temperature is very small.

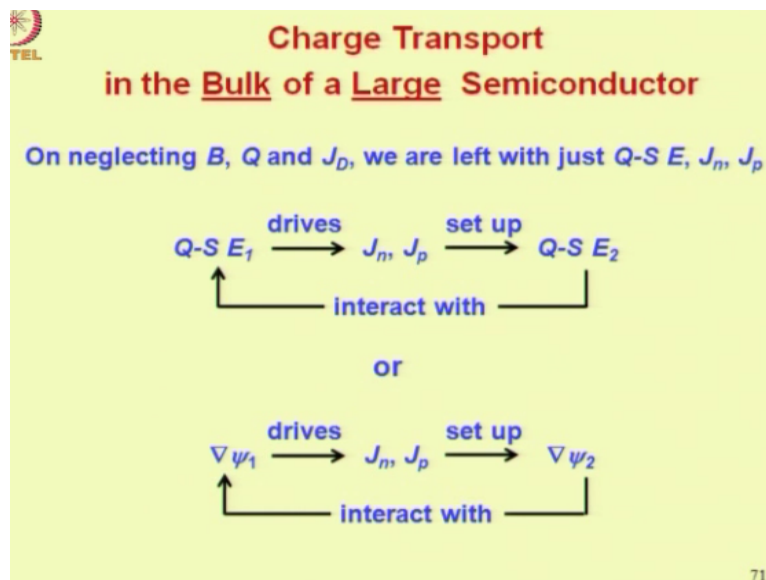
Now at this point, let me clarify what lattice temperature means. As they are mentioned earlier we shall talk about 2 temperatures, the temperatures of the carrier and the temperature of the lattice. Temperature of the lattice is nothing but the temperature of the phonons. So

now let us look at the displacement current. Once we assume E is the quasistatic, we have already said that the displacement current is negligible. So the implication of assuming the JD to be negligible is that E is quasistatic.

Now, there is a note here at the bottom of the table which talks about how the JD and the magnetic field or flux B come into play as you go on changing the rate of change of dou E or rate of change of electric field. So dou E/dou T rises, first JD will come into play and there after you will get the effect of dou B/dou T. So we will have to consider effect of the displacement current first.

So we can continue to neglect the magnetic field for some high frequencies and that very high frequencies, we will have to take into account magnetic field, that is the meaning. So what we are talking about here is as you increase dou E/dou T, how does the approximation that we have made fail and how we need to incorporate the effects.

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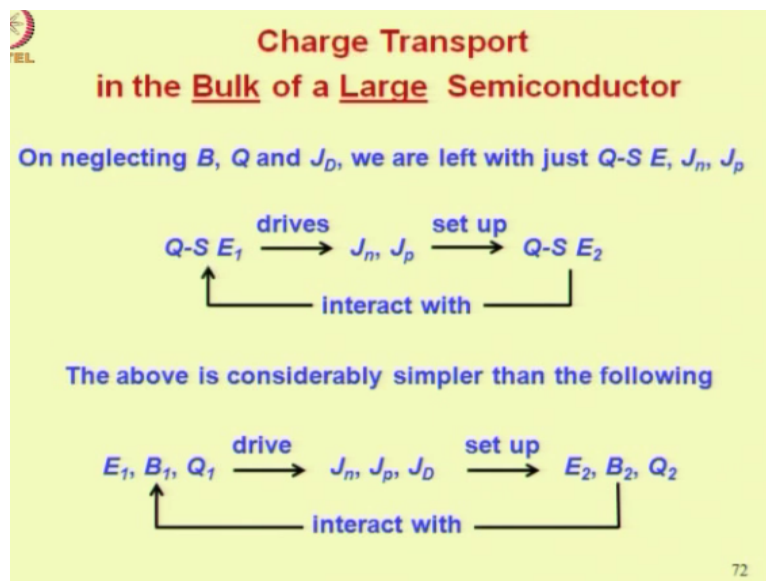
So now our picture is as follows, on neglecting B, Q and JD, we are left with just the fluxes namely, fluxes or flows, flows is a more appropriate term here, because E is not a flux but the field. So we are left with quasistatic electric field E, then Jn and Jp, and the interaction between these 3 flows is as shown namely that the quasistatic electric field drives the directed motion, creates Jn and Jp which in turn set up their own quasi-static electric field.

Because during motion they may redistribute carriers and positive and negative charges may be created. So those are responsible for this particular electric field and this electric field

interacts with driving field, okay, so that is the picture. Now you can see that this is considerably simplified picture from 6 flows, we have come to just 3 flows. So another alternative way of visualising the same thing is instead of the quasistatic electric field, we can talk in terms of the potential gradient.

So potential gradient drives the current, which in turn sets up its own potential gradient, because of positive and negative charges coming from redistribution of carriers and there is an interaction of these 2 potential gradients.

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So we have now got a simplified picture of the directed motion, superimposed or random motion and the directed motion being only involving 3 flows. Similarly, this is considerably simpler than E , B , Q driving J_n , J_p , J_D which in turn setting up their own fluxes and so on. So this picture which is the accurate picture valid for all frequencies and all conditions namely all temperature gradients, a simplified for our course we are going to use this picture only, okay.

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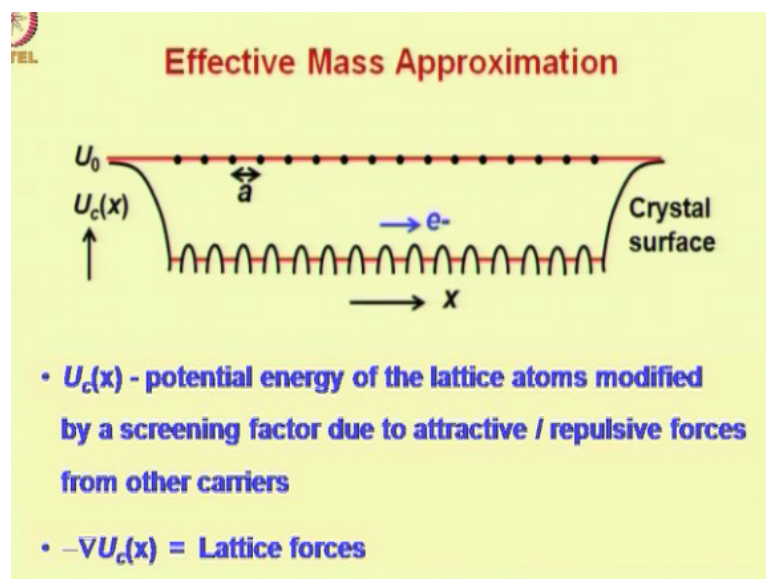
Charge Transport in the Bulk of a Large Semiconductor

Flow	Approximations	Implications
J_n	• Particle approximation	Carrier flow can be visualized as a randomly moving population (like dust particles in air) subjected to a directed motion by E_1, B_1, Q_1
J_p	• Effective mass approximation	
		Tunneling current is neglected

Let us now look at approximations related to the current densities, J_n and J_p . So the important approximations related to these flows are the particle approximation and the effective mass approximation. Now, the implications of these approximations are that carrier flow can be visualised as the randomly moving population just like dust particles or insects hovering on a sweet box, subjected to a directed motion by electric flux or electric field, or magnetic flux and heat flux.

We have neglected the heat flux and magnetic flux but in general it can be visualised as a result of these driving forces. Now we are going to neglect the tunneling currents, because if you like the particle approximation, then you cannot explain the tunneling current. For explaining tunneling current, you have to regard the electron as a wave.

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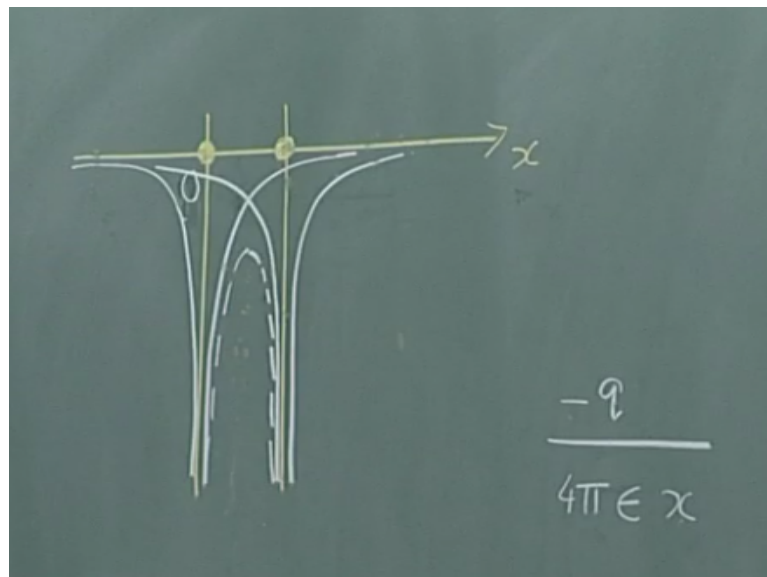


Now let us discuss the effective mass approximation. First let us focus on the diagram, now what is shown here is the electronic potential $U_c(x)$, on the y axis as a function of distance inside the crystal. For simplicity, we have assumed a one-dimensional crystal, so the electronic potential means this is the kind of potential and the electron will see. So if you are deep inside the semiconductor, then you will only see these periodic potentials because of the atoms.

These points here indicate the atoms, location of the atoms. The distance between the atoms is a , and this line, this red line, indicates the potential corresponding to an electron far away from all forces. This is called the vacuum energy or vacuum potential. These 2 ends here and here are the ends of the semiconductor sample of crystal surface. So as an electron comes nearer the end it will experience a potential barrier, it will not be able to escape easily.

Now we will be concentrating on the electron deep inside so that it experiences only this periodic potential. Let us explain how this periodic potential that is shown here is obtained, how this kind of a shape is obtained for the potential.

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Suppose I consider an atom which is positively charged at this location, this is let us say x . Now I want to plot the potential that will be experienced by an electron, so as I move of electron along this x axis what potential will it experience. Now the formula for electronic potential is $-q$ upon $4\pi\epsilon_0 x$, where let us say this is origin. So x is the distance of the electron from the atom.

So the conventional potential is $+q$ divided by $4\pi\epsilon_0 x$, where ϵ_0 is the dielectric constant of the medium, whereas for the electrons you will put a negative sign. Now if I sketch a curve representing this function, potential function it would look like this. So for large x this goes to 0 and for small x it goes to $-\infty$. So your potential function looks something like this.

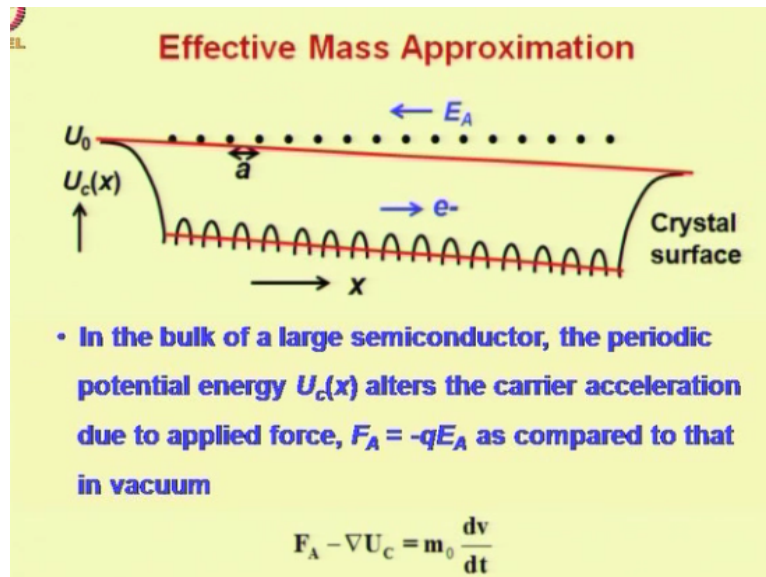
Now similarly I can plot the potential function if I am coming from this side, it would be just a curve that is reflected on this so symmetric, something like this. Now let us bring another atom nearby so suppose I put another atom somewhere here, now if you sketch the potential for this atom it would look like this. So between these 2 atoms if I sum up these potentials, this line and this line, and I show it by a dotted line it would look something like this.

Now if I put an additional atom here, then the potential due to that atom will get added to this. Similarly, if I put an atom here its potential will also get added here, however since the potential is rapidly reducing to 0 the effect of the other atoms on this potential would not be that much. So this is how you get this particular curve for potential between any 2 atoms.

Now if you put a series of atoms like this between every 2 atoms you will get curves similar to this. Now in an atom, in a crystal you not only have positively charged atoms, you also have a large number of carriers. We are looking at the picture for a single electron or a single hole, let us say we look at the picture for a single electron, then the electron is under the influence of the potentials of all the atomic nuclei and it is also affected by the potentials due to other carriers, other electrons and holes.

So therefore in this diagram that is shown on the slide this $U_c(x)$ includes not only the periodic potentials of the atoms but also the effect of other carriers. Now that is what is written here on the slide, so $U_c(x)$ is the potential of the lattice atoms modified by a screening factor due to attractive or repulsive forces from other carriers, also the negative gradient of this potential $U_c(x)$ would give you the lattice forces. So the forces exerted by the atomic nuclei and other carriers on the electron.

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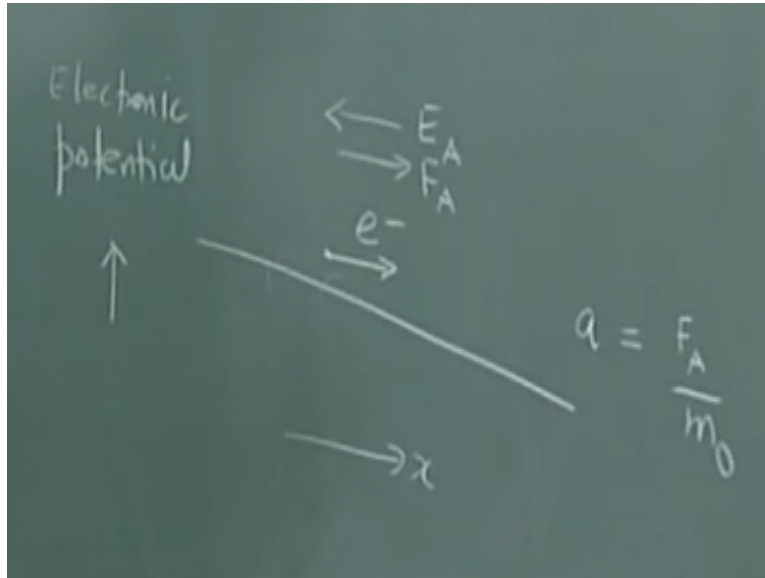


Now let us apply an electric field so that the electron is accelerated. This diagram shows the application of electric field; the moment we apply the electric field what you find is that all these atomic potentials have started decreasing with distance. So the new potential is super imposition of an externally applied potential or field and the periodic potentials. So clearly it gives you the feeling that the single electron that we are considering is now on a potential slope.

So there is potential slope like this and definitely this potential slope is going to accelerate the electron. The field applied is from right to left, and that is exerting a force from left to right. So as shown on this slide, in the bulk of the large semiconductor, the periodic potential $U_c(x)$ alters the carrier acceleration due to the applied force, $F_A = -qE_A$ which is the applied electric field, as compared to that in vacuum.

So what we are saying is, if the electron was in vacuum, then the picture would have been like this.

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So in vacuum the potential picture is something like this, so you have this line, this is again electronic potential and this is x and you have an electron here so it will accelerate in this direction. So the potential variation like this implies an electric field in this direction and a force in this direction. So this is a potential picture in vacuum, you do not have these curve crystal potentials, periodic potential.

So in such a situation if the electrons are accelerating, then this acceleration would be given by $A = \frac{F_A}{m_0}$ the force by the mass in vacuum that is m_0 . Now let us look at the picture in the crystal, so here the equation has got modified to $F_A - \text{gradient of } U_c$, the so called periodic potential = $m_0 \frac{dv}{dt}$, in vacuum this - gradient of U_c would not be there, the equation would be $F_A = m_0 \frac{dv}{dt}$.

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Effective Mass Approximation

- In the bulk of a large semiconductor, the periodic potential energy $U_c(x)$ alters the carrier acceleration due to applied force, $F_A = -qE_A$ as compared to that in vacuum

$$F_A - \nabla U_c = m_0 \frac{dv}{dt}$$

Moving on to the next slide, the modification of the carrier acceleration by $U_c(x)$ is modelled by assigning the carrier in a semiconductor and effective mass m_n assuming electrons, the effective mass for holes would be m_p , and what is the equation for m_n , so that is written out here FA, which is $m_0 \frac{dv}{dt} + \text{gradient of } U_c$, this is the periodic potential experienced when the electron is moved in a crystal rather than in vacuum, then this is actually set = m_n into $\frac{dv}{dt}$.

So this is how the effect of ΔU_c is captured in m_n . The next slide points out that m_n is different from m_0 and can even be negative. Now this is the important point that m_n is always different from m_0 because it captures the effect of electronic potentials and it turns out that when you do a detailed quantum mechanical derivation, for some situations the effective mass can even be negative.

One more point about the effective mass approximation. $U_c(x)$ varies over a small length scale. So the interatomic distance a is much less than the thermal average de Broglie wavelength of a carrier at any temperature, which is given by $\frac{h}{m_n v_{\text{thermal}}}$. So since this a is very small, then the de Broglie wavelength, notice here that $m_n v_{\text{thermal}}$ is the momentum of electron at any temperature because of random thermal motion.

So because of this condition, you have to treat the situation quantum mechanical. You cannot regard the electron as a particle in this case. For example, in silicon, at 300 K the interatomic distance are lattice constant A is 5.43 Angstrom whereas the thermal average wavelength of the electrons at 300 K is 120 Angstrom, so you can see that 5.43 is much $<$ 120. So therefore, m_n , which captures effect of $U_c(x)$ on carrier acceleration has to be derived from Schrodinger's equation rather than Newton's law.

So this tells you why we need to derive the effective mass quantum mechanics. So when we read on the equation for effective mass earlier, when we wrote the force equal to effective mass into acceleration, it appeared to be like a Newton's law, but when you want to find out that effective mass m_n , you have to do a quantum mechanical rx. This is because the potential variations are very rapid over very small length scales.

Small length scale meaning length scales much less than the de Broglie wavelength of the electron.

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Effective Mass Approximation

Thus, m_n is

- a quantum-mechanical concept
- valid in the presence of applied fields only

Some more important points about the effective mass approximation, thus m_n is a quantum mechanical concept and very importantly it is valid in the presence of applied fields only. So please look at the situation in which we have derived this m_n , so we are applying an electric field and then finding out the acceleration and that acceleration in a crystal, we are using to get the effective mass.

So it is a concept that is only applicable under applied electric fields or magnetic fields or applied forces. Now, if there is no applied force, what effective mass would the carrier have? We can only make a guess, so for simplicity, use it as a limiting case of zero applied force and use the same effective mass that we use in the presence of the applied force.

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Effective Mass Approximation

In a 3-dimensional crystal, m_n is

- non-isotropic, due to different potential variations in different directions
- a tensor, since the carrier acceleration is not collinear but at an angle with the direction of the driving force

$F \rightarrow a = (m_0)^{-1} F$ $F \rightarrow a_j = (m_{ij}^*)^{-1} F_j$

vacuum semiconductor

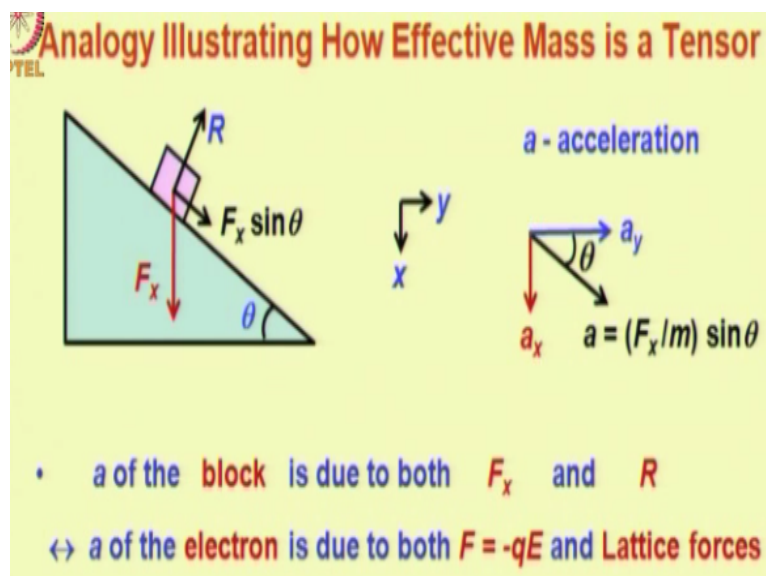
Let us discuss the effective mass approximation further. In a 3-dimensional crystal, Mn is non-isotropic due to different potential variations in different directions. So here is a 2-dimensional array of atoms and 2 directions or shown. Supposing you move in this direction, then the distance between the atoms is less than the distance between the atoms that you get if you move in this direction.

Therefore, the crystal potential in this direction would be different than the crystal potential in this direction. So potential variations would be different depending on the direction, because the arrangement of atoms is different, so this is why the effective mass would not be same in a crystal in all directions. Next important point about effective mass is that it is a tensor. Since the carrier acceleration is not collinear, but at an angle with the direction of the driving force.

So diagram below illustrates the situation in vacuum, suppose force is in this direction, the acceleration would be along the direction of the force and the equation would be $a = F / M_0$. However, because in a semi-conductor the electron is influenced by the crystal potential. It turns out that analysis shows that if the force is in this direction, the acceleration would be in a different direction, say something like this.

And now, this is a tensor, because for force in this direction, you are producing an acceleration along the direction of the force as well as in a direction perpendicular to the direction of the force. So here, a_j is a matrix, m_{ij} is also a matrix. Now let us illustrate this idea of the tensor nature of the effective mass with the help of an analogy.

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Let us look at a situation where the acceleration is not in the direction of the force. So here is a block on an inclined plane and suppose we apply a force vertically downwards, now this force could be because of gravity, but let us now not restrict ourselves to gravity, it could be any reason. Suppose a force is applied in the downward direction, now the acceleration would be along the inclined plane in this direction.

So you see the force and acceleration are not collinear. Now what is the reason, the reason is inclined plane guides the motion. So it is this force R , which is preventing the mass from accelerating in the direction of the force entirely. Now let us write an expression for this acceleration, how would it look? We will choose vertically downward direction as x and the horizontal direction towards right as y .

Now the acceleration can be shown using this diagram. The actual acceleration along the inclined plane is $a = F_x/M \sin \theta$ where θ is the angle of the plane. Now if you divide it into 2 components, one in the direction of the force, that is a_x and the perpendicular direction a_y , then we can say as shown in the slide here a of the block is due to both F_x and R .

Similarly, or analogously a of the electron is due to both $F = -qE$ that is applied force and lattice forces. So lattice forces are analogous to R . As this slide shows, we can model a of the block as the effect of x alone absorbing the effect of R in an effective mass. So we are trying to apply the effective mass concept to this particular situation, right. Now this is analogous to the statement, we can model a of the electron as the effect of applied force F alone absorbing the effect of lattice forces in an effective mass.

Now let us carry out this exercise for the inclined plane. Effects causes both $a_x = F_x/M \sin^2 \theta$ and a_y , which is $= F_x/M \sin \theta \cos \theta$. How do we get this? So $F_x/M \sin \theta$ is the acceleration and you want to get a_x component, it would be $a \sin \theta$ and a_y would be $a \cos \theta$. So that is what is shown here.

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Analogy Illustrating How Effective Mass is a Tensor

a - acceleration

$a = (F_x/m) \sin \theta$

- F_x causes both $a_x = (F_x/m) \sin^2 \theta$ and $a_y = (F_x/m) \sin \theta \cos \theta$
- Extending the above to a general 2-D case and rewording, when a is not collinear with F , each of a_x and a_y can be caused by both F_x and F_y leading to

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \underbrace{\begin{bmatrix} m_{xx}^{-1} & m_{xy}^{-1} \\ m_{yx}^{-1} & m_{yy}^{-1} \end{bmatrix}}_{\text{Inverse effective mass tensor}} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

Now you can extend the above to a general 2-dimensional case and rewrite the statement as when a is not collinear with F , each of a_x and a_y can be caused by both F_x and F_y , so leading to an equation such as that shown here, a matrix type equation. So a_x is caused by both F_x and F_y . So the mass that comes here is a tensor. This is actually the inverse effective mass tensor, because you have reciprocal of the mass.

So these 4 entities here are represented as F_{xx} because this mass helps you to find out x due to F_x and this is M_{xy} because this mass helps you to find out x due to F_y . So that is how you have the other 2 elements also. Now, this gives you an idea as to why, when the acceleration is not in the direction of the force, the effective mass concept leads you to a tensor like representation for the effective mass.

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Analogy Illustrating How Effective Mass is a Tensor

Assignment-2.2

In the 'inclined plane' analogy of the previous slide, determine the expressions for m_{xx} , m_{xy} , m_{yx} , m_{yy} .

Now here is an assignment for you on this analogy. In the inclined plane analogy of the previous slide determines the expressions for M_{xx} , M_{xy} and M_{yx} and M_{yy} . Moving on to the next slide:

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Fluxes	Approximation	Implication
J_n J_p	Particle attributes, namely - <ul style="list-style-type: none"> • density or concentration • momentum density • energy density of carriers <u>within a local volume</u> * are uniform at their average values (their distributions are ignored)	

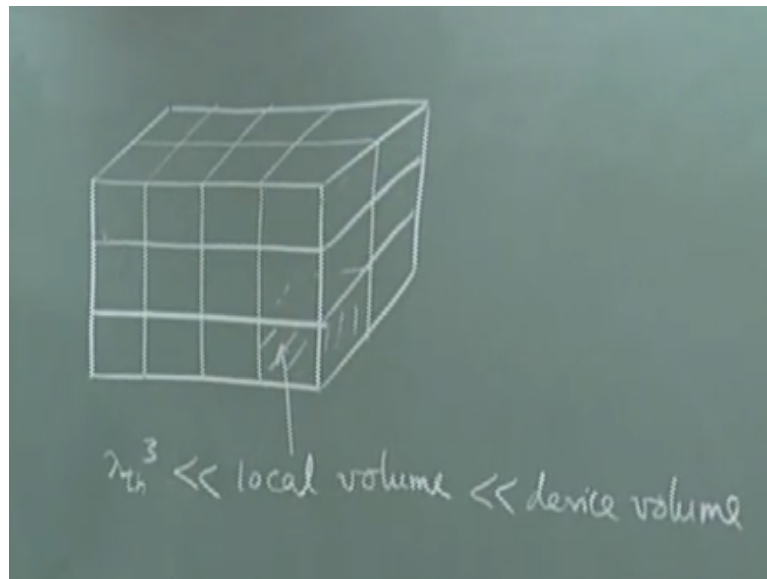
* $\lambda_{th}^3 \ll \text{local volume} \ll \text{device volume}$

Let us discuss some further approximations for J_n and J_p . Now this approximation is that particle attributes namely density or concentration of carriers, momentum density of carriers and energy density of carriers within a local volume. There is an underline here, we will shortly see what local volume means. It is given in the foot note. So these 3 quantities of carriers, carrier density, momentum density, and energy density.

So within a local volume are uniform at their average values. In other words, the distributions are ignored. Now, we will shortly explain this in detail, but before that what is the meaning of local volume, let us see. So it is a volume, which is much $>$ a volume with a linear dimension equal to the thermal average wavelength of the electron. Because only then, you can treat the electron as a particle.

So local volume should be much more than the volume derived from the de Broglie wavelength, but at the same time, if it is too large then you will not be able to capture any variations in these quantities, like carrier density, momentum density and so on. Therefore, the volume that you choose that is the local volume should be much less than the device volume. So now what we are saying is the following:

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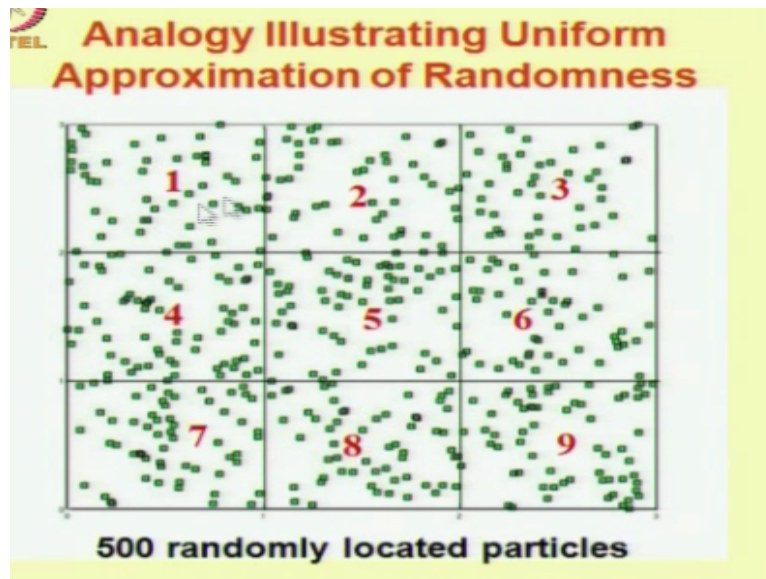


Suppose this is the device that you want to model. In this the quantities namely, the carrier density, momentum density and energy density and so on all associated with the carrier can vary, spatially. Now you want to capture those variations because that variation will affect the current flow for any given voltage. So since you want to capture the variation, you would like to divide this device region into parts, which are sufficiently small.

So that in this region, if I assume a constant parameter, constant value for the carrier density, momentum density and energy density, then this is another constant here, another constant here, so even if I take these constant values, the changes in this value should reflect the overall variation in the device sufficiently accurately. Now what we are saying is each of these volumes however, should remain much greater than lambda thermal cube.

So that you can treat the electron as a particle. In 3-dimensions, it would look something like this. This is the meaning of the statement; local volume is much $>$ lambda thermal cube. So this is the local volume. But this quantity should be much less than device volume. So device volume is this whole volume of this device. Now let us illustrate the statement how the various quantities can have distribution and what is the meaning of the average of this distribution.

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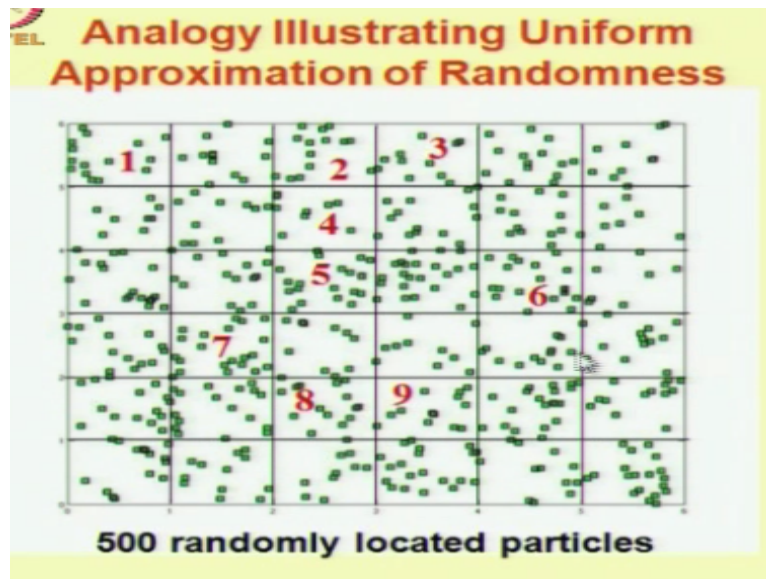
Now here is an analogy illustrating uniform approximation of randomness. So here we have taken 500 randomly located particles and we have divided this area in which this 500 particles are contained into 9 smaller areas. So you can say these are local areas analogous to local volumes. Now, let us count the number of particles in each of these local areas, 9 local areas. Now if you do that exercise, picture would be something like this as shown in the table.

So local area number 1, number of particles is 48, local area number 2, it is again 48, however, in area 3, it is 44. Area 4, here you have many more, it is 65. It goes on like that. Why because this random location. The particle location is random. Now let us take the average of all these 9 areas. So I get an average of 55.6, so average particle count in these 9 areas is 55.6.

Now if you calculate the standard deviation of all these 9 quantities, you get standard deviation of 7.5. The ratio of standard deviation to average is about 0.13 that is 13%. So in other words here, the standard deviation is fairly small compared to the average value. Now, if such as situation is there, then I could assume the average value to represent the particle count in every local area.

So particle density in a large local area, example areas 1 to 9, has small standard deviation by average and so can be approximated to uniform at the average value. Now let us consider an example where this kind of an approximation will fail. In other words, it will not be good. So let us take the same picture of 500 particles located in random fashion, but now let us choose a smaller local area.

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So here, the same area has been divided into a very large number of local areas. So if you count the local areas now here, 1, 2, 3, 4, 5, 6, so 36 local areas, out of which let us pick some 9 local areas and let us do the same exercise of calculating the average and standard deviation for these 9. Now the picture would be something like this. So local area 1, you now have a smaller number of particles than the previous example. They are only 15.

Local area 2, here 14, and so on. Local area number 5, that this, there are many particles about 20. Now the average of all these is 15.3, whereas standard deviation of these numbers is 3.86. The ratio of standard deviation average is now much higher, 25%. Now that is why particle density in a small local area as compared to the previous case, has large standard deviation by average and so cannot be approximated to be uniform at the average value.

So this should explain to you the concept of using an average for quantities, which are randomly distributed. We can extend this idea to random distribution of momentum, random distribution of carrier energy, and those quantities can also be replaced by their averages. So why is the distribution random, well that is how the nature is. When you have a large number of people, all of them will not have the same height.

Similarly, if you take a large number of electrons in a crystal, all of them will not have the same speed, at a given temperature. So-called thermal velocity or thermal speed is an average value. You know it is root mean square average. Similarly, momentum will also be distributed and the energy also will be distributed.

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Uniform Approximation of Randomness

The randomness in distribution of the dopants in semiconductors is such that if N is the average number of dopants in a given volume, the standard deviation in the number of dopants is \sqrt{N} , i.e. the actual number of dopants in the volume considered may range between $N \pm \sqrt{N}$.

Here is some example of uniform approximation of randomness. The randomness in distribution of the dopants in semi-conductors is such that if N is the average number of dopants in a given volume, the standard deviation in the number of dopants is square root N . That is the actual number of dopants in the volume considered may range between $N \pm \sqrt{N}$. So if you look at the diagram here, if you take this local volume, which looks something like this.

And in this, if you count and you have N impurity atoms, now you take another local volume of the same magnitude, somewhere else, and you count the number here, it would not be the same N . Now what could this value be? So it would lie most likely between $N + \sqrt{N}$ or $N - \sqrt{N}$.

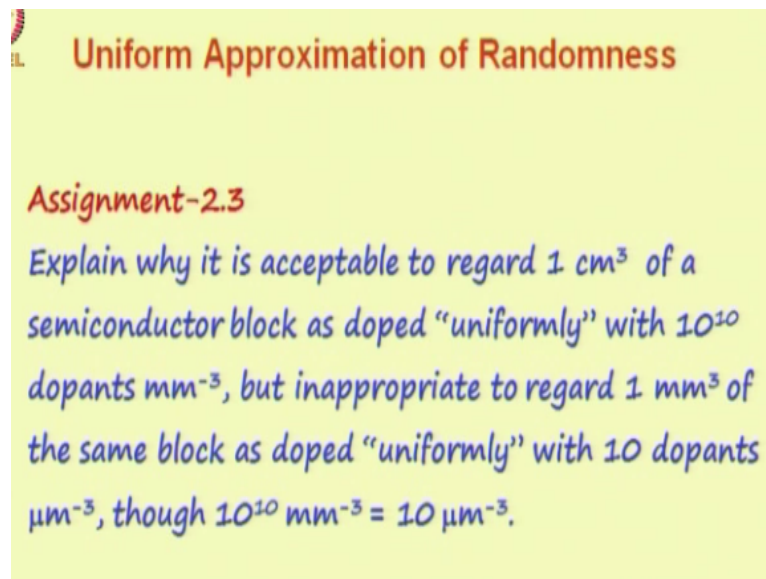
So you can see if N is 10^{15} , then 10^{15} average value means that different parts of the device of 1 cm cube volume, so 10^{15} per centimetre cube, if that is the doping that we are talking about, then we are actually saying that if you consider 1 cm cube volumes of the semiconductor of different volumes, actually the value could be anywhere between $10^{15} + \sqrt{10^{15}}$ and $10^{15} - \sqrt{10^{15}}$.

Now square root 10^{15} is very small compared to 10^{15} , right. On the other hand, if this number is changed to say 25 in some volume, a much smaller volume i take, then different regions of the device having the same small volume will have number of dopant

atoms between $25 + \sqrt{25}$ and $25 - \sqrt{25}$ that is 25 ± 5 . Now you see this 5 now is not negligible as compared to 25.

In fact, your particle counts or impurity count is anywhere between 20 and 30, right that is how you see you are getting a much larger variation, if you consider small volumes. So that is why the randomness is very important. When your device size is large, we can ignore the randomness. Whereas when the device size is small, you have to take into account fluctuations in the different regions of the device, because of randomness.

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Uniform Approximation of Randomness

Assignment-2.3

Explain why it is acceptable to regard 1 cm^3 of a semiconductor block as doped "uniformly" with 10^{10} dopants mm^{-3} , but inappropriate to regard 1 mm^3 of the same block as doped "uniformly" with 10 dopants μm^{-3} , though $10^{10} \text{ mm}^{-3} = 10 \mu\text{m}^{-3}$.

Here is an assignment. Explain why it is acceptable to regard 1 cm cube of a semiconductor block as doped uniformly with 10^{10} dopants per mm cube, but inappropriate to regard 1 mm cube of the same block as doped uniformly with 10 dopants per micron cube, though 10^{10} per mm cube = 10 per micron cube. Let us move on to the next topic.

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**Charge Transport
in the Bulk of a Large Semiconductor**

Fluxes	Approximation	Implication
J_n J_p	Particle attributes, namely - <ul style="list-style-type: none"> • density or concentration • momentum density • energy density of carriers <u>within a local volume</u> * are uniform at their average values (their distributions are ignored)	Device behaviour is visualized as the result of conservation or balance of each of these attributes during carrier flow

* $\lambda_{th}^3 \ll \text{local volume} \ll \text{device volume}$

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What is the implication of our approximation that you can replace the actual distribution of momentum density and energy density by an average value. The implication, now is that the device behaviour is visualized as a result of conservation or balance of each of these attributes during carrier flow. So now, we can analyse the device in terms of the balances of the carrier concentration, the balance of the momentum density, and the balance of the energy density during flow.

So it allows you to invoke conservation laws that is the very important achievement of this approximation.

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Now we are towards the end of the lecture. So let us make a summary of the important points. So in this lecture, we talked about how driving forces can superimpose a directed motion over

random motion. The actual picture of this carrier motion is complex because it involves the interplay of 6 different flows, the electric field, the magnetic flux, the heat flux, the electron flux and the whole flux, and displacement current density.

So we discussed a series of approximations, which help in simplifying the problem of the flows for the purpose of our course. So the approximations we discussed were that we will neglect the magnetic flux B , the heat flux Q and we will assume that the electric field varies very slowly with time. In other words, we make the quasistatic approximation for the electric field. Consequently, the displacement current also goes out of the picture.

So once the flows namely B , JD and Q go out of the picture, we are left with a very simple situation where we need to consider the interplay of electron flux JN , the whole flux or current JP and the quasistatic electric field. So the modelling problem becomes reasonably simple. Then we discussed some important approximations related to the modelling of the fluxes JN and JP .

So we said that one important approximations that we make is the particle approximation. We regard the electrons and holes as particles. Now this approximation was discussed in detail in early lectures. In this lecture, we spent considerable amount of time discussing about the concept of effective mass of electrons and holes. So between 2 collisions, a carrier behaves as though it had a mass different from that in vacuum.

If you apply a force, the carrier accelerates between 2 collisions as though it had a mass different from that in vacuum, because it is influenced by the crystal forces or crystal potential. Now, we explained why this effective mass has to be derived from quantum mechanics and it cannot be derived from Newton's law. So we need to derive the effective mass from Schrodinger's equation because the effective mass is a result of potential variations, which are very rapid in small length scales of the interatomic distance.

And this interatomic distance is much smaller than the de Broglie wavelength of the electron. In fact, we now have an explanation why the carrier transport, which involves superimposition of directed motion or random motion is called semi-classical transport, because though between 2 collisions, the movement of the electron can be treated by Newton's law.

The collision itself or the scattering event has to be treated quantum mechanically as discussed in the previous lecture and the effective mass of the electron, which helps you to get the acceleration because of applied force between 2 collisions also has to be determined quantum mechanically. So you cannot have a completely classical treatment of carrier motion. We discussed some more important points about the effective mass.

Namely that it is non-isotropic that is depending on the direction of motion of the electron in a crystal, it changes, because the potential variation of the atoms is different in different directions, interatomic distances are different. Then, effective mass is a tensor because when you apply a force, the electron does not move in the same direction as the force. So since the force and the electron motion are non-collinear, therefore, the effective mass becomes a tensor.

Also, the effective mass concept is only applicable under applied fields because it helps you to find the acceleration. As a limiting case, we use the effective mass also for the situation when there is no electric field or magnetic field. Finally, we discussed the important approximation related to JN and JP namely that the carrier density, the momentum density of the carriers and the kinetic energy density of the carriers is assumed to be uniform in a local volume, even though these quantities are randomly distributed.

So we are ignoring the random distribution of the density, the momentum density and the energy density and we are assuming these parameters to be uniform over local volumes. So we are dividing the device into smaller local volumes for the purpose of analysis. Now what is the great advantage of this approximation, that it allows us to treat the carrier motion or analyse the carrier motion in terms of conservation of carrier concentration, conservation of the momentum density and conservation of the energy density.

We are very familiar with treating phenomena in terms of Newton's law and these conservation principles. So mass conservation, energy conservation and momentum conservation, which is exactly what we are going to do in semiconductor devices.