

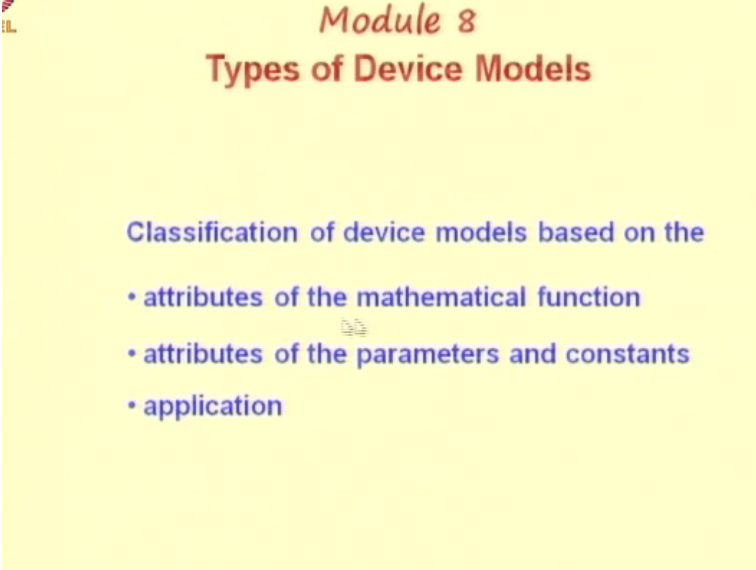
**Semiconductor Device Modeling**  
**Prof. Shreepad Karmalkar**  
**Department of Electrical Engineering**  
**Indian Institute of Technology - Madras**

**Lecture - 38**  
**Types of Device Models**

In the previous lecture, we began a discussion of the Types of Device Models because the situation in a device is very complex multiple approaches to modelling exist, in the previous lecture we have considered classification of the device models based on the rate of time variation of the voltage or current, so we talked about quasi-static and non-quasi-static models, then we talked about classification based on the frequency of the applied voltage or current signal.

We talked about low frequency and high frequency models in this context, then we spoke about rigorous and phenomenological models based on the derivation and finally we talked about analytical and numerical solution techniques.

**(Refer Slide Time: 01:14)**



*Module 8*  
**Types of Device Models**

Classification of device models based on the

- attributes of the mathematical function
- attributes of the parameters and constants
- application

In this lecture we shall consider further types of models namely classification of device models based on attributes of the mathematical function, attributes of the parameters and constants and based on the application.

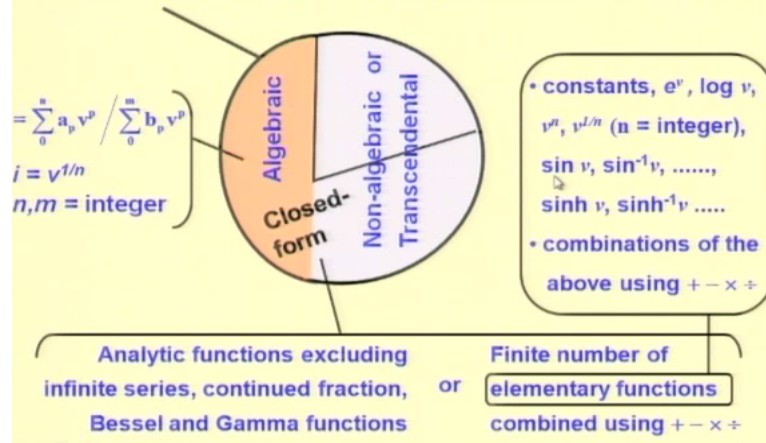
**(Refer Slide Time: 01:31)**



## Classification Based on Attributes of the Function

(Applies to analytical models)

Analytic function: a function expressible as Taylor's series



Let us consider classification based on attributes of the function, this classification applies to analytical models, let us look at the advantages knowing the mathematical forms of the model equation, as we have remarked in the very introductory lectures models are for gaining insight and computation in design applications, one mathematical form may be superior to another from the computational point of view.

Similarly, an awareness of the various mathematical forms of the model solution can give us ideas about what kind of model equation may result from a modelling exercise, we may be able to anticipate the mathematical form of the model solution from the beginning of the device modelling procedure when we are applying it to any particular device. First let us define what is an analytical function, it is a function expressible as Taylor's series.

So analytical models can be formulated in terms of analytic functions, now let us see what are the types of analytical functions there are possible for analytical models, so one broad classification of analytic functions is in terms of algebraic and non-algebraic or transcendental functions, now let us look at the forms of algebraic functions, so one form of algebraic functions is ratio of polynomials.

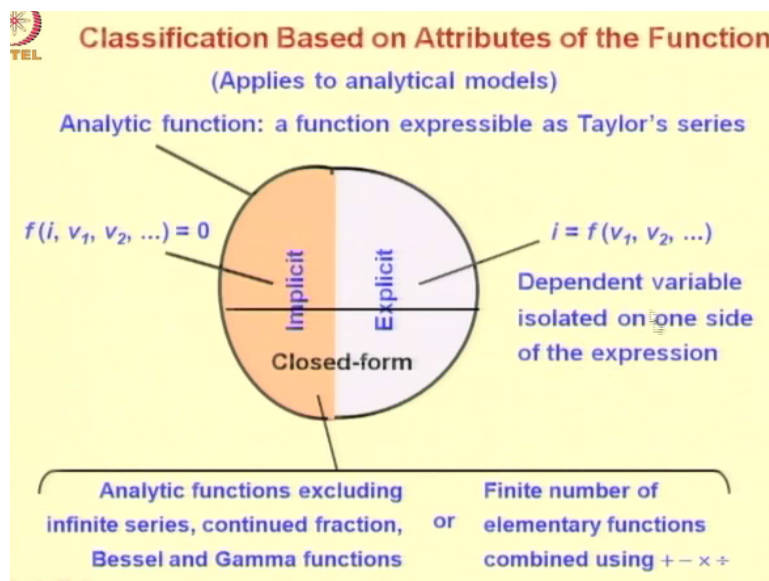
So here  $i$  can be regarded as a current and  $v$  can be regarded as a voltage then  $i = \frac{\sum a_p v^p}{\sum b_p v^p}$  where  $p$  varies from 0 to  $n$  divided by  $\sum b_p v^p$  where  $p$  varies from

0 to m, here n and m are integers, so the form  $i = v$  power  $1/n$  where n is an integer also represents an algebraic function, now the algebraic functions and a class of non-algebraic and transcendental functions constitute what is called the closed-form type of functions.

So closed-form functions are analytic functions excluding infinite series, continued fraction, Bessel and Gamma functions, so infinite series, continued fraction, Bessel and Gamma functions, infinite number of calculations are required in exact evaluation of the function, another way of identifying closed-form functions is that this function consists of a finite number of elementary functions combined using the arithmetic operations namely +, -, multiplication and division.

The elementary functions are constants  $e$  power v, log v, v power n, v power  $1/n$ , where n is an integer and then trigonometric functions sine v and inverse trigonometric functions sine inverse v and cosine and so on and similarly hyperbolic sine and hyperbolic inverse sine functions and similarly you also include cosine hyperbolic and cosine inverse tan hyperbolic and tan hyperbolic inverse functions and also combinations of the above using +, -, multiplication and division.

**(Refer Slide Time: 05:39)**



Another way you can classify analytic functions is implicit and explicit functions, the closed-form functions can be are both implicit and explicit type, so an explicit function is of the form  $i =$  function of the various voltages, so in this form the dependent variable is isolated on one side of

the expression, whereas in the implicit form the dependent and independent variables both are on the same side of the equation.

**(Refer Slide Time: 06:19)**

**Examples of Various Types of Analytical Models**

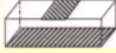
$I = I_s \left[ \exp\left(\frac{V - IR_t}{\eta V_t}\right) - 1 \right]$     **Transcendental, Implicit**

$V = IR_t + \eta V_t \ln\left(\frac{I}{I_s} + 1\right)$     **Transcendental, Explicit**

**Not Closed-form if  $I$  is the dependent variable**

$I_D = K \left[ (V_{GS} - V_T) V_{DS} - 0.5 V_{DS}^2 \right]$     **Explicit, polynomial, closed-form**  
 for  $V_{GS} \geq V_T$      $V_{DS} \leq V_{GS} - V_T$     **(for given  $V_{GS}$  or  $V_{DS}$ )**

$\frac{R}{R_0} = \frac{1}{1 + \Delta / G} + \frac{1}{(1 + \Delta / G)(T / G)} \sum_{n=1}^{\infty} \frac{\tanh [2n(T / G)] \sin^2 n}{n^3} \quad n = \frac{m\pi}{1 + \Delta_G / G}$


**[Lindsted, IEEE TED, p. 41, 1972]**    **Explicit, transcendental, infinite series**

Let us look at examples of various types of analytical models, consider the model of a real diode  $I = I_s \exp(V - I R_t / \eta V_t) - 1$ , the same equation written in a different form with  $V$  on one side or the left hand side and the current  $I$  on the right hand side, now this form is transcendental and this form too is transcendental because it involves logarithmic functions.

However, this is an implicit form because it contains both the dependent and independent variables on the same side whereas the same equation has been recast here with voltage on the left hand side and current on the right hand side, in other words the variables have been isolated independent and dependent variables have been isolated on the 2 sides of the equation and therefore this form is explicit.

Now these 2 functions can also be regarded as closed-form if  $V$  is the dependent variable, so if this is dependent variable then you can be regarded as close-form because if  $I$  substitute the independent variable  $I$  value of  $I$ ,  $I$  can easily solve for  $V$  from here as well as from here.

On the other hand, it is not closed-form so both of these are not closed-form if I is dependent variable for example if I give you the value of V and ask you to find out I you cannot find out I by finite number of calculations either in this equation or in this equation, consider another mathematical function that of this ID VDS characteristics of a MOSFET, now this is an explicit function then it is polynomial or algebraic form and it is also closed-form for given values of VGS or VDS.

Here is another model function where R is a resistance of the top small contact with respect to the bottom larger contact in this geometry, R0 is the resistance between the contacts when both are of the same size as the top contact, delta is the extension of the bottom contact as compared to the top contact and G is the length of the top contact resistance in this direction, so this R/R0 expression is given here now this is an explicit form of function, it is transcendental.

Because it involves hyperbolic tangent and so on and it is an infinite series form you can see here that the number of terms is infinity m varies from 1 to infinity, the n here is given in terms of m according to this formula.

(Refer Slide Time: 09:55)

**Example of a Numerical Model**

$$I = \int \mathbf{J} \cdot d\mathbf{S} \quad \mathbf{J} = \mathbf{J}_n + \mathbf{J}_p$$

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	$\partial_t n = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)$
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	$\partial_t p = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)$
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s \quad \rho = q(p + N_a^+ - n - N_s^-)$

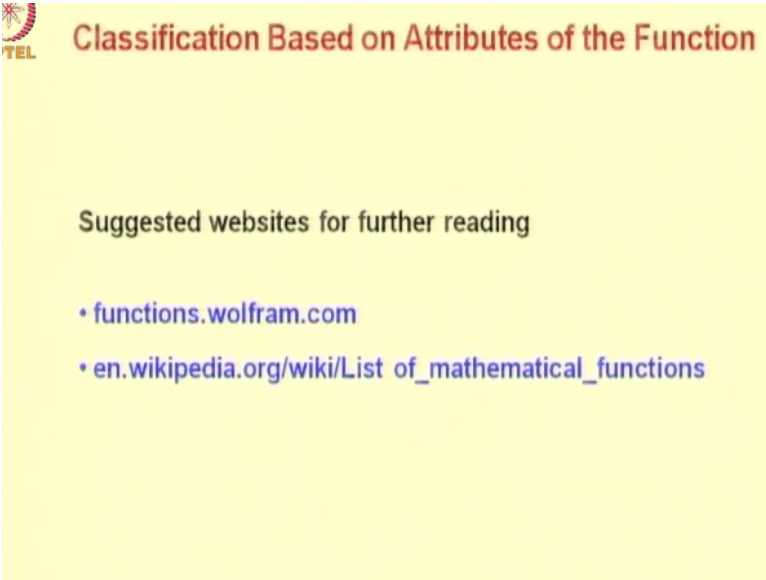
$$\psi = -\int \mathbf{E} \cdot d\mathbf{l} \quad \delta p = p - p_0 \quad \delta n = n - n_0 \quad \tau \equiv \tau_{\text{minority}}$$

**Drift-Diffusion Model**

Having consider the example of several analytical models, let us look at an example of numerical models this is our drift diffusion model all these situations can be solved simultaneously with all

their terms for a general device only by numerical techniques, so that is why this is referred to as a numerical model.

**(Refer Slide Time: 10:20)**



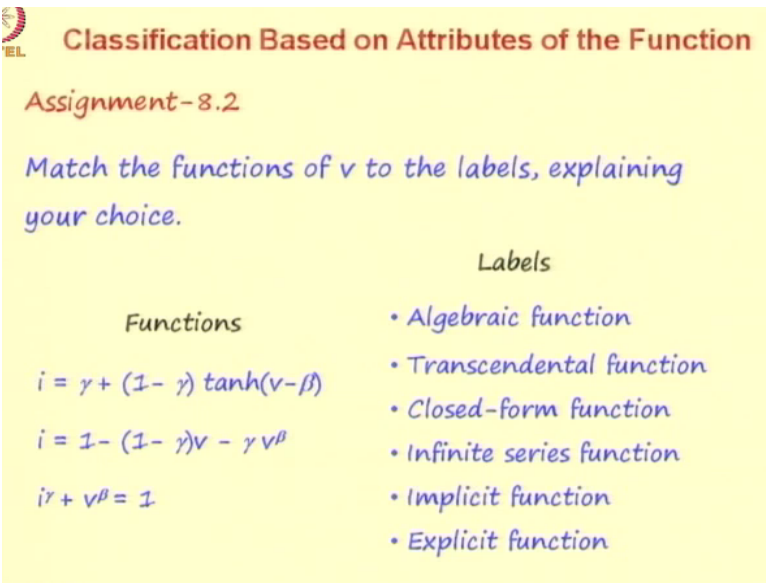
**Classification Based on Attributes of the Function**

Suggested websites for further reading

- [functions.wolfram.com](https://functions.wolfram.com)
- [en.wikipedia.org/wiki/List\\_of\\_mathematical\\_functions](https://en.wikipedia.org/wiki/List_of_mathematical_functions)

Since this topic is very interesting you can read up these websites to understand more details of various mathematical functions.

**(Refer Slide Time: 10:42)**



**Classification Based on Attributes of the Function**

*Assignment-8.2*

*Match the functions of  $v$  to the labels, explaining your choice.*

<i>Functions</i>	<i>Labels</i>
$i = \gamma + (1 - \gamma) \tanh(v - \beta)$	• Algebraic function
$i = 1 - (1 - \gamma)v - \gamma v^\beta$	• Transcendental function
$i\gamma + v^\beta = 1$	• Closed-form function
	• Infinite series function
	• Implicit function
	• Explicit function

An assignment, match the functions of  $v$  to the labels, explaining your choice, so these are the functions of  $v$ ,  $i$  as a function of  $v$ , so  $\gamma$  and  $\beta$  here are parameters okay, the labels are algebraic functions, transcendental functions, closed-form function, infinite series function, implicit function and explicit function.

(Refer Slide Time: 11:21)

**Classification Based on Attributes of the Function**

**Assignment-8.3**

Proper graphing or sketching of any function  $i = f(v)$  involves showing the following features:

- $i$ - and  $v$ - intercepts
- tangents
- maxima and minima
- cusps
- points of inflection
- jump discontinuities
- asymptotes
- holes, corners, isolated points

Another assignment regarding graphing or sketching of a function, proper graphing or sketching of any function  $i$  as a function of  $v$  involves showing the following features,  $i$  and  $v$  intercepts, maxima and minima, points of inflection, asymptotes, tangents, cusps, jump discontinuities, holes, corners and isolated points.

(Refer Slide Time: 11:50)

**Classification Based on Attributes of the Function**

**Assignment-8.3**


Sketch the following functions, showing as many of the above features as may be applicable:

a)  $i = \gamma + (1 - \gamma) \tanh(v - \beta)$   
where  $0 < \gamma < 1, \beta > 0$

b)  $i = 1 - (1 - \gamma)v - \gamma v^\beta$   
where  $0 < \gamma < 1, \beta > 1$

Now sketch the following functions showing as many of the above features as maybe applicable, so first function is  $i = \gamma + (1 - \gamma) \tanh(v - \beta)$  where  $\gamma$  lies between 0 and 1 and  $\beta > 0$ , another function is  $i = 1 - (1 - \gamma)v - \gamma v^\beta$  where  $\gamma$  lies between 0 and 1 and  $\beta > 1$ .

(Refer Slide Time: 12:28)



**Classification Based on Attributes of the Parameters and Constants**

Empirical model	Semi-physical / Semi-empirical model	Physical model
-----------------	--------------------------------------	----------------

**Predictive or scalable:** A model which correctly predicts the effects of parameter variations on the characteristics.

- For such a model, the parameters extracted at one  $T$  or set of dimensions, work for another  $T$  or set of dimensions.
- In the context of MOSFETs, a scalable model is one which is predictive with respect to  $W, L$

Now let us consider another classification that is based on attributes of the parameters and constants, if all the parameters have no physical meaning or none of the parameters have physical meaning then the model is said to be empirical, on the other hand if all parameters in the model have a physical meaning then it is a physical model, however most models belong to the category of semi physical or semi empirical.

This is because it is difficult to model all physical effects in an equation which is computationally efficient, therefore what one does is one captures the dominant physical phenomenon in an equation which is simple and then introduces empirical adjustments of this basic equations to improve its accuracy, however results is constantly on to make models more and more physical.

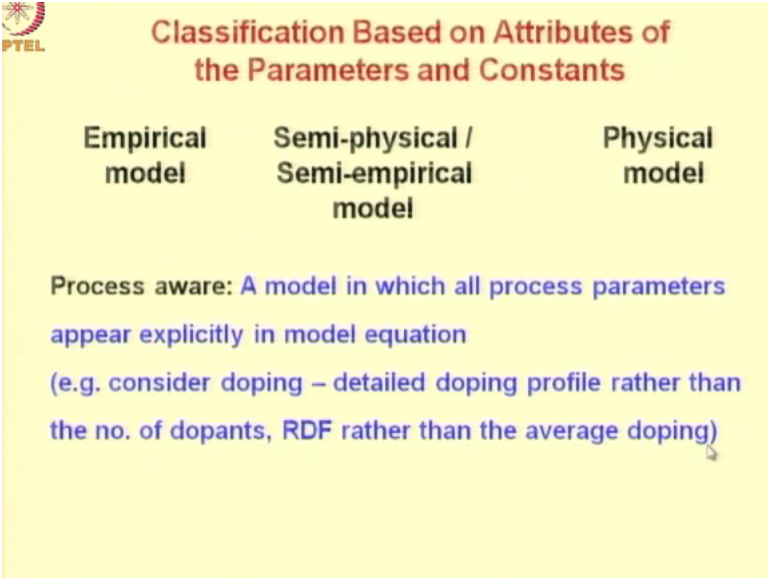
And therefore you have various terms associated with models such as predictive or scalable model, now this refers to a model which correctly predicts the effects of parameter variations on the characteristics, for such a model the parameters extracted at one temperature or one setup dimensions work for another temperature or another set of dimensions, consider the example of a diode model.



If the diode model is in terms of series resistance  $R_s$  we have seen an equation for Ariel diode in earlier modules that resistance if you extract for a set of measured data at 1 temperature you may not be able to use the same resistance value for predicting the measured data at another temperature, you can however make the model scalable or predictive with respect to temperature, if you replace the resistance  $R_s$  by a model of  $R_s$  in terms of temperature.

In that model of  $R_s$  in terms of temperature there will be some constants or parameters and those parameters if you extract at some temperature they may even work at another temperature because the model expression for  $R_s$  consists of temperature as a variable, in the context of MOSFETs a scalable model is one which is predictive with respect to  $W$  and  $L$ , this is how a scalable MOSFET model is understood then common parlance.

**(Refer Slide Time: 15:30)**



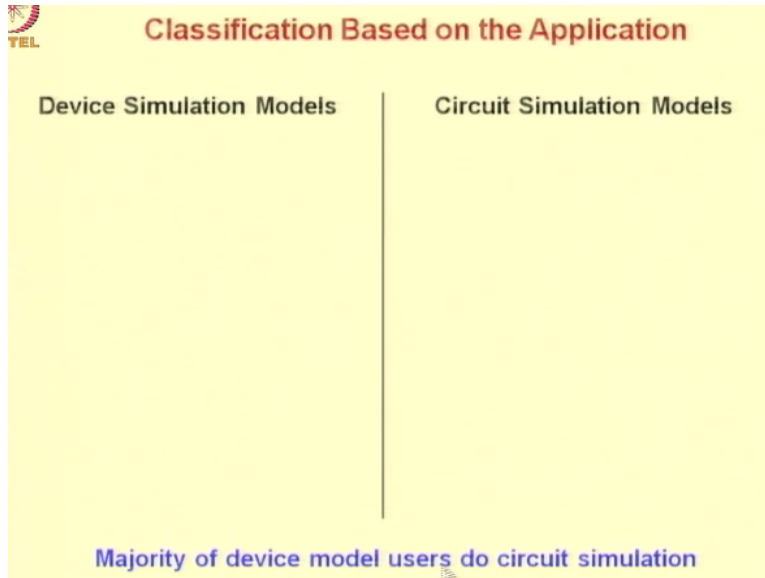
**Classification Based on Attributes of the Parameters and Constants**

Empirical model	Semi-physical / Semi-empirical model	Physical model
-----------------	--------------------------------------	----------------

**Process aware:** A model in which all process parameters appear explicitly in model equation  
(e.g. consider doping – detailed doping profile rather than the no. of dopants, RDF rather than the average doping)

The term process aware models, a process aware model is one in which all process parameters appear explicitly in model equation, for example if you consider doping as a process parameter then process aware model will have a detailed doping profile information rather than just the number of the dopants or it will have information about the random dopant fluctuation rather than just the average value of doping.

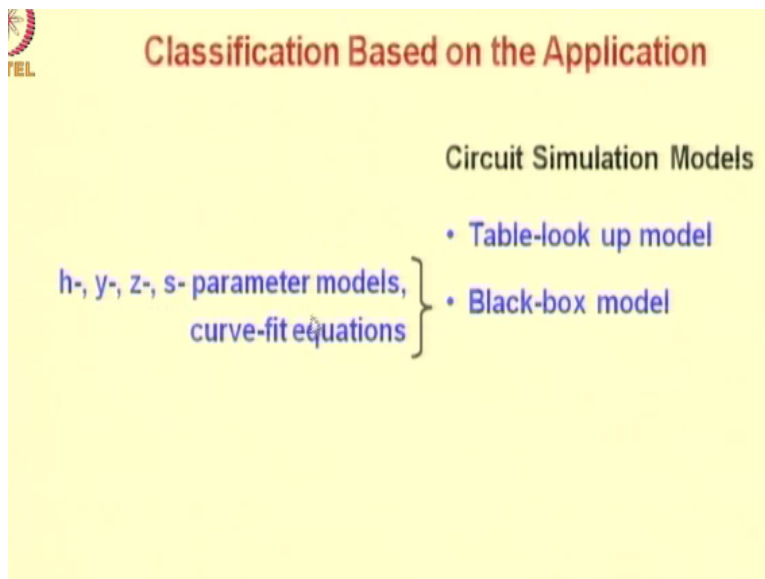
**(Refer Slide Time: 16:10)**



Let us look at classification based on application, based on application models can be broadly classified into Device simulation models and Circuit simulation models, majority of the device model users do circuit simulations, in order that the model will be suitable for circuit simulation it should satisfy several constraints, one of them is computational efficiency.

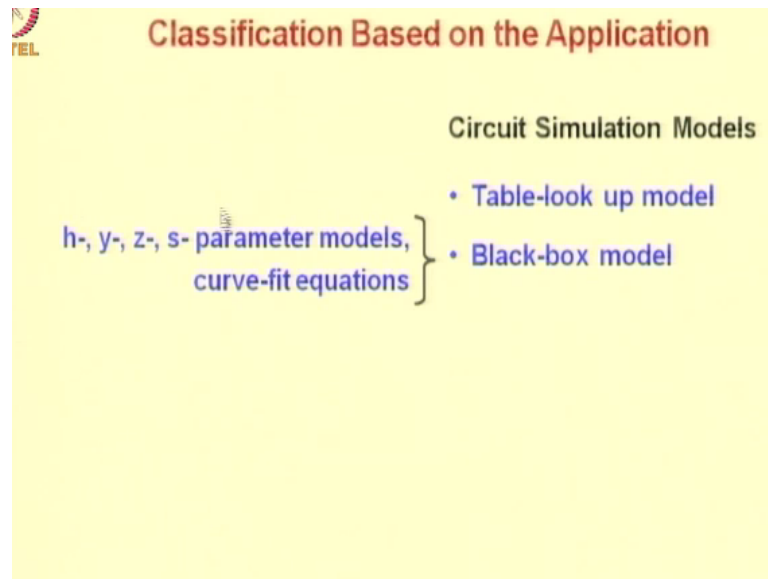
Then there is also the issue of mathematical properties of the model such as continuity asymptotic correctness and so on, now because of these constraints there are a variety of models which can be used for circuit simulation.

**(Refer Slide Time: 16:57)**



The model suitable for circuit simulation which has absolutely no physical basis is the Table-lookup model it is just a table of currents or small-signal parameter values for a large number of combinations of bias voltages, so you replace the device in a circuit by a table of current voltage values and this current voltage values are used for calculating the parameters of the circuit, this is very fast model but has no physical basis we shall discuss models with increasing physical basis okay, one by one.

**(Refer Slide Time: 17:36)**



The Black-box model examples of these are h, y, z, or s parameter models or curve-fit equations, here the devices treated as black-box and its terminal are input output characteristics are modelled in terms of a few parameters.

**(Refer Slide Time: 18:01)**



## Classification Based on the Application

### Circuit Simulation Models

A simple model which captures the device behaviour but is not necessarily physical

- Table-look up model
- Black-box model
- Behavioral model

Behavioral model it is a simple model which captures the device behaviour but is not necessarily physical, so an empirical model which captures the behaviour can be called a behavioral model it is not necessary that the has to be empirical for it to be behavioral but the point is the physical content in the model is not an issue, the main purpose of this model is to capture the input output characteristics, a particular model may be referred to by several different terms, so the model can be empirical it can also be behavioral and so on.

**(Refer Slide Time: 18:43)**



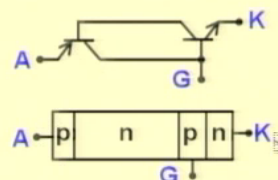
## Classification Based on the Application

### Circuit Simulation Models

A complex device structure modeled as an interconnection of simpler devices

- Table-look up model
- Black-box model
- Behavioral model
- Sub-circuit model
- Macro-model

e. g. Power MOSFET, thyristor, Insulated Gate Bipolar Transistor (IGBT), model for latch-up problem in CMOS ICs, etc.

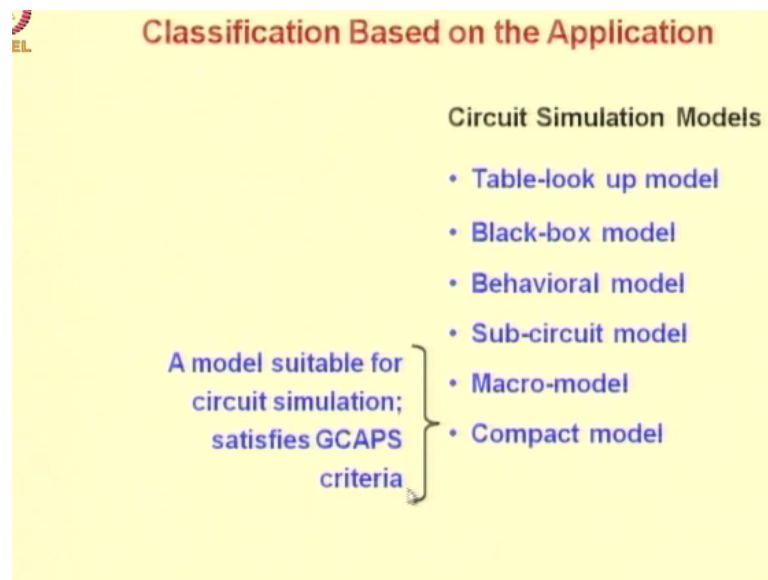


Another class of models is Sub-circuit or Macro-model, a complex device structure modeled as and interconnection of simpler devices, let us look at some examples of these type of models you can look at the internet and find macro-models or sub-circuit models for power MOSFET,

thyristor, Insulated Gate Bipolar Transistor and model for latch up problem in CMOS ICs, by way of example we will show here a macro-model of a thyristor.

So this is the 4 layer device structure called thyristor, left hand contact is the anode, right-hand contact is the cathode and this is the gate, so this 4 layer structure can be regarded as a combination of PNP and NPN transistors connected in a cyclic fashion as shown in this circuit, this circuit consisting of 2 transistors can be regarded as the Macro-model of this device schematic.

**(Refer Slide Time: 20:06)**



The final type of circuit simulation model and probably the most important type of circuit simulation model is the Compact model, it is the model suitable for circuit simulation which satisfies the GCAPS criteria, in our course we shall be discussing about compact models of devices, so the models which will be derived for MOSFET and any other device would be of the compact variety.

**(Refer Slide Time: 20:38)**



## Compact Models

- These are of great interest to the large number of IC designers
- These illustrate the various conflicting GCAPS requirements and challenges in modeling  
e.g. compact models try to achieve physical and mathematical properties at the same time, and so their derivation is sufficiently challenging

Let us look at a few features of compact models these are of great interest to the large number of IC designers and this illustrate the various conflicting GCAPS requirements and challenges in modelling, for example compact models try to achieve physical and mathematical properties at the same time and so their derivation is sufficiently challenging, since we are going to discuss compact models of various devices in this course and compact models are important.

Let us look at some features why there are some constraints such as continuity or any other constraints on compact models, let us look at a model required for simulating circuits.

**(Refer Slide Time: 21:31)**

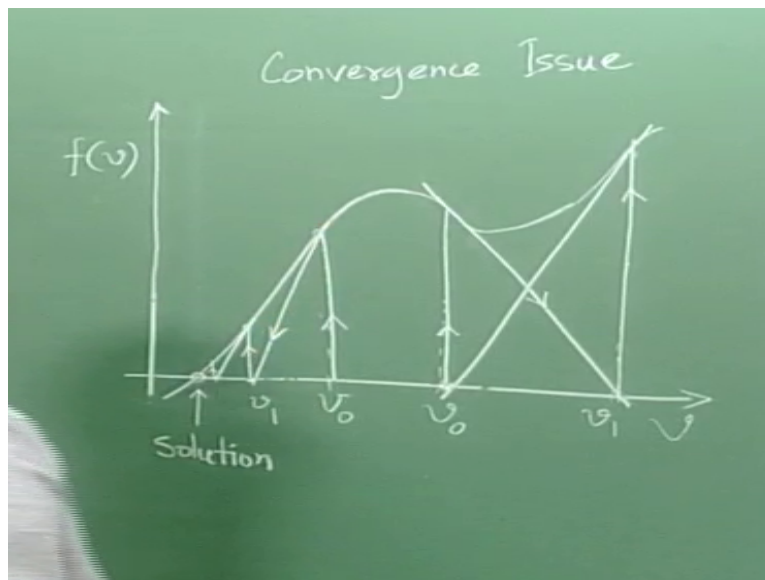
The image shows a handwritten circuit diagram and equations on a green chalkboard background. The circuit diagram depicts a resistor  $R$  connected between a supply voltage  $V_{DD}$  and the drain terminal of an nMOS transistor. The gate of the transistor is connected to a gate voltage  $V_{GS}$ , and its source is connected to ground. The drain voltage is labeled  $V_{DS}$ . Below the diagram, the following equations are written:

$$\frac{V_{DD} - V_{DS}}{R}$$
$$= K \left\{ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right\}$$
$$f(V_{DS}) = 0$$

Take a simple circuit like this which consists of resistance in series with a MOSFET, suppose I want to find out what is the drain to source voltage or voltage at this point the power supply voltage is  $V_{DD}$  and the input voltage to the gate is  $V_{GS}$ , the equation that will give you a  $V_{DS}$  is shown here, the currents with the resistance  $R$  given by  $V_{DD} - V_{DS}/R$ , this voltage - this voltage by  $R = K$  times  $\{(V_{GS} - V_T) \text{ into } V_{DS} - V_{DS}^2/2\}$ , so this represents the current through the MOSFET.

So you equate the current through the resistance to the current through the MOSFET Kirchhoff's current law and you get this equation, now this equation can be cast in the form of a  $f(V_{DS}) = 0$ , swift all the terms to the left hand side and then you get this form of the equation, now the route of this equation will give you the value of  $V_{DS}$  for this circuit, so this is the kind of situation we encounter in circuit simulation.

**(Refer Slide Time: 22:50)**



Now we have shown a simple situation in practice the function of  $V_{DS}$  can be fairly complex, now let us say in some circuit the function of  $v$  had this particular shape for some reason, here we have shown  $f(v)$  as a function of  $v$ , the small  $v$  can be regarded as a voltage like the  $V_{DS}$ , now how do we get this solution of this problem that is where does the function go to 0, so this is the solution.

Normally circuit simulation programs use the Newton-Raphson technique to arrive at the solution, so what is this approach you start with an initial guess, now suppose for example I start with an initial guess let me call it  $v_0$  then based on the value of the function here, I take the next guess as follows so I take the slope of this curve which brings me to this point and I call this the first iteration guess.

Now to get the next better guess again find out the value of  $f(v)$  at this point and at this point I take a slope of this curve and extend the tangent to the  $v$  axis to get the next guess, so this is  $v_2$  and then I go on doing like this so ultimately I will reach here okay after a few iterations very close to this point infinite number of iterations maybe required to actually get to the point.

But we can assume certain level of accuracy and for that accuracy we can terminate the process after that level of accuracy is achieved, so now you can see that the solution according to this Newton-Raphson technique depends on the initial guess okay the number of iterations required depends on initial guess and it depends on the slope of the curve we can see that clearly, supposing my initial guess was this in some case.

After all when complex circuits is solved I do not have much control on the values that are going to be present at the various nodes in each iteration and I do not have that much control over initial guess let us say and it turned out to be this for this kind of a function, following the same approach how will I get the next guess, so I go up find out the point  $f(v)$  at that point I take a tangent and this turns out to be something like this.

So now this is my next guess to get the next better guess I go up I am on this side here and I take tangent at this point and who knows I might find that this tangent passes through this point and so I am back to  $v_0$  and then I will go up to get the next guess I will come back here, so I will keep oscillating between  $v_1$  and  $v_0$ , in other words I have encountered what is called convergence problem, I am not able to converge.

Therefore, the shape of this  $f(v)$  is quite critical in the solution and as is the initial guess, now if your model equation is not with right properties I might end up with the situation like this



because of this the demand for model suitable for circuit simulation imposes constraints, even if you take your voltage across the device to unrealistically very high values which can happen during the circuit simulation when you are trying to solve the circuit mathematically.

Because we do not know at any instant of time what are the voltages at various nodes, so for very large voltages much beyond the practical range your model current voltage model should not be unrealistic and should not for example result in shapes such as shown here for example if your model equation is such that it gives negative conductance beyond some voltage range, in the range in which the device works it may be alright.

But beyond the range it may give negative conductance or other such problems and they in turn will give rise to this kind of situation, so that is why asymptotic correctness is a very important criterion for compact models, when we talked about the physical basis we did talk about the asymptotic correctness.

When we talked about the spreading resistance model okay the very simple model without improvement which was based on a constant angles spreading current was not asymptotically correct please recall the discussion and then we did make some changes in the model to bring about asymptotic correctness that is the prediction of right value of the quantity being modeled even for very very high values of the independent variable.

So this discussion should tell you why the derivatives of the function or derivatives of the model are also important if for example here that any of the points or derivatives is discontinuous you may get into convergence problems, now you may find in a compact model some additional empirical factors to improve convergence properties.

**(Refer Slide Time: 29:08)**

The image shows a green chalkboard with a handwritten equation for the generation and recombination current,  $I_{GR}$ . The equation is written in white chalk and is as follows:

$$I_{GR} = I_{SR} \left[ \left( 1 - \frac{V_{di}}{V_J} \right)^2 + 0.001 \right]^{M/2} \times \left[ \exp\left( \frac{V_{di}}{NR V_{th}} \right) - 1 \right]$$

A hand is visible at the bottom of the chalkboard, pointing towards the equation.

Here is an example of generation and recombination model employed in modelling of a diode in SPICE, so you find a factor 0.001 here in this expression where generation recombination current is expressed in terms of the voltage across the diode  $V_{di}$  is the voltage across the depletion layer of the diode,  $V_J$  is the contact potential and this is an idealization factor okay or ideality factor, this is thermal voltage.

Now this factor has been introduced to improve convergence properties we will not discuss how the convergence properties are improved, we will just point out how the demands of circuit simulation impose certain restrictions in the type of model equations or introduced certain features in the type of equations of a compact model.

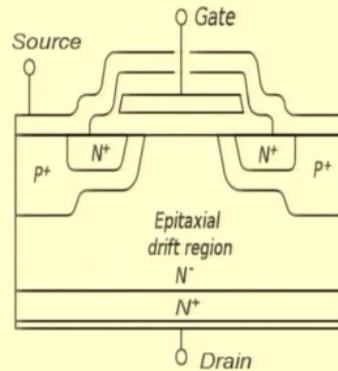
**(Refer Slide Time: 30:08)**



## Classification Based on the Application

### Assignment-8.4

Wikipedia gives the macro-model or sub-circuit model of an Insulated Gate Bipolar Transistor (IGBT), whose cross-section is obtained by replacing the bottom  $n^+$  substrate of the Power MOSFET shown here, by a  $p^+$  substrate.



Now here is an assignment related to the macro-model, Wikipedia gives the macro-model or sub-circuit model of an Insulated Gate Bipolar Transistor (IGBT) whose cross section is obtained by replacing the bottom  $n^+$  substrate of the power MOSFET shown here by a  $p^+$  substrate, this is the structure of a power MOSFET if you replace the  $N^+/P^+$  this becomes an Insulated Gate Bipolar Transistor.

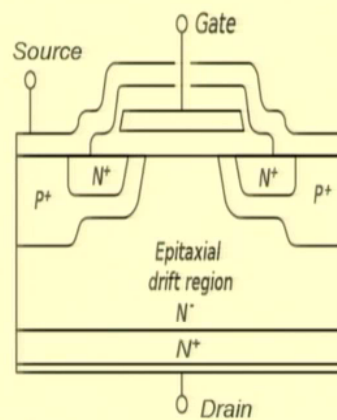
(Refer Slide Time: 30:36)



## Classification Based on the Application

### Assignment-8.4

Modify this macro-model to represent the Power MOSFET between its source, drain and gate terminals.



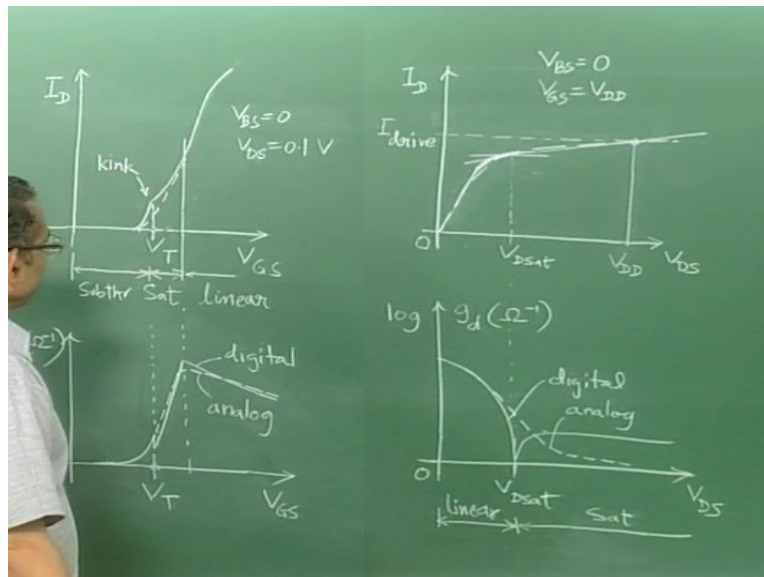
Now modify macro-model that you find in the Wikipedia to represent the power MOSFET between its source, drain and gate terminals, so Wikipedia gives the MOSFET when this  $N^+$  is replaced by  $P^+$  using that model and using appropriate modification of the model you are expected to represent this power MOSFET structure.

(Refer Slide Time: 30:57)

Classification Based on the Application	
<b>Digital Model of a MOSFET</b> <ul style="list-style-type: none"> <li>• Goal is prediction of inverter switching speed, for which, accurate prediction of drive current and parasitic capacitance sufficient</li> <li>• Regional equations for sub-threshold, linear, saturation, depln, accum</li> <li>• More but simple equations <math>\Rightarrow</math> few model parameters</li> </ul>	<b>Analog Model of a MOSFET</b> <ul style="list-style-type: none"> <li>• Both DC and small-signal characteristics – <math>g_d</math>, <math>g_m</math>, <math>g_{mb}</math> should be predicted accurately</li> <li>• Single equation smoothly connecting all regions</li> <li>• Single but complex equation <math>\Rightarrow</math> many model parameters</li> </ul>
Majority of ICs are digital CMOS	

Let us look at a final classification based on application, namely Digital model and Analog model of a MOSFET, digital model the goal is prediction of inverter switching speed for which accurate prediction of drive current and parasitic capacitance is sufficient.

(Refer Slide Time: 31:25)



So what is the drive current look at the  $I_D$   $V_{DS}$  characteristic of a MOSFET, the current corresponding to  $V_{DS} = V_{DD}$  that is a maximum current in the device this is called the drive current, the analog model on the other hand both DC and small-signal characteristics such as  $g_d$ ,  $g_m$  and  $g_{mb}$  that is transconductance due to body effect should be predicted accurately, so that is the difference between the goals of the 2 models.

Consequently, for a digital model the model equations are regional for sub-threshold, linear, saturation, depletion and accumulation regions and so on right, so you have equations which are valid in these various regions of operations of the MOSFET or the MOS capacitor and these equations which are valid in different segments are put together to get the overall equation, whereas in analog MOSFET you need a single equation which smoothly connects all regions.

Because the conductance's which depend on the derivative of the current voltage curve are to be predicted accurately, as a consequence of these features the digital model of a MOSFET has more number of equations which are all very simple therefore the model has few parameters so more equations but all simple equations therefore few parameters that is the characteristic of digital models.

Analog model on the other hand you have a single equation but it is fairly complex and therefore it has many model parameters, now let us illustrate the difference between analog and digital models, consider the  $I_D$  vs  $V_{GS}$  curve of a MOSFET for a given bulk source bias and drain source bias, assume a small drain source bias of 0.1 volt, generally  $I_D$  vs  $V_{GS}$  curves are plotted for small values of  $V_{DS}$ .

Now a digital model will consist of model equations for sub-threshold, saturation and linear regions okay, so you can see that you have one equation for the sub-threshold which is having these kind of a graph and then you can see that there is a kink here, you have another equation for the super threshold region okay which covers the saturation and linear regions, now this is okay for a digital application.

But for analog purposes we need a smooth curve which is shown by dotted or dash line, the consequences of these different  $I_D$  vs  $V_{GS}$  equations will be evident in the  $g_m$  vs  $V_{GS}$  regions curve, so you can see that in the  $g_m$  vs  $V_{GS}$  curve you have a discontinuity because of this kink, the slope of the  $I_D$  vs  $V_{GS}$  curve is different on this side and on this side of the threshold voltage, and that is what is reflected in the discontinuity here in the  $g_m$  vs  $V_{GS}$  curve.

The analog model on the other hand is continuous it has no discontinuity anywhere okay, you see another sharp point in a digital model at this point which is not there in analog curve, so the solid is digital model and dash line is a results from analog models in all these diagrams that we are going to show. Look at ID VDS curve for digital model a curve such as this is shown by the solid line where you can see here the slopes are different on the 2 sides.

Here the slope is this and beyond the saturation voltage the slope is different okay, you can see the difference in slope, whereas analog model no such difference in slope is exist the slope is continuous, consequences of this on the drain conductance versus VDS characteristics which is obtained by differentiating this is very clear, so here the differences are much more enhanced, this solid line is digital model and this dash line is analog.

So you can see that for the analog model the curve is continuous for the digital model since this is a semi log plot  $g_d$  is plotted on the log scale the modeled curve is going to in fact 0, if this goes to 0 on at this and okay, so even if it does not go to 0, the slope is changing abruptly and therefore here the slope is changing abruptly and therefore you have this kind of a behaviour.

And you can see that in the saturation region the conductance is not going to 0 as in analog model but is saturating at a constant value because the slope of this line in the digital model is the same it is a straight line whereas analog model you can see that progressively the slope is becoming are approaching 0, majority of the IC or digital CMOS and therefore digital models are widely prevalent.

**(Refer Slide Time: 37:42)**



## Inverse Modeling

This can mean:

- extraction of device structure from measured data  
e.g. extraction of 2-D doping profile in a MOSFET from measured C-V data
- modeling of the  $I_D - V_{DS}$  characteristics of an N-channel MOSFET for  $V_{DS} < 0$  or P-channel MOSFET for  $V_{DS} > 0$ .

Let us look at the meaning of word Inverse modelling it can mean 2 things, one meaning is extraction of device structure from measured data example extraction of 2-dimensional doping profile in a MOSFET from measured C-V data, another meaning is modeling of the ID VDS characteristic of an N-channel MOSFET for  $V_{DS} < 0$  or P-channel MOSFET for  $V_{DS} > 0$ , so normally we will talk about  $V_{DS} > 0$  for n-channel MOSFET so all these curves are for n-channel MOSFET.

So inverse modeling involved will involve predicting the curves for  $V_{DS}$  negative for n-channel MOSFET or  $V_{DS}$  positive for a p-channel MOSFET.

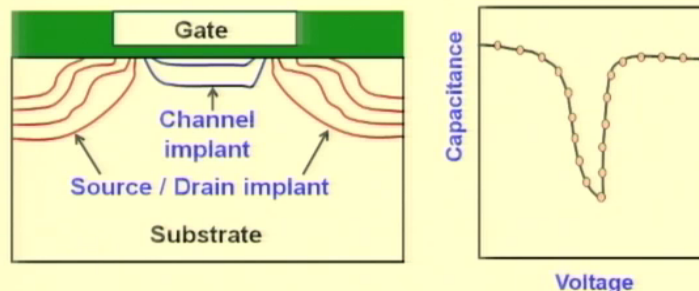
**(Refer Slide Time: 38:36)**



## Inverse Modeling

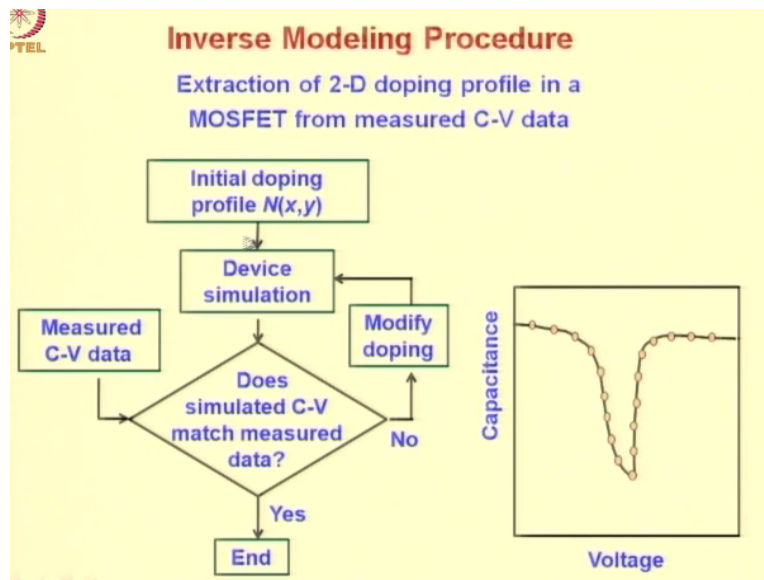
This can mean:

- extraction of device structure from measured data  
e.g. extraction of 2-D doping profile in a MOSFET from measured C-V data



We just let us illustrate quickly how does inverse modeling extract the device structure from measured data, suppose we want to extract these 2 dimensional doping profile of this MOSFET from C-V measurements here is the capacitance versus voltage between the gate and the substrate, now from this data you want to extract this information about the 2 dimensional doping profile.

**(Refer Slide Time: 39:05)**



So broadly the flowchart of this activity would be as follows, you assume some initial doping profile and do a device simulation using that doping profile, so the doping profile you should assume initially would be reasonable based on your experience of various MOSFET, now do a device simulation to get the C-V characteristic for this doping profile.

Does simulated C-V match measured data? If no then you modified your doping and do the device simulation again and go on doing this until the simulated and measure data match at that point you can come out and then the resultant doping profile is the actual doping profile of the device, so measured C-V data is being input into this step for matching with the simulation data. With that we have come to end of this module on types of device models.

**(Refer Slide Time: 40:10)**



**Jargon**

AC	Implicit	Predictive
Analog	Infinite series	Process aware
Behavioural	Inverse	Quasi-static (QS)
Black-box	Large-signal	Rigorous
Closed-form	Low frequency (LF)	Scalable
Compact	Macro	Small signal
DC	Non-Quasi Static (NQS)	Static
Digital	Numerical	Sub-circuit
Empirical	Parametric	Table look-up
Explicit	Phenomenological	Transcendental
High Frequency (HF)	Physical	Transient

So let us make summary of important points, first we presented a number of terms that are used in the context of device modeling and we explained many of these terms as the types of device models based on various types of classifications of the device models.

(Refer Slide Time: 40:34)

**Classification Based on the Time Rate of Change of Voltage / Current**

In the context of the DD Model

Static or DC Model:  $\partial_t n, \partial_t p, \partial_t E = 0$

Quasi-static (QS) Model:  $\partial_t \delta n \ll \delta n / \tau, \partial_t \delta p \ll \delta p / \tau, \partial_t Q_i \ll Q_i / \tau_{tr}, \partial_t E \ll E / \tau_d$

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	$\cancel{\partial_t n} = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)$
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	$\cancel{\partial_t p} = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)$
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s, \rho = q(p + N_d^+ - n - N_a^-)$

$J = J_n + J_p + \epsilon \partial_t E$

The first classification we considered was the based on time rate of change of voltage or current in the context of DD model, the static or DC model was a model that used conditions of electron concentration hole concentration electric field as constant with time and therefore  $dn/dt$  and  $dp/dt$  and  $dE/dt$  were 0.

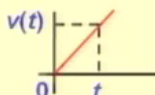
The quasi-static model on the other hand assume that the variation of the excess carrier concentration such as electron and hole concentrations was rather slow on the time scale of the minority carrier lifetime the variation of inversion charge in a MOSFET was rather slow on the scale of a transit time and the variation of the electric field was rather slow on the scale of dielectric relaxation time.

Under such conditions you can neglect the  $\frac{dn}{dt}$  and  $\frac{dp}{dt}$  terms in the continuity equations of electrons and holes and the  $\frac{dE}{dt}$  term in the expression for the current density which consists of the electron current due to drift and diffusion, hole current due to drift and diffusion and displacement current represented by the  $\frac{dE}{dt}$  term.

**(Refer Slide Time: 41:44)**

**Classification Based on the Time Rate of Change of Voltage / Current**

Quasi-static approximations yield a reasonably accurate model for rapidly time varying device phenomena. How ?

- $i(t)$  due to  $v(t)$  is modeled starting from the approximation: 

$p(x), n(x), E(x)$  at  $t \approx$  steady state conditions corresponding to  $v(t)$

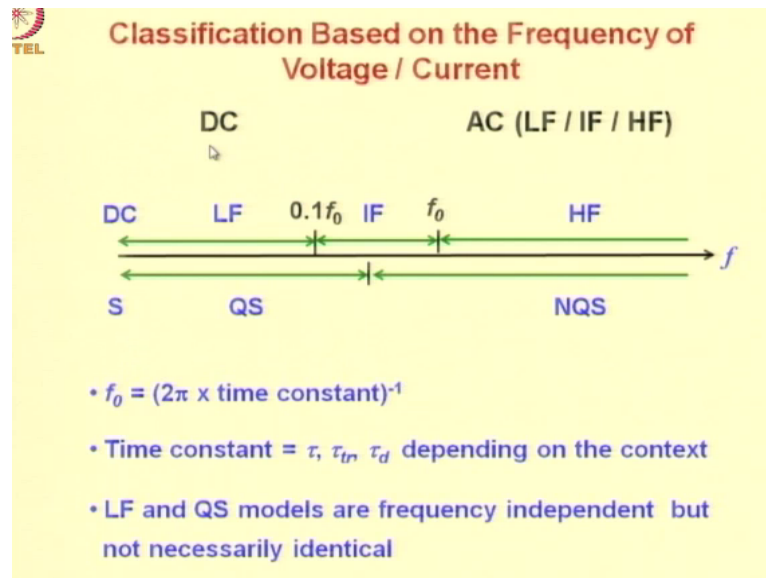
- The model is of the form  $i_{qs}(t) = I[v(t)] + d_t Q[v(t)]$  where  $I(V)$  is the current and  $Q(V)$  is the electron / hole charge stored in the device due to steady state bias  $V$ .

The quasi-static approximations yield a reasonably accurate model for rapidly varying device phenomena, now how do you do that well they get the current instantaneous current as a function of instantaneous voltage starting from the approximation that the distribution of holes, electrons and electric field at any time is approximately = the steady state conditions corresponding to the voltage at that instant of time.

And the result of this derivation from these conditions is a model of the form  $I = I$  of  $v(t)$  where capital  $I$  of  $v(t)$  is the DC current voltage model +  $d/dt$  of  $Q$  of  $v(t)$  where  $Q$  as a function of  $v$  is

the DC charge voltage model, so where  $I(V)$  is the current and  $Q(V)$  is the electron, hole charge stored in the device due to steady state by bias  $V$ .

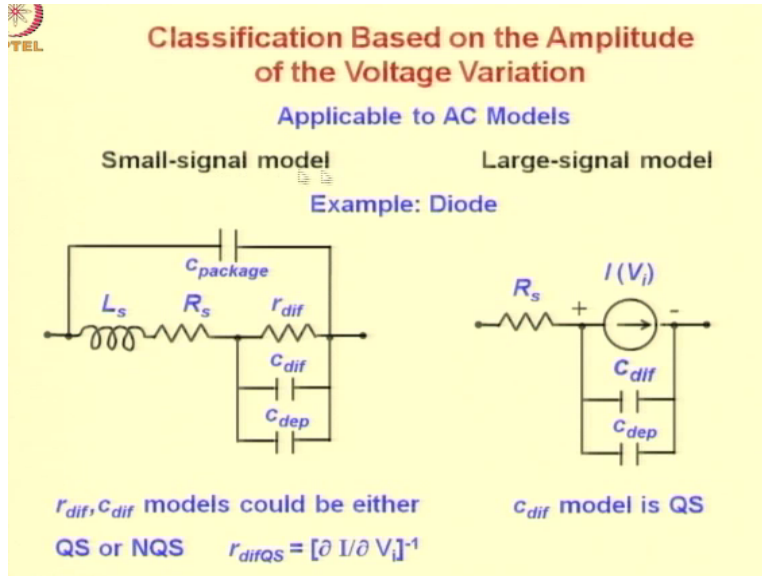
**(Refer Slide Time: 42:46)**



Next we considered classification based on the frequency of voltage or current the classification was DC and AC where AC can be further classified into low frequency, intermediate frequency and high frequency as shown in the diagram, since this is a logarithmic scale of frequencies, the DC or static models can only be represented for  $f = 0$  which will go which will be located at -infinity and that is why this either of these is shown to be extending towards -infinity.

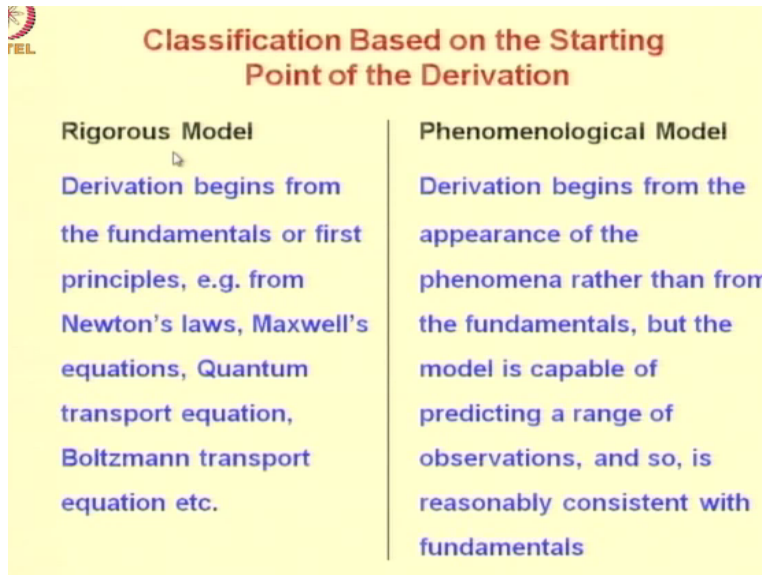
The transition frequency  $f_0$  beyond which the high frequency modeling becomes necessary is given by  $1/2 \pi$  into time constant where the time constant can be minority carrier lifetime transit time or dielectric relaxation time depending on the context, low frequency and quasi-static models are frequency independent but not necessarily identical we explain this or demonstrated this with the help of model of the diffusion capacitance of a diode.

**(Refer Slide Time: 44:00)**



Then we consider classification based on the amplitude of the voltage variation, small-signal models and large-signal models, in large-signal models you have the device represented by a dependent current source, voltage dependent current source where this current  $I$  is a function of  $V_i$  follows the DC model.

**(Refer Slide Time: 44:26)**



Classification based on starting point of the derivation, so you have rigorous models and phenomenological models in this class, in rigorous model derivation begins from the fundamentals or first principles for example from Newton's Law, Maxwell's equations, Quantum transport equation or Boltzmann transport equation.

While in phenomenological model the derivation begins from the appearance of the phenomena rather than from the fundamentals, but the model is capable of predicting a range of observations and so is reasonably consistent with fundamentals.

**(Refer Slide Time: 45:00)**

Classification Based on the Solution Technique		
	Analytical Model	Numerical Model
S	• Includes device specific simplifications of the "EB" by neglecting some effects	• No simplifications of the "EB"
Q		⇒ model works for a wide range of devices/bias; model physically more accurate.
EB	⇒ model is device specific; model physically less accurate	• Transforms the "E"s using finite element, finite difference or monte carlo methods
(A)		(⇒ numerical inaccuracy present but controllable) so that purely arithmetic operations yield values of current / charge for specific values of voltage
(S)	• Uses techniques such as calculus, algebra, geometry, trigonometry to yield the current / charge as an analytic function of the voltage	• Computationally intensive
T		
I		
P	• Computationally economical	

Classification based on the solution technique, you have analytical models and numerical models, so analytical model tends to be device specific and physically less accurate than numerical models which work for a wide range of device or bias conditions, though there is numerical inaccuracy in numerical model because of the method of solution, the inaccuracy can be controlled by increasing the amount of computation.

Numerical model is computationally intensive while the analytical model is computationally economical, so this is how you are trading of the feature of the wide range of applicability of the model and physical accuracy with computation.

**(Refer Slide Time: 45:55)**

## Classification Based on the Solution Technique

	Analytical Model	Numerical Model
<b>S</b>	• Includes device specific simplifications of the "EB"	• No simplifications of the "EB"
<b>Q</b>	by neglecting some effects	⇒ model works for a wide range of devices/bias; model physically more accurate.
<b>EB</b>	⇒ model is device specific; model physically less accurate	• Transforms the "E"s using finite element, finite difference or monte carlo methods
<b>(A)</b>		(⇒ numerical inaccuracy present but controllable)
<b>(S)</b>	• Uses techniques such as calculus, algebra, geometry, trigonometry	so that purely arithmetic operations yield values of current / charge for specific values of voltage
<b>T</b>	to yield the current / charge as an analytic function of the voltage	⇒ model provides no insight
<b>I</b>		
<b>P</b>	⇒ model provides insight	

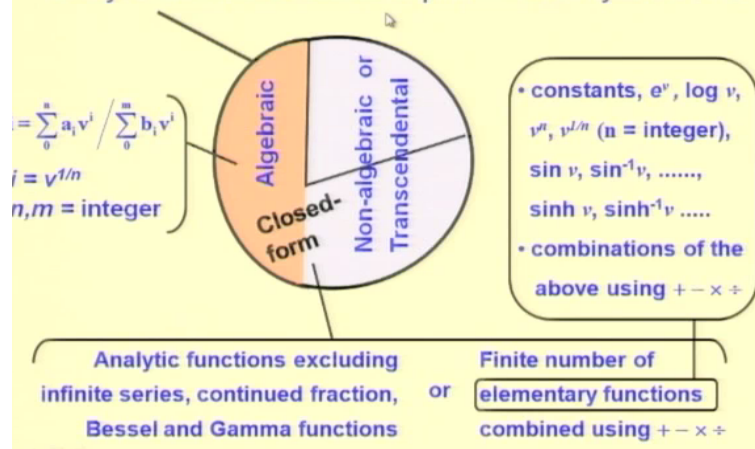
Analytical model provides insight while numerical model provides does not provide insight.

(Refer Slide Time: 46:03)

## Classification Based on Attributes of the Function

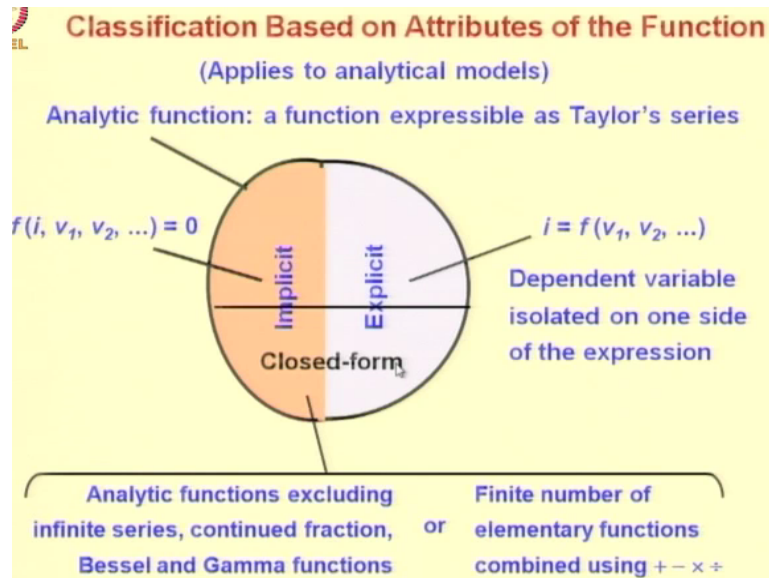
(Applies to analytical models)

Analytic function: a function expressible as Taylor's series



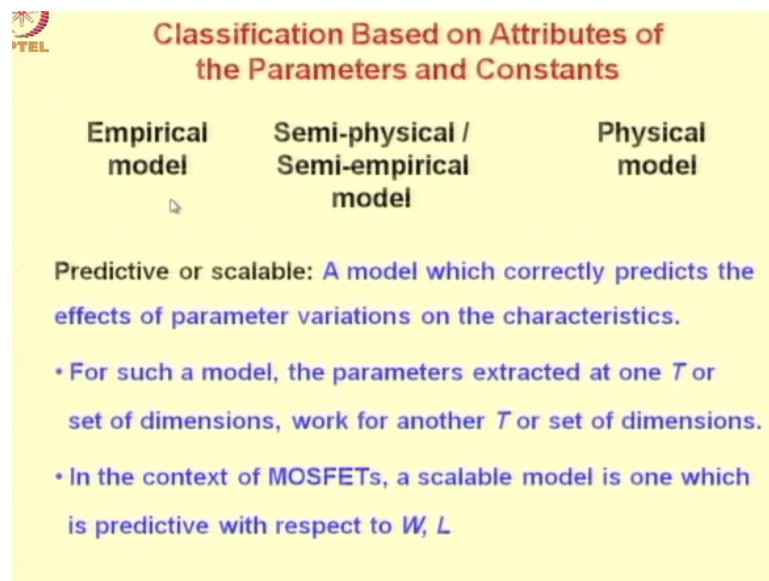
Then we consider classification of models based on attributes of the function, for analytical models we said that the mathematical function could be either algebraic or non-algebraic or transcendental and further it could be closed-form, we explained what is the meaning of algebraic closed-form and non-algebraic or transcendental functions.

(Refer Slide Time: 46:28)



Similarly, another form of classification for analytical model is implicit and explicit closed-form model could be either of implicit variety or the explicit variety, it all depends on how many iterations or how much computations are required to calculate the quantity of interest, if you need infinite number of iterations are extremely large computations to calculate the quantity then the model is not closed-form.

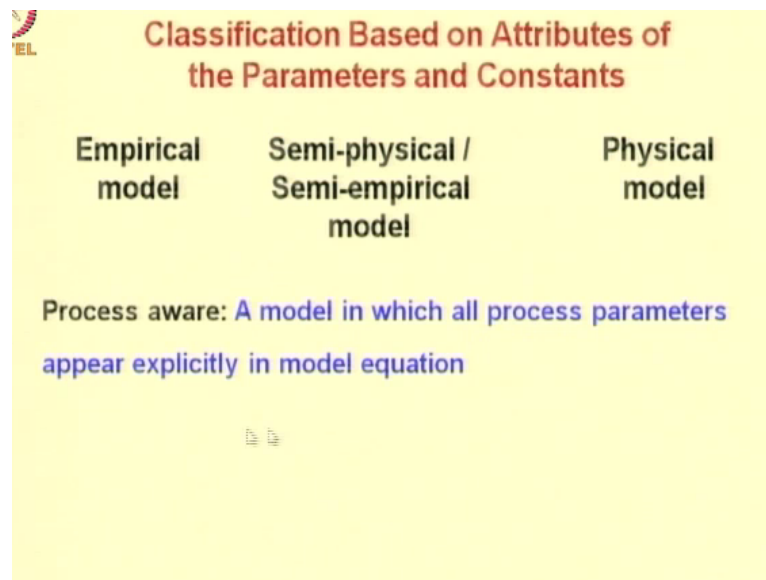
(Refer Slide Time: 46:58)



Classification based on attributes of the parameters and constants, so models would be empirical in which case none of the parameters have physical meaning or they could be physical in which case all parameters have physical meaning, most of the models fall in the semi physical or semi

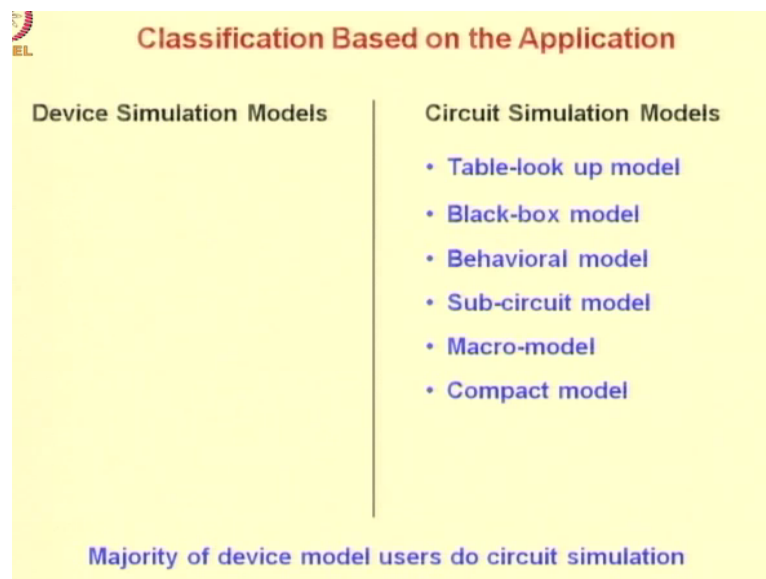
empirical model variety however efforts are constantly on to increase the physical content of the model and therefore you talked about making models more predictive or scalable.

**(Refer Slide Time: 47:27)**



We also talk about models being made process aware, so that the model contains all process parameters explicitly in the model equation.

**(Refer Slide Time: 47:42)**

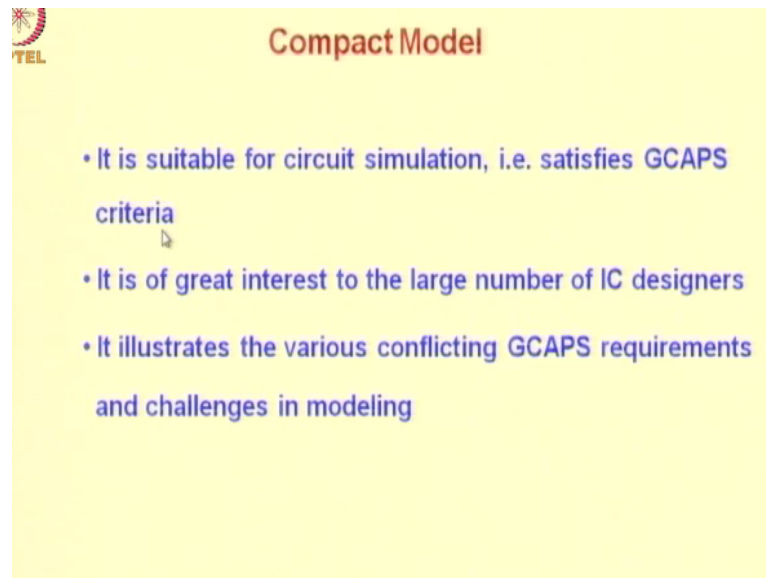


Finally, we can consider classification based on the application, so here one classification was device simulation models and circuit simulation models, we said that because of the kind of constraints imposed by circuit simulation a variety of models exist, computation, convergence



and so on are important considerations in a circuit simulation model, so we you have a variety of models which try to achieve these goals.

**(Refer Slide Time: 48:15)**



The slide features a yellow background with a red 'TEL' logo in the top left corner. The title 'Compact Model' is centered at the top in red. Below the title, there are three bullet points in blue text:

- It is suitable for circuit simulation, i.e. satisfies GCAPS criteria
- It is of great interest to the large number of IC designers
- It illustrates the various conflicting GCAPS requirements and challenges in modeling

We are going to discuss compact models in this course for various devices and therefore we pointed out a few features of these models it is suitable for circuit simulation that satisfies GCAPS criteria, so here continuity physical basis, in physical basis you have the asymptotic correctness being a very important criterion accuracy, simplicity and generality. The continuity is an important requirement because we discussed the method of solution used.

For example, Newton-Raphson where derivatives of the model functions becoming important, the derivatives of the model function also become important in accurate modeling of the distortion, where the shape of the waveform is very crucial in modeling after the particular signal goes to the device, the shape of the signal gets distorted supposing input sinusoid and you get a small modification of the sinusoid out.

The modified sinusoid can be expressed as for example a Taylor's series or you can expect it as a 4ier series and this is where the various derivatives will come into play for accurate modeling of distortion that is why a compact model should have all derivatives continuous, this model is of great interest to the large number of IC designers and it illustrates the various conflicting GCAPS requirements and challenges in modeling.

(Refer Slide Time: 49:53)

Classification Based on the Application	
<b>Digital Model of a MOSFET</b>	<b>Analog Model of a MOSFET</b>
<ul style="list-style-type: none"><li>• Goal is prediction of inverter switching speed, for which, accurate prediction of drive current and parasitic capacitance sufficient</li><li>• Regional equations for sub-threshold, linear, saturation, depln, accum</li><li>• More but simple equations ⇒ few model parameters</li></ul>	<ul style="list-style-type: none"><li>• Both DC and small-signal characteristics – <math>g_d</math>, <math>g_m</math>, <math>g_{mb}</math> should be predicted accurately</li><li>• Single equation smoothly connecting all regions</li><li>• Single but complex equation ⇒ many model parameters</li></ul>
Majority of ICs are digital CMOS	

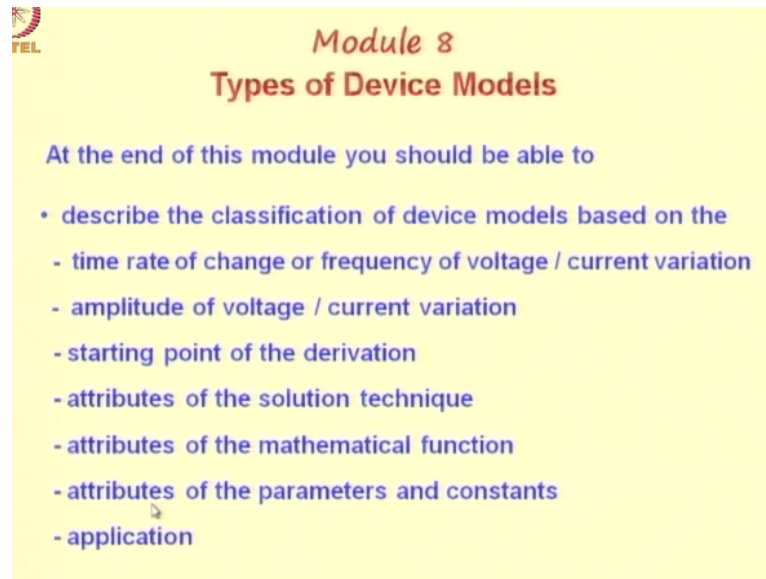
Another classification based on the application we discussed was digital model and analog model, digital model has simple equations and few model parameters, though the number of equations is large they are simple and therefore number of parameters is few, on the other hand the analog model because of the requirements that it must predict the conductance's accurately it has a complex equation though it is a single equation but has many model parameters.

(Refer Slide Time: 50:26)

Inverse Modeling
This can mean:
<ul style="list-style-type: none"><li>• extraction of device structure from measured data e.g. extraction of 2-D doping profile in a MOSFET from measured C-V data</li><li>• modeling of the <math>I_D-V_{DS}</math> characteristics of an N-channel MOSFET for <math>V_{DS} &lt; 0</math> or P-channel MOSFET for <math>V_{DS} &gt; 0</math>.</li></ul>

Finally, we talked about inverse modeling which could mean extraction of device structure from measured data or modeling of the  $I_D V_{DS}$  characteristics of an n-channel MOSFET for  $V_{DS} < 0$  or p-channel MOSFET for  $V_{DS} > 0$ .

(Refer Slide Time: 50:41)



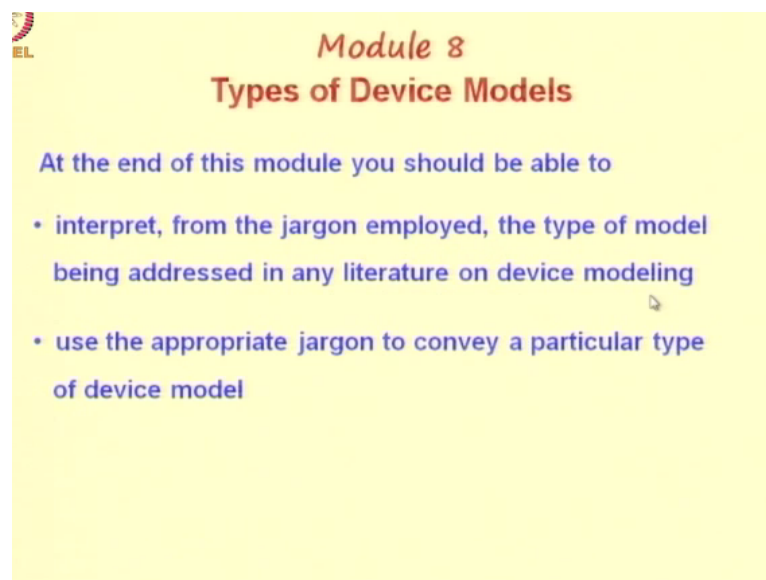
**Module 8**  
**Types of Device Models**

At the end of this module you should be able to

- describe the classification of device models based on the
  - time rate of change or frequency of voltage / current variation
  - amplitude of voltage / current variation
  - starting point of the derivation
  - attributes of the solution technique
  - attributes of the mathematical function
  - attributes of the parameters and constants
  - application

Let us recapitulate the learning outcomes, I hope that at the end of this particular module you are able to describe the classification of device models based on the time rate of change or frequency of voltage or current variation, amplitude of voltage current or voltage variation, based on time starting point of the derivation, classification based on attributes of the solution technique, attributes of the mathematical function or attributes of the parameters and constants and finally, classification based on the application.

(Refer Slide Time: 51:15)



**Module 8**  
**Types of Device Models**

At the end of this module you should be able to

- interpret, from the jargon employed, the type of model being addressed in any literature on device modeling
- use the appropriate jargon to convey a particular type of device model

You should be able to interpret from the jargon employed, the type of model being addressed in any literature on device modeling and finally use the appropriate jargon to convey a particular

type of device model. With that we have come to the end of this module, hereafter we shall consider the devices one by one for modeling, we will start with the MOSFET.