Semiconductor Device modeling
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Lecture - 35
SQEBASTIP: Nine Steps of Deriving a Device Model

In the previous lecture, we have introduced the 9 steps of device modeling. These 9 steps are abbreviated as SQEBASTIP where S stands for structure and characteristics to scale Q for qualitative model E and B for equations and boundary conditions A and S for approximations and solution, T for testing of the solution and I for improvement and finally P for parameter extraction.

We said that we illustrate these 9 steps using the example of modeling of a spreading resistance. In the previous lecture, we detailed the 2 steps namely structure and characteristics to scale and qualitative modeling. We emphasized why it is important to visualize the structure and characteristics to scale. In qualitative modeling we mentioned that is important to take into account all the different effects.

Because if you want the model to be accurate it is not intrinsic mathematical accuracy of the solution. But the fact that we are taking into account all the different effects this is more important. We also said that qualitative modeling begins with a set of approximations and that is why in an earlier module you will recall we have said that modeling is art of making approximations.

In this lecture, we will proceed further to detail some of the other steps of device modeling.

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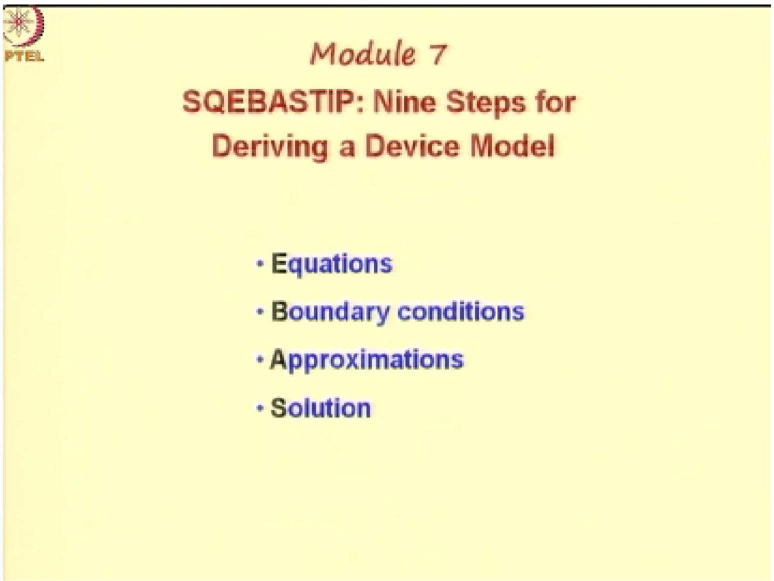

Module 7

SQEBASTIP: Nine Steps for Deriving a Device Model

- Equations
- Boundary conditions
- Approximations
- Solution

These steps are equations, boundary conditions, approximations and solution.

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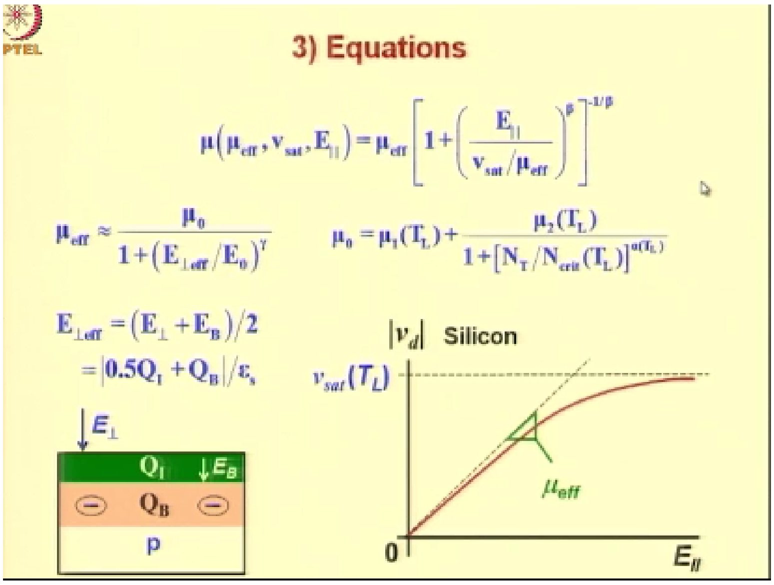

3) Equations

$$\mu\left(\mu_{eff}, v_{sat}, E_{||}\right) = \mu_{eff}\left[1+\left(\frac{E_{||}}{v_{sat}/\mu_{eff}}\right)^{\beta}\right]^{-1/\beta}$$

$$\mu_{eff} \approx \frac{\mu_0}{1+\left(E_{\perp eff}/E_0\right)^{\gamma}} \qquad \mu_0 = \mu_1(T_L) + \frac{\mu_2(T_L)}{1+\left[N_T/N_{crit}(T_L)\right]^{\alpha(T_L)}}$$

$$E_{\perp eff} = \left(E_{\perp}+E_B\right)/2$$
$$= \left|0.5Q_I + Q_B\right|/\varepsilon_s$$

Let us begin with equations. We have remarked earlier that in this course we will be dealing with models based on the drift diffusion equations. Let put this equation down here on the slide. I did not explain them because we have explained these equations a number of times earlier. The current is obtained by integrating the current density over the contact area and the potential between the 2 terminals is obtained by the integrating the electric field over distance.

Now in the drift diffusion model the important parameters to be known are mobility, lifetime and details of generation recombination mechanism. These details we have already given in the module on drift diffusion model just by way of example let me reproduce here the equation for the mobility which is a very important parameter. So the mobility is a function of the effective mobility which depends on the perpendicular electric field.

That is electric perpendicular to the direction of the flow of carriers then the saturation velocity and the parallel electric field which is the electric field in the direction of motion of the carriers. Now this particular formula for the mobility is applicable for both electrons as well as holes. The effective mobility depends on the bulk mobility for low perpendicular electric field.

Divided by 1 + an effective value of perpendicular field by a critical field parameter E 0 raise to the power gamma. The effective mobility is the slope of this drift velocity versus parallel electric field curve for small values of electric field. The effective value of perpendicular electric field is related to the mobility in the inversion charge. So this is the diagram of MOS capacitors or MOSFET in inversion in which you have the inversion charge and below that the depletion charge.

So this electric field the effective value is average of the perpendicular electric field at this surface which is the interface between silicon and silicon dioxide and at the end of the inversion layer this electric field is given the symbol E suffix B. So average of these 2 electric field is what this inversion charge experiences and that is what is given here as the effective value of perpendicular electric field.

And this formula can be converted into charges where Q1 is inversion charge and QB is the depletion charge marked here. Now the bulk mobility mu 0 is a function of temperature that is a lattice temperature and the total doping that is sum total of acceptor and donor type doping. So you can review the expressions for lifetime and generation recombination mechanisms from that module on drift diffusion transport.

And these equations will be used for device modeling.

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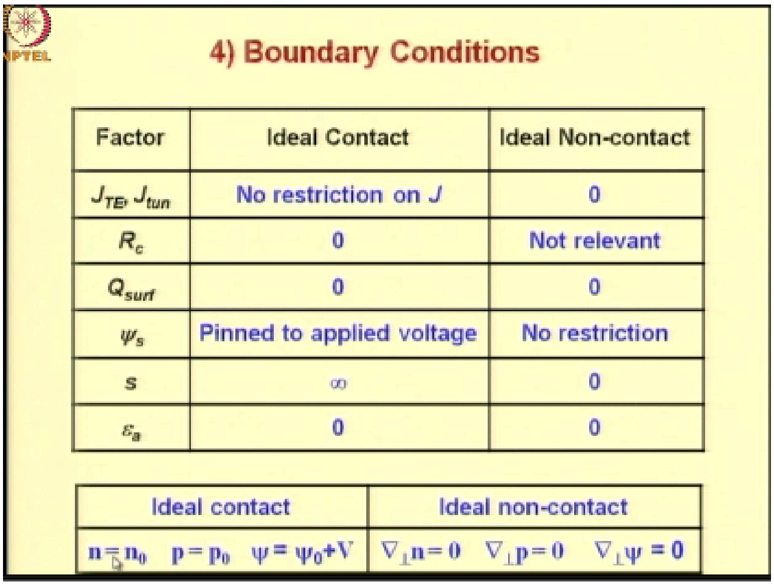

4) Boundary Conditions

Factor	Ideal Contact	Ideal Non-contact
J_{TE}, J_{tun}	No restriction on J	0
R_c	0	Not relevant
Q_{surf}	0	0
ψ_s	Pinned to applied voltage	No restriction
s	∞	0
ε_a	0	0

Ideal contact	Ideal non-contact
$n = n_0$ $p = p_0$ $\psi = \psi_0 + V$	$\nabla_\perp n = 0$ $\nabla_\perp p = 0$ $\nabla_\perp \psi = 0$

Let us look at the boundary conditions. In the module, on drift diffusion transport we have explained that the boundary conditions depend on all this factors. Now we had said that often we use the so called ideal conditions for contact and non contact and ideal contact conditions the description of these factors are given as in this column. Similarly, for ideal noncontact the descriptions are as given here.

I am not repeating these descriptions because we have already done it in the previous lecture when we discuss qualitative modeling. Now these descriptions of the factors which govern the boundary condition lead you to the ideal contact and non contact boundary conditions. Now after this general comments about equations and boundary conditions to be used for device modeling.

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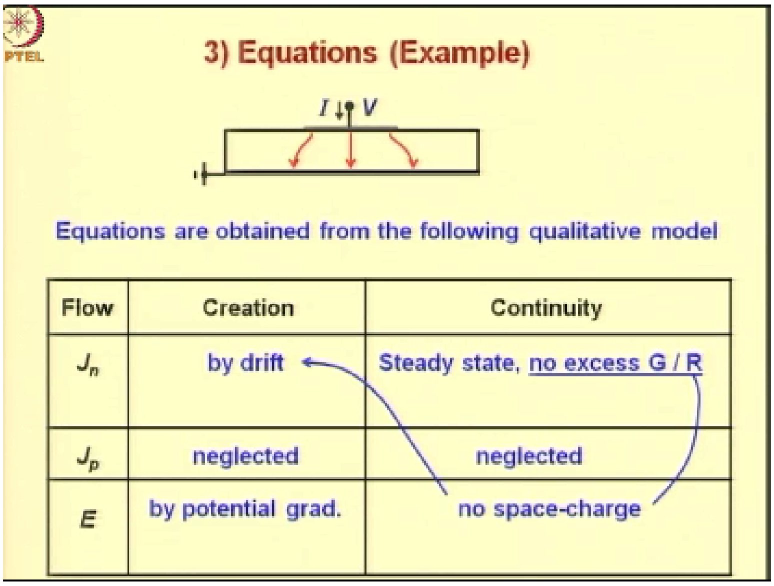

3) Equations (Example)

Equations are obtained from the following qualitative model

Flow	Creation	Continuity
J_n	by drift	Steady state, no excess G / R
J_p	neglected	neglected
E	by potential grad.	no space-charge

Let us illustrate the ideas using the example of spreading resistance. So the equations for spreading resistance follow from the qualitative model which is repeated here. So this was the table that we had given in the qualitative model for the spreading resistance. Now each of these descriptions in this cells of the table will be replaced by corresponding equations. So the arrows here indicate the following.

The no space charge assumption leads to the fact that the current can only be because of drift because the diffusion current is 0. This is the uniform semiconductor and no space charge means the electron concentration is equal to the doping concentration which is uniform and electron concentration being uniform there cannot be a diffusion current. We are neglecting the effect of holes here.

Similarly, no space charge also leads to the condition that there is no excess generation or recombination because the carrier concentration is equal to the equilibrium value.

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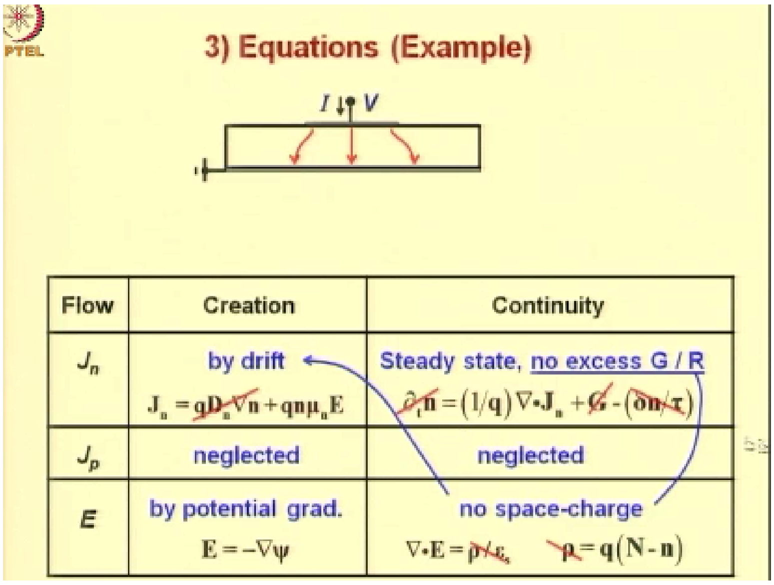

3) Equations (Example)

Flow	Creation	Continuity
J_n	by drift $J_n = \cancel{qD_n\nabla n} + qn\mu_n E$	Steady state, no excess G / R $\cancel{\partial_t n} = (1/q)\nabla\bullet J_n + \cancel{G} - \cancel{(\delta n/\tau)}$
J_p	neglected	neglected
E	by potential grad. $E = -\nabla\psi$	no space-charge $\nabla\bullet E = \cancel{\rho/\varepsilon_s}$ $\quad \cancel{\rho} = q(N-n)$

So converting these 2 equations no space charge means diversions of E is 0 and Q into N which is the doping concentration - electron concentration is 0. This means the electron concentration is equal to the doping concentration. We have neglected the holes here. Electric field creation by potential gradient the corresponding equation is E = - grad psi. JN = QN mu N into E. qDn grad N is neglected.

And the continuity equation for electrons we neglect dou N/dou T because it is steady state and we remove the terms namely G and delta N/tau because there is no excess generation or recombination and therefore diversions of JN would be 0.

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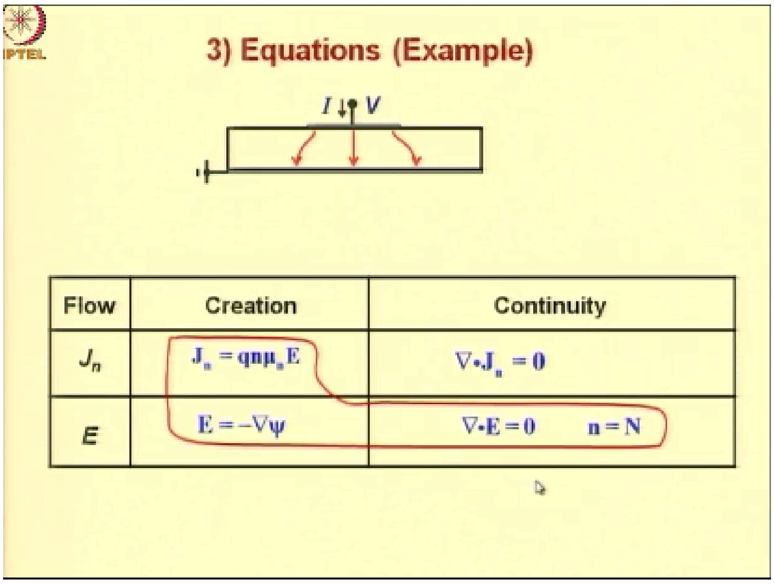

3) Equations (Example)

Flow	Creation	Continuity
J_n	$J_n = qn\mu_n E$	$\nabla\cdot J_n = 0$
E	$E = -\nabla\psi$	$\nabla\cdot E = 0$ $n = N$

So here is the summary of the equations. Now one thing to be noted here is that if I use this information about the electron concentration being equal to capital N here and then substitute this equation for Jn into the diversions of Jn = 0 equations. I will end up getting this equation namely diversion of E = 0. So these 2 equations are not really independent. This is because our space charge = 0 approximation has the corollary that diffusion is neglected and generation recombination is also not there.

Therefore, one can use any one of these 2 equations in solutions. So I can either use these 4 equations or I can use these 4 equations. Now let me explain how these equations are used. I can substitute E = - grad psi into diversions of E and I will get the equation del square psi = 0. So you will recall that we had said that in drift diffusion model there are 3 coupled equations essentially in Np and psi.

So in this particular example of spreading resistance we have neglected the holes. Therefore, we have only 2 equations in n and psi. So these are the 2 equations. So small n = capital N already because of our assumption of negligible space charge and combining this equation and this equation gives you del square psi = 0. So del square psi = 0 and n = N are the 2 equations for us which will lead us to the model.

The equation Jn = Qn mu N into E is required to find out the current once we have solved the electric field from the equation del square psi = 0. So this is the summary of the equations.

Here we have used diversions of E = 0 in place of diversions of Jn = 0. Later on we will show that while deriving a closed form model diversions of Jn = 0 will be more convenient to use that diversions of E =0. Both are equations of different forms for the same phenomena in this case of spreading resistance.

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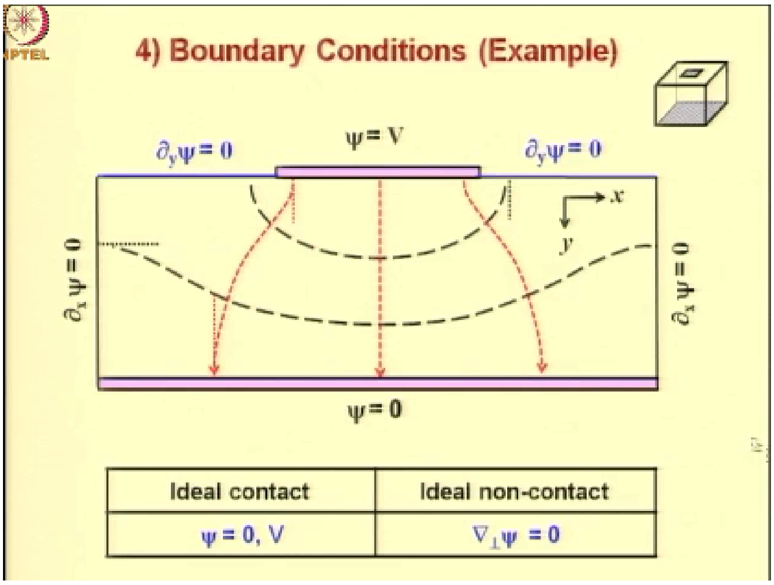

4) Boundary Conditions (Example)

Ideal contact	Ideal non-contact
$\psi = 0, V$	$\nabla_\perp \psi = 0$

Let us look at the boundary conditions. So we use ideal contact and non contact boundary conditions which are given here. Note that we have not put any conditions on holes because we are neglecting the holes. So translating these equations here to the boundaries first contact boundaries. So at the bottom contact psi = 0 and N = N0 and at the top contact psi = V and N = N0.

At the top non contact boundaries you have the gradient of psi in the Y solution and gradient of N in the Y direction = 0. The same thing applies here as well. So this is the Y direction and this is the X direction for us. So in other words you have no perpendicular electric filed nor do you have any diffusion current in this direction. In fact, there is no current in this direction because no field in this direction means no drift current and dou/dou Y of N = 0 means no diffusion current in this direction.

So no drift and no diffusion together mean no current in the perpendicular direction. Following the same approach on the 2 sides noncontact boundaries you have the gradients of psi and N in the X direction as 0 this is our X direction. So no electric field escapes normal to this surface no current escapes normal to this surface. Now since we are already using the equation electron concentration = capital N everywhere and N is uniform.

Therefore, we really do not have to use the boundary conditions on the electron concentration because already the electron concentration is equal to equilibrium value everywhere. Therefore, the information regarding the electrons or electron concentration at the boundaries has been removed here because you only have to solve this equation del square psi = 0. The equation for electron concentration is decoupled from the del square psi = 0 equation.

So this is actually the simple picture that we have to solve. Now just by way of information it is this boundary conditions at the contact and non contact which give rise to the equi-potential lines we had drawn earlier in the qualitative modeling and the current flow lines. So the current flow lines emanate perpendicular to the contact you can see that or terminate perpendicular to the contact and the equi-potential lines are perpendicular to the non contact boundaries as you can see here.

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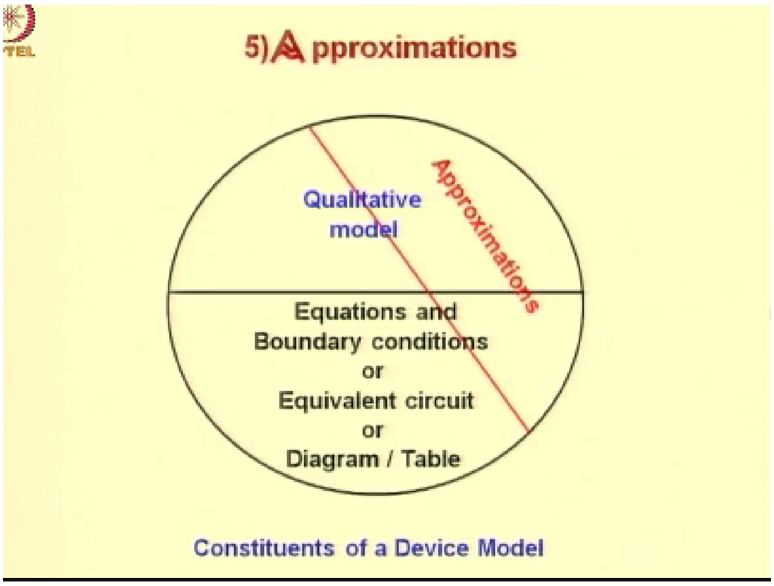

5) Approximations

Qualitative model

Approximations

Equations and
Boundary conditions
or
Equivalent circuit
or
Diagram / Table

Constituents of a Device Model

Now let us move on the approximations. Let me reproduce a slide from the second module that is a module on introduction wherein we talked about the constituents of a device model. So we said that the device model consists of a qualitative part and a quantitative part where the quantitative part could be equations in boundary conditions or equivalent circuits or diagram or table where the diagram also includes graphs.

And then we had remarked that there are a set of approximations associated both with qualitative and quantitative part. Now let us focus on these approximations here. So since there are approximations belonging to qualitative model as well as to quantitative model.

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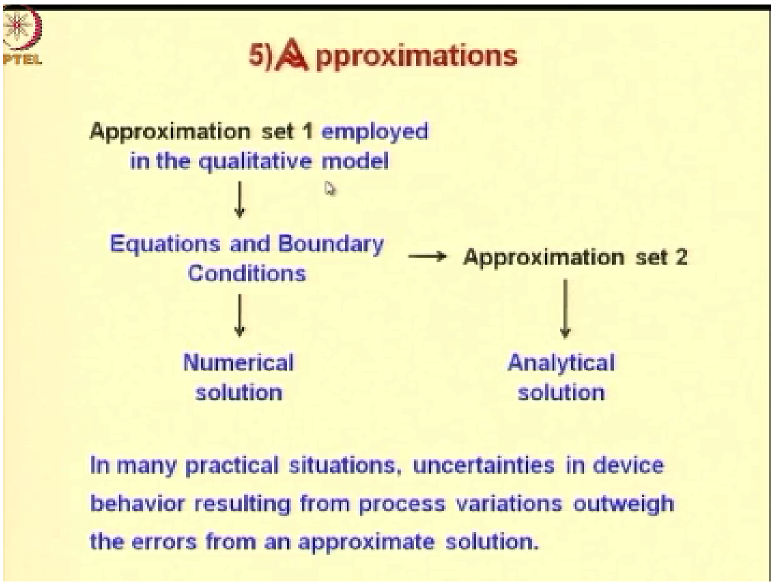

5) Approximations

Approximation set 1 employed in the qualitative model

Equations and Boundary Conditions

Approximation set 2

Numerical solution

Analytical solution

In many practical situations, uncertainties in device behavior resulting from process variations outweigh the errors from an approximate solution.

Let us describe the relationship between these approximations. So the approximations set 1 is employed in the qualitative model and these lead to the equations and boundary condition which can be solved numerically. It may not be always possible to solve these equations analytically because even after these approximations the equations can be fairly involved. And only a numerical solution may be possible.

So if you want analytical solution then you make further approximations of this equations in boundaries conditions to get the analytical solutions. So therefore you have 2 sets of approximations. So the approximation set 1 is the approximations associated with the qualitative model and the approximation set 2 is the approximations associated with the equations and boundary conditions or the quantitative model.

In many practical situations uncertainties in device behavior resulting from process variations, outweigh the errors from an approximate solution and in fact this is the motivation for going in for an analytical solution even if it involves approximations of the equations in boundary conditions after you already have made approximations in the qualitative stage.

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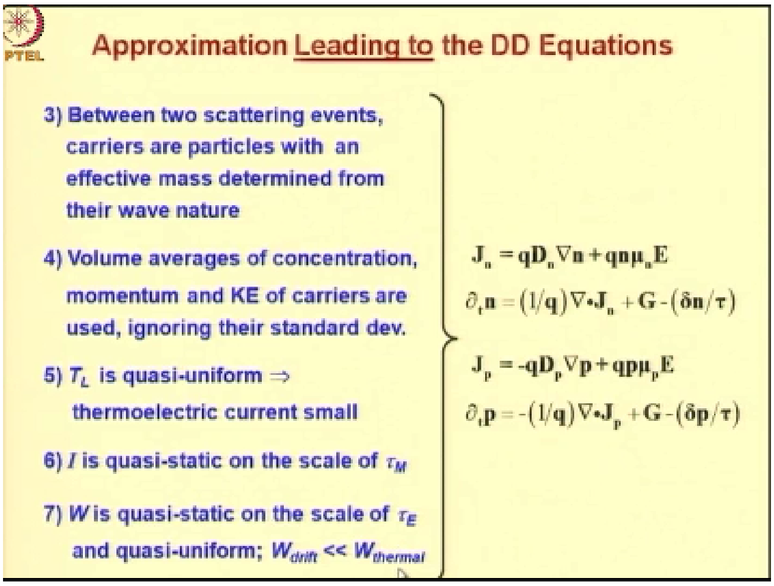

Approximation Leading to the DD Equations

3) Between two scattering events, carriers are particles with an effective mass determined from their wave nature

4) Volume averages of concentration, momentum and KE of carriers are used, ignoring their standard dev.

5) T_L is quasi-uniform ⇒ thermoelectric current small

6) I is quasi-static on the scale of τ_M

7) W is quasi-static on the scale of τ_E and quasi-uniform; $W_{drift} << W_{thermal}$

$$\mathbf{J}_n = q\mathbf{D}_n\nabla n + qn\mu_n\mathbf{E}$$

$$\partial_t n = (1/q)\nabla\bullet\mathbf{J}_n + G - (\delta n/\tau)$$

$$\mathbf{J}_p = -q\mathbf{D}_p\nabla p + qp\mu_p\mathbf{E}$$

$$\partial_t p = -(1/q)\nabla\bullet\mathbf{J}_p + G - (\delta p/\tau)$$

So let us look at the approximations which lead to the drift diffusion equations. So these are the approximations made in the qualitative stage. So I am just repeating something that we have done earlier. So for the electrostatic equations you know that the approximation is that magnetic field is neglected and electric field has no circulating component that is it arises from static charges only.

When you write J = Jn+Jp you are assuming that E is quasi-static on the scale of dielectric relaxation time. And therefore displacement current is small. So otherwise you will have the displacement current adding on to Jn and Jp which are due to drift diffusion and thermoelectric currents. Now these are the current density and continuity equations for electrons and holes.

A number of approximations are associated with these namely between 2 scattering events carriers are particles with an effective mass determined from their wave nature. Volume average of concentration, momentum and kinetic energy of carriers are used ignoring their standard deviations. The lattice temperature is quasi-uniform as a function of space that is thermoelectric current are small.

The current is quasi-static on the scale of momentum relaxation time and the kinetic energy density W is quasi-static on the scale of energy relaxation time and quasi-uniform over the space. Further the drift component of kinetic energy of carriers is much less than the thermal component of kinetic energy. Thermal component of kinetic energy also means the random component of kinetic energy.

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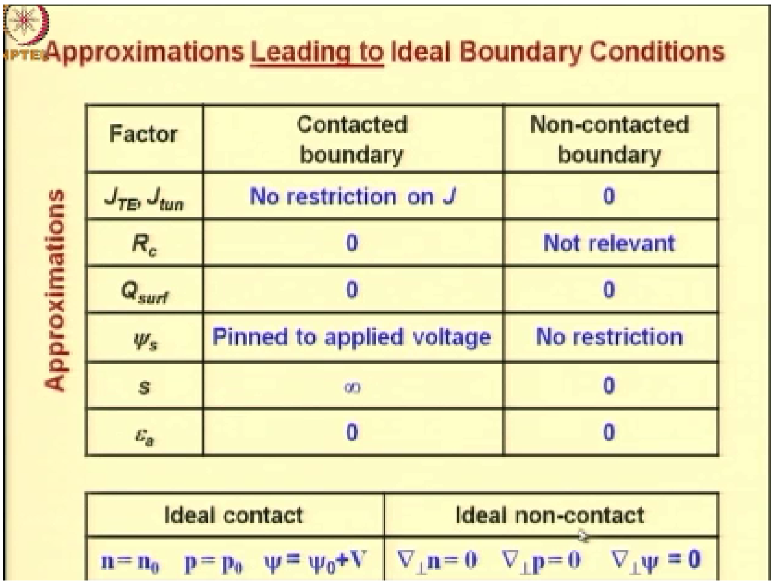

Approximations Leading to Ideal Boundary Conditions

Approximations

Factor	Contacted boundary	Non-contacted boundary
J_{TE}, J_{tun}	No restriction on J	0
R_c	0	Not relevant
Q_{surf}	0	0
ψ_s	Pinned to applied voltage	No restriction
s	∞	0
ε_a	0	0

Ideal contact	Ideal non-contact
$n = n_0$ $p = p_0$ $\psi = \psi_0 + V$	$\nabla_\perp n = 0$ $\nabla_\perp p = 0$ $\nabla_\perp \psi = 0$

Now let us look at the approximations leading to the ideal boundary conditions. Again we will quickly reproduce from our earlier discussion. So these approximations pertain to the factors which affect the boundaries which are listed in this column. And these are the approximations which we normally make to get ideal boundary conditions. Now if you want more realistic boundary conditions.

You can refer to the module on drift diffusion transport where the boundary conditions in the presence of all this factors are described for contacts and non-contacts. Now here we are talking about the ideal boundary conditions. So these approximations lead to these boundary conditions on ideal contact and ideal non-contacts.

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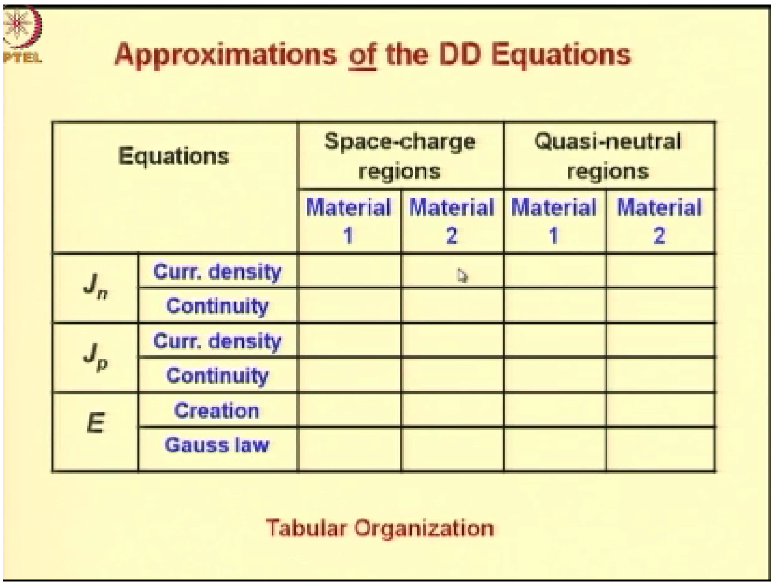

Approximations of the DD Equations

Equations		Space-charge regions		Quasi-neutral regions	
		Material 1	Material 2	Material 1	Material 2
J_n	Curr. density				
	Continuity				
J_p	Curr. density				
	Continuity				
E	Creation				
	Gauss law				

Tabular Organization

Now let us look at approximations of the drift diffusion equations. These approximations are used to derive analytical models. So these set of approximations are the approximations set 2 which we refer to in an earlier slide. The best way to remember all this approximations of the set 2 or the approximations of the drift diffusion equations is in the form of a table shown here.

We have listed in this rows the electron current density, the hole current density and electric field for each of this flows you have 2 equations as we know in drift diffusion model. So each row here is an equation of the drift diffusion model. The columns on the other hand are space charge and neutral regions of the device. Now the space charge region could exist in different materials of the device.

So your device may have different materials. Similarly, quasi-neutral regions also may exist in different materials of the device. So this is a general form of the table which is applicable to any device. So you can list what approximations you make for each equation of the drift diffusion model in each region. We will illustrate this idea using an example.

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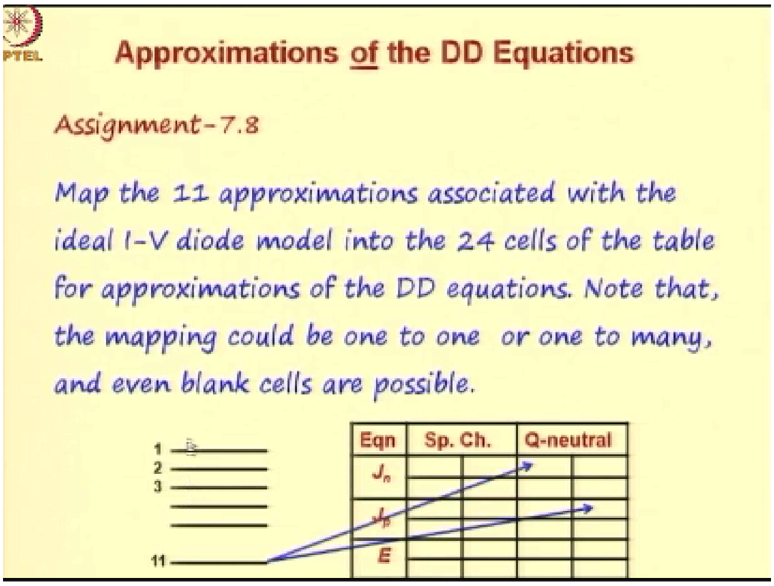

Approximations of the DD Equations

Assignment-7.8

Map the 11 approximations associated with the ideal I-V diode model into the 24 cells of the table for approximations of the DD equations. Note that, the mapping could be one to one or one to many, and even blank cells are possible.

1
2
3
11

Eqn	Sp. Ch.		Q-neutral	
J_n				
J_p				
E				

So here is an assignment for you as an illustration. Map the 11 approximations associated with the ideal IV diode model into the 24 cells of the table for approximations of the drift diffusion equation. Note that the mapping could be 1 to 1 to 1 to many and even blank cells are possible. So here is the diagrammatic representation of what you are expected to do so these are the 11 approximations.

We had referred to these 11 approximations in the introduction module and we will shortly repeat those for your convenience just after description of this particular slide. So these 11 approximations each of them have to be mapped into this cells of the table.

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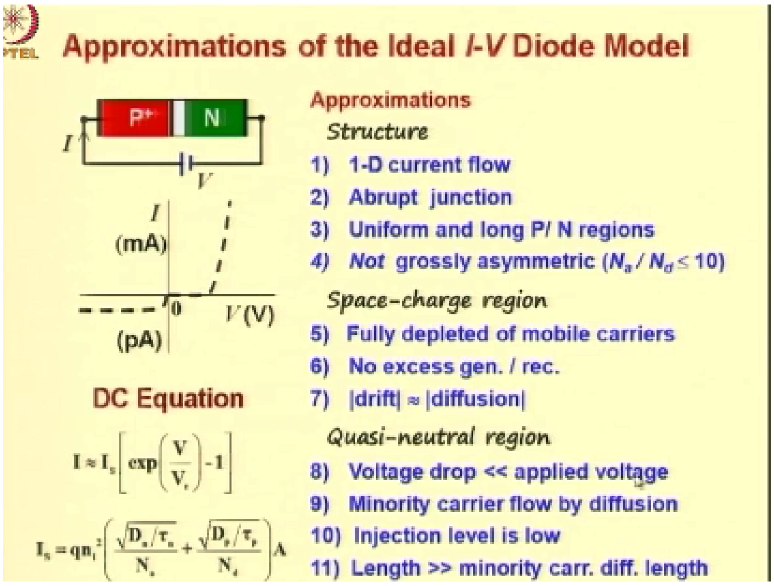

Approximations of the Ideal *I-V* Diode Model

Approximations

Structure

1) 1-D current flow
2) Abrupt junction
3) Uniform and long P/ N regions
4) *Not* grossly asymmetric ($N_a / N_d \leq 10$)

Space-charge region

5) Fully depleted of mobile carriers
6) No excess gen. / rec.
7) |drift| ≈ |diffusion|

Quasi-neutral region

8) Voltage drop << applied voltage
9) Minority carrier flow by diffusion
10) Injection level is low
11) Length >> minority carr. diff. length

DC Equation

$$I \approx I_s\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

$$I_s = qn_i^2\left(\frac{\sqrt{D_n/\tau_n}}{N_a} + \frac{\sqrt{D_p/\tau_p}}{N_d}\right)A$$

So these are the 11 approximations reproduced here from the slide presented in the introduction section of this course.

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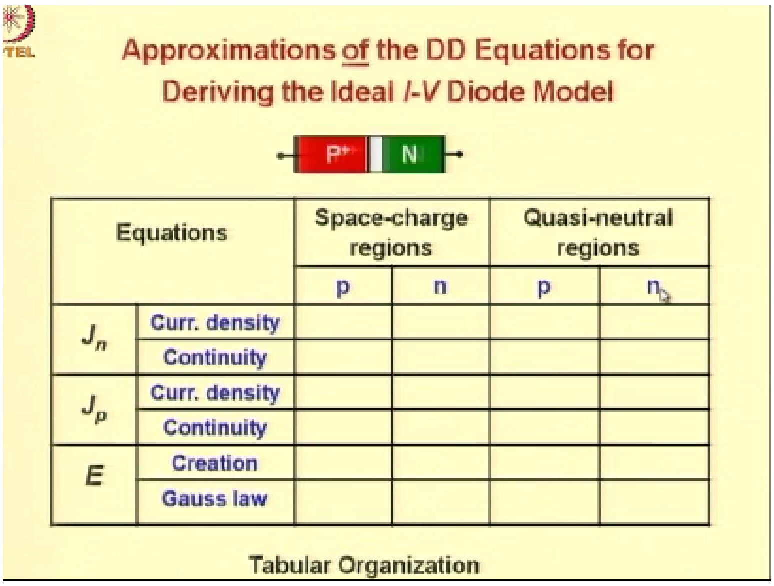

Approximations of the DD Equations for Deriving the Ideal *I-V* Diode Model

Equations		Space-charge regions		Quasi-neutral regions	
		p	n	p	n
J_n	Curr. density				
	Continuity				
J_p	Curr. density				
	Continuity				
E	Creation				
	Gauss law				

Tabular Organization

And these are your 24 cells of the table in which you have to put the approximations of the drift diffusion equations for deriving the ideal IV diode model. So note here the space charge region as P and N sections. So this P column corresponds to the space region on the P side and this N column corresponds to space charge region on the N side. Similarly, this quasi-neutral region the details will appear in this column and the P type quasi-neutral region that is a red portion the details will appear in this column.

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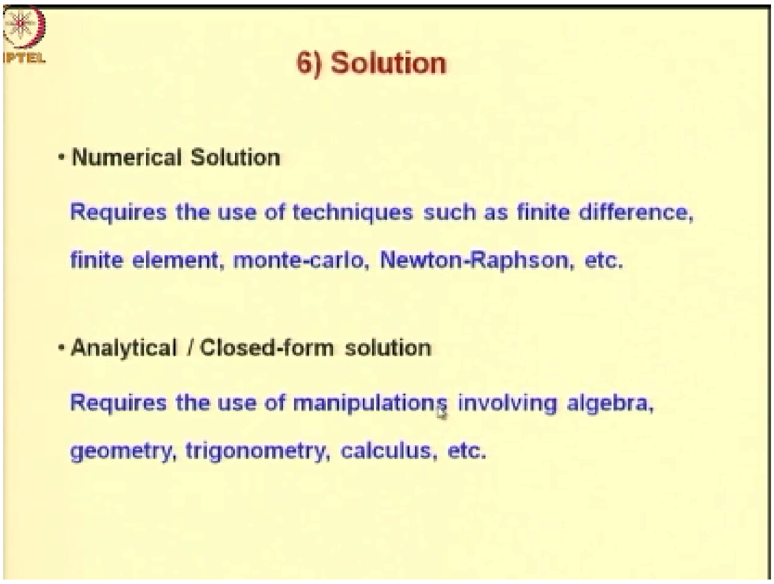

6) Solution

- Numerical Solution

  Requires the use of techniques such as finite difference, finite element, monte-carlo, Newton-Raphson, etc.

- Analytical / Closed-form solution

  Requires the use of manipulations involving algebra, geometry, trigonometry, calculus, etc.

Let move on to the solution step of device modeling. 2 types of solutions are possible. Numerical or analytical or closed form. Let us distinguish between these 2 types of solutions. So we often come across this terms numerical, analytical closed form etcetera. So we need to understand these terms properly. Here, we will give a brief description of each of these 2. In the next module where we discussed the types of models.

We will develop further on some of the nomenclatures used for different types of device model. A numerical solution requires a use of techniques such as finite difference, finite element, Monte-Carlo and Newton Raphson. On the other hand, analytical or closed form solution involves the use of manipulations involving algebra, geometry, trigonometry, calculus etcetera.

Now in this course we are concerned with analytical or closed form type of solutions.

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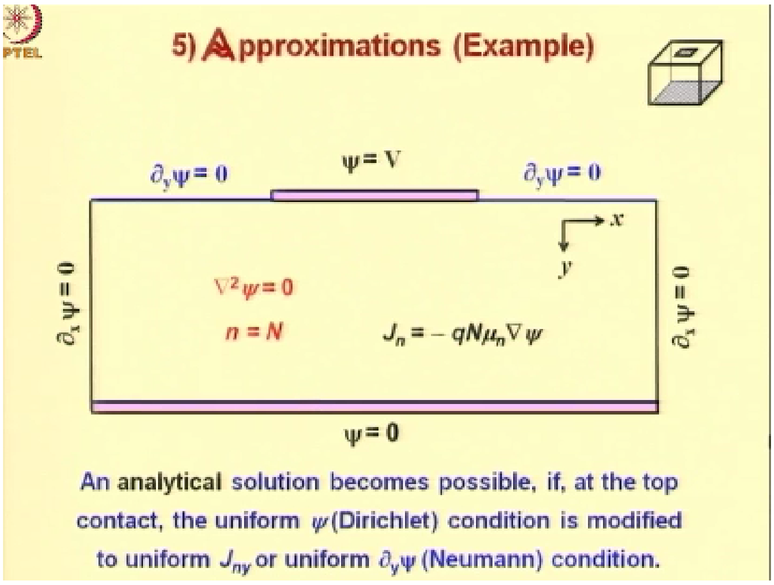

5) Approximations (Example)

An analytical solution becomes possible, if, at the top contact, the uniform ψ (Dirichlet) condition is modified to uniform J_{ny} or uniform $\partial_y\psi$ (Neumann) condition.

Let us illustrate the steps of approximations and solution with an example that of the spreading resistance. So here are the equations and these are the boundary conditions. Note that the actual resistance is 3 dimensional and we are showing here a cross section only where you have shown the boundary conditions in the X and Y direction. You will also have boundary conditions in the Z directions that is along this direction here in this 3D view or along the perpendicular to this slide that is the Z direction.

So I leave it to you to write the boundary conditions on dou psi/dou Z etcetera yourself. The above equation in boundary conditions are based on the approximations associated with the drift diffusion model and the ideal boundary conditions listed in the preceding slides. Now it turns out that for the boundary conditions assumed that is which are shown here the equations can only be solved numerically.

So a number of researchers have attempted this problem. I have not been able to find an analytical solution for this problem in terms of the equations and boundary conditions that we have defined. They have only been able to obtain a numerical solution and analytical solution becomes possible if at the top contact that is here the uniform psi which is a Dirichlet boundary condition is modified to uniform Jny or a uniform dou psi/dou Y which means a uniform electric field in the Y direction over the contact which is a Neumann condition.

So if you change the uniform potential to uniform electric field uniform normal electric field or uniform normal current density then it is possible to get an analytical solution it has been found.

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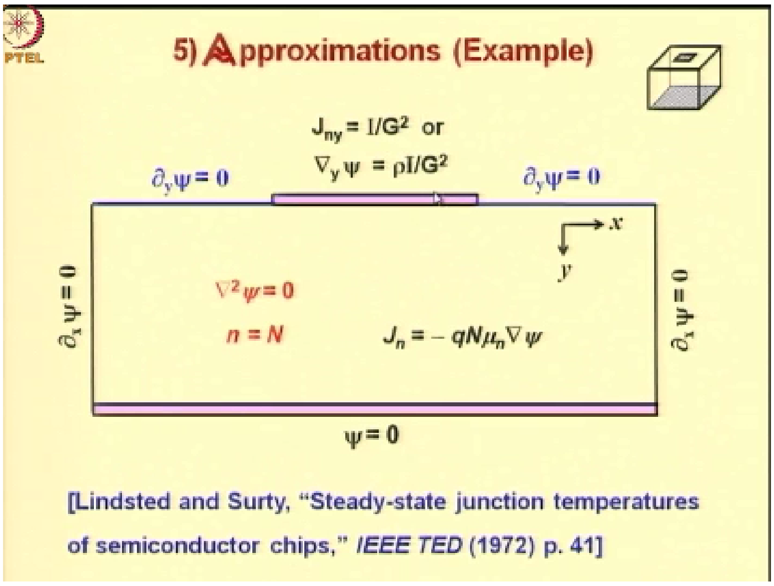

5) Approximations (Example)

$J_{ny} = I/G^2$ or

$\nabla_y \psi = \rho I/G^2$

$\partial_y \psi = 0$

$\partial_y \psi = 0$

$\partial_x \psi = 0$

$\partial_x \psi = 0$

$\nabla^2 \psi = 0$

$n = N$

$J_n = -qN\mu_n \nabla \psi$

$\psi = 0$

[Lindsted and Surty, "Steady-state junction temperatures of semiconductor chips," *IEEE TED* (1972) p. 41]

Now this is a reference which discusses the analytical solutions. So the boundary conditions has been modified to dou psi/dou Y remains constant at rho is the resistivity of the spreading resistance, resistivity of these material here. So this rho is nothing but one/Q times capital N into mu N. So rho multiplied by I divided by J square. So for any given current I that would be the electric field over this contact.

Alternately, you can talk in terms of current density normal to the contact that would be simply the current divided by the area of the contact which is G square because it is a square contact of dimension G. So this is the square contact.

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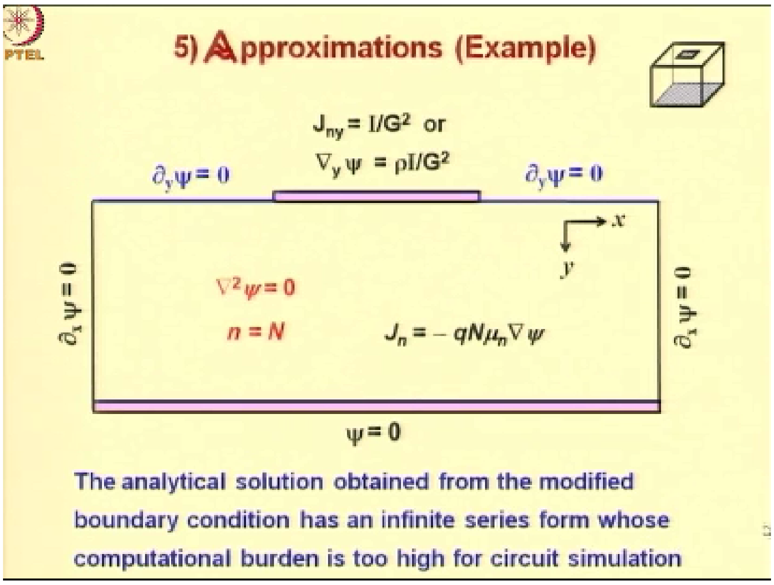

5) Approximations (Example)

$J_{ny} = I/G^2$ or

$\nabla_y \psi = \rho I/G^2$

$\partial_y \psi = 0$

$\partial_y \psi = 0$

$\partial_x \psi = 0$

$\partial_x \psi = 0$

$\nabla^2 \psi = 0$

$n = N$

$J_n = -qN\mu_n \nabla \psi$

$\psi = 0$

The analytical solution obtained from the modified boundary condition has an infinite series form whose computational burden is too high for circuit simulation

Now the analytical solution obtained from the modified boundary conditions has an infinite series form whose computational burden is too high for circuit simulation. In the next module we shall see various forms of analytical solution. Here what we are saying is that when you modify the boundary condition at the top contact from Dirichlet to Neumann type then numerical solution can be replaced by an analytical solution.

We are not giving analytical solution here you can refer to the paper that I have given in the previous slide if you are interested in it, but we are remarking is the property of the analytical solution that it has an infinite series form which mean you have to take a large number of terms to get an accurate value of the resistance. So computational burden becomes high particularly for circuit simulation where you have a large number of devices and for each device you cannot make so many calculations to get the current voltage characteristics.

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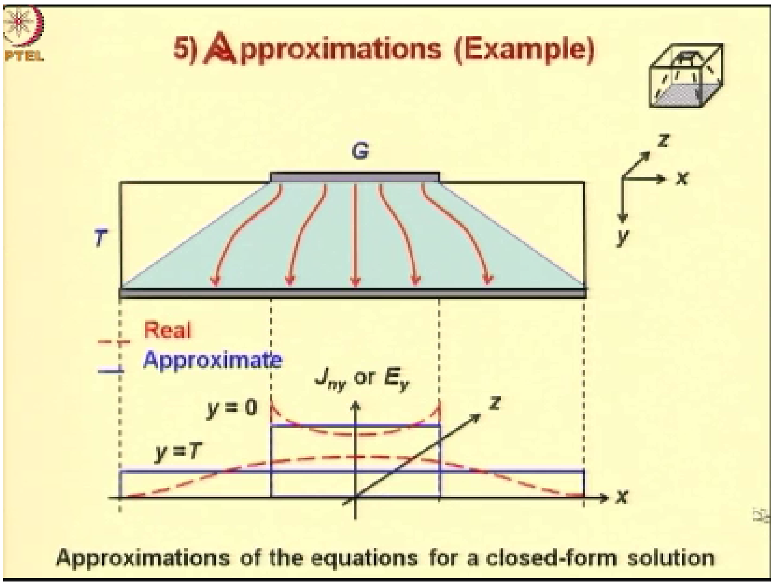

5) Approximations (Example)

Approximations of the equations for a closed-form solution

Therefore, let us look at another method which can be used to get a closed form solution. So this is the picture of a spreading resistance this is a cross section view and this is the 3D view. So current flow lines in the cross section are shown here based on this current lines one makes the following approximations about the current flow. So you assume that the current flow expands at a constant angle from the top contact on either side.

So a current flow is restricted to this pyramidal region. In the cross section it is a trapezoid in 3 dimensional view it is a pyramid okay so this is the current flow pattern. So the current flow is expanding at a constant angle. Now how do you decide what is the angle that is decided by the fact that the pyramidal current flow pattern the area of the top of the pyramid is same as the top contact area and bottom of the pyramid is same as bottom contact area.

Now let us see if you assume this kind of an approximations for the current flow pattern how do we proceed further. Now the real picture of the distribution of the current density over the contact area is as shown here Y = 0 that is a top contact. Your current density is high at the edges and low at the center. This is because at corners you have high electric fields.

So that is why you have high current density. So what we are plotting here in the vertical axis is the Y component of the electron current density or the Y component of the electric field. At the bottom contact however we have discussed this point here the electric field or the current density normal to the contact is maximum at the center and goes on falling to 0 as you move away from the center.

Now this is because the length of the electric field or flow lines goes on increasing as you move away from the center in the horizontal direction and therefore the electric field goes on falling at the bottom contact. This is in contrast to the fact at the top contact where electric field increases near the edges of the contact. Now the area under these current density distributions represents the total current.

Please note that the current density is per unit area whereas we are plotting that as a function of distance. So when I integrate this graph over distance I will not get the total current, but I will get current per unit length because this is current density which is current per unit area and when we integrate over length you will get current per unit length. So this is not really the total current. The area under these graphs do not represent total current directly. However, they are a measure of the total current.

Now we make the following approximations regarding the current flow. So after we have assumed that the current flow expands at a constant angle as shown here in a pyramidal fashion. The additional approximation we make is that the current density is uniform over any horizontal cross section of the resistor. If I want the area under this rectangle to be equal to the area under this curved portion.

Evidently my constant value has to be more than the value at the center otherwise the area under the rectangle and the area under the curl portion will not be same. So whatever area you are gaining in your approximation here is being lost from these corners. Similar comments apply here whatever area you are gaining at this corners at the approximations you are losing from these regions.

So in very simple terms therefore in your approximation the electric field or current density is higher than at the center at the top contact whereas it is lower than the values at the center at the bottom contact.

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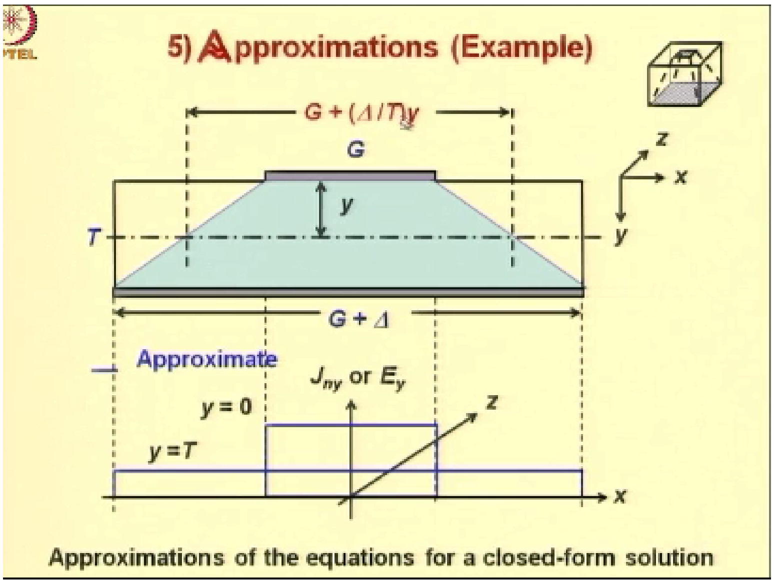

5) Approximations (Example)

Approximations of the equations for a closed-form solution

So this is your approximate picture of the current flow pattern and the current density over the contacts as well as at any cross section, horizontal cross section of the resistor. So what is the cross section of current flow at any Y? So this is your Y axis this is the origin here the center of the top contact is the origin. So as you move downward from this contact to a distance Y your area will be G+delta/T into Y.

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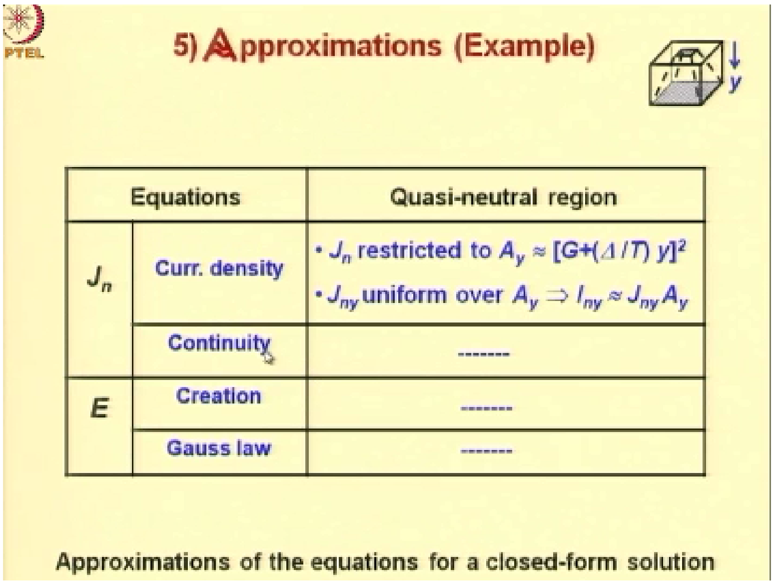

5) Approximations (Example)

Equations		Quasi-neutral region
J_n	Curr. density	• J_n restricted to $A_y \approx [G+(\Delta/T)\,y]^2$ • J_{ny} uniform over $A_y \Rightarrow I_{ny} \approx J_{ny} A_y$
	Continuity	-------
E	Creation	-------
	Gauss law	-------

Approximations of the equations for a closed-form solution

Now let us put down those approximations in the form of this table. What are the approximations of the drift diffusion equation? So the first approximation related to the current density Jn is Jn is restricted to Ay given by G+delta/T into Y the whole thing square. You can see that this formula is correct if I put Y = 0 that is the top contact I get area as G square which is the area of the top contact.

When I put Y = T which is the distance between the 2 contacts then I get G+delta whole square which is the area of the bottom contact. And the next approximation related to Jn is that Jny that is a normal component of the current for electrons is uniform over the area of cross section Ay which means I of Ny which is the total current I at any Y across the horizontal plane is given by the current density over that plane Jny which is uniform multiplied by the area of cross section Ay.

Now these are the only approximations. So we are not making any approximations related to the continuity equations, the creation of the electric field that is E = - grad psi equation or the Gauss Law.

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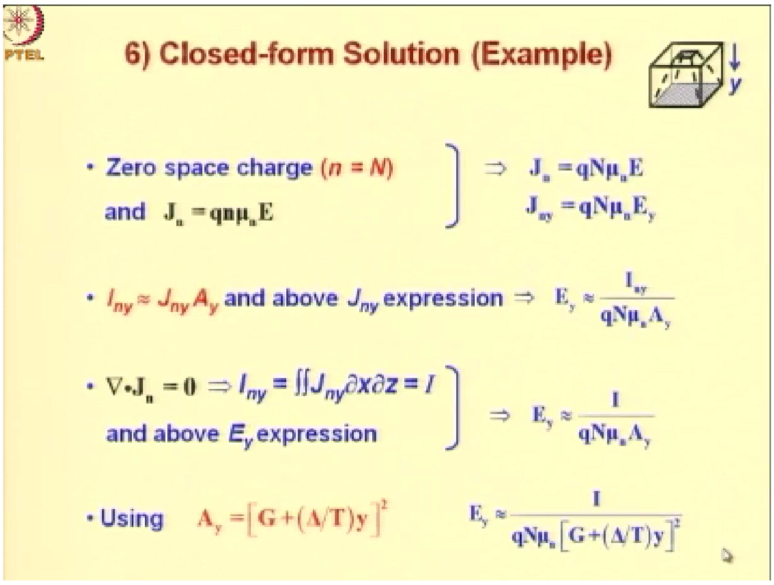

6) Closed-form Solution (Example)

- Zero space charge ($n = N$) and $J_n = qn\mu_n E$ $\Rightarrow$ $J_n = qN\mu_n E$, $J_{ny} = qN\mu_n E_y$
- $I_{ny} \approx J_{ny} A_y$ and above J_{ny} expression $\Rightarrow$ $E_y \approx \frac{I_{ny}}{qN\mu_n A_y}$
- $\nabla \cdot J_n = 0 \Rightarrow I_{ny} = \iint J_{ny} \partial x \partial z = I$ and above E_y expression $\Rightarrow$ $E_y \approx \frac{I}{qN\mu_n A_y}$
- Using $A_y = \left[G + (\Delta/T)y\right]^2$ $\quad E_y \approx \frac{I}{qN\mu_n \left[G + (\Delta/T)y\right]^2}$

Not let us look at the closed form solution itself. So 0 space charge that is N electron concentration equal to the doping concentration capital N and the electron current density equation together lead you to these equations for Jm and Jny. So here we have to substitute a electron concentration equal to capital N. Whatever we are showing in red are the approximations and whatever we are showing in black here are the equation of the drift diffusion model being used.

Similarly, the approximation at any Y the electron current is equal to the electron current density over that horizontal cross section at Y multiplied by the Ay and using this above expression for Jny we get the electric field Y component = Iny divided by Q times N mu N into Ay. Next use the diversions Jn = 0 formula of the drift diffusion equations which amounts to saying that Iny = I at the contact because it is given by this formula.

I leave it to you as an assignment to show that Iny given by this formula is = I at the contact as a consequence of this diversions of Jm = 0. Intuitively it is obvious that if diversions of the current density is 0 then if I take the current across any horizontal cross in the spreading resistance at any Y. Then that current should be the same as the current at the contact because there is no generation or recombination.

There is no kinetic generated or lost and it is steady state. So use this fact and use the above Ey expression so this expression for Ey. Then we get the Ey formula as shown here all that we have done here is replaced the electron current at any Y we have replaced by the current at the contact. Now using the formula for the area of cross section of the current flow at Ay this equation gets modified to this equation. All that we have done is we have replaced Ay by this formula here.

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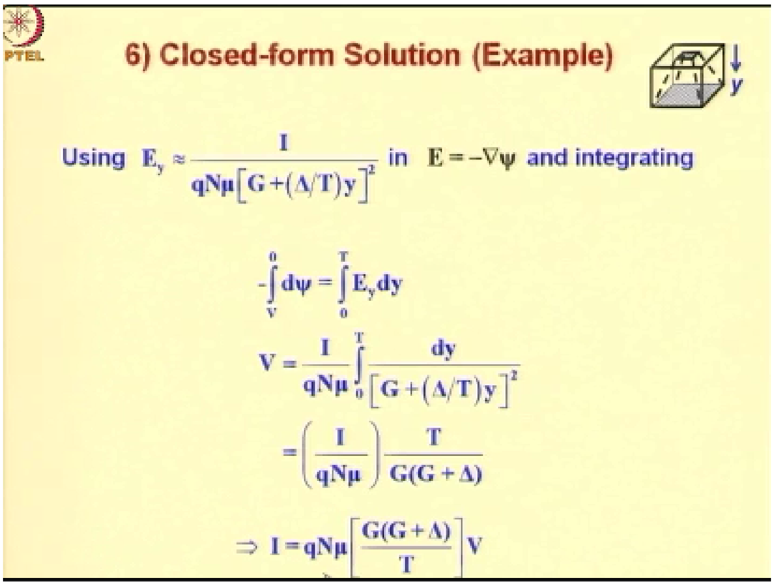

6) Closed-form Solution (Example)

Using $E_y \approx \frac{I}{qN\mu\left[G+(\Delta/T)y\right]^2}$ in $E = -\nabla\psi$ and integrating

$$-\int_V^0 d\psi = \int_0^T E_y dy$$

$$V = \frac{I}{qN\mu}\int_0^T \frac{dy}{\left[G+(\Delta/T)y\right]^2}$$

$$= \left(\frac{I}{qN\mu}\right)\frac{T}{G(G+\Delta)}$$

$$\Rightarrow I = qN\mu\left[\frac{G(G+\Delta)}{T}\right]V$$

Now using this formula for Ey in terms of the current I and the area at Ay in the formula E = - gradient of psi/integrating you can get a relation between the applied voltage and the current as follows. So we integrate both sides so you get integral D psi from V to 0 with a negative sign = integral Ey Dy from 0 to T. So you are integrating in the Y direction from this contact to this contact on both sides of this equation and you are substituting for Ey this formula.

So left hand side becomes V you can see that. Right hand side I/Q times doping concentration to mu this is mu suffix N. Here also this should be mu suffix N. So these are constant so they have come out of the integral and inside you have D/G+delta/T into Y the whole square. When you integrate that you get I/Q times N into mu again this should be mu suffix N into T/G into G+delta.

So this integral over this limits gives you this result. Consequently, you can rearrange this equation to give I as a function of V. So this is your multiplying constant.

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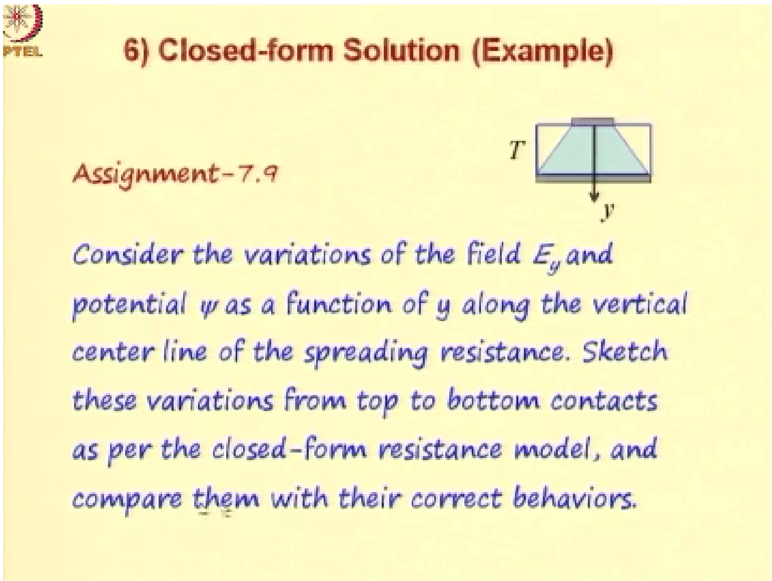

6) Closed-form Solution (Example)

Assignment-7.9

Consider the variations of the field E_y and potential ψ as a function of y along the vertical center line of the spreading resistance. Sketch these variations from top to bottom contacts as per the closed-form resistance model, and compare them with their correct behaviors.

Before moving on further let us give you an assignment because you have just finished the closed form solution of the problem under some approximations. Consider variations of the field Ey and potential psi as a function of Y along the vertical center line of the spreading resistance. So these are vertical center line of the spreading resistance. Sketch these variations from top to bottom contacts as per the closed form resistance model and compare them with their correct behaviors.

So when we say the correct behavior we are not expecting you to do a numerical solution or anything like that. We are remarking here the correct qualitative behaviors which you can derive based on the current density distributions at the top and bottom contact or the normal component of the electric field distributions at the top and bottom contacts we have discussed earlier.

So at the top contact the normal electric field distribution is something like this and at the bottom contact it is something like this so you can use that and the approximate results which are constant.

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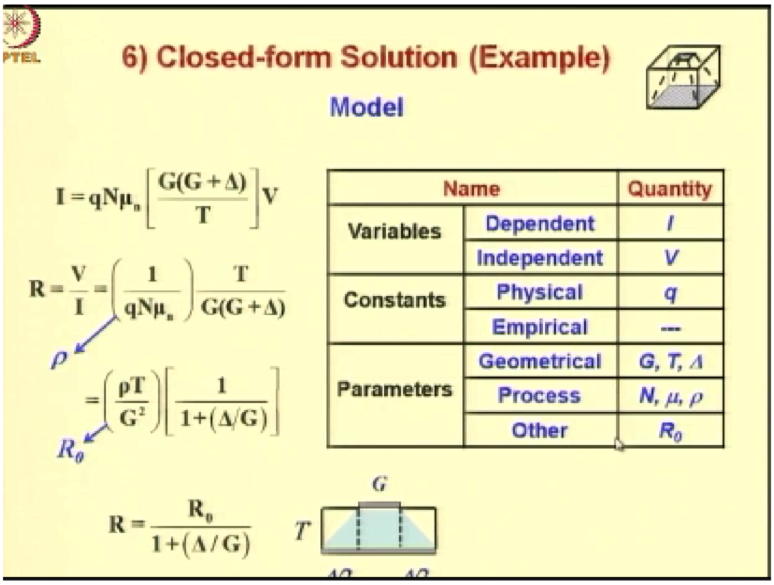

6) Closed-form Solution (Example)

Model

$$I = qN\mu_n \left[\frac{G(G+\Delta)}{T} \right] V$$

$$R = \frac{V}{I} = \underbrace{\left(\frac{1}{qN\mu_n} \right)}_{\rho} \frac{T}{G(G+\Delta)}$$

$$= \underbrace{\left(\frac{\rho T}{G^2} \right)}_{R_0} \left[\frac{1}{1+(\Delta/G)} \right]$$

$$R = \frac{R_0}{1+(\Delta/G)}$$

Name		Quantity
Variables	Dependent	*I*
	Independent	*V*
Constants	Physical	*q*
	Empirical	---
Parameters	Geometrical	*G, T, Δ*
	Process	*N, μ, ρ*
	Other	R_0

Now let us look at the closed form solution little further as to how you write the closed form solution this is very, very important. After obtaining the solution it has to be arrange in a form that appeal to physical intuitions and which one can appreciate directly. So we want to convert this into an expression for the resistance. So R = V/I from this equation your V/I turns out to be of this formula where one/Q times N into mu is the resistivity. Again here mu is the mobility of electrons.

You can rewrite this same formula in this form which is physically more appealing where you recognize the fact that rho into T/G square represents the resistance of the spreading resistor. When the size of the bottom contact is equal to the size of the top contact when delta is 0. You have 1- dimensional resistor with contact area equal to the top contact area so that is why it is represented as R suffiX0 where 0 implies the value delta = 0.

So what we have done is we have taken the G outside this brackets here. So inside we have the term delta/G. So now your resistance equation is written in this form. R = R0/one+delta/G. So this form is physically very appealing because it shows that the resistance of a spreading resistor decreases as you increase the length of the bottom contact laterally. Delta is the extension of the bottom contact from the top contact.

So when the extension is 0 the resistance = the 1-dimensional value otherwise it falls as reciprocal of 1 + the extension by the size of the top contact. Now this form is physically very appealing and you can easily see that you prefer this form of the model to the form written here. Mathematically, this form is same as this form, but physically this is more appealing.

The form of this resistance expression shows that it is not the absolute value of delta, but the value of delta relative to G that matters and plays a role in reducing the resistance. Now let us look at variables, constants and parameters of our model. The dependent and independent variables are current and voltage. The physical constant is Q there are no empirical constants G, T and delta are the geometrical parameters and doping concentration mobility and resistivity are the process parameters.

Now you would recall that in our table of variables, constants and parameters there was no R0 earlier. This R0 is another parameters that has entered into the model at this point of time. Now this parameters has been listed in the other category because if you see the expression for R0 which is rho T/G square in this T and G are geometrical parameters and rho resistivity is a process parameters.

Therefore, this term rho T/G square cannot be classified either purely and geometrical parameters or purely as a process parameter. That is why it has been listed in the other category.

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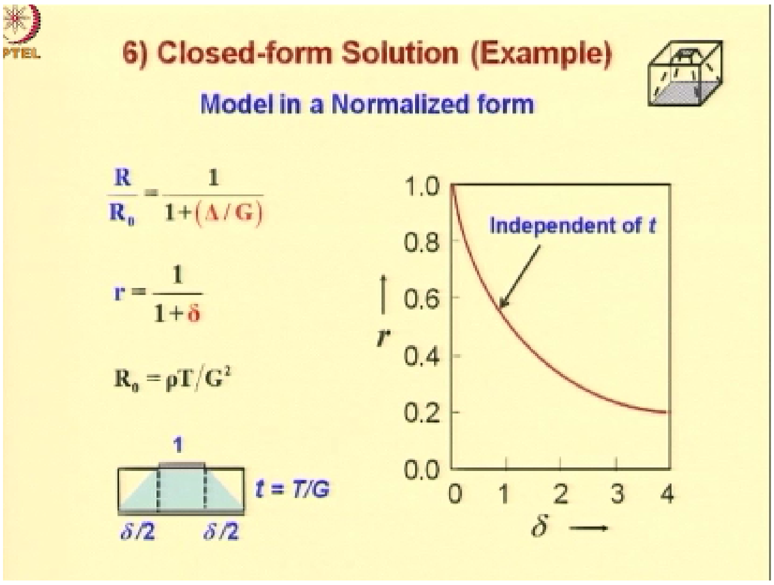

6) Closed-form Solution (Example)

Model in a Normalized form

$$\frac{R}{R_0} = \frac{1}{1+(\Delta/G)}$$

$$r = \frac{1}{1+\delta}$$

$$R_0 = \rho T/G^2$$

Let us look at the model equation further. Let write it in a form that is referred to as normalized form which is very compact form. So R/R0 = 1/1+delta/G can be written as small R which represents the dimensional less ratio of the resistance to the value of the resistance when the extension of the bottom contact is 0. So small R = 1/1 + small delta which is the normalized value of capital delta to the dimension of the top contact.

So this again a dimensionless quantity. So here in this form which is very, very compact the resistance has been expressed in terms of dimensionless quantities. If you recall the form of the resistance equation that we wrote down earlier it was a function of T, G and delta, but when you write in normalized form it is not a function of T at all. It is only a function of delta/G and R/R0.

So a single line represents in a compact form the results for all the different values of T delta G and also resistivity. Now here R0 is given by rho T/G square. Now this is the resistance geometry in a normalized form where the top contact dimension is taken as unity because you are normalizing that with respect to its own dimension then the bottom contact is unity + delta/2 on either side where this is small delta/2.

A small delta is normalized value of delta to G and similarly this dimension the distance between the 2 contacts is normalized with respect to G. This normalized form is very, very powerful and compact. Since the writing expressions in a normalized form is very important.

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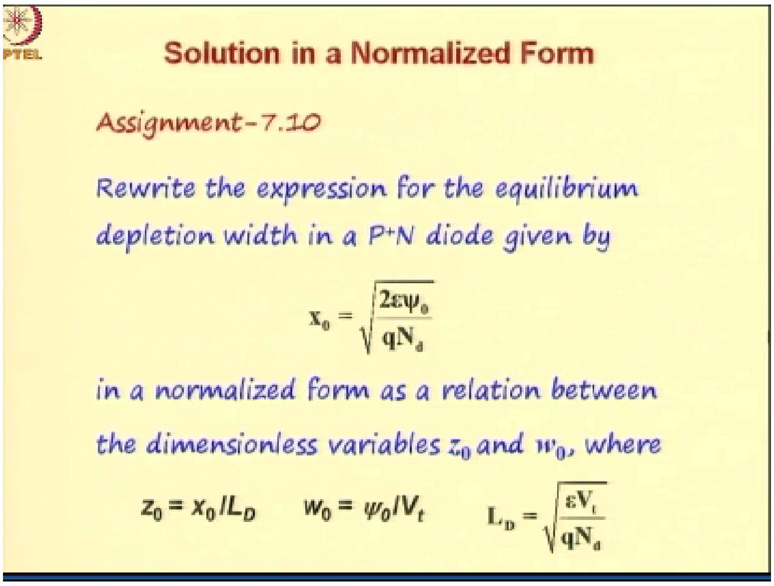

Solution in a Normalized Form

Assignment-7.10

Rewrite the expression for the equilibrium depletion width in a P+N diode given by

$$x_0 = \sqrt{\frac{2\varepsilon\psi_0}{qN_d}}$$

in a normalized form as a relation between the dimensionless variables z_0 and w_0, where

$z_0 = x_0/L_D$ $\quad w_0 = \psi_0/V_t$ $\quad L_D = \sqrt{\frac{\varepsilon V_t}{qN_d}}$

We will give you a few assignments before we close this lecture. Rewrite the expression for the equilibrium depletion width in a P+N diode given by X0 = square root of 2 epsilon psI0/Qnd in a normalized form as a relation between the dimensionless variables Z0 and W0 which are given below. So Z0 is the depletion with X0 normalized to the Debye length Ld of the lighted outside. W0 is the normalized potential.

The built in potential normalized to the thermal voltage and Ld = the Debye length of the region having a doping Nd. So please write it in terms of Z0 and W0 and see how this expression becomes really very compact.

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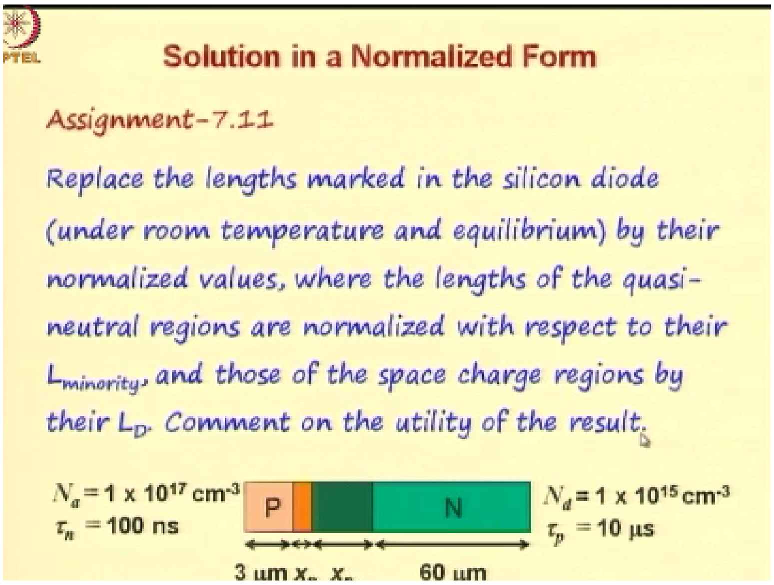

Solution in a Normalized Form

Assignment-7.11

Replace the lengths marked in the silicon diode (under room temperature and equilibrium) by their normalized values, where the lengths of the quasi-neutral regions are normalized with respect to their $L_{minority}$, and those of the space charge regions by their L_D. Comment on the utility of the result.

$N_a = 1 \times 10^{17}$ cm^{-3}
τ_n = 100 ns

$N_d = 1 \times 10^{15}$ cm^{-3}
τ_p = 10 µs

Another assignment related so this is a PN junction in which Xp and Xn are width of the space charge region of the P and N psi and width of the PN is 3 micron and width of the N region is 60-microns. Now this ND and the minority carrier lifetime in the N region are given here and Na and the minority carrier lifetime in the P region are having these values.

Replace the lengths marked in the silicon diode under room temperature and equilibrium by their normalized value where the lengths of the quasi-neutral regions are normalized with respect to their minority carrier diffusion length L minority. So 3-micron and 60-micron should be normalized with respect to their minority carrier diffusion lengths and those of the space charge regions by their Debye lengths.

So you should normalize Xp with respect to the Debye length in this region and Xn with respect to the Debye length in that region and see, you know, what kind of dimensionless numbers you get for these lengths. Comment on the utility of the result.

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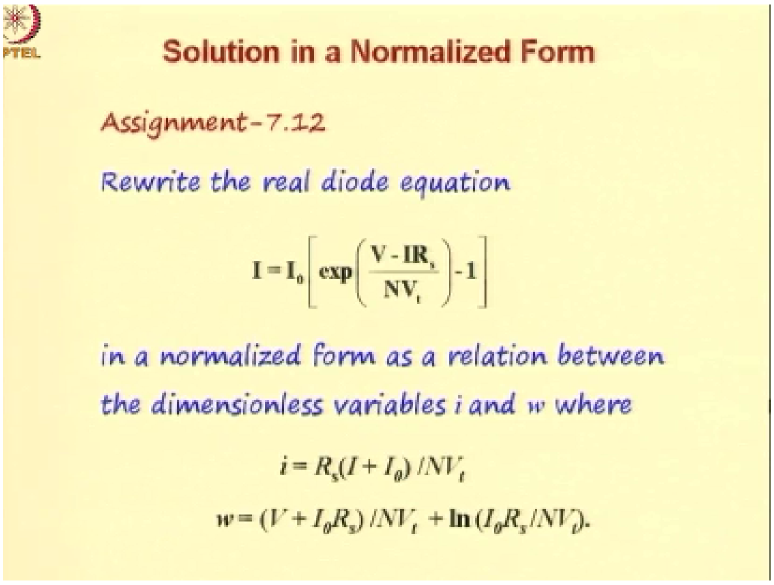

Solution in a Normalized Form

Assignment-7.12

Rewrite the real diode equation

$$I = I_0\left[\exp\left(\frac{V - IR_s}{NV_t}\right) - 1\right]$$

in a normalized form as a relation between the dimensionless variables i and w where

$$i = R_s(I + I_0)/NV_t$$

$$w = (V + I_0R_s)/NV_t + \ln(I_0R_s/NV_t).$$

Rewrite the real diode equation I = I0 into exponential of V - the current I into the series resistance Rs divided by the ideality factor into thermal voltage - 1 in a normalized form as a relation between the dimensionless variables small I and small W given below. Small I = series resistance Rs into I+I0 where I0 is the reverse saturation current divided by NVt.

And W = V+I0 Rs in brackets divided by N times Vt + natural logarithm of I0 RS divided by N times Vt. So please rewrite these equations as I as a function of W and you will see that this equation with so many terms comes out in a very beautiful compact form with very few terms. With that, we have come to the end of the lecture. Let us quickly summarize the important points.

So in this lecture we detailed 4 steps of the device modeling procedure namely equations, boundary conditions, approximations and solutions. We said that our equations for various device models are going to be based on the drift diffusion equation and we are going to use as far as possible the ideal boundary conditions. In approximations we pointed out there were 2 sets of approximations those leading to the equations and boundary conditions.

And those which are done after you write the equations in boundary conditions to get the analytical type of solution. So for analytical model the set of approximations will consist of 2 sets. One made during the qualitative modeling stage to derive the equations in boundary conditions and another set which are the approximations of the equation and boundary conditions to enable us to derive an analytical model.

For a numerical model however for numerical solution you only have the approximations made during the qualitative stage which leads to the equations and boundary conditions. The equations are solved using numerical techniques. In this solutions we remarked that in this course, we are going to as far as possible limit ourselves to analytical solution or analytical model and we empathized why these solutions should be cast in a physically appealing form.

And we pointed out one particular form of writing an equation namely the normalized form which allows us to represent a reasonably complex model equation in a very compact form. So that the compact form represents with a few terms the results for a wide range of parameter values of the model. We shall continue with the remaining steps of modeling procedure in the next lecture.