Semiconductor Device Modeling
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Lecture - 26
Characteristic Times and Lengths

In the previous lecture, we have begun a discussion of the utility of characteristic times and lengths. Specifically, we discussed how these scaling times and lengths can be used to explain device phenomenon. We gave examples of p-n junction, MOS junction and capacitor and MOSFET. Then we showed how the characteristic times and lengths can be useful in device simulation and characterization.

For instance, we showed that you can simulate surface recombination by volume recombination and to come up with simulation scheme how the scaling times and lengths are useful. Characterization of devices, how measurement of doping profile, the characteristic length namely Debye length comes into picture, okay, how the doping if it varies rapidly over a Debye length, then it cannot be resolved.

You cannot get the variation of that doping that occurs very rapidly over Debye length scale. Then we began a discussion of the various approximations, how appropriations can be validated based on characteristic times and lengths. Since modeling is the art of making approximations, we want to discuss the validation of many important approximations and we shall continue this discussion in the present lecture.

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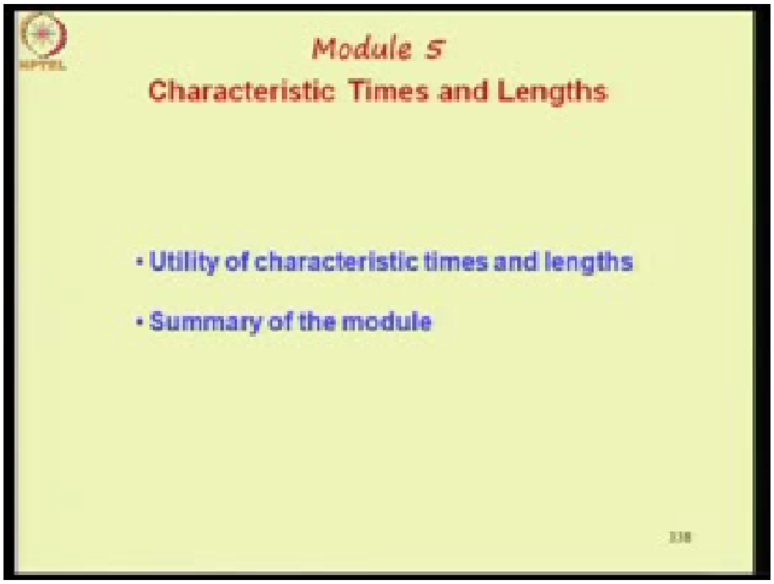

Module 5

Characteristic Times and Lengths

- Utility of characteristic times and lengths
- Summary of the module

We will also summarize the module after our discussion.

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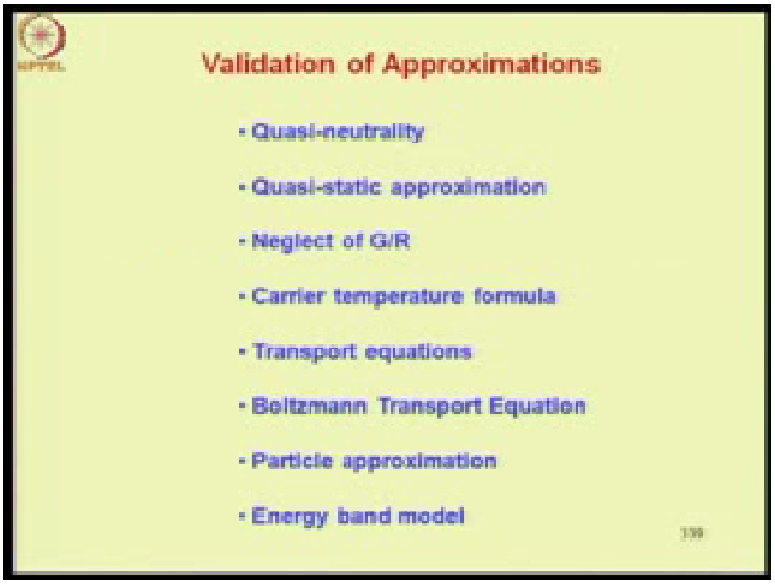

Validation of Approximations

- Quasi-neutrality
- Quasi-static approximation
- Neglect of G/R
- Carrier temperature formula
- Transport equations
- Boltzmann Transport Equation
- Particle approximation
- Energy band model

In the previous lecture, we discussed Quasi neutrality, Quasi-static approximation and neglect of generation recombination. In present lecture, we will discuss the carrier temperature formula, Transport equations, Boltzmann Transport Equation, Particle approximation and energy band model.

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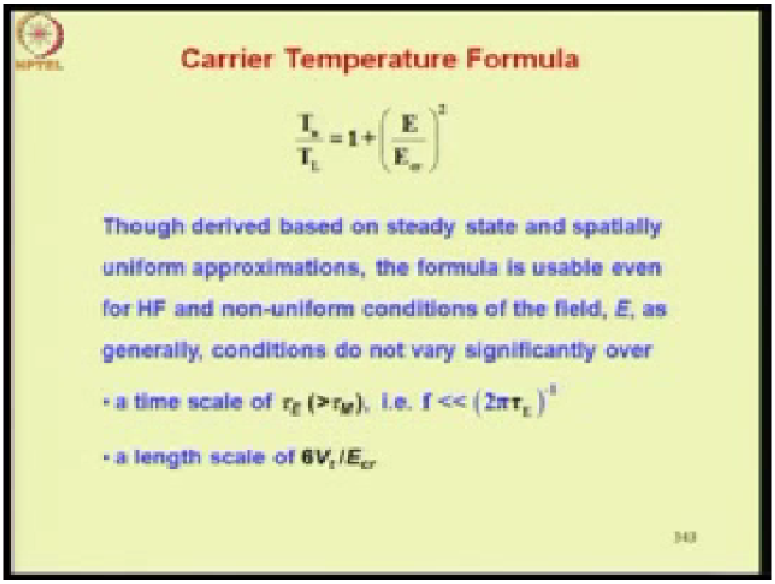

Carrier Temperature Formula

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

Though derived based on steady state and spatially uniform approximations, the formula is usable even for HF and non-uniform conditions of the field, *E*, as generally, conditions do not vary significantly over

- a time scale of τ_E ($>\tau_M$), i.e. $f << (2\pi\tau_E)^{-1}$
- a length scale of $6V_t/E_{cr}$

Let us begin with the carrier temperature formula which is given by, for electrons T suffix n which is the temperature of electrons, divided by the lattice temperature, so this ratio = 1+the electric field divided by the critical electric field, the whole square. Though derived based on steady state and spatially uniform approximations, the formula is usable even for high-frequency and non-uniform conditions of the field E.

This is because generally conditions do not vary significantly over a time scale of energy relaxation time, tau E which is more than the momentum relaxation time, that is the frequency is much < 1/2 pi tau E. Now while deriving this formula, we use both the momentum and energy balance equations, so both tau M and tau E come into play. However, since tau E is more than tau M, tau E becomes the governing factor, okay.

The formula is also valid if the conditions do not vary significantly over a length scale of 6 times thermal voltage by the critical electric field. Now, how do you get this length scale. This is left to you as an assignment where I have given some guidance on how to derive this length scale.

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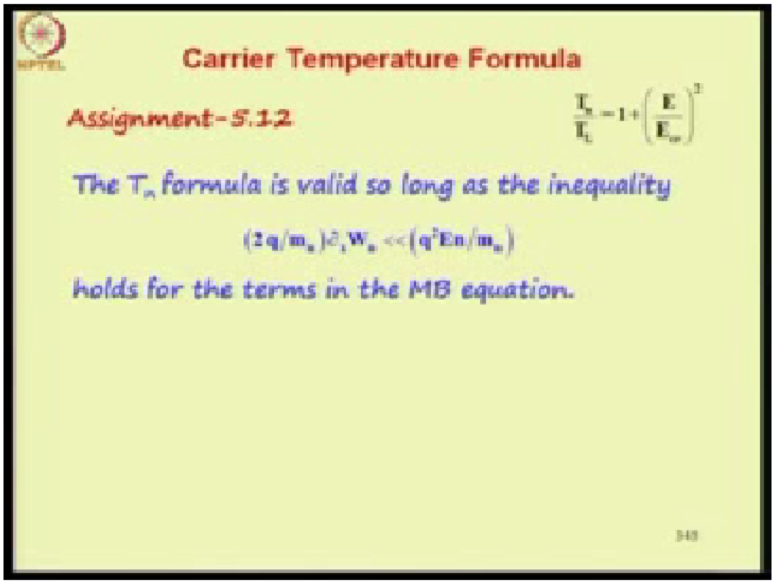

Carrier Temperature Formula

Assignment-5.12

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

The T_n formula is valid so long as the inequality

$$(2q/m_n)\partial_t W_n << (q^2 En/m_n)$$

holds for the terms in the MB equation.

So the assignment is as follows: The Tn formula is valid so long as the inequality 2q/m suffix n which is the effective mass of electrons into the spatial gradient of Wn is much < q square E into n which is the electron concentration divided by effective mass of electrons. So long as this inequality holds for the terms in the momentum balance equation. So if you recall, the formula for carrier temperature was derived based on steady-state formulations of momentum balance and energy balance equations.

So in the momentum balance equation, on the right-hand side, you have these 2 terms which are listed out here, so the spatially varying term, if it is shown to be much < this q square En/mn term, then the formula is valid. Now let us see under what conditions this inequality holds.

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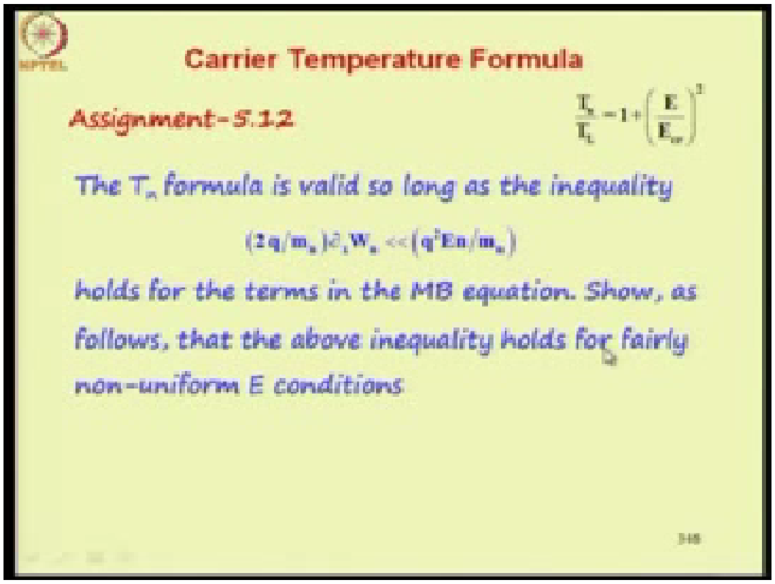

Carrier Temperature Formula

Assignment-5.12

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

The T_n formula is valid so long as the inequality

$$(2q/m_n)\partial_z W_n \ll (q^2 E n/m_n)$$

holds for the terms in the MB equation. Show, as follows, that the above inequality holds for fairly non-uniform E conditions

Show as follows that the above inequality holds for fairly non-uniform electric field E conditions.

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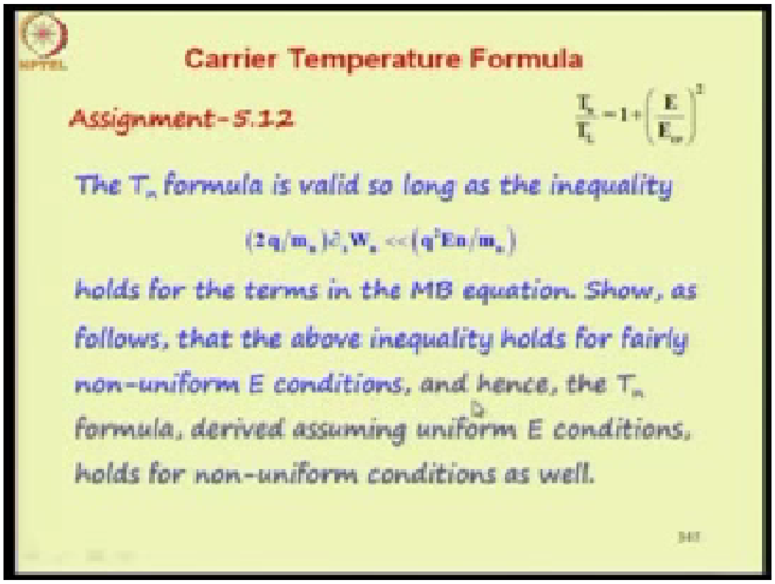

Carrier Temperature Formula

Assignment-5.12

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

The T_n formula is valid so long as the inequality

$$(2q/m_n)\partial_z W_n \ll (q^2 E n/m_n)$$

holds for the terms in the MB equation. Show, as follows, that the above inequality holds for fairly non-uniform E conditions, and hence, the T_n formula, derived assuming uniform E conditions, holds for non-uniform conditions as well.

And hence the Tn formula that is this formula derived assuming uniformly E conditions holds for non-uniform conditions as well. So again recall that the carrier temperature formula was derived based on steady-state and uniform electric field and carrier concentration conditions.

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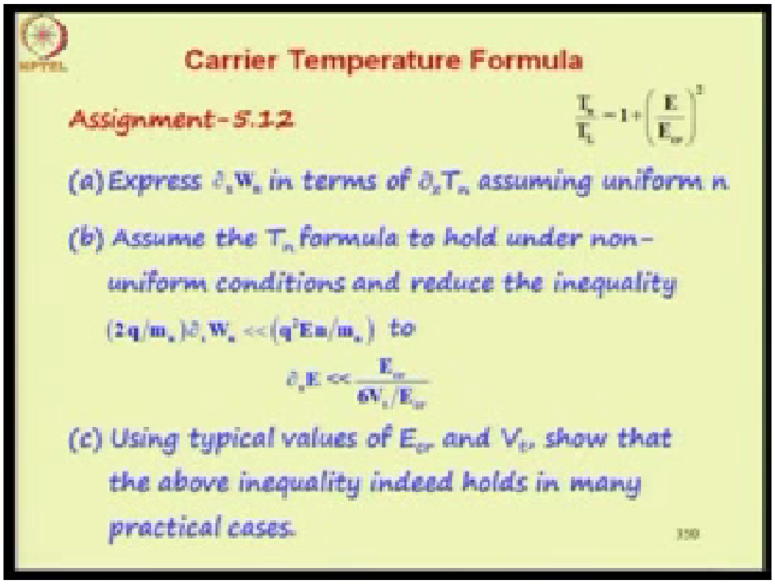

Carrier Temperature Formula

Assignment-5.12

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

(a) Express $\partial_x W_n$ in terms of $\partial_x T_n$ assuming uniform n

(b) Assume the T_n formula to hold under non-uniform conditions and reduce the inequality $(2q/m_n)\partial_x W_n << (q^2En/m_n)$ to

$$\partial_x E << \frac{E_{cr}}{6V_t/E_{cr}}$$

(c) Using typical values of E_{cr} and V_t, show that the above inequality indeed holds in many practical cases.

Now how do you show that. So the step 1 express dou Wn/dou x in terms of dou Tn/dou x assuming uniform electron concentration n. So here you will have to express Wn in terms of the carrier temperature. So you know that formula, use that. Then assume the Tn formula to hold under non-uniform conditions and reduce the inequality 2q/mn into dou Wn/dou x much < q square En/mn to dou E/dou x much less than critical electric field divided by 6 times Vt/critical electric field.

So you will have to show that left-hand side can be replaced by this formula, this term. So dou Wn/dou x can be expressed in terms of dou E/dou x and right-hand side can be expressed as a field term divided by a length term. Now this is the characteristic length for our situation here. So what this formula shows is that so long as your change in electric field over x is much less than critical electric field over 6 Vt/E critical length. So long as this variation is less than this, your formula will hold.

Now using typical values of critical electric field and thermal voltage, show that the above inequality indeed holds in many practical cases. So you will have to substitute the typical values and show that this length scale is really very small and if the right-hand side of this term of the inequality, if it allows you a change of critical electric field over that small length scale. So this is really very high rate of change and most practical dou E/dou x conditions would be less than this.

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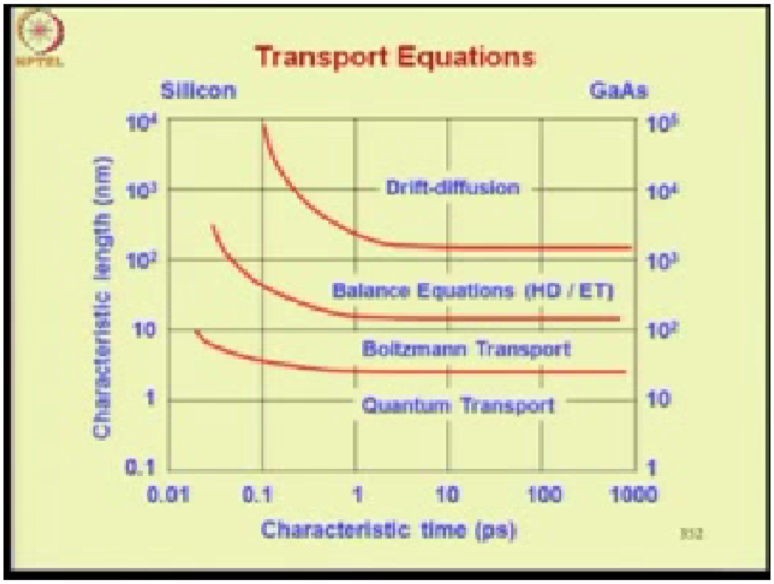

Transport Equations

Let us move on to transport equations. We have discussed the various forms of transport equation, the Drift diffusion, Balance equations, which can be expressed in 2 different formats, Hydrodynamic and Energy Transport and then Boltzmann Transport equation and Quantum Transport equations. What is the validity range of each of these, that is what is shown in this diagram.

Let us take this boundary here, the vertical line here. You see that this corresponds to a characteristic time of 0.1 picoseconds. The boundary between Drift Diffusion and Balance Equations 0.1 picoseconds is the value of momentum relaxation time. So once your signal changes very rapidly over this time scale, the Drift Diffusion formula is no more valid, okay. So you will have the go to Balance equations.

You recall that the Drift Diffusion formulation assume a steady-state version of Momentum Balance equation. So the steady-state version of Momentum Balance equation is no more valid, okay if your signal varies rapidly over time scale of momentum relaxation time. So this is what explains this boundary here between Drift Diffusion and Balance equations. Now let us look at this horizontal boundary between Drift Diffusion and Balance equations.

So you find a length value of a little more than hundred nanometers. So at this length scales, the electric field can be large and therefore, no matter what your signal frequency is, in fact even under steady-state conditions, one will have to move from Drift Diffusion to Balance equations formulation because under such high fields, you can have effect such as velocity saturation and velocity overshoot.

So to model those affects, one will have to use energy balance. Now let us move further down, when do you go from Balance equations to Boltzmann Transport. If your device sizes of the order of 10 nanometers or little more, which happens to be the range in which the mean free path of carriers in silicon or the de Broglie wavelength of carriers in silicon, so they are around this length value.

So once your length scale becomes even smaller than the electric field variation and so on is so high that 3 balance equations are not sufficient, namely the Carrier Balance, the Momentum Balance and energy balance. So you also have to go to further Balance equations, more particularly, the variation of the energy flux with distance, okay. That term becomes important.

So that is why instead of using more balance equations, you directly aim at solving the Boltzmann Transport equation. And if the device size is really a very small, it becomes much less than the de Broglie wavelength, then you have go to Quantum Transport. The right-hand side here is the length scale for gallium arsenide and because effective mass of electrons in gallium arsenide is much smaller.

The order of magnitude of this length scale is 10 times that for silicon. So here the transition from Drift Diffusion to Balance equations sets in for even larger devices, right, for gallium arsenide. So even for larger size gallium arsenide devices, one will have to use Balance equations because velocity overshoot, velocity saturation, all those effects will be strong.

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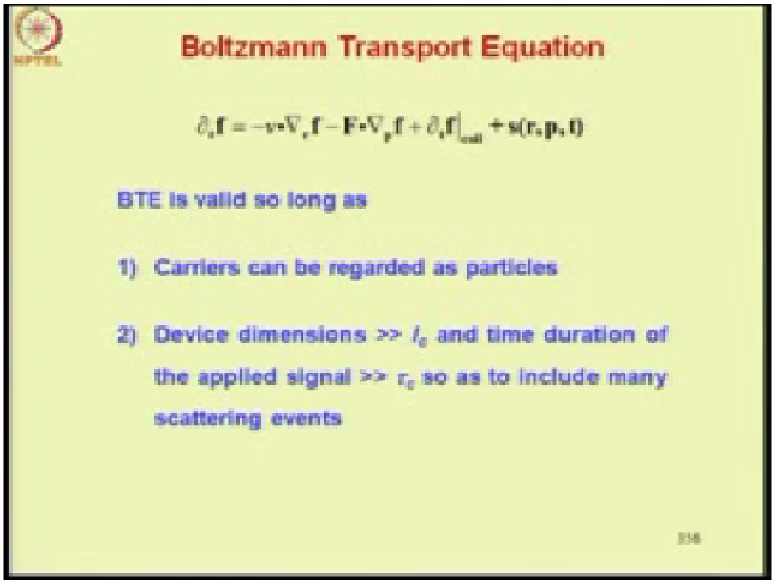

Boltzmann Transport Equation

$$\partial_t \mathbf{f} = -\mathbf{v}\cdot\nabla_r \mathbf{f} - \mathbf{F}\cdot\nabla_p \mathbf{f} + \partial_t \mathbf{f}\big|_{coll} + \mathbf{s}(\mathbf{r},\mathbf{p},\mathbf{t})$$

BTE is valid so long as

1) Carriers can be regarded as particles
2) Device dimensions >> l_0 and time duration of the applied signal >> τ_0 so as to include many scattering events

Let us look at the Boltzmann Transport equation. What is the range of its validity? So BTE is valid, so long as carriers can be regarded as particles for which the conditions will be discussed shortly and 2 device dimensions are much greater than the mean free path and time duration of the applied signal is much more than means free time between collisions, so as to include many scattering events.

So this is important because you are using the concept of a distribution function unless you have sufficient number of carriers unless you have sufficient number of scattering events, the concept of distribution function, which is a statistical concept, will not hold. So that is why you need many scattering events which are insured if your length scale of the device is much more than the mean free path or the time scale of your signal variation.

So the signal varies over the time scale which is much more than the mean free time between collisions, that ensures many scattering events are available for in the duration or region of analysis.

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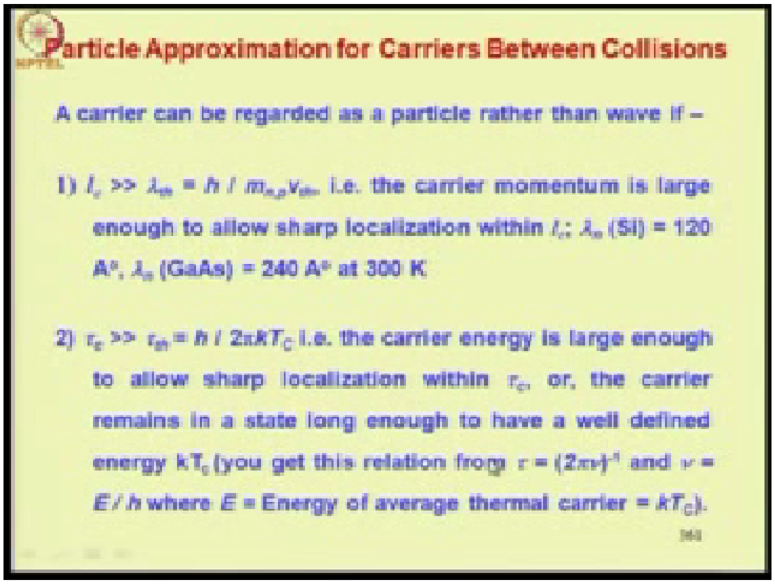

Particle Approximation for Carriers Between Collisions

A carrier can be regarded as a particle rather than wave if –

1) $l_c >> \lambda_{th} = h / m_{n,p} v_{th}$, i.e. the carrier momentum is large enough to allow sharp localization within l_c; λ_n (Si) = 120 A°, λ_n (GaAs) = 240 A° at 300 K

2) $\tau_c >> \tau_{th} = h / 2\pi k T_C$ i.e. the carrier energy is large enough to allow sharp localization within τ_c, or, the carrier remains in a state long enough to have a well defined energy kT_c (you get this relation from $\tau = (2\pi\nu)^{-1}$ and $\nu = E / h$ where E = Energy of average thermal carrier = kT_C).

Now let us look at the particle approximation of carriers between collisions. A carrier can be regarded as a particle rather than wave if: Lc is much more than the thermal average wavelength of the electron which is given by the Planck's constant h divided by effective mass of electrons or holes whichever is the particle under question multiplied by the thermal velocity.

So this is the formula here. That is the carrier momentum is large enough to allow sharp localization within lc, so if the momentum is large, de Broglie wavelength is small. And that means the particular carrier can be localized in a length which is much smaller than lc. So the lambda for electrons and silicon is 120 Angstroms and gallium arsenide is 240 Angstroms at 300 K. Now this can be calculated using the formula. We have already calculated this, then we calculated the orders of magnitude of various characteristic lengths.

Now the second condition for the particle approximation is that, the mean free time between collisions should be much greater than this time constant tau thermal which is given by h/2 pi kTc, that is the carrier energy is large enough to allow sharp localization within tau C or the carrier remains in a state long enough to have a well-defined energy, kTc, okay. So this is the condition.

So if you want to regard the carriers of particle between 2 collisions, you should be able to localize the carrier within a short time or a short region. So the distance between collisions is lc, if you can localize a particle within a very small fraction of lc, then you can regard it as a particle or if you can localize a particle within a time interval which is much shorter than the mean free time between collisions, then also you can regard it as a particle. Some other conditions.

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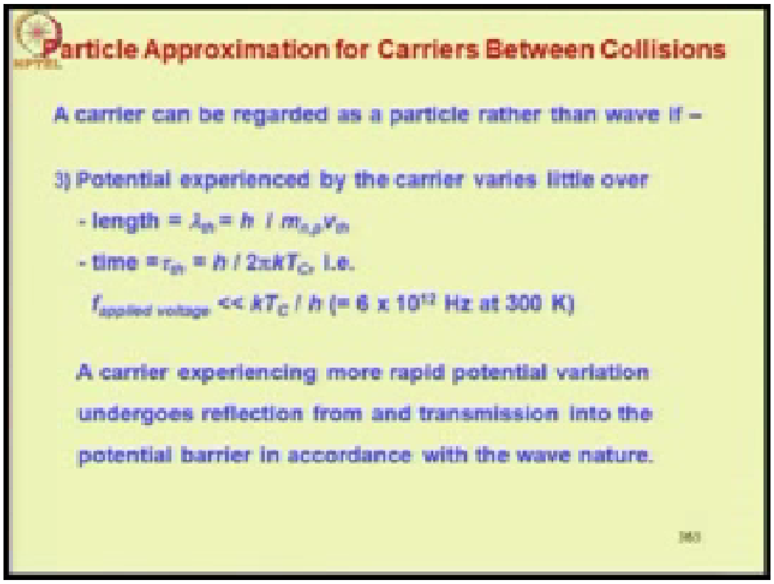

Particle Approximation for Carriers Between Collisions

A carrier can be regarded as a particle rather than wave if –

3) Potential experienced by the carrier varies little over

- length = $\lambda_{th} = h / m_{n,p} v_{th}$
- time = $\tau_{th} = h / 2\pi k T_C$, i.e.

$f_{\text{applied voltage}} << kT_C / h$ (= 6×10^{12} Hz at 300 K)

A carrier experiencing more rapid potential variation undergoes reflection from and transmission into the potential barrier in accordance with the wave nature.

Many times carriers are subjected to potential variation. So what is a condition on potential variation. The potential experience by the carrier where is little over the length de Broglie wavelength lambda thermal. Or the potential varies little over the time scale, tau thermal, right, which is h/2 pi kTc which can be converted into frequency and the inequality would then be, the frequency of applied voltage should be much less than kTc/h and that is 6 into 10 power 12 hertz at 300 K.

Now you see this frequency is really very high and therefore for most practical frequencies, the particle approximation will hold for carriers. A carrier experiencing more rapid potential variation undergoes reflection from and transmission into the potential barrier in accordance with the wave nature, okay.

So supposing you have a potential variation and this potential variation is very rapid over the de Broglie wavelength, then effect of that potential variation cannot be treated by particle means because the carrier will undergo reflection, okay and even transmission, tunneling, right through the barrier. So, then you will have to invoke the wave nature.

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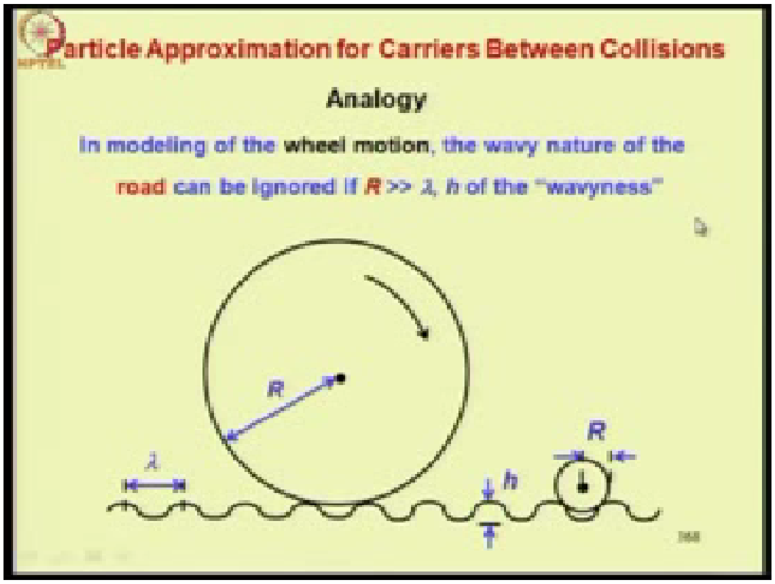

Particle Approximation for Carriers Between Collisions

Analogy

In modeling of the wheel motion, the wavy nature of the road can be ignored if $R >> \lambda, h$ of the "wavyness"

Let us explain the particle approximation for carriers between collisions using an analogy because this is really a very, very important idea. Now we say that in modeling of the device current, the wavy nature of electron can be ignored if mean free path between collisions is much greater than the wavelength lambda of the wavyness, that is the thermal average wavelength. Now, this is very much like the following situation.

Consider a wheel moving over a road that is wavy, lambda is the wavelength of the road and h is the height of this hump. Now if the radius of your wheel are is much greater than this lambda, then you really do not have to consider the wavy nature of the road, you can regard the road as flat. On the other hand, if you radius is really small and is of the order of the wavelength lambda, then you see that the wheel will try to follow this wavy nature, right, hump.

And therefore this wavy would become important. So putting this in the form of a statement in modeling of the wheel motion, the wavy nature of the road can be ignored if radius are, of the wheel is much greater than lambda or h, this dimension of the wavyness. So just as the relative values of 2 length dimensions decides here whether the wheel can be regarded to be on a flat road or a wavy road.

Similarly, the relative comparison between 2 length dimensions for electron, namely the mean free path and the de Broglie wavelength, this criterion helps you to decide whether the particle approximation for electron is valid.

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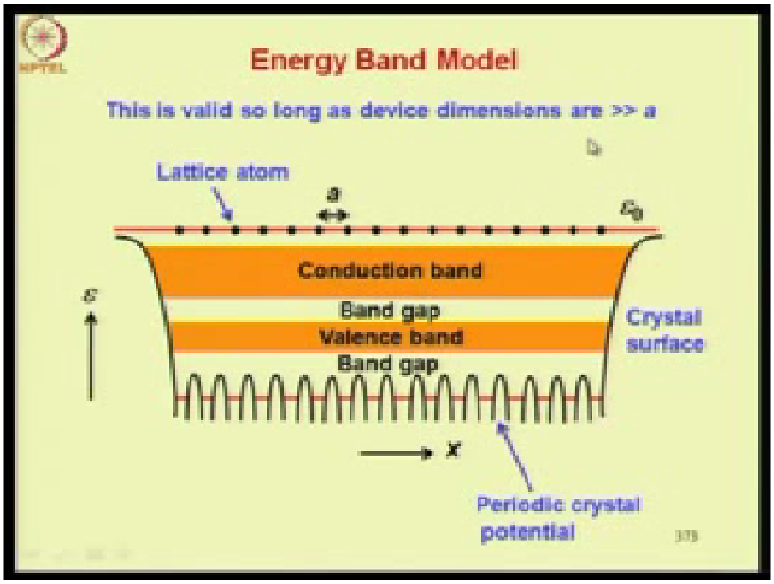

Energy Band Model

This is valid so long as device dimensions are >> a

Finally let us look at the energy band model. In a crystal, allowed energy levels of electrons are grouped into bands separated by forbidden gaps. You know this from the first level course. To help you, recall the following diagram. This is electronic energy plotted as a function of x. This is so-called energy band diagram, EX diagram. E knot is a vacuum level and these 2 orange bands are the allowed energy bands, outermost energy band is called the conduction band and valence band, they are separated by band gap.

You similarly have other bands which are narrower. For example, this is a band, but which looks very thin and it is separated from the valence band from a band gap. So this particular model of allowed energies for electrons in a crystal, what is the range of validity. So this model is valid, so long as device dimensions are much more than the lattice constant a, which is the distance between the atoms.

So to further appreciate the nature of this approximation, let me show you in the picture the lattice constant a. So this is a one-dimensional lattice for simplicity we are considering and this is atomic arrangement, regular arrangement of atoms, these are the atomic potentials, okay. Now you see that these potentials are varying with a period equal to the distance between 2 atoms and that is a.

So long as your device dimensions, here, so this length is much more than a, in other words you should have numerous atoms, here, only then the idea of a continuous allowed energy band is valid. If the number of atoms become small because the device dimensions are small, okay, compared to a, then you will no more have bands, but rather you will have discrete levels, that is what will happen and the energy band model in that case will not hold.

Now these are some of the important approximations in which you use the concept of scaling lengths and times to decide whether the approximation is valid or not. So with that we have completed our discussion of this module, the topics in this module and so we move on to summarize the important points of the module.

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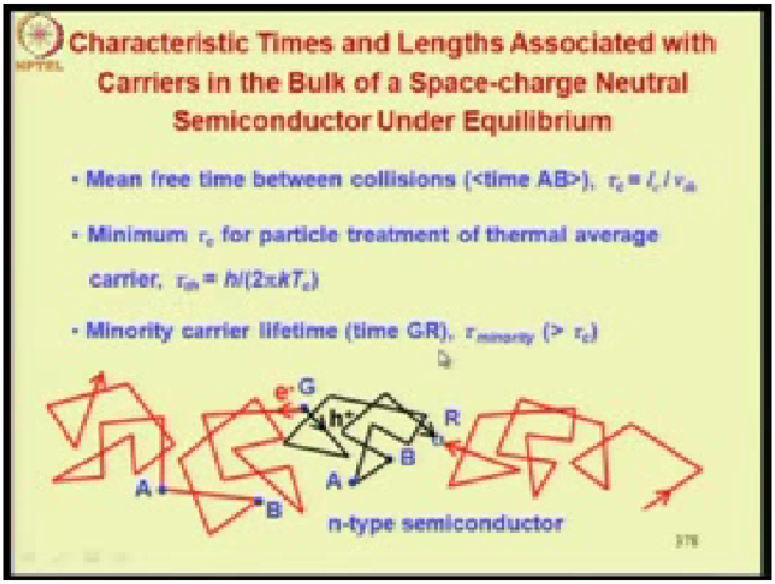

Characteristic Times and Lengths Associated with Carriers in the Bulk of a Space-charge Neutral Semiconductor Under Equilibrium

- Mean free time between collisions (<time AB>), $\tau_c = l_c / v_{th}$
- Minimum τ_c for particle treatment of thermal average carrier, $\tau_{th} = h/(2\pi k T_c)$
- Minority carrier lifetime (time GR), $\tau_{minority}$ (> τ_c)

Let us start with characteristic times and lengths associated with carriers in the bulk of a space charge neutral semiconductor under equilibrium. This is a picture of random thermal motion of electrons and holes in a n type semiconductor. This point represents generation process where electron hole pair is being generated and this point here shows the recombination process where the generated hole is recombining with an electron found nearby, okay.

There are numerous number of electrons. Now for this picture under equilibrium and space charge neutral conditions because we are considering a uniform semiconductor, we have the following important parameters. The RMS velocity or thermal velocity, mean free path between collisions which is the average of length AB, that is this length AB, okay. We are saying an average because you see the length AB is different, this is long, this is short and so on.

And you similarly have an average length AB for holes as well. Then you have the length parameter De Broglie wavelength of thermal average carrier which is given by h by effective mass into thermal velocity. The characteristic times associated with this equilibrium picture, mean free time between collisions that is average of time AB, so the time taken by the carrier to undergo 2 collisions, right?

The time interval between 2 collisions, that is given by lc mean free path divided by thermal velocity. The minimum time between collisions for particle treatment of thermal average carrier is given by this characteristic time, h/2 pi kTc. Then comes minority carrier lifetime. That is the time interval between events G and R, generation and recombination, this is G here and this is R.

So the time interval in which so many collisions are taking place, so many scattering events. So this time interval is called minority carrier lifetime because this is a time for which a minority carrier lives after a generation before it recombines and generally this is much more than the mean free time between collisions because there are several scattering events before a carrier that is generated recombines.

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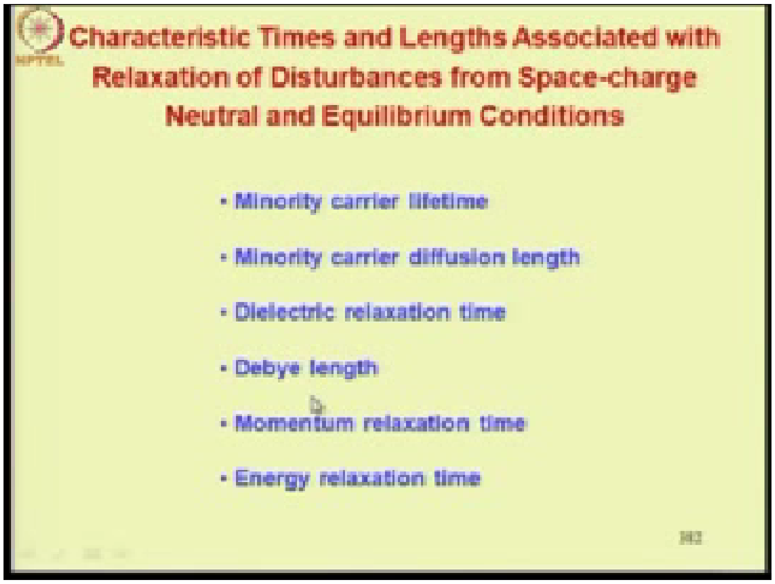

Characteristic Times and Lengths Associated with Relaxation of Disturbances from Space-charge Neutral and Equilibrium Conditions

- Minority carrier lifetime
- Minority carrier diffusion length
- Dielectric relaxation time
- Debye length
- Momentum relaxation time
- Energy relaxation time

Now let us look at characteristic times and lengths associated with relaxation of disturbances from space charge neutral and equilibrium conditions. So, so far we discussed the equilibrium condition and charge neutral conditions. Now suppose you disturb the equilibrium or the spatial neutrality, then the system will try to return to equilibrium or charge neutral condition. Now what are the lengths and times associated with this return, okay, to charge neutrality or equilibrium.

So in this context, we discussed the minority carrier lifetime, the minority carrier diffusion length. Dielectric relaxation time and Debye length. Momentum relaxation time and energy relaxation time.

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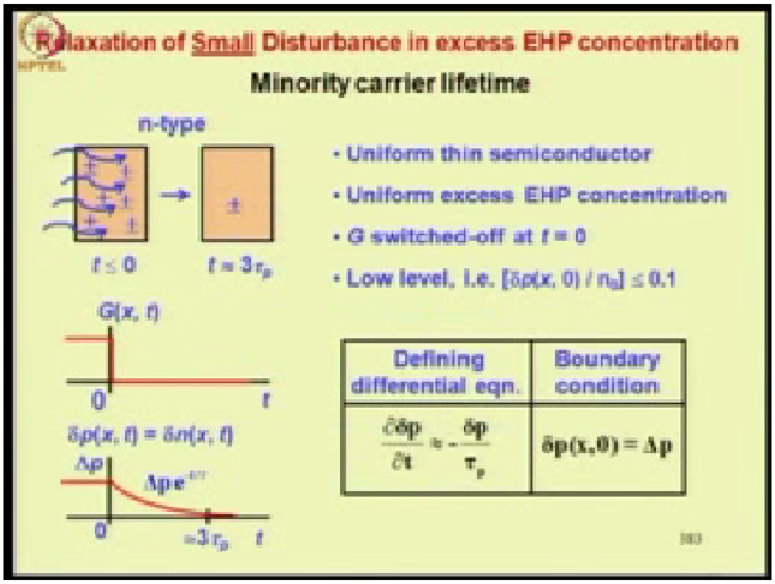

Relaxation of Small Disturbance in excess EHP concentration

Minority carrier lifetime

- Uniform thin semiconductor
- Uniform excess EHP concentration
- G switched-off at $t = 0$
- Low level, i.e. $[\delta p(x, 0) / n_0] \le 0.1$

Defining differential eqn.	Boundary condition
$\frac{\partial \delta p}{\partial t} = -\frac{\delta p}{\tau_p}$	$\delta p(x,0) = \Delta p$

Let us look at relaxation of small disturbance in access Electron Hole Pair concentration, Minority carrier lifetime. Now this time was introduced using this physical situation where for t < 0, you have uniform generation of electron hole pairs, access electron hole pairs inside the volume, for whatever reason. In this case, the reason is illumination and then at t = 0, the source is just cut off and then the system returns to equilibrium, so excess carriers decay.

It takes 3 times the minority carrier lifetime, a duration of this value, 3 times tau p for the system to return to equilibrium. So this time constant tau p is valid for conditions of uniform thin semiconductor. The thin semiconductor here, the thinness is because we are concerned one-dimensional situation for analysis. Uniform access electron hole pair concentration, the source of access electron hole pair is switched off at t = 0 and low-level conditions prevail, that is g sufficiently small.

So that the excess hole concentration is much less than the equilibrium majority carrier concentration. So under these conditions, you have an exponential decay of excess carriers with time once the impulse is switched off and the time constant here is the minority carrier lifetime. This is the Defining differential equation and this is the boundary condition under which it is solved.

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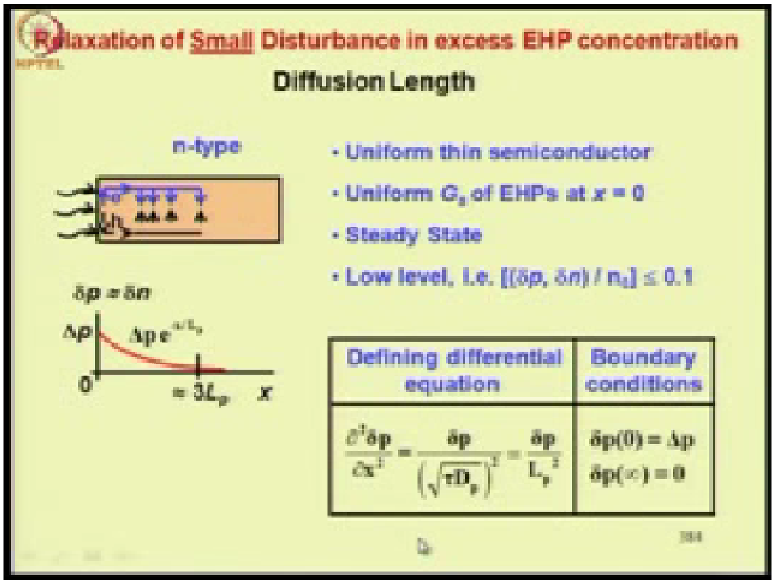

Relaxation of Small Disturbance in excess EHP concentration

Diffusion Length

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_o] \leq 0.1$

Defining differential equation	Boundary conditions
$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^2}$	$\delta p(0) = \Delta p$ $\delta p(\infty) = 0$

Now repeating analogous discussion for diffusion length, this length is introduced in the physical situation shown here, this is the uniform thin semiconductor, there is uniform surface generation of electron hole pairs at x = 0, over here. So which helps you to elevate the excess carrier concentration at this boundary alone and then there is steady state and low-level conditions. So under this conditions, it was shown that from the elevated excess carrier concentration at x = 0, a decay happens which is exponential as a function of distance and the excess carriers decay.

And the system returns to equilibrium over a length of 3 times the diffusion length of minority carriers. The defining equation or differential equation for the diffusion length is shown here, it is a second order differential equation in space and there are 2 boundary conditions because it is a second order differential equation, one boundary condition on excess carrier concentration and x = 0 and another one at x very long or infinite.

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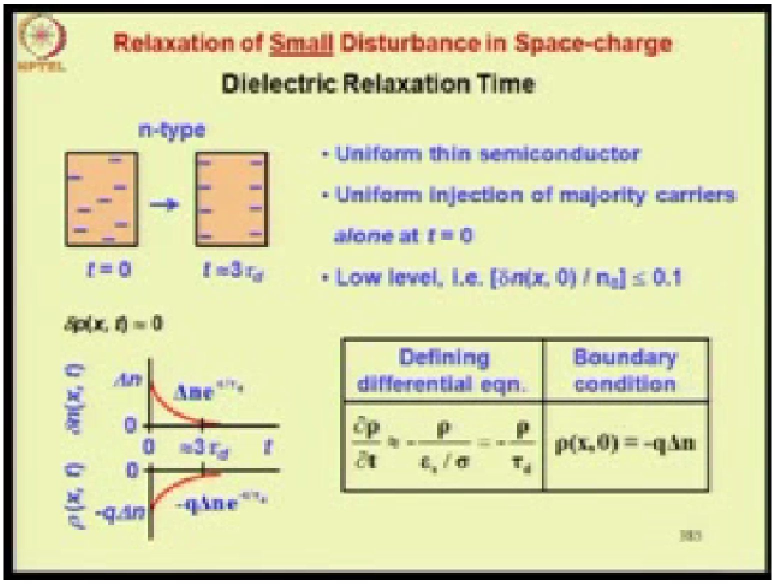

Relaxation of Small Disturbance in Space-charge

Dielectric Relaxation Time

- Uniform thin semiconductor
- Uniform injection of majority carriers alone at $t = 0$
- Low level, i.e. $[\delta n(x, 0) / n_o] \leq 0.1$

Defining differential eqn.	Boundary condition
$\frac{\partial \rho}{\partial t} \approx -\frac{\rho}{\varepsilon_s / \sigma} = -\frac{\rho}{\tau_d}$	$\rho(x, 0) = -q\Delta n$

Relaxation of small disturbance in space charge. Dielectric relaxation time is a time constant related to this relaxation. Now this time constant was introduced under the conditions of a uniform thin semiconductor, then uniform injection of majority carriers alone at t = 0, so we considered an injection of electrons in an n type semiconductor and we assume the electrons are uniformly distributed at t = 0 and then the system relaxes to charge neutral conditions. So we assumed low-level conditions.

Under these conditions, the decay of access electron concentration is exponential and the time constant here is called the dielectric relaxation time. The space charge associated with the electrons also decay exponential. Now the holes are not involved here because we did not consider any injection of holes; otherwise, we would not have had a space charge. So only if you inject only one type of carrier then you can have space charge and in this case we are injecting majority carriers.

So defining differential equation is a first-order equation in time and the dielectric relaxation time is given by epsilon S/sigma and this equation is solved under the boundary condition of space charge at t = 0 at any x, because the conditions are uniform is given by = q times the excess electron concentration that is injected.

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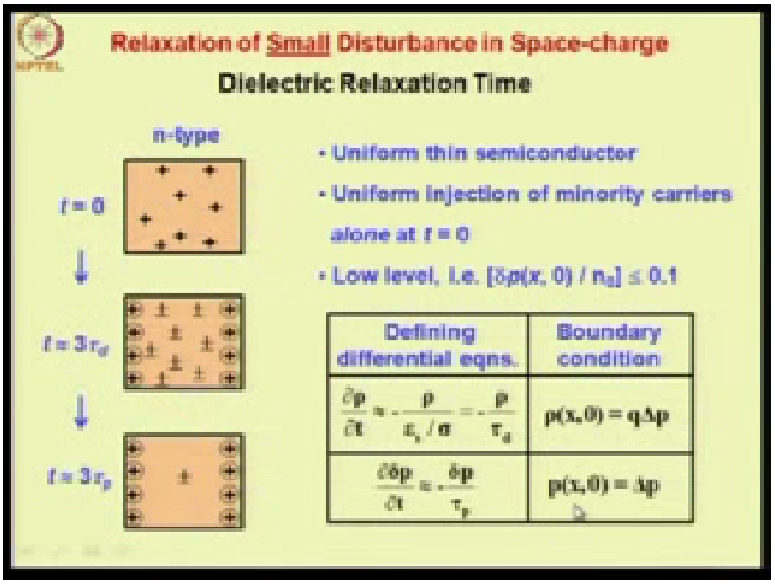

Relaxation of Small Disturbance in Space-charge

Dielectric Relaxation Time

- Uniform thin semiconductor
- Uniform injection of minority carriers alone at $t = 0$
- Low level, i.e. $[\delta p(x, 0) / n_o] \le 0.1$

Defining differential eqns.	Boundary condition
$\frac{\partial \rho}{\partial t} = -\frac{\rho}{\varepsilon_s / \sigma} = -\frac{\rho}{\tau_d}$	$\rho(x, 0) = q\Delta p$
$\frac{\partial \delta p}{\partial t} = -\frac{\delta p}{\tau_p}$	$p(x, 0) = \Delta p$

The dialect relaxation time associated with injection of minority carriers. So here again you have uniform thin semiconductor and instead of majority carriers, you are injecting minority carriers at t = 0, low-level conditions. Then you have 2 parallel processes operating here. One is neutralization of the space charge which happens in 3 times the dielectric relaxation time and second is the recombination of minority carriers which happens over a time scale of 3 times the lifetime of holes.

So both these parallel processes are operating here in contrast to the situation for majority carrier injection where you did not have any recombination over there because majority carriers are large in number, they find difficult to locate holes for recombining whereas minority carriers are very small in number and are surrounded by large number of majority carriers and therefore they can be lost by recombination also apart from movement because of the field created, right, which neutralizes the space charge.

So the defining differential equations here are once associated with the space charge and the one associated with recombination, both are first-order equations and the time constants are here and they are solved subjected to a single boundary condition in each case.

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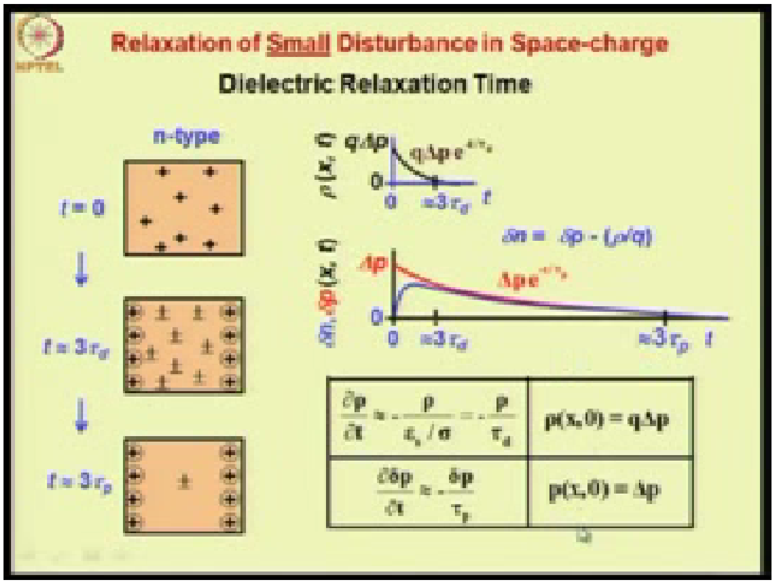

Relaxation of Small Disturbance in Space-charge

Dielectric Relaxation Time

$\frac{\partial \rho}{\partial t} = -\frac{\rho}{\varepsilon_s / \sigma} = -\frac{\rho}{\tau_d}$	$\rho(x, 0) = q\Delta p$
$\frac{\partial \delta p}{\partial t} = -\frac{\delta p}{\tau_p}$	$p(x, 0) = \Delta p$

The excess hole concentration decays exponentially but over a time scale of 3 times tau p whereas the space charge decays over a much, much shorter period over a time scale of 3 times tau d and this space charge decay happens because of drawing in of electrons from the surface into the volume, okay. So the electron concentration rises rapidly and then thereafter once the space charge is neutralized, it falls slowly over a period of 3 times the minority carrier lifetime. So both these decays are exponential.

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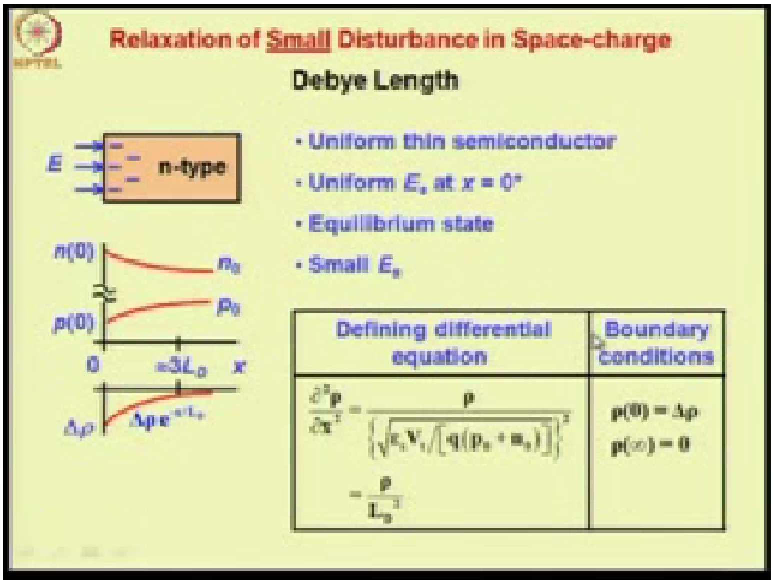

Relaxation of Small Disturbance in Space-charge

Debye Length

- Uniform thin semiconductor
- Uniform E_s at $x = 0^+$
- Equilibrium state
- Small E_s

Defining differential equation	Boundary conditions
$\frac{\partial^2\rho}{\partial x^2} = \frac{\rho}{\left\{\sqrt{\varepsilon_s V_t/\left[q(p_0+n_0)\right]}\right\}^2} = \frac{\rho}{L_D^2}$	$\rho(0) = \Delta\rho$ $\rho(\infty) = 0$

Debye Length - Relaxation of small disturbance in space charge over distance. This length was introduced under the conditions of uniform thin semiconductor. A uniform surface electric field at x = 0+. So we are talking about 0+ here because the electric field inside the semiconductor is different from the electric field outside because the ambient dielectric constant is different from the dielectric constant in the semiconductor.

We are assuming an equilibrium state and we assume that the electric field is small. So in all this characteristic times and length definitions, the disturbance is always assumed to be small because only under that condition, we get simple differential equations. So in this case, the electron concentration which is elevated at the surface because of attraction of electrons by the electric field decays exponentially to equilibrium electron concentration and not in the bulk, okay.

Under charge neutral conditions and because it is equilibrium, pn product is NI square and the hole concentration rises to the concentration under space charge neutral conditions, p knot. This also happens exponentially. The space charge which arises because of elevation of electrons and depletion of holes, the space charge varies exponentially. Now the holes really do not contribute much to the space charge because their number is really small.

So this exponential behavior of the space charge mostly follows the exponential behavior decay of the electron concentration. The defining differential equation for this condition is a second-order differential equation for space charge over distance and this is where you get the Debye length which is given by epsilon s Vt/q times the sum of hole and electron concentration. So you see this formula is valid for p as well as n type semiconductors is and there is a square root sign over this.

So this second-order differential equation is solved using 2 boundary condition, one for space charge at 0, x = 0 and other one at space charge at infinity which is 0 because the conditions are relaxing to charge neutral conditions. You can alternatively express the conditions also in terms of potential. So for potential also you have a second-order differential equation here under these conditions and you can introduce Debye length even using this second-order differential equation for the potential.

Here you have 2 boundary conditions and the potential varies exponentially, so it decays from an elevated value of delta psi to 0, it is assumed to be 0 in psi, okay. So this decay happens over 3 times the Debye length as in the case of the space charge which decays over 3 times the Debye length.

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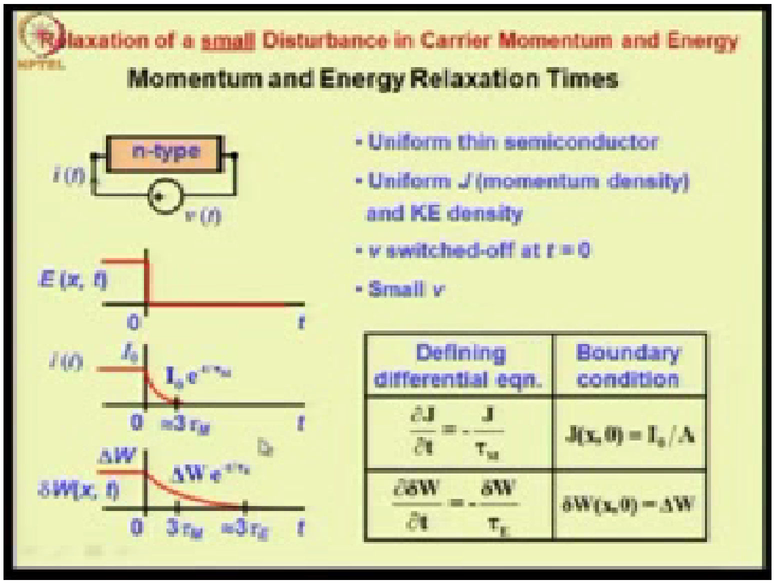

Relaxation of a small Disturbance in Carrier Momentum and Energy

Momentum and Energy Relaxation Times

- Uniform thin semiconductor
- Uniform J (momentum density) and KE density
- v switched-off at $t = 0$
- Small v

Defining differential eqn.	Boundary condition
$\frac{\partial J}{\partial t} = -\frac{J}{\tau_M}$	$J(x, 0) = I_0 / A$
$\frac{\partial \delta W}{\partial t} = -\frac{\delta W}{\tau_E}$	$\delta W(x, 0) = \Delta W$

Then the considered relaxation of a small disturbance in carrier momentum and energy. 2 time constants related to this disturbance where or relaxation of the disturbance where momentum and energy relaxation times. Now these 2 time constants were discussed together considering single situation because whenever you disturb momentum, you disturb energy as well. So relaxation of momentum and energy, right, happen in the same situation.

So whenever you disturb momentum, you disturb energy also. So the conditions with which we introduced this relaxation time was a uniform thin semiconductor and a uniform current density which is equivalent to imposing a uniform momentum density in the semiconductor for carriers and uniform kinetic energy density. So you apply a signal, a voltage to a uniform semiconductor and this voltage is suddenly stepped from a non 0 value to 0 value at t = 0. So v is switched off at t = 0.

So it is this v that creates momentum and energy for carriers, okay, dielectric momentum and energy that is more than the equilibrium value. Now when you switch off the impulse, the system tries to return to equilibrium. We assume small value of voltage so that we get simple equation to characterize this situation. So the electric field is uniform over x, so at instant t = 0, it goes to 0 because the v is switched off.

Now what we find in the situation is that the current I, here, will decay exponentially as a function of time. So you might imagine if you did not know the concept of momentum and energy relaxation time that moment you switch off the voltage, the current will abruptly fall to 0, no that is not what is going to happen. So the current is going to decay to 0 over a time period which is 3 times the momentum relaxation time.

Now what about the energy. You see the energy of the carriers consists of a dielectric component and around random component. Now while the dielectric component is decaying because the current is decaying, the random component takes longer time to decay and therefore the energy of the carriers which includes dielectric and random components. So that energy decays exponentially over a much longer time scale than the current decay and this is 3 times the energy relaxation time.

The defining differential equations for momentum and energy relaxations are all first-order differential equations in time and therefore you have only one boundary condition, the so-called initial condition on current and kinetic energy.

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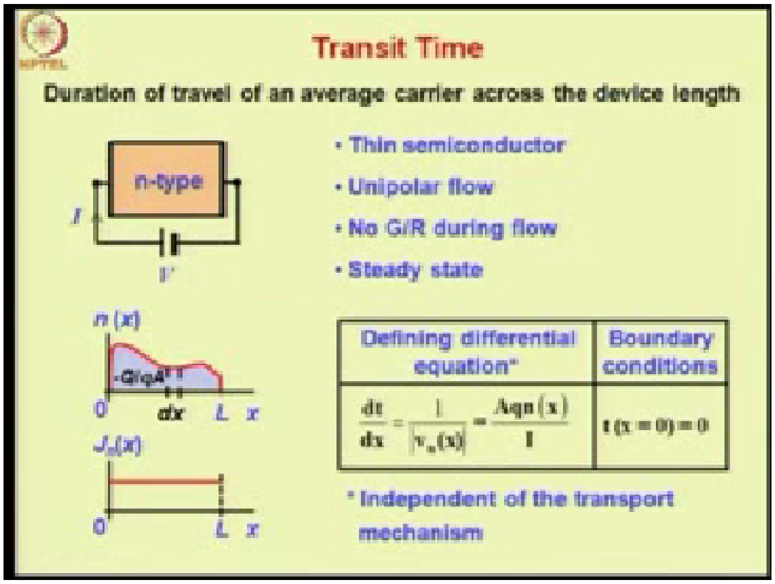

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Defining differential equation*	Boundary conditions
$\frac{dt}{dx} = \frac{1}{\lvert v_n(x) \rvert} = \frac{Aqn(x)}{I}$	$t(x=0)=0$

* Independent of the transport mechanism

We talked about the Transit time which is duration of travel of an average carrier across the device length. Now the transit time was not associated with return of some disturbed situation to equilibrium or charge neutrality. So therefore the transit time discussion, there was a little different than the discussion and the equations associated with momentum and the relaxation times, dielectric relaxation time, minority carrier lifetime, diffusion length time, Debye length.

Now this situation, this time constant was introduced using a thin semiconductor, a unipolar flow, no generation or recombination during flow and steady state conditions. Note carefully that the semiconductor is thin but not necessarily uniform. It could be non-uniform. So this is in fact shown here. So you have some arbitrary variation of the carrier concentration and it is unipolar conditions.

Therefore, we bother only about electrons or about holes. So in this case, we talked about electrons. Now because there is steady state and there is no generation recombination, the current density of carriers would remain constant over x and under this conditions, no matter what is the cause of the current, it could be diffusion, it could be drift, it could be combination of the 2, whatever, you can define a time associated with the travel of the electron from one point to another in the device.

So the defining differential equation here is a time taken dt to travel a distance dx in the device and dt/dx is nothing but 1/reciprocal of the electron velocity and we consider only the modulus because we are going to take the time to be positive and this is equal A times the charge of the electron into the electron concentration at x divided by the current which is constant throughout the device. The boundary condition is t at x = 0, is 0. So we assume that the time instant when the electron leaves the left terminal is 0.

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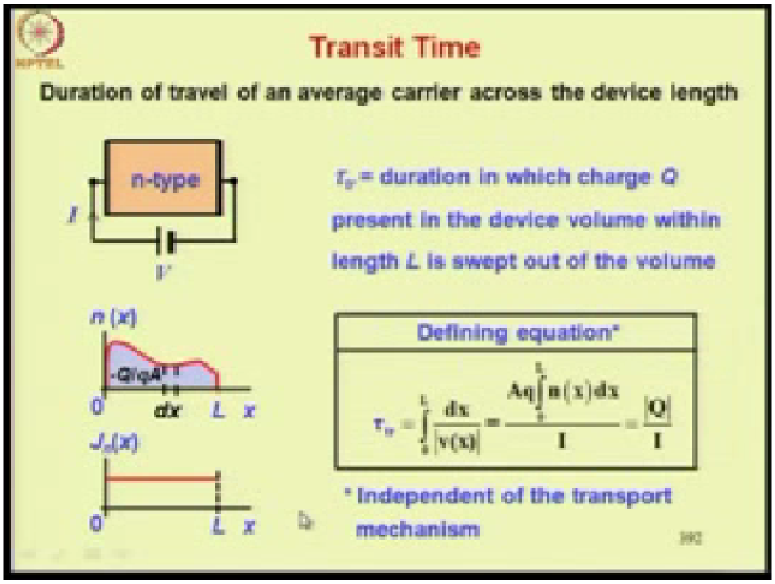

Transit Time

Duration of travel of an average carrier across the device length

τ_{tr} = duration in which charge Q present in the device volume within length L is swept out of the volume

Defining equation*

$$\tau_{tr} = \int_0^L \frac{dx}{|v(x)|} = \frac{Aq\int_0^L n(x)\,dx}{I} = \frac{|Q|}{I}$$

* Independent of the transport mechanism

So when you saw the differential equation, you end up with a result of transit time equal to the charge within the device divided by the current independent of the transport mechanism and this allows you an alternate interpretation of transit time, namely, that it is a duration in which charge q present in the device volume within length l, so this is the length l, is swept out of the volume.

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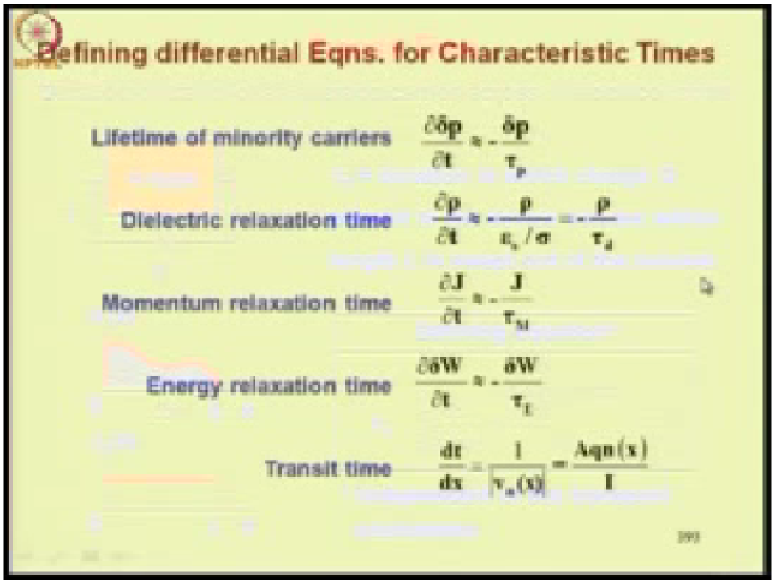

Defining differential Eqns. for Characteristic Times

Lifetime of minority carriers $\frac{\partial \delta p}{\partial t} \approx -\frac{\delta p}{\tau_p}$

Dielectric relaxation time $\frac{\partial \rho}{\partial t} \approx -\frac{\rho}{\varepsilon_s / \sigma} = -\frac{\rho}{\tau_d}$

Momentum relaxation time $\frac{\partial J}{\partial t} \approx -\frac{J}{\tau_M}$

Energy relaxation time $\frac{\partial \delta W}{\partial t} \approx -\frac{\delta W}{\tau_E}$

Transit time $\frac{dt}{dx} = \frac{1}{v_n(x)} = \frac{Aqn(x)}{I}$

Now if you collect whether the defining differential equations for characteristic times except the transit time equation, all other equations you can see here are of first-order differential equations which introduced the time constant, okay, on the right-hand side of this equation. The transit time equation, however, is, this is also a first-order differential equation, however, the difference between the transit time differential equation and other equations is that all other equations are of the form which lead to an exponential variation of the quantity that is being relaxed.

Whereas the transit time is not associated with any relaxation process and therefore, it does not have any such form of the differential equation, though it is first-order.

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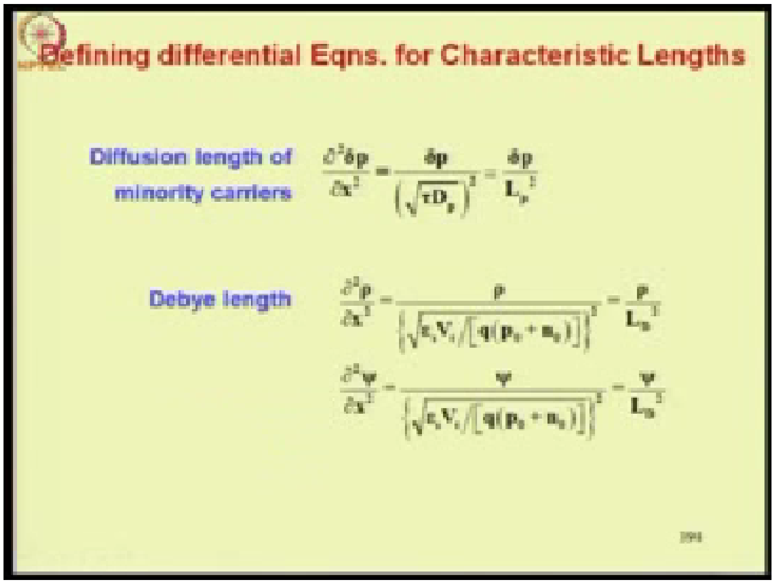

Defining differential Eqns. for Characteristic Lengths

Diffusion length of minority carriers $\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^2}$

Debye length $\frac{\partial^2 \rho}{\partial x^2} = \frac{\rho}{\left\{\sqrt{\varepsilon_s V_t / [q(p_0 + n_0)]}\right\}^2} = \frac{\rho}{L_D^2}$

$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi}{\left\{\sqrt{\varepsilon_s V_t / [q(p_0 + n_0)]}\right\}^2} = \frac{\psi}{L_D^2}$

In contrast, the defining differential equations for characteristic lengths are second-order as can be seen here.

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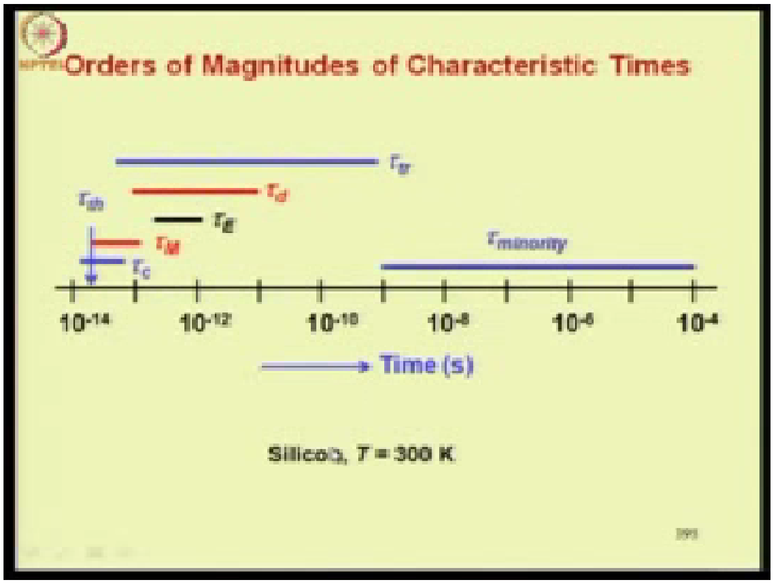

Orders of Magnitudes of Characteristic Times

Silicon, $T = 300$ K

The orders of magnitude of various characteristic times and lengths, we calculate it the maximum and minimum values of the characteristic times and we plotted on a logarithmic scale because we found that they span a very wide range, several orders of magnitude and we showed that the minority carrier lifetime is to be the highest among all the time constants and the time calculated from quantum mechanics for the minimum time between collisions for which particle (()) (44:07) can hold, that time constant happens to be the minimum on the scale.

Other things lie in between generally for a large number of devices, the increasing order of time is tau thermal, tau C, tau M, tau E, tau d, tau transit, that is transit time and finally minority carrier lifetime.

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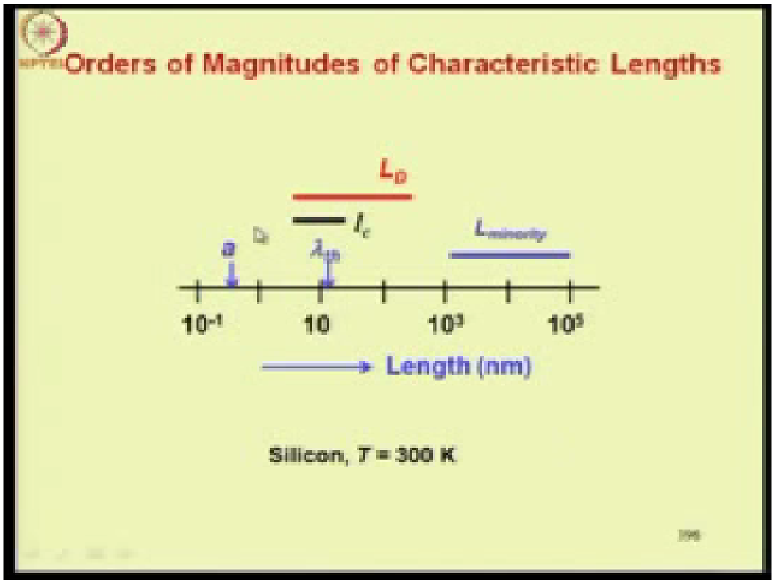

Orders of Magnitudes of Characteristic Lengths

Silicon, $T = 300$ K

This is orders of magnitudes of characteristic lengths. Here, we found that the minority carrier diffusion length is towards the upper limit of the length scale and the distance between the atoms is at the lower end. So generally the minority carrier diffusion length is more than the Debye length is more than the mean free path and which is more than the thermal average wavelength and which in turn is more than the distance between the atoms. So in many semiconductor, this order would hold.

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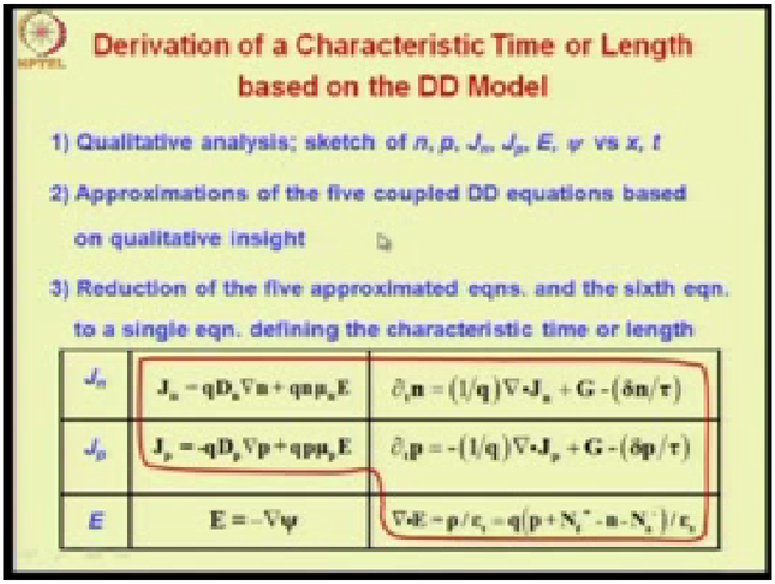

Derivation of a Characteristic Time or Length based on the DD Model

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

2) Approximations of the five coupled DD equations based on qualitative insight

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic time or length

J_n	$\mathbf{J}_n = qD_n\nabla n + qn\mu_n\mathbf{E}$	$\partial_t n = (1/q)\nabla\cdot\mathbf{J}_n + G - (\delta n/\tau)$
J_p	$\mathbf{J}_p = -qD_p\nabla p + qp\mu_p\mathbf{E}$	$\partial_t p = -(1/q)\nabla\cdot\mathbf{J}_p + G - (\delta p/\tau)$
E	$\mathbf{E} = -\nabla\psi$	$\nabla\cdot\mathbf{E} = \rho/\varepsilon_s = q(p + N_d^+ - n - N_a^-)/\varepsilon_s$

Now derivation of a characteristic time or length based on the drift diffusion model, we set out certain steps in which the characteristic times and lengths are derived. So these are derived based on the 6 equations and therefore there are 6 quantities here. So the first that is a qualitative analysis and sketch of the 6 quantities involved in these equations, namely electron concentration, hole concentration, electron current density, hole current density, electric field and potential as a function of x and t.

If it is a steady state situation, there is no variation with time. There is variation with distance only. In a transient situation, however, there can be variation with distance and time; however, in all our analysis, we assume uniform conditions whenever transient situations were involved. So uniform as a function of x. The next step is approximations of the 5 coupled drift diffusion equations based on qualitative insight. So out of this 6 equations, these 5 equations are coupled.

So we try to approximate these based on qualitative insight gained in the first step. In the third step, we reduce the 5 approximated equations and the 6th equation, that is this equation to a single equation defining the characteristic time or length.

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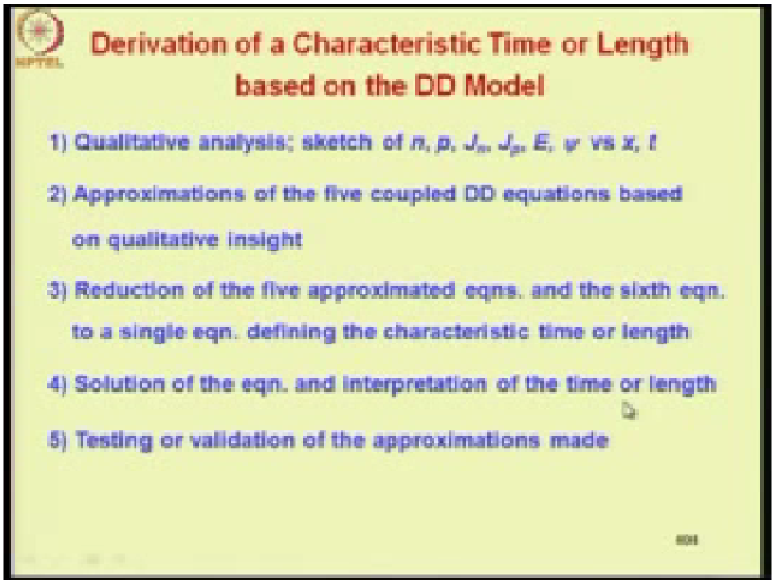

Derivation of a Characteristic Time or Length based on the DD Model

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t
2) Approximations of the five coupled DD equations based on qualitative insight
3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic time or length
4) Solution of the eqn. and interpretation of the time or length
5) Testing or validation of the approximations made

Step 4 involves solution of the defining differential equation and interpretation of the time or length. And finally, it is very important to test or validate the approximations made. So there are the 5 steps in which we derive the characteristic time or length and incidentally these are the 5 steps in which any device analysis is carried out.

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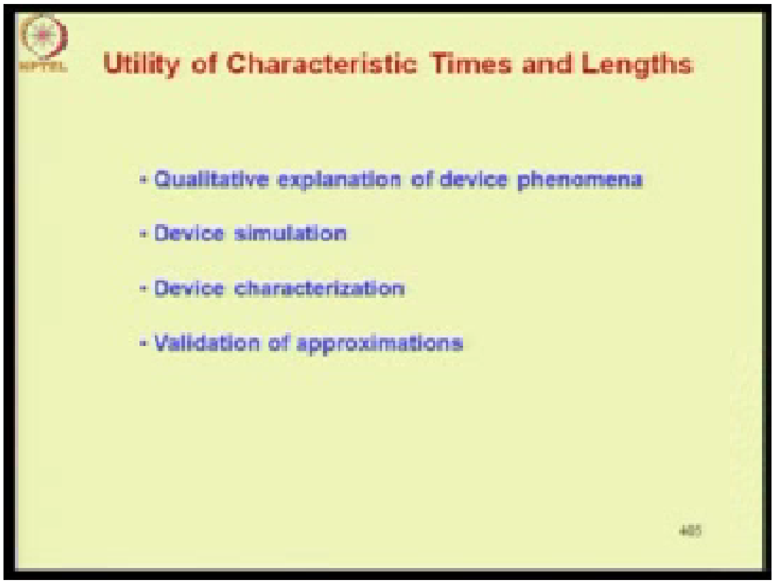

Utility of Characteristic Times and Lengths

- Qualitative explanation of device phenomena
- Device simulation
- Device characterization
- Validation of approximations

Then we discuss at length, Utility of Characteristic Times and Lengths. Specifically, we considered 4 aspects, qualitative explanation of device phenomena where the length and timescales are useful, device simulation, device characterization and validation of approximations. So as far as qualitative explanation of device phenomena is concerned, we gave examples of p-n junction operation, MOS junction operation and MOSFET operation and showed how characteristic time and lengths are involved in the explanation.

For device simulation, we considered simulation of surface recombination by volume recombination and for device characterization we considered measurement of the doping profile and showed how Debye length is coming in in the type of doping profiles that can be characterized. Validation of approximation was done.

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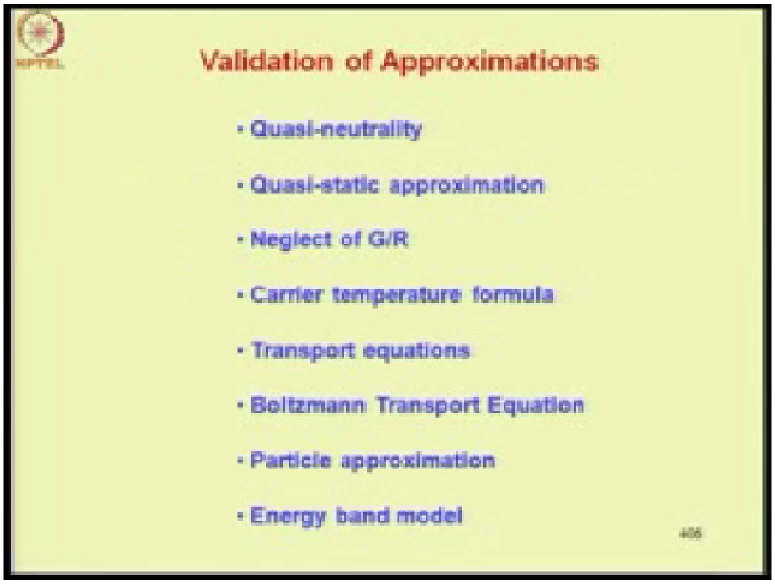

Validation of Approximations

- Quasi-neutrality
- Quasi-static approximation
- Neglect of G/R
- Carrier temperature formula
- Transport equations
- Boltzmann Transport Equation
- Particle approximation
- Energy band model

This topic was discussed at great length and we discussed all these approximations, their validations, quasi-neutrality, quasi-static approximation, neglect of generation or recombination, carrier temperature formula, Transport equations, Boltzmann Transport equation, particle approximation and energy band model. Now specifically, we showed that in many situations, quasi-neutrality holds in regions which are high mobile carrier concentration and for many time varying situations, quasi-static approximation can be made and in many VLSI devices,

This device size is so small compared to the diffusion length that we can neglect generation and recombination. Similarly, we showed that carrier temperature formula derived under steady state and uniform conditions in the device is also valid for many practically non-uniform and high-frequency conditions.

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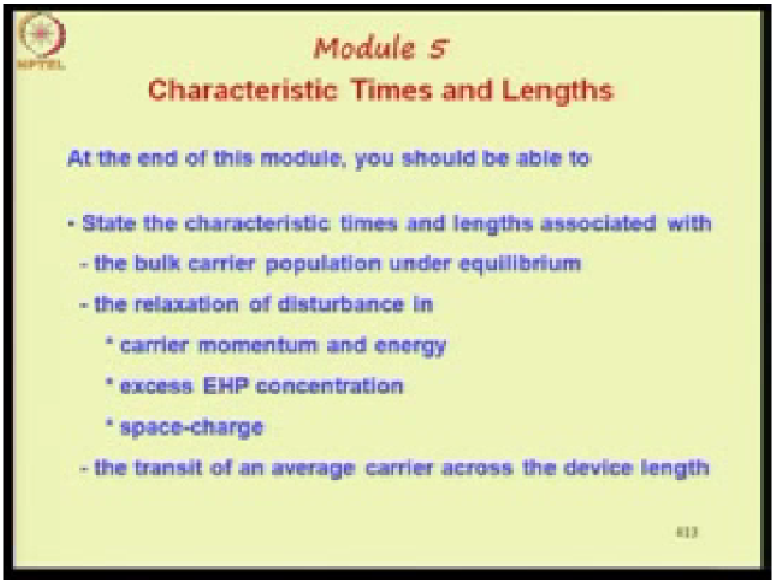

Module 5

Characteristic Times and Lengths

At the end of this module, you should be able to

- State the characteristic times and lengths associated with
  - the bulk carrier population under equilibrium
  - the relaxation of disturbance in
    - carrier momentum and energy
    - excess EHP concentration
    - space-charge
  - the transit of an average carrier across the device length

Towards the end, let us recapitulate what we have achieved in terms of the learning outcomes. At the End of this module, you should be able to state the characteristic times and lengths associated with the bulk carrier population under equilibrium. The relaxation of disturbance in carrier momentum and energy. Relaxation of disturbance in access electron hole pair concentration. Relaxation of disturbance in space charge.

Finally, you should be able to state time constant associated with transit of an average carrier across the device length.

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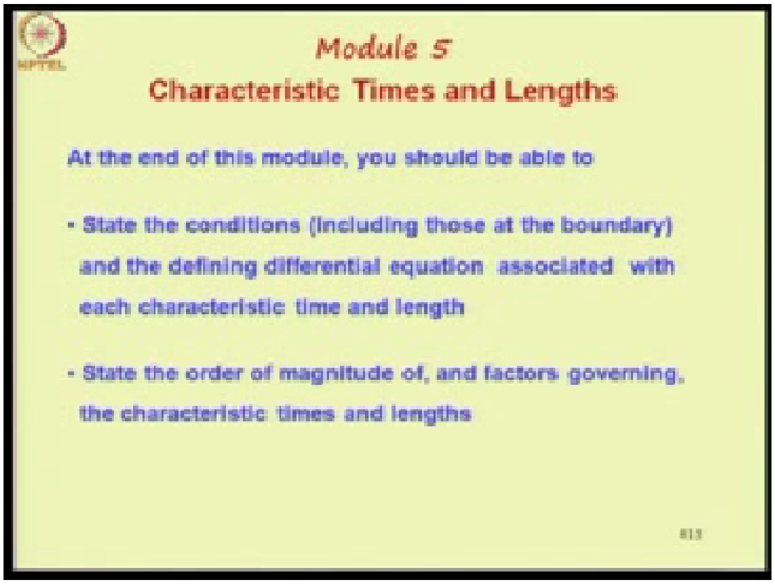

Module 5

Characteristic Times and Lengths

At the end of this module, you should be able to

- State the conditions (including those at the boundary) and the defining differential equation associated with each characteristic time and length
- State the order of magnitude of, and factors governing, the characteristic times and lengths

You should be able to state the conditions, including those at the boundary and the defining differential equation associated with each characteristic time and length. You should be able to state the order of magnitude of, and factors governing the characteristic times and lengths.

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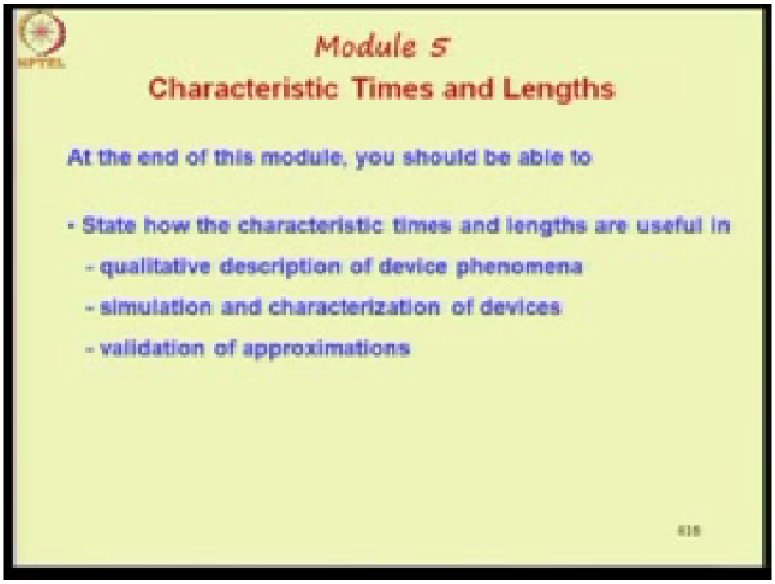

Module 5
Characteristic Times and Lengths

At the end of this module, you should be able to

- State how the characteristic times and lengths are useful in
  - qualitative description of device phenomena
  - simulation and characterization of devices
  - validation of approximations

You should be able to state how the characteristic times and lengths are useful in qualitative description of device phenomena, simulation and characterization of devices and validation of approximations.

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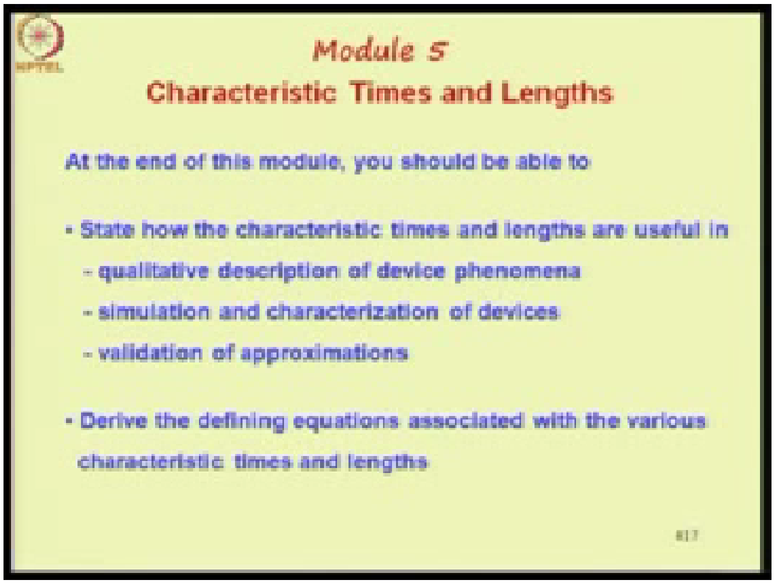

Module 5
Characteristic Times and Lengths

At the end of this module, you should be able to

- State how the characteristic times and lengths are useful in
  - qualitative description of device phenomena
  - simulation and characterization of devices
  - validation of approximations
- Derive the defining equations associated with the various characteristic times and lengths

You must be able to derive the defining equations associated with the various characteristic times and lengths.

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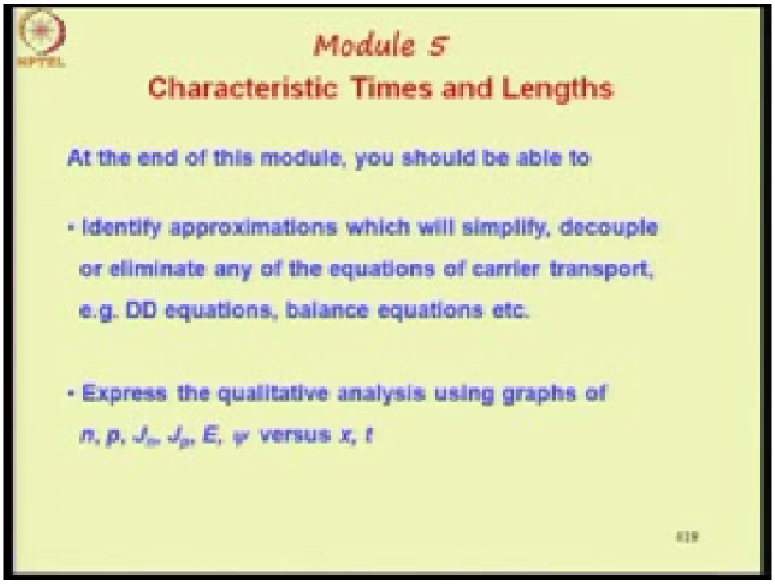

Module 5

Characteristic Times and Lengths

At the end of this module, you should be able to

- Identify approximations which will simplify, decouple or eliminate any of the equations of carrier transport, e.g. DD equations, balance equations etc.
- Express the qualitative analysis using graphs of n, p, J_n, J_p, E, ψ versus x, t

You should be able to identify approximations which will simplify, decouple, or eliminate any of the equations of carrier transport such as the Drift Diffusion equations, Balance equations, et cetera. This approximation method is very important for device analysis. Finally, you must be able to express the qualitative analysis using graphs of electron concentration, hole Concentration, electron current density, hole current density, electric field and potential as a function of distance and time.

So with that, we have come to the end of this module. We shall begin a fresh topic in the next module.