Semiconductor Device Modeling
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Lecture - 24
Characteristic Times and Lengths

In the previous lecture, we discussed the transit time and diffusion length. The transit time is the time duration taken by an average carrier to traverse the device length. This transit time was given by modulus of Q/I where I is the current flowing in to the device and the Q is the charge within the device volume. This formula is valid independent of the mechanism of carrier transport. It could be drift, it could be diffusion, it could be combination of different diffusion and so on.

As far as diffusion length is concerned, the diffusion length is the average distance a carrier moves by diffusion before it recombines in a situation where the carriers are diffusing and recombining. This diffusion length is associated with minority carriers because it is only a minority carriers which travel purely by diffusion or mostly by diffusion, not by drift.

While deriving the diffusion length, we make 3 crucial approximations, namely the quasi-neutrality approximation, the diffusion approximation for minority carriers and the low-level injection approximation. The low-level approximation, we have in making in the context of all other characteristic times as well where there was a disturbance of the carrier concentration. The formula for diffusion length is square root of diffusion coefficient into the lifetime of the minority carriers.

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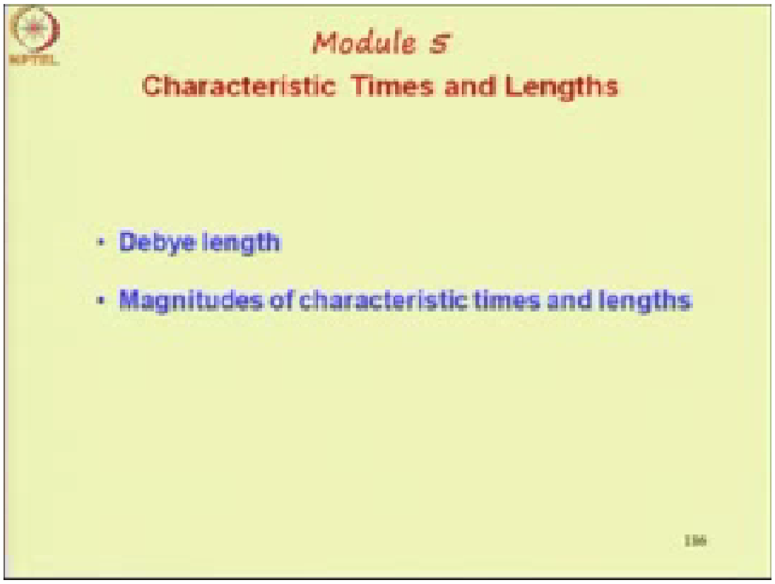

Module 5

Characteristic Times and Lengths

- Debye length
- Magnitudes of characteristic times and lengths

In this lecture, we will consider the Debye length and the magnitudes of the characteristic times and lengths. So the Debye length is associated with relaxation of space charge regions and since it is very important for us to know what are the magnitudes of the various characteristic times and lengths, we will do a calculation for each of the times and lengths and see what is the range of values that these times and lengths take.

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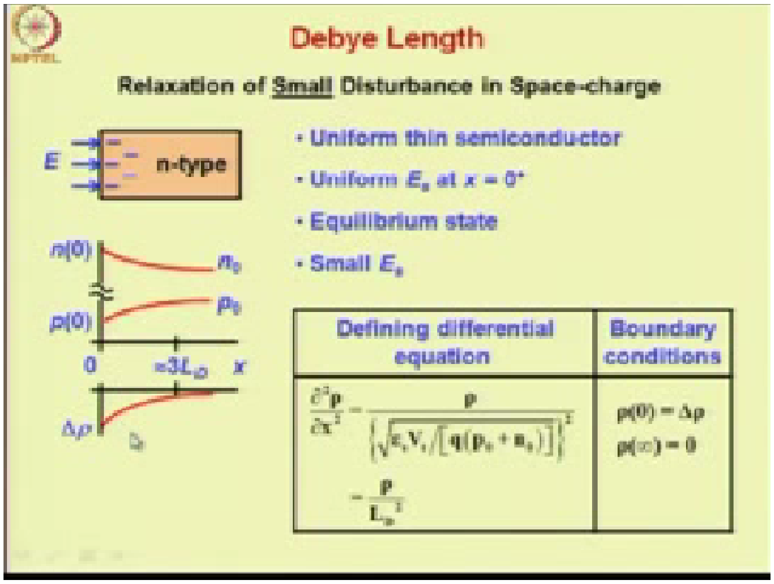

Debye Length

Relaxation of Small Disturbance in Space-charge

- Uniform thin semiconductor
- Uniform E_s at $x = 0^+$
- Equilibrium state
- Small E_s

Defining differential equation	Boundary conditions
$\frac{\partial^2 p}{\partial x^2} = \frac{p}{\left\{\sqrt{\varepsilon_s V_t / \left[q(p_0 + n_0)\right]}\right\}^2} = \frac{p}{L_D^2}$	$p(0) = \Delta p$ $p(\infty) = 0$

The physical situation which introduces the Debye length is shown here. You have an electric field impinging on the surface of a semiconductor which was assumed to be n-type. Since the field is inward, it draws the mobile electrons towards the surface and we are talking about the width over which the space charge created by mobile electrons and as we shall see, there will be some very small contribution of holes also, so the space charge region decays to charge neutral region.

So the conditions are summarized here, a uniform thin semiconductor, thin because we want to make one-dimensional assumption. Uniform Es at x=0+, so this is the x axis and 0+ is the right-hand side of this vertical line, so Es represents the field in the semiconductor. You know that the field in the semiconductor at the surface will be different from the electric field E that is in the ambient because of the difference in dielectric constants of the ambient and the semiconductor.

Third you will notice here that we have the equilibrium state because there is no excess generation or recombination. Please do not confuse these electrons which are drawn into the sample, drawn towards the surface as excess carriers because you know that when we talk about excess carriers, you always have electrons and holes paired up. So you have access electrons as well as holes and the generation rate is disturbed, okay. So here you are not disturbingly the generation rate.

So where are these electrons coming from. Well they are coming from the terminal here, okay, because after all you are applying an electric field in a practical situation, how you do it. Ss it would be something like this.

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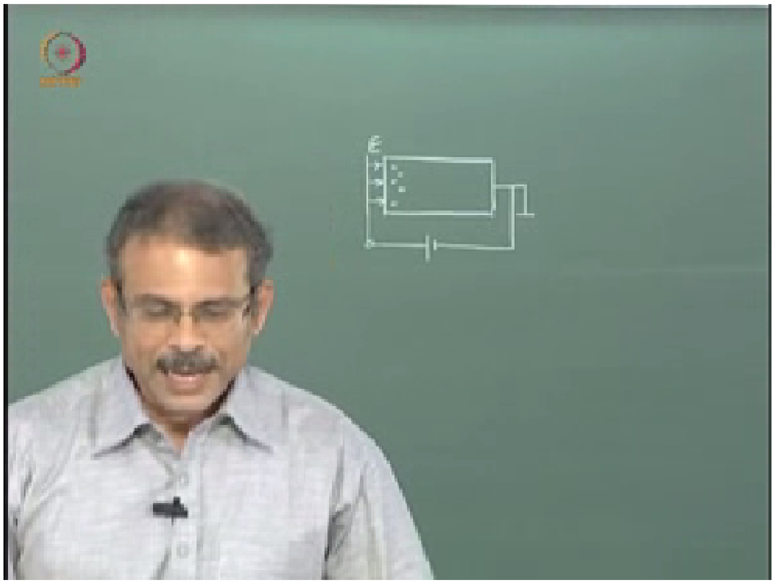

So you have a terminal here, let us say. This is a reference terminal and here, you have an electrode which is creating the field, so this electrode is connected to a cell with reference to this point. So the electrons which are coming here, will come from this terminal, okay. That is what is happening and the positive charges pile up creating this electric field E.

So you will find in fact that the whole concentration reduces here, okay. So this is one more indication that these are not excess carriers, okay. Why do the whole concentration reduce, because it is equilibrium state N into P is NI square. So if electron concentration increases, hole concentration falls. We will discuss some details of this situation shortly. Now 3 times Ld, where Ld is the Debye length is approximately the region over which the space charge decays to 0.

The space charge due to disturbance in electron concentration and hole concentration is what is shown here. The initial value of the space charge, that is the value at x=0 is delta ro. The defining differential equation, we will establish is the second order differential equation in x, very much like the second order differential equation in x for the diffusion length, okay.

So it is interesting to see that while defining differential equations for characteristic times are first-order differential equations in time except for the case of transit time for which also we can write a first-order equation but we did not discuss the situation or transit time because we said that is a little bit more complex. So far characteristic lengths, diffusion length and Debye length, the differential equation is second order in X. So this d square rho/dx square=rho divided by this quantity.

Here also in analogy to the diffusion length you have a square root sign in which you have the dielectric constant of silicon, the thermal voltage coming in here and here in the denominator, you have the equilibrium concentration of holes and electrons summed up. So you see that this formula is valid both for p type as well as the n type semiconductor. So this square root quantity is represented as a characteristic length called the Debye length.

The boundary conditions would be at x=0, rho is delta rho which is the value shown here and at x=infinity, the rho goes to 0 because the space charge has decayed. There is another way we can write the same differential equation that is in terms of the potential okay. So in fact as we will see, it is easier to write equation in terms of the potential which can ultimately be converted into an equation in terms of the space charge.

So the equation in terms of the potential is again a second order differential equation because we will show that for small disturbances, for small electric fields, the potential and space charge are linearly related. So the boundary conditions here are Delta psi is a potential at psi equal to 0 here and finally for x very large, the potential goes to 0. So we are a assuming a 0 reference for extend into infinity for the potential. So that is what is shown here.

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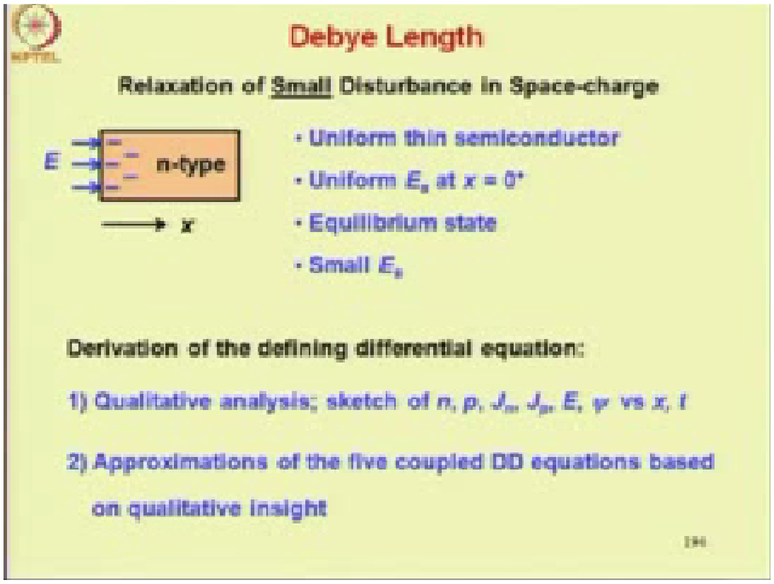

Debye Length

Relaxation of Small Disturbance in Space-charge

- Uniform thin semiconductor
- Uniform E_s at $x = 0^+$
- Equilibrium state
- Small E_s

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

2) Approximations of the five coupled DD equations based on qualitative insight

Let us begin with a qualitative analysis in terms of sketch of N P Jn Jp E psi versus x. There is no variation with time because this is an equilibrium state which also implies a steady state.

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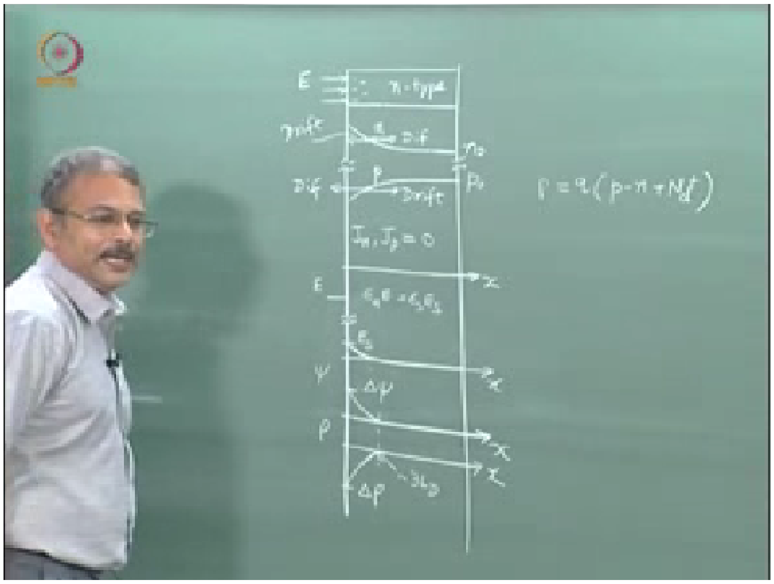

So these are the sketches of these various quantities. Not let us look at this sketch and understand this shape. So because there is an electric field, the mobile electrons are drawn towards the surface. They are piling up at the surface as compared to the bulk. We have said that these electrons have come from the terminal here, right.

Now, how do you account for the shape. At each point, there is a tendency for electrons to diffuse to the right. However, because this electric field which gets into the semiconductor also, this electric field pulls the electrons towards the surface. So, there is a drift. This drift and diffusion are in balance.

So they ensure that there is no net current flow for electrons because this is equilibrium state. Now that is how you get the shape. You can explain similarly for holes. For holes, the tendency for diffusion is in this direction. So this is diffusion.

On the other hand, electric field is rightward and this will tend to push the holes away in this direction, okay. So that is how you get the shape, okay and now you can easily establish. In fact, if Jn and Jp, each of them is individually 0, then the Pn product is NI square. I will leave it to you as an assignment. Coming to Jn and Jp, each of them is 0 because for electrons and for holes separately, the drift and diffusion are exactly balance, okay and they are opposite to each other.

So no current flow. This is again a sign of equilibrium state. Electric field - so you have an electric field outside that is this electric field E. Now as you get in, the electric field will fall, okay to value Es inside the semiconductor and thereafter it will decay just as the electron concentration is falling to equilibrium and the whole concentration is rising to equilibrium. So these variations are there.

Similarly there is a variation of the electric field over the same length and it goes to 0 because faraway there are no charges here. Now what is a ratio of E to Es. Well, the formula is Epsilon ambient into E=Epsilon silicon into Es and Epsilon Ambient, if it is vacuum here, is 1 and there is vacuum here and Epsilon S, if it is silicon, it is 12. So that is how you know that this quantity will be 1/12th of this quantity. Because we are not drawing it to scale, I have shown a cut here.

Similarly, here also you notice that I have shown cuts because electron and hole concentrations are both shown on a linear scale. I really cannot use a single scale to show both electron and hole concentrations. Electron is majority carrier which will be of the order of 10 power 16, 17, 18, 19 and so on and hole concentration is minority carrier, it would be something like 10 power 4, 10 power 3, 10 power 2, and so on. So I cannot show both of them on the same linear scale. That is why there is a cut.

Now why not I show it on a log scale. You see what will happen is, we have said that the disturbance is small, the electric field we are applying is small; therefore, the variation in the electron concentration or the amount of electrons which pile up here would be small in number. That is why on a log scale, I will really not be able to show any variation for majority carriers. For minority carriers, I may be able to show the variation. However, we would like to see the variations for both because it is a variation in electron concentration that is really contributing to the space charge as we shall see.

Minority carrier concentration is so small that any variations there do not contribute to the space charge. Coming to the potential psi, so if I integrate the electric field, I will get the potential because electric field is directed from left to right, the potential is more positive on the left and it is falling. We have chosen the reference as 0 for the right end, okay. The space charge is derived from the electric field by Gauss' law taking the derivative of the field picture. So you can see that the slope here is negative that is why the space charge is negative, okay.

Alternately we can also get the space charge from n and P because you know that the space charge is given by q times P-n+Nd+. These are n type semiconductors. So I can take the variation in p and n and from here also, I can get the space charge. I will get the same shape. Now based on this understanding of what is happening inside the semiconductor for the situation, we can write equations and derive the Debye length.

Now the Debye length is something that is characterizing this width, right. The width over the space charge region is changing, potentially is changing, or electric field is changing or these concentrations are changing. So basically this is the length that we are talking about. Now we will show that this length is what is approximately 3 times that particular characteristic length called Ld.

So let us move on in that is no one to approximate the 5 coupled drift-diffusion equations based on qualitative insight gained in the first step.

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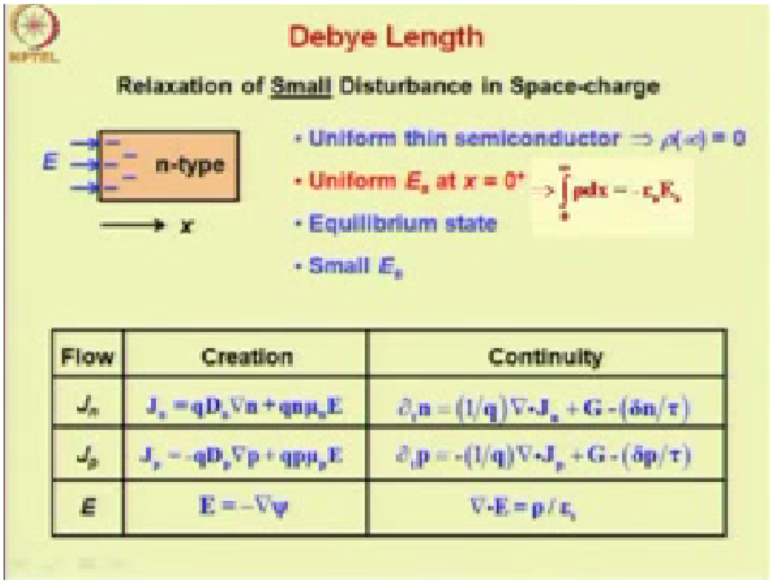

Debye Length

Relaxation of <u>Small</u> Disturbance in Space-charge

- Uniform thin semiconductor $\Rightarrow \rho(x) = 0$
- Uniform E_s at $x = 0^+$ $\Rightarrow \int_0^{\infty} \rho dx = -\varepsilon_s E_s$
- Equilibrium state
- Small E_s

Flow	Creation	Continuity
J_n	$J_n = qD_n \nabla n + qn\mu_n E$	$\partial_t n = (1/q) \nabla \cdot J_n + G - (\delta n/\tau)$
J_p	$J_p = -qD_p \nabla p + qp\mu_p E$	$\partial_t p = -(1/q) \nabla \cdot J_p + G - (\delta p/\tau)$
E	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \varepsilon_s$

These are our 5 equations. Let us begin by considering the conditions of uniform semiconductor. So because the semiconductor is uniform, for extending to infinity, your space charge is going to

0. So this particular condition is imposing the boundary condition. Rho at infinity=0. Let us look at the next condition. Uniform Es means the Es is uniform over this surface. Note that E is the field in ambient, Es is a field in the semiconductor at the surface. So at x=0+.

Now this condition puts this particular condition on the space charge that is the integral of the space charge over the length from 0 to infinity is Epsilon S, Es. This is by Gauss' law. So what we are saying here is, if the field is Es, this is the field in side here is Es, so x=0 + his right-hand side of this vertical line. Then this Es is related to the space charge picture as follows. So this area is Epsilon S, Es, okay. That is how the field applied controls the space charge.

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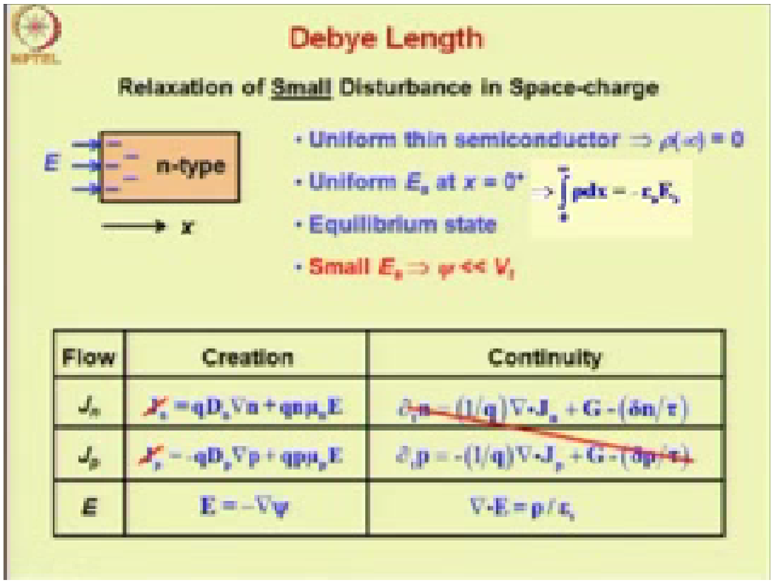

Debye Length

Relaxation of Small Disturbance in Space-charge

- Uniform thin semiconductor $\Rightarrow \rho(\infty) = 0$
- Uniform E_s at $x = 0^+$ $\Rightarrow \int_0^\infty \rho dx = -\varepsilon_s E_s$
- Equilibrium state
- Small $E_s \Rightarrow \psi << V_t$

Flow	Creation	Continuity
J_n	$\cancel{J_n} = qD_n \nabla n + qn\mu_n E$	$\cancel{\partial_t n = (1/q)\nabla \cdot J_n + G - (\delta n/\tau)}$
J_p	$\cancel{J_p} = -qD_p \nabla p + qp\mu_p E$	$\cancel{\partial_t p = -(1/q)\nabla \cdot J_p + G - (\delta p/\tau)}$
E	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \varepsilon_s$

The equilibrium state means very importantly Jn is 0 and Jp is 0. So that is why we are striking off these terms. This means that here we will substitute 0. Another very important consequence of the equilibrium state is we do not have to solve the continuity equations because you can see at each of the terms of this equation is 0, okay. Dou n/dot is 0 because equilibrium means steady state carrier concentrations do not change with time, the current density is 0, there is no excess generation, there is no excess recombination.

Small Es, small E, so what does it imply. This allows us to make the approximation that psi, the potential disturbance inside the semiconductor is much < thermal voltage.

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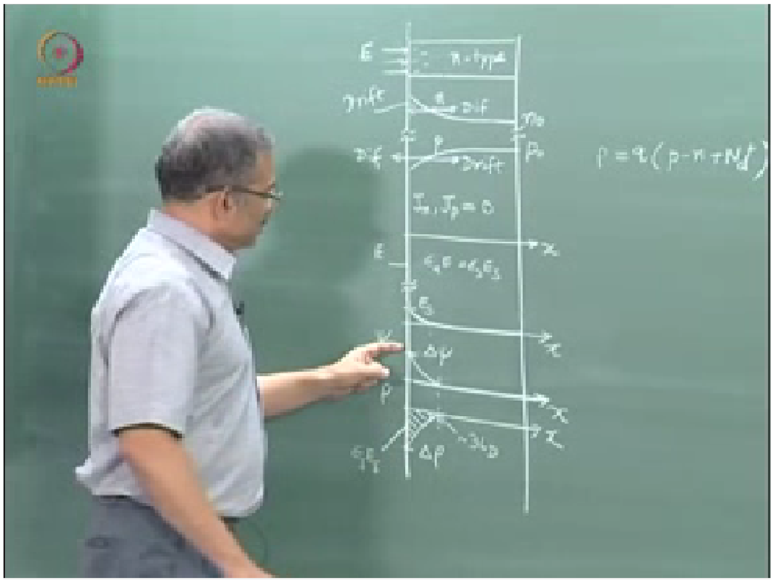

That is this is the potential variation. Under equilibrium, everywhere potential is 0, when you do not apply an electric field. When you apply small electric field, we are saying that the electric field is so small that the maximum value of the potential here is much less than thermal voltage. Now we will understand why we are using thermal voltage as a metric over there to decide the smallness of psi. This will become clear shortly.

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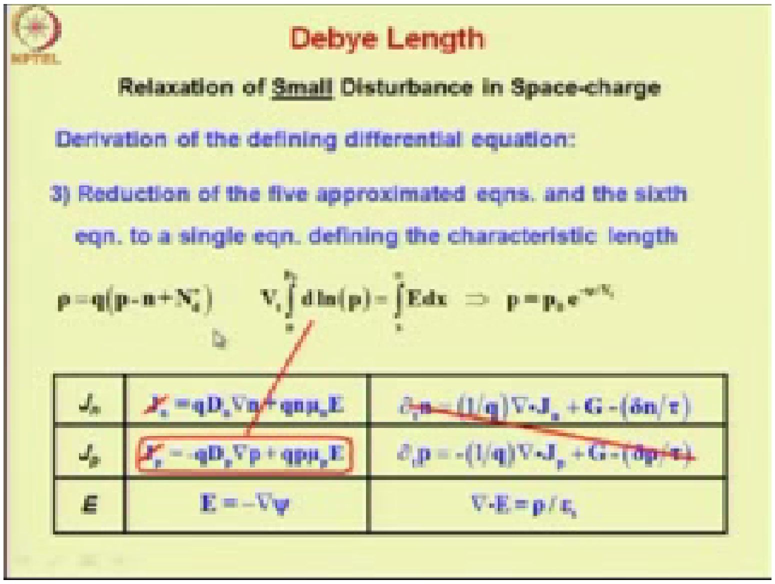

Debye Length

Relaxation of Small Disturbance in Space-charge

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$\rho = q(p - n + N_d^+)$ $\quad V_t \int_{p_0}^{p} d\ln(p) = -\int_0^x E dx \Rightarrow p = p_0 e^{-\psi/V_t}$

J_n	$\cancel{J_n} = qD_n\nabla n + qn\mu_n E$	$\cancel{\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)}$
J_p	$\boxed{\cancel{J_p} = -qD_p\nabla p + qp\mu_p E}$	$\partial_t p = -(1/q)\nabla\cdot J_p + G - \cancel{(\delta p/\tau)}$
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

The next step is reduction of the 5 approximated equations and the 6th equation to a single equation defining the characteristic length. Now it is a space charge that we are concentrating on; therefore, let us begin with the equation for the space charge and look at what are the expressions for p and n. How do we get the expressions for p and n.

Well clearly from here we can see that the expression for p would be obtained from this equation by setting this Jp=0 and similarly the equation for n will be obtained from this equation. Now let us take the equation for p. It can be easily shown that this equation reduces to Boltzmann relation that is shown here. That is, it leads to the exponential relation between the carrier concentration at any x and the potential at x.

Now how do you get this equation. Well very simple, let us put that down on the board.

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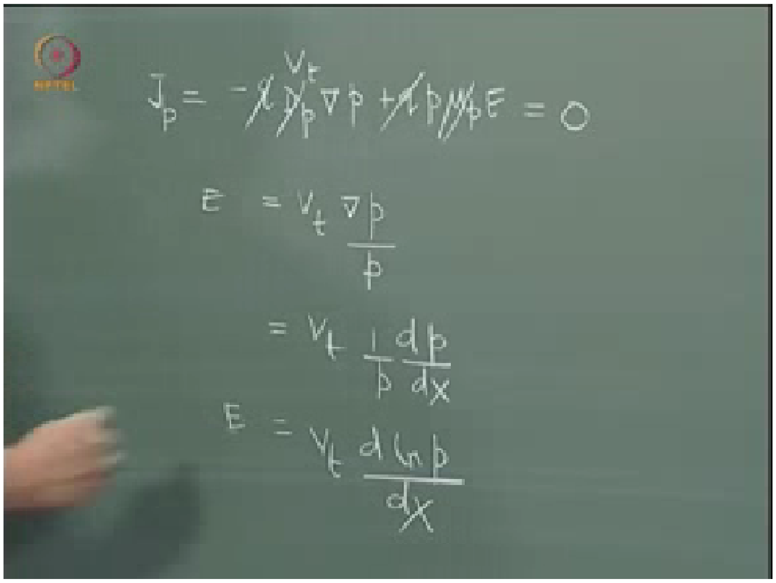

$$J_p = -\cancel{q}\overset{V_t}{\cancel{D_p}}\nabla p + \cancel{q}p\cancel{\mu_p}E = 0$$

$$E = V_t \frac{\nabla p}{p}$$

$$= V_t \frac{1}{p}\frac{dp}{dx}$$

$$E = V_t \frac{d\ln p}{dx}$$

So -qDp grad p+qp mu p into E. This is your Jp but this is 0 because it is equilibrium. So you strike of q/mu P, this will become VT here, so you are left with VT gradient of p is a - sign. So I can shift this on right-hand side and p into E. Now I can divide by p here, so that I will get this. Now gradient of p/p in one-dimension what is this, this is nothing but VT into dp/dx into 1/p. Now dp/p, you are writing it as d of [0:20:38] p, right. So this is how you get the relation electric field is equal to this and this is what you are integrating.

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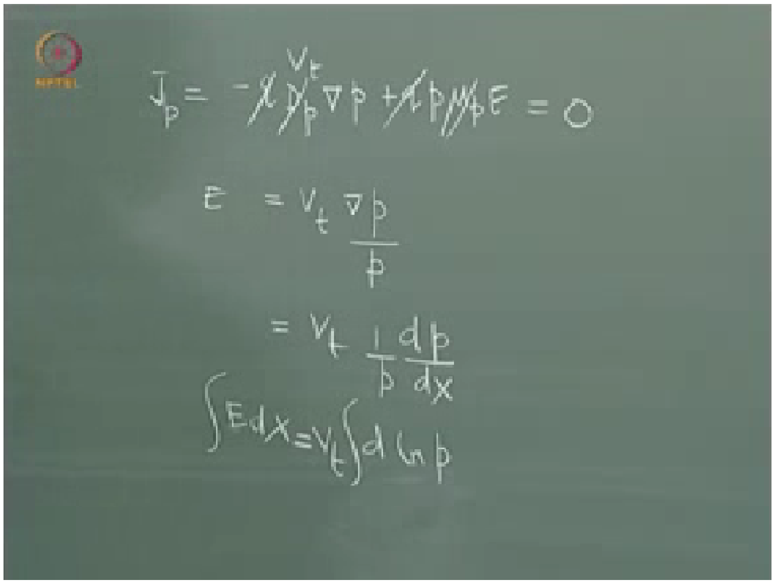

$$\bar{J}_p = -\cancel{q}\overset{V_t}{\cancel{D_p}}\nabla p + \cancel{q}\, p\, \cancel{\mu_p} E = 0$$

$$E = V_t \frac{\nabla p}{p}$$

$$= V_t \frac{1}{p}\frac{dp}{dx}$$

$$\int E\, dx = V_t \int d \ln p$$

Now you can put the dx on the left hand side, remove it from there and you can now integrate okay. So this is what is being done. So at any x, the hole concentration is p and reduction into infinity, it is p0. Those are the limits. So p here at any x and p0 for extending to infinity.

So when you integrate, left hand side becomes Vt into [0:21:35] of p0/p and right hand side becomes psi.

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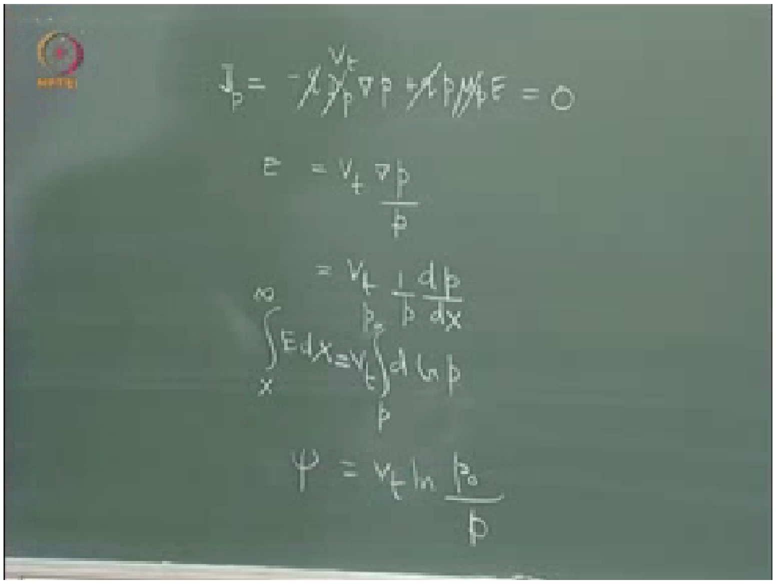

$$\bar{J}_p = -\cancel{q}\overset{V_t}{\cancel{D_p}}\nabla p + \cancel{q}\, p\, \cancel{\mu_p} E = 0$$

$$E = V_t \frac{\nabla p}{p}$$

$$= V_t \frac{1}{p}\frac{dp}{dx}$$

$$\int_x^{\infty} E\, dx = V_t \int_p^{p_0} d \ln p$$

$$\psi = V_t \ln \frac{p_0}{p}$$

So you can see that we are integrating from x to infinity, this is p to p0. So this is Vt ln p0/p and left side is psi at infinity is 0. Psi at any x is psi. So that is why psi is - of integral Edx. So this quantity on the left hand side will be - of psi infinity -psi x and that is - of - psi that is + psi, okay, so this is the formal. So this is alright because p < p0, okay, that is why you get psi here. So this is the result that is shown here. So p=p0 and exponential - psi/Vt, okay.

So you can see that as the potential is rising here, the p is falling. That is why the negative sign in the exponential.

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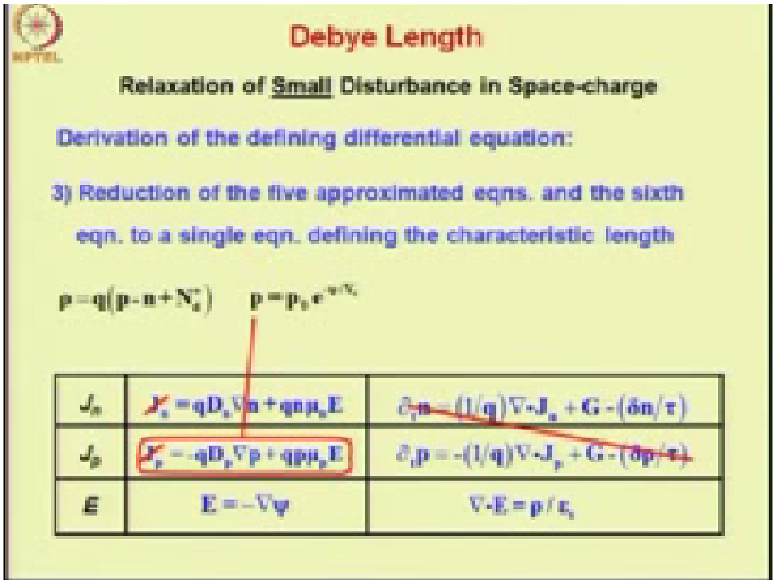

Debye Length

Relaxation of Small Disturbance in Space-charge

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$\rho = q(p - n + N_d^+)$ $p = p_0 e^{-\psi/V_t}$

J_n	$\cancel{J_n} = qD_n\nabla n + qn\mu_n E$	$\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)$
J_p	$\cancel{J_p} = -qD_p\nabla p + qp\mu_p E$	$\partial_t p = -(1/q)\nabla\cdot J_p + G - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

So p is p0 into exponential - psi/Vt.

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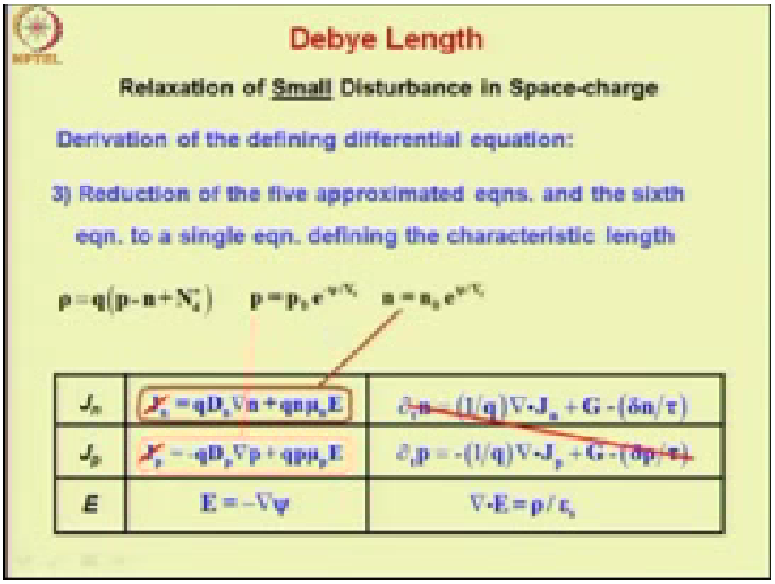

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$\rho = q(p - n + N_d^+)$ $p = p_0 e^{-\psi/V_t}$ $n = n_0 e^{\psi/V_t}$

J_n	$\cancel{J_n} = qD_n\nabla n + qn\mu_n E$	$\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)$
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E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

Now let us do a similar thing for electrons and you will find, you can do the manipulation yourself, n is n0 exponential psi/Vt. So here you have a positive sign because n increases as psi increase. Look at this here. Your n is increasing when the psi is increasing. Now substituting those exponentials, your expression for space charge is as shown here, q into p0 exponential -psi/Vt-n0 into exponential psi/Vt+Nd+.

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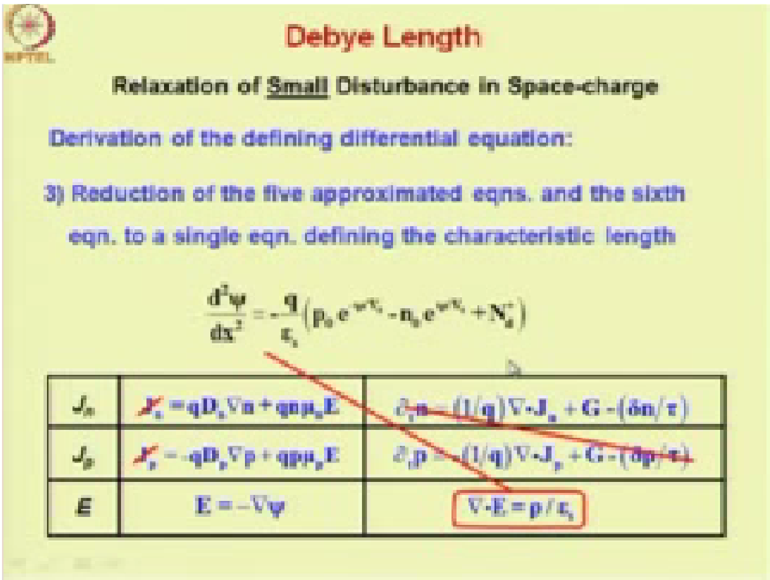

Debye Length

Relaxation of Small Disturbance in Space-charge

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_s}\left(p_0 e^{-\psi/V_t} - n_0 e^{\psi/V_t} + N_d^+\right)$$

J_n	$\cancel{J_n} = qD_n\nabla n + qn\mu_n E$	$\cancel{\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)}$
J_p	$\cancel{J_p} = -qD_p\nabla p + qp\mu_p E$	$\partial_t p = -(1/q)\nabla\cdot J_p + G - \cancel{(\delta p/\tau)}$
E	$E = -\nabla\psi$	$\boxed{\nabla\cdot E = \rho/\varepsilon_s}$

Now rho is nothing but diversions of E into Epsilon S or rho/Epsilon S is diversions of E. So you know that Poisson's equation if you convert diversions of E to psi, it becomes d square psi/dx square and there is a negative sign because e is -d psi/dx. So that is why you are getting a negative sign. So we have converted the Gauss law into Poisson's equation here.

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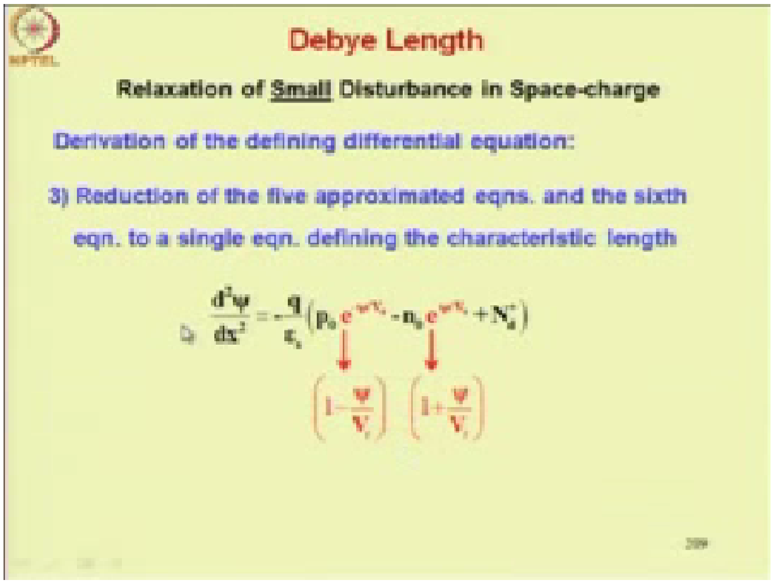

Debye Length

Relaxation of Small Disturbance in Space-charge

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_s}\left(p_0 \underset{\left(1-\frac{\psi}{V_t}\right)}{\underset{\downarrow}{e^{-\psi/V_t}}} - n_0 \underset{\left(1+\frac{\psi}{V_t}\right)}{\underset{\downarrow}{e^{\psi/V_t}}} + N_d^+\right)$$

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Now, this is where you find we are using the smallness approximation. Because the applied electric field is small, your potential which is the consequence of electric field, is small and it is so small that psi/Vt here is much < 1 and therefore we can approximate E power- psi/Vt as 1-psi/Vt and we can approximate E over psi/Vt as 1+psi/Vt.

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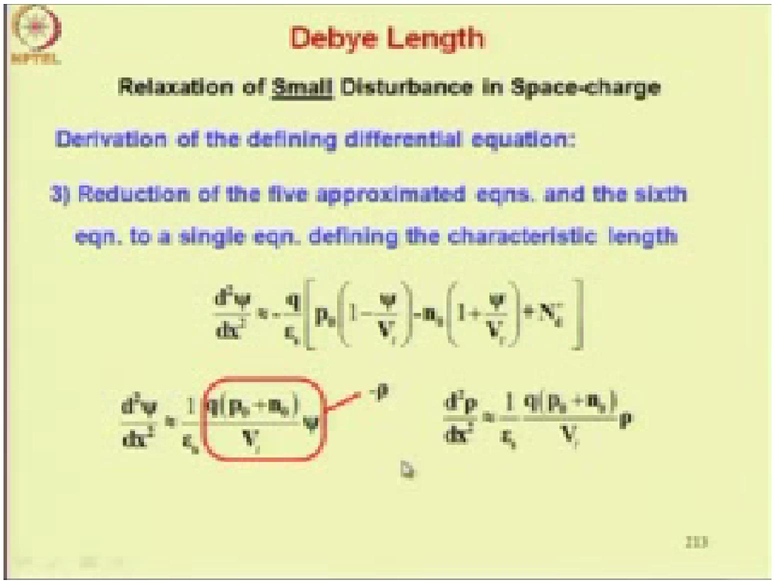

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Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{d^2\psi}{dx^2} \approx -\frac{q}{\varepsilon_s}\left[p_o\left(1-\frac{\psi}{V_t}\right)-n_o\left(1+\frac{\psi}{V_t}\right)+N_d^+\right]$$

$$\frac{d^2\psi}{dx^2} \approx \frac{1}{\varepsilon_s}\boxed{\frac{q(p_o+n_o)}{V_t}}\psi \quad -\rho \qquad \frac{d^2\rho}{dx^2} \approx \frac{1}{\varepsilon_s}\frac{q(p_o+n_o)}{V_t}\rho$$

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Substituting those approximations here, this is your equation. Now, one can easily show that this equation reduces or simplifies to the equation shown here. Now what has happened is the term p0 into 1-n0 into this 1+Nd+ has been set = 0 because you know that the semiconductor sample far away from the surface, here, is neutral.

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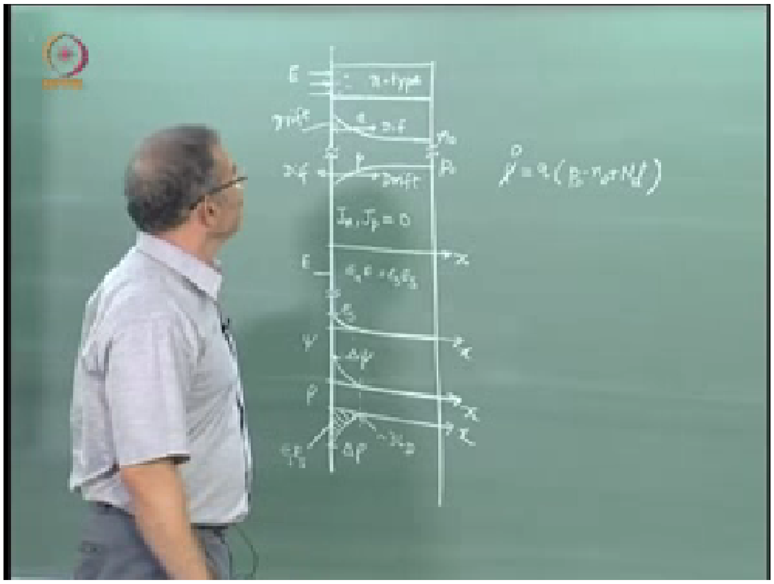

So therefore, rho when you put p0 n0 is 0, okay. Submitted sample is neutral. At x = infinity. Now comparing this equation with the Poisson's equation you can easily establish that this term in the red box represents - of rho because Poisson's equation I dou square psi/dou x square or d square psi/dx square here because there is no time variation, = - of rho/Epsilon S. Therefore, this term can be identified as - of rho. Now since according to this rho & psi are linearly related, I could easily replace the psi on the left-hand side also by rho and the result will be.

This equation in row, okay because psi is some constant into rho because of this particular relation. If I put that on both sides, the constant gets cancelled on both sides and you get this equation. So you see that both psi as well as rho are characterized by the same spatial, very similar spatial variation.

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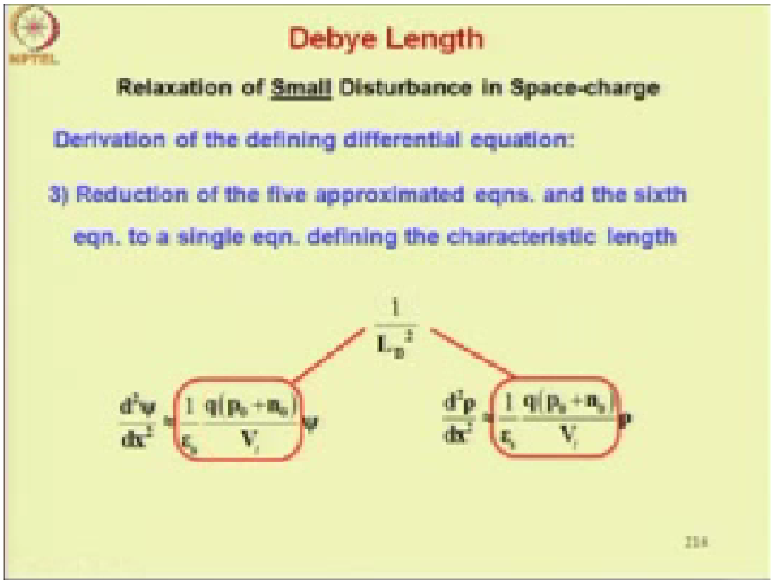

Debye Length

Relaxation of Small Disturbance in Space-charge

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{1}{L_D^2}$$

$$\frac{d^2\psi}{dx^2} = \boxed{\frac{1}{\varepsilon_s}\frac{q(p_0+n_0)}{V_t}}\psi \qquad \frac{d^2\rho}{dx^2} = \boxed{\frac{1}{\varepsilon_s}\frac{q(p_0+n_0)}{V_t}}\rho$$

Now using the fact that dimensionally this quantity can identified as 1/Ld square because left hand side, you have d square psi/dx square, 1/length square and that length we are calling as Debye length in memory of the scientist who proposed this length, Debye. So now we need to solve the equation and interpret this length.

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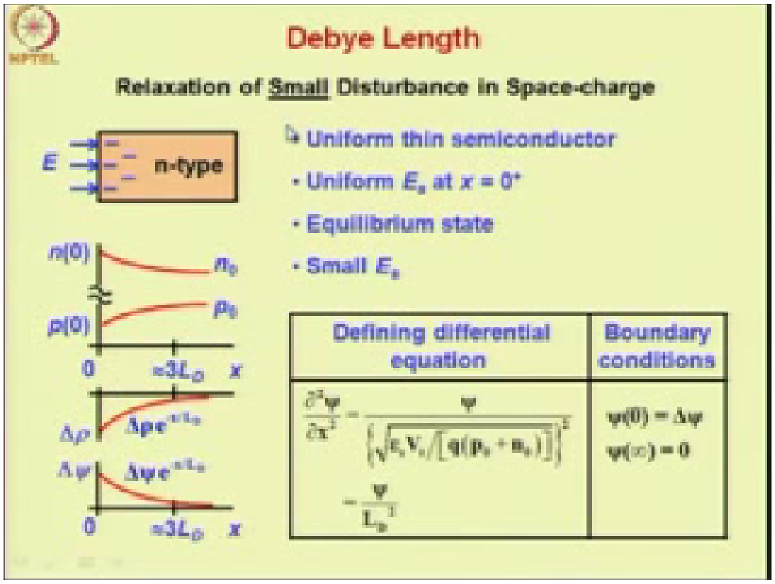

Debye Length

Relaxation of **Small** Disturbance in Space-charge

- Uniform thin semiconductor
- Uniform E_s at $x = 0^+$
- Equilibrium state
- Small E_s

Defining differential equation	Boundary conditions
$\frac{\partial^2\psi}{\partial x^2} = \frac{\psi}{\left\{\sqrt{\varepsilon_s V_t/\left[q(p_0+n_0)\right]}\right\}^2}$ $= \frac{\psi}{L_D^2}$	$\psi(0) = \Delta\psi$ $\psi(\infty) = 0$

So the equation is given here. This is a second order differential equation subjected to boundary conditions. You can easily do the solution yourself and the result would be rho is given by Delta rho into E power-x/Ld. So it is an exponential decay, okay. Now this is interesting, the decay associated with excess minority carriers which introduce a diffusion length was also sequential.

Similarly the decay of the various quantities which introduced the characteristic times, except for the transit time, those were also exponential. Now these exponential variations are valid only for small disturbances. This is very important to note that is why in each of the slides, the word small is being underlying.

One can analogously write an equation for psi, it is Delta psi into E power-x/Ld which is solution of this differential equation and it is from this exponential behavior that you come to the conclusion that by 3 times the length constant entering into the exponential, the quantity should fall to 5% of its value at the surface by 3 times the length constant.

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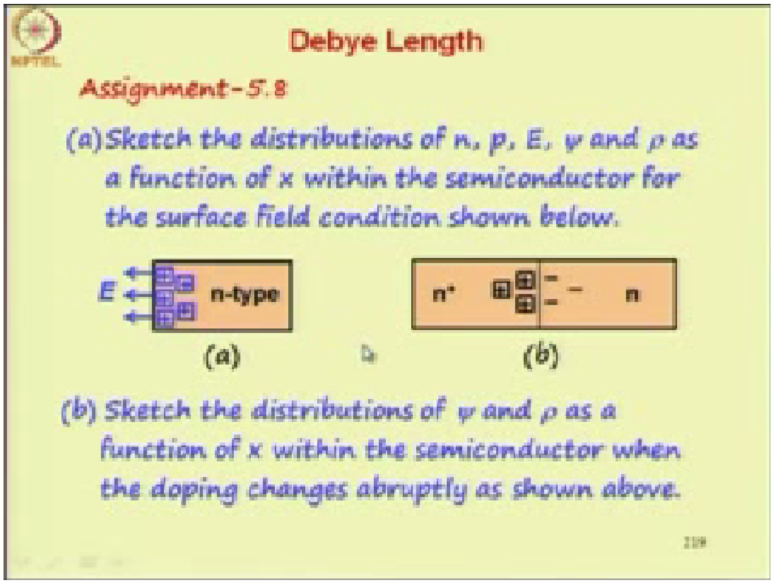

Debye Length

Assignment-5.8

(a) Sketch the distributions of n, p, E, ψ and ρ as a function of x within the semiconductor for the surface field condition shown below.

(b) Sketch the distributions of ψ and ρ as a function of x within the semiconductor when the doping changes abruptly as shown above.

Now let us do an assignment. Sketch the distributions of n, p, E, and psi. Jn and Jp are not given here because they are 0 and rho, the space charge as a function of x within the semiconductor for the surface field condition shown below. So here your field is output from the surface, that is a difference between the physical situation we have discussed just now for introducing the Debye length and this situation shown here. So in this case, the space charge will be due to bound donors.

Another assignment - sketch the distributions of psi and rho as a function of x within the semiconductor when the doping changes abruptly as shown above. So here doping is changing from n+n suddenly at this point and there will be a space charge region in which left hand side, you will have some ionized donors being exposed because of electrons being transferred from the high electron concentration region to low electron concentration region.

So these are the transferred electrons from n+. So this assignment, you have to sketch the quantities qualitatively.

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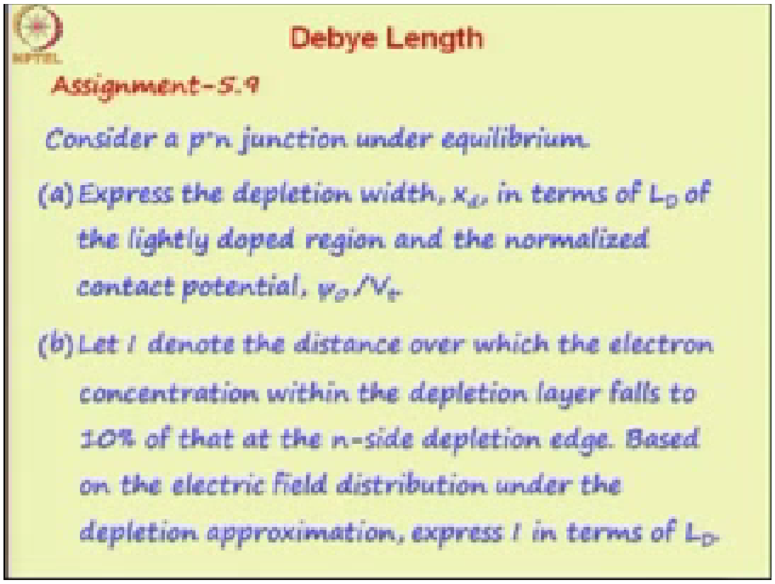

Debye Length

Assignment-5.9

Consider a p^+n junction under equilibrium.

(a) Express the depletion width, x_d, in terms of L_D of the lightly doped region and the normalized contact potential, ψ_0/V_t.

(b) Let l denote the distance over which the electron concentration within the depletion layer falls to 10% of that at the n-side depletion edge. Based on the electric field distribution under the depletion approximation, express l in terms of L_D.

One more assignment. Consider a p+n junction under equilibrium. Express the depletion width Xd in terms of the Debye length Ld of the lightly doped region and the normalized contact potential psi0/Vt. Part B, let l denote the distance over which the electron concentration within the depletion layer falls to 10% of that at the n side depletion edge, okay. Based on the electric field distribution under the depletion approximation, express this l in terms of the Debye length. Let me give you a diagram.

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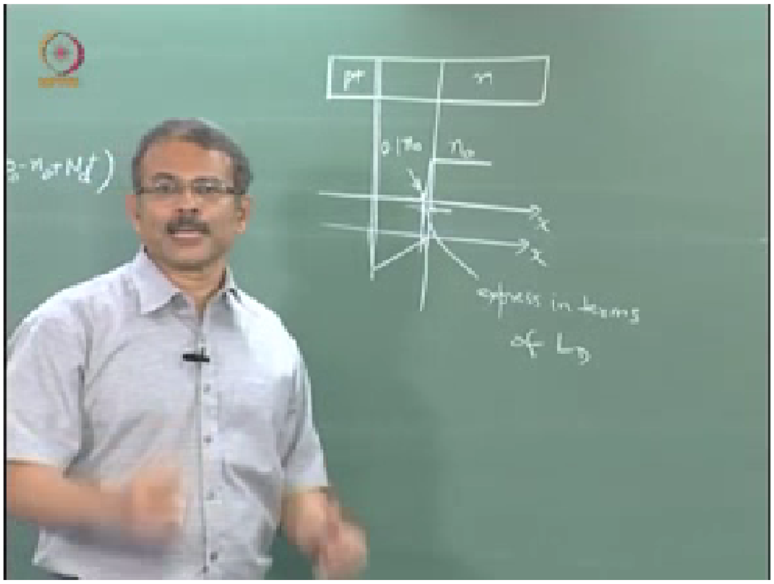

So this is your p+n junction. Let us say this is your space charge region, most of which will be on the n side and if you sketch your electron concentration, on a linear scale, it would look something like this, right. It will fall rapidly. The electric field picture would be something like this under equilibrium, right. So now you have to show that if this is the point where the concentration falls to 0.1 n0, 10% of the value here, so what is the distance.

This is the distance you have to calculate and this you should express in terms of the Debye length. So you must relate this distance to this potential drop in this small triangle okay, that is what you have to do and use Boltzmann relation.

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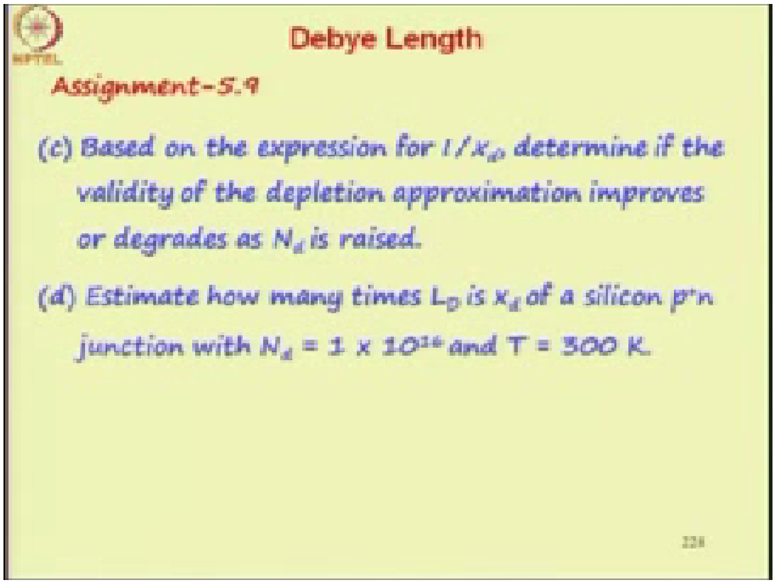

Debye Length

Assignment-5.9

(c) Based on the expression for l/x_d, determine if the validity of the depletion approximation improves or degrades as N_d is raised.

(d) Estimate how many times L_D is x_d of a silicon p^+n junction with $N_d = 1 \times 10^{16}$ and $T = 300$ K.

Based on the expression for ratio l/Xd, so here we are calling this distance as l and Xd is entire depletion width, however, since the depletion on this side is small, on the p side is small, you could regard this distance as Xd. So we are talking about the ratio of this length l and this length Xd, that is related to the area under this triangle to the area under this triangle. You can neglect the area of the small triangular portion here.

So based on the expression for ratio l/Xd, determine if the validity of the depletion approximation improves or degrades as Nd is raised. Estimate how many times Ld is Xd of a Silicon p+n junction with Nd = 1 into 10 power 16 and temperature is 300K.

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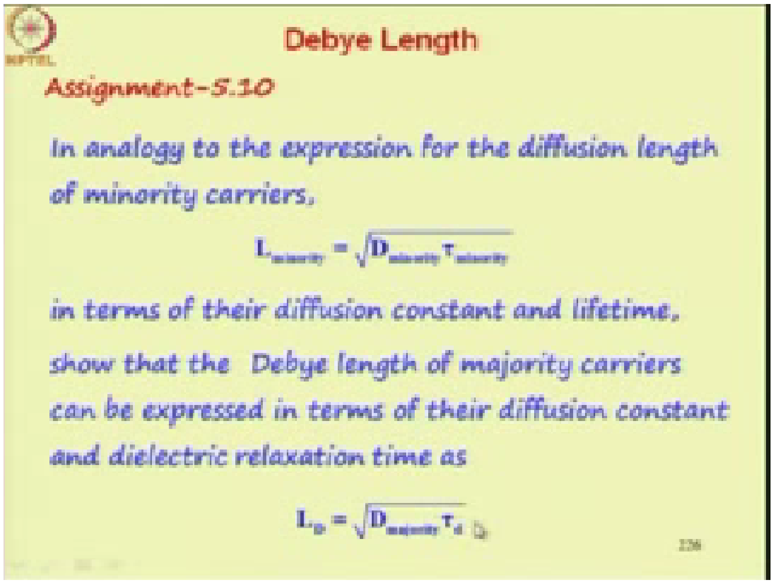

Debye Length

Assignment-5.10

In analogy to the expression for the diffusion length of minority carriers,

$$L_{minority} = \sqrt{D_{minority}\tau_{minority}}$$

in terms of their diffusion constant and lifetime, show that the Debye length of majority carriers can be expressed in terms of their diffusion constant and dielectric relaxation time as

$$L_D = \sqrt{D_{majority}\tau_d}$$

Final assignment on Debye length, assignment 5.10. In analogy to the expression for the diffusion length of minority carriers given by L minority = square root of the D minority into Tau minority, in terms of the diffusion constant and lifetime, show that the Debye length of majority carriers can be expressed in terms of their diffusion constant and dielectric relaxation time as Ld = square root of diffusion constant of majority carriers into dielectric relaxation time, that is interesting.

So minority carrier lifetime here and dielectric relaxation time here, right. These are the analogous time constants for the lengths. So minority carrier diffusion length and minority carrier lifetime, both of these are related to the same physical situation and Debye length and dielectric relaxation time are related to the same physical situation, okay, involving space charge decay.

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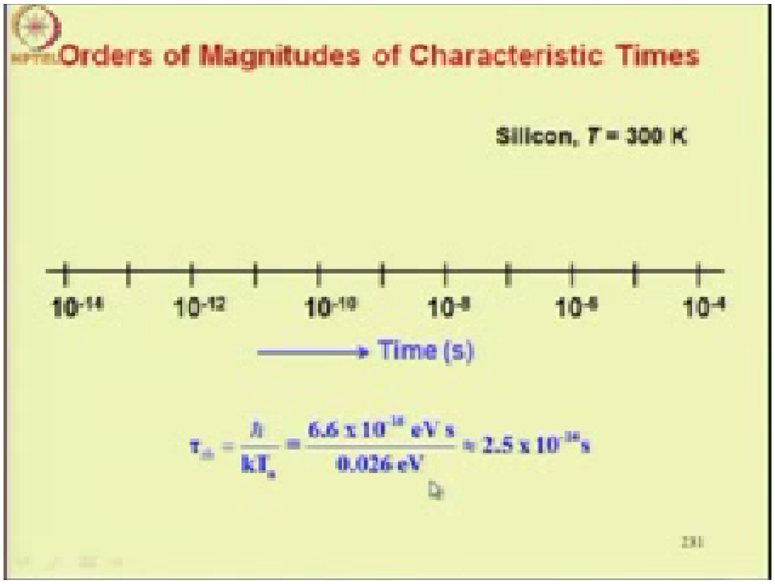

Orders of Magnitudes of Characteristic Times

Silicon, $T = 300$ K

$$\tau_{th} = \frac{\hbar}{kT_n} = \frac{6.6 \times 10^{-16}\ \text{eV s}}{0.026\ \text{eV}} \approx 2.5 \times 10^{-14}\ \text{s}$$

Now we come to the important issue of finding out orders of magnitudes of characteristic times and lengths. So first we shall begin with the characteristic times and we shall do this calculation for silicon at temperature of 300 K. Now as engineers, we must be aware of numerical values of various lengths, times and so on and other parameters which we use. This is very, very important. Let us start with the characteristic times.

So since our time constants over a very wide range of orders of magnitude, we have chosen the logarithmic scale here. So the minimum on the scale is 10 power-14 seconds, maximum is 10 power-4 seconds. Let us start with the time constant that emerges from the equation E = h nu, the Planck's relation.

So this time constant is analogous to the wavelength that emerges from the equation P=h/lambda, okay. What does the time constant tau thermal denote, it denotes the average time for which electron should remain, should be alive, so that you can assign a definite energy to the electron, okay. So this is something that is emerging from quantum mechanics. What is the minimum amount of time for which an entity should remain so that you can assign a definite energy, so we can regard it as a particle.

So tau thermal is given by h cross/KT where T is the carrier temperature. Since we are talking about electrons, it is represented as Tn. This temperature is 300 K in our case. So putting down the value for h cross in electron volt seconds and KTn at room temperature is 0.026 electron volts.

You get the value to be about 2.5 into 10 power-14 seconds. This will be the smallest for time constants. So in order for the particular approximation to evaluate, an electron should remain without any disturbance from any external sources for at least this much amount of time. If you want to regard it as a particle. For example, this time between 2 collisions should not be very close to this time constant if you want to regard the electron as a particle. So this falls here on the time scale.

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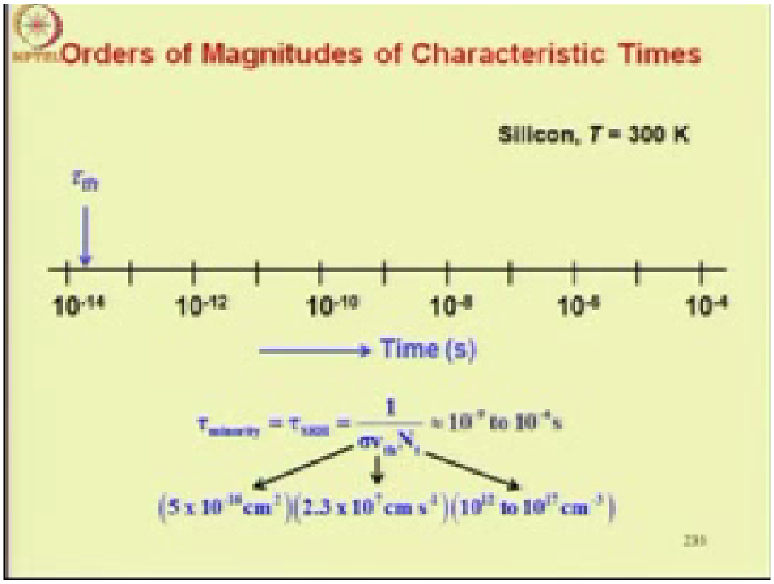

Orders of Magnitudes of Characteristic Times

Silicon, $T = 300$ K

$$\tau_{minority} = \tau_{SRH} = \frac{1}{\sigma v_{th} N_t} \approx 10^{-9} \text{ to } 10^{-4}\,\text{s}$$

$$(5 \times 10^{-16}\,\text{cm}^2)(2.3 \times 10^7\,\text{cm s}^{-1})(10^{12} \text{ to } 10^{17}\,\text{cm}^{-3})$$

Minority carrier lifetime - for silicon, it will be the time constant given by the SRH recombination theory and that is = 1 by the capture cross-section into thermal velocity into trap concentration per unit volume. Now the capture cross-section is 5 into 10 power-16 centimeter square. The thermal velocity is 2.3 into 10 power 7 centimeters per second.

You will shortly get an assignment to calculate this velocity and the time concentration ranges between 10 power 12 to 10 power 17 per centimeter cube. Now by way of example, this time concentration can represent gold atoms which have been introduced in silicon to kill the lifetime. So the 10 power 12 and 10 power 17 per centimeter cube represents the concentration of gold atoms in this example.

So the value turn out to be in the range of 10 power-9 to 10 power -4 seconds. So on a timescale, this is really a very wide range for minority carrier lifetime.

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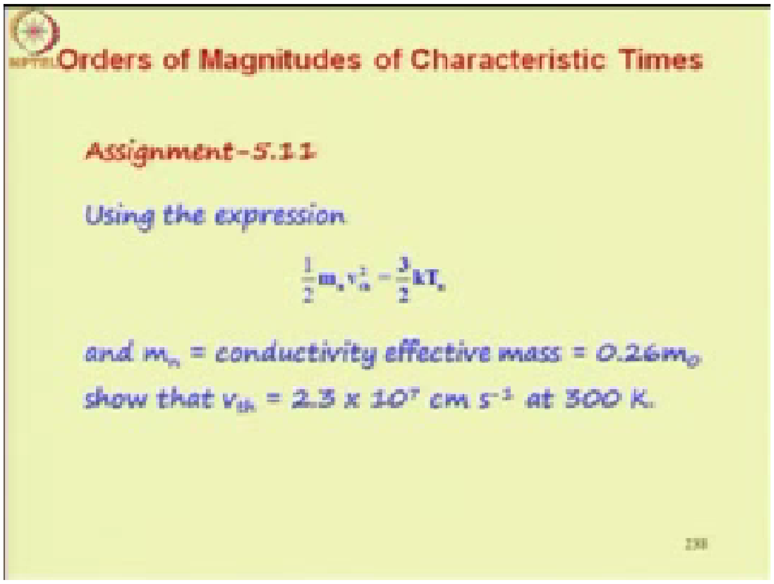

Orders of Magnitudes of Characteristic Times

Assignment-5.11

Using the expression

$$\frac{1}{2}m_n v_{th}^2 = \frac{3}{2}kT_n$$

and m_n = conductivity effective mass = $0.26m_0$

show that $v_{th} = 2.3 \times 10^7$ cm s^{-1} at 300 K.

Here is the assignment on thermal velocity. Using the expression half of effective mass into thermal velocity square = 3/2 KTn and Mn = conductivity effective mass which is 0.26 times m0 for electrons. Show that thermal velocity is given by this value at 300 K.

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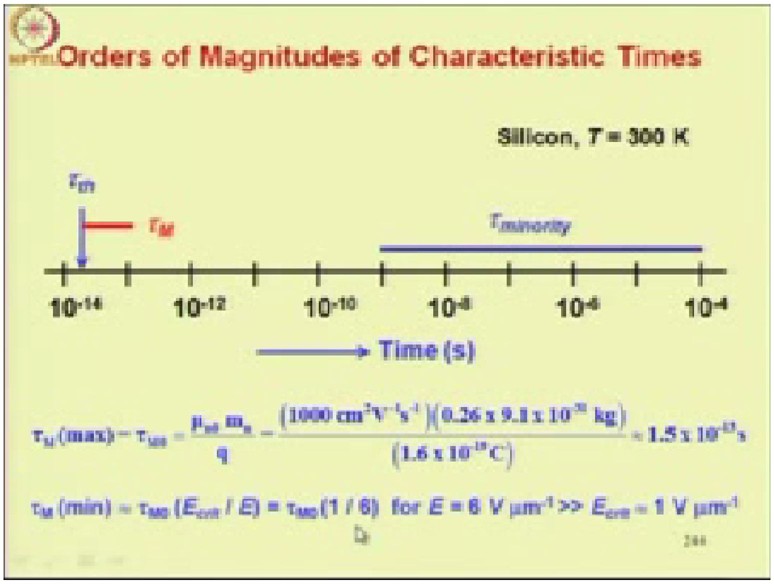

Orders of Magnitudes of Characteristic Times

$$\tau_M(\max) = \tau_{M0} = \frac{\mu_{n0} m_n}{q} = \frac{(1000\ cm^2V^{-1}s^{-1})(0.26 \times 9.1 \times 10^{-31}\ kg)}{(1.6 \times 10^{-19}C)} = 1.5 \times 10^{-13}s$$

$\tau_M(\min) = \tau_{M0}(E_{crit}/E) = \tau_{M0}(1/6)$ for $E = 6\ V\ \mu m^{-1} >> E_{crit} = 1\ V\ \mu m^{-1}$

Let us move on to momentum relaxation time. The maximum value of momentum relaxation time is Tau M0 that is close to the equilibrium. When you apply electric field, the momentum relaxation time goes on decreasing. So, the minimum value of momentum relaxation time will be obtained using this formula which is an approximation of the relation tau M=tau M0 into square root of TL, the lattice temperature by the electron temperature. So you will recall that we had presented this formula in our earlier module and we have said that this result is obtained from quantum mechanics.

We did not really derive this result, right. We simply accepted the result from quantum mechanics. So this ratio of lattice temperature to electron temperature can be cast in terms of the electric field and the formulae is, if you recall, Tn/Tl=1+ electric field by critical electric field square, right and therefore when you take a square root, you have square root of 1+E/E critical square and for large E, it simply becomes E/E critical. So that is what is used here.

E critical is about 1 V per micron. Now, tau M0, we can readily obtain from the measured values of mobility because you know the formula mu n0 is q times tau M0/effective mass, Mn. So now I can rearrange that same result to express tau M0 in terms of mu M0 and effective mass and here I can use measured values of mobility.

Let me use the value of mobility = 1000 centimeter square per volt second which corresponds to doping level of about 10 power 16 per centimeter cube of phosphorus atoms in silicon. So this multiplied by effective mass which is the conductivity effective mass that is why the factor of 0.26/q.

This results in the value 1.5 and 10 power-13 seconds whereas the minimum value is calculated for say E = 6 volts per micron, okay, this is a very high field and since this is much more than 1 volt per micron, we can use this approximation. So it is simply tau M0/6. So the maximum value is about 0.15 picoseconds and minimum value is one 6th of that. So on the time scale; it appears like this, here.

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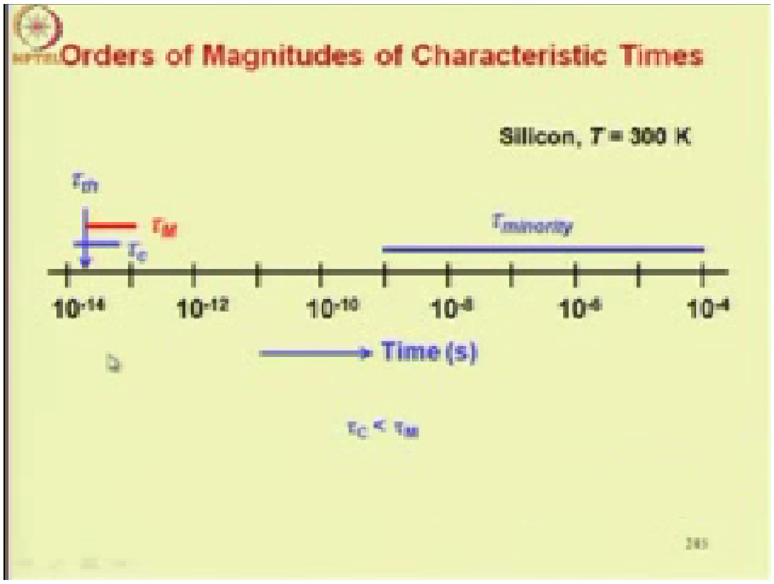

Orders of Magnitudes of Characteristic Times

$\tau_C < \tau_M$

Now the mean free time or the time between collisions is always less than momentum relaxation time. So the range of tau C would be something as shown here as compared to the Red Line or tau M.

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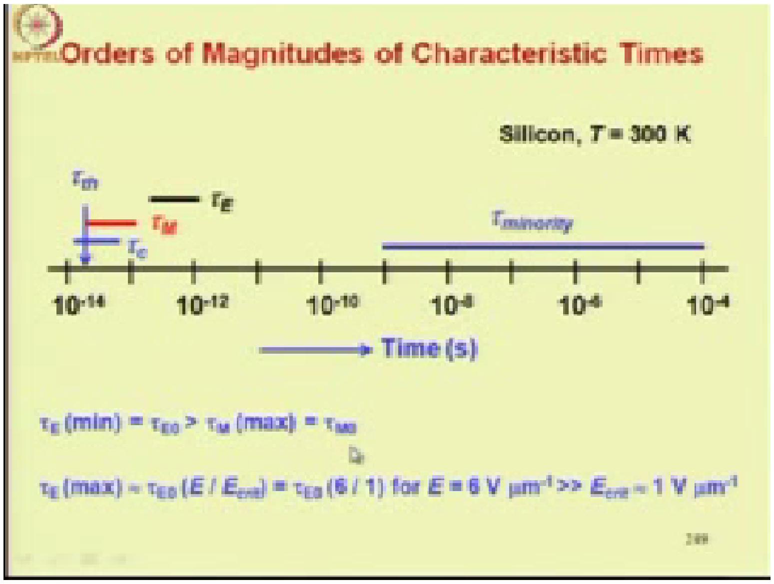

Orders of Magnitudes of Characteristic Times

$\tau_E\,(\text{min}) = \tau_{E0} > \tau_M\,(\text{max}) = \tau_{M0}$

$\tau_E\,(\text{max}) \sim \tau_{E0}\,(E / E_{crit}) = \tau_{E0}\,(6 / 1)$ for $E = 6\ \text{V}\ \mu\text{m}^{-1} >> E_{crit} \sim 1\ \text{V}\ \mu\text{m}^{-1}$

Let us look at the energy relaxation time. In this case, the minimum value of energy relaxation time would be tau is 0, while the maximum value would be given by a formula analogous to that we used for tau M for high electric fields while tau M decreases, tau E increases according to this relation, okay. Now the minimum tau E should be more than the momentum relaxation time because you recall that energy relaxation time is always more than momentum relaxation time, okay. Energy relaxes later than momentum.

So the maximum value of tau E, we will choose at 6 times the minimum value which is slightly more than tau M0. So based on this considerations, you place the energy relaxation time here, okay. So minimum value of tau E is more than maximum value of tau M.

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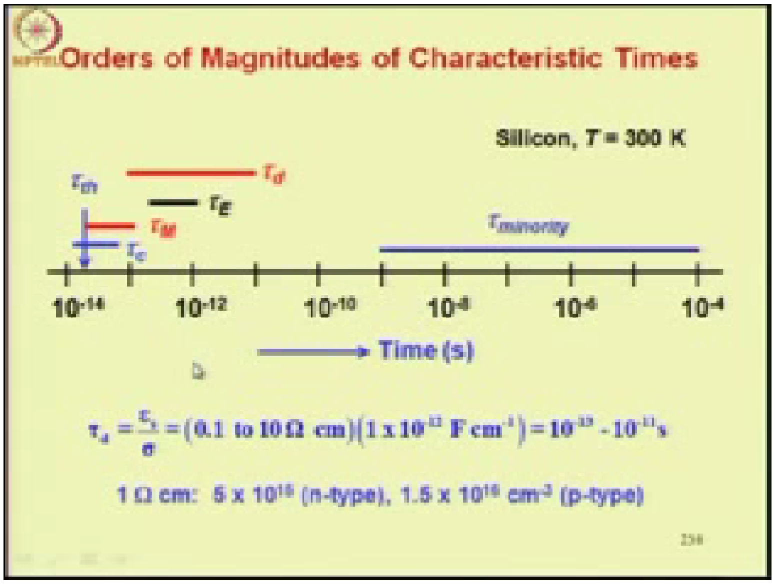

Orders of Magnitudes of Characteristic Times

Silicon, $T = 300$ K

$$\tau_d = \frac{\varepsilon_s}{\sigma} = (0.1 \text{ to } 10\,\Omega\text{ cm})(1 \times 10^{-12}\text{ F cm}^{-1}) = 10^{-13} - 10^{-11}\text{ s}$$

1 Ω cm: 5×10^{15} (n-type), 1.5×10^{16} cm^{-3} (p-type)

Dielectric relaxation time is given by epsilon S/sigma, so 1/sigma is nothing but rho. Now you know that the regions of interest in a device have resistivities ranging from 0.1 to 10 ohm centimeter. So 1/sigma varies from 0.1 to 10 ohm centimeter multiplied by dielectric constant and permittivity of silicon. The range is between 10 power -13 to 10 power-11 seconds. So that is somewhere here.

Just to give you a feel in terms of the doping level, 1 ohm centimeter of silicon has a doping of 5 into 10 power 15 phosphorus atoms, okay if it is n type and if it is p type, doping boron, then the number of atoms is 1.5 into 10 power 16. So 1 ohm centimeter lies between 0.1 and 10, so from here you can figure out what would be the doping levels corresponding to these extremes.

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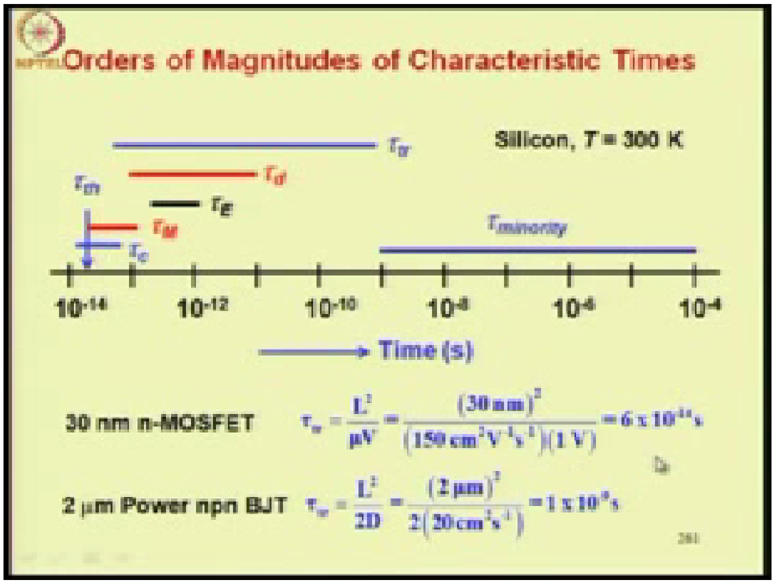

Orders of Magnitudes of Characteristic Times

Silicon, $T = 300$ K

30 nm n-MOSFET $\tau_{tr} = \frac{L^2}{\mu V} = \frac{(30\ \text{nm})^2}{(150\ \text{cm}^2\text{V}^{-1}\text{s}^{-1})(1\ \text{V})} = 6 \times 10^{-14}\ \text{s}$

2 μm Power npn BJT $\tau_{tr} = \frac{L^2}{2D} = \frac{(2\ \mu\text{m})^2}{2(20\ \text{cm}^2\text{s}^{-1})} = 1 \times 10^{-9}\ \text{s}$

Let us come to the final characteristic time, namely the transit time. So the shortest transit time will be for drift which is given by L square/mu V and the longest transit time will be for the diffusion which is L square/2D. Now let us take practical cases. Let us take a 30 nanometer n type MOSFET in which the electrons will flow by drift and let us take a 2 micron power npn BJT.

The 2 micron refers to the base width, okay whereas 30 nanometers here refers to the channel length. So this 30 nanometer is L over here and this 2 micron is the base width which is also represented by L, so that we can compare this 2 relations. So putting the mobility of about 150 centimeter square per volt second and the voltage across the MOSFET of about 1-volt drain to source voltage.

You get a transit time of about 6 in 10 power-14 seconds. On the other hand, for the diffusion across the base of a Power BJT, for electrons, you are using your diffusion coefficient of 20 centimeter square per second, okay. This corresponding to doping level of approximately 10 power 16. This results in a long time of 1 in 10 power-9 seconds. Now you might just wonder why should you call 10 power-9 seconds which is nanoseconds as long.

Well, you must compare this with the time constant, the transit time you obtained here which is of the order of 10 power-13 or -14. So you see the transit time in various devices ranges over a very wide range. So the minority carrier lifetime and transit times are both varying over a wide range. So it is the dielectric relaxation time also. However, from here you can easily see the relative magnitudes of the various time constants.

Generally for many devices, minority carrier lifetimes would be the largest of all the time constants, the transit time would be shorter, so would be dielectric relaxation time and the dielectric relaxation time would be longer than momentum and energy relaxation times or the meantime between collisions.

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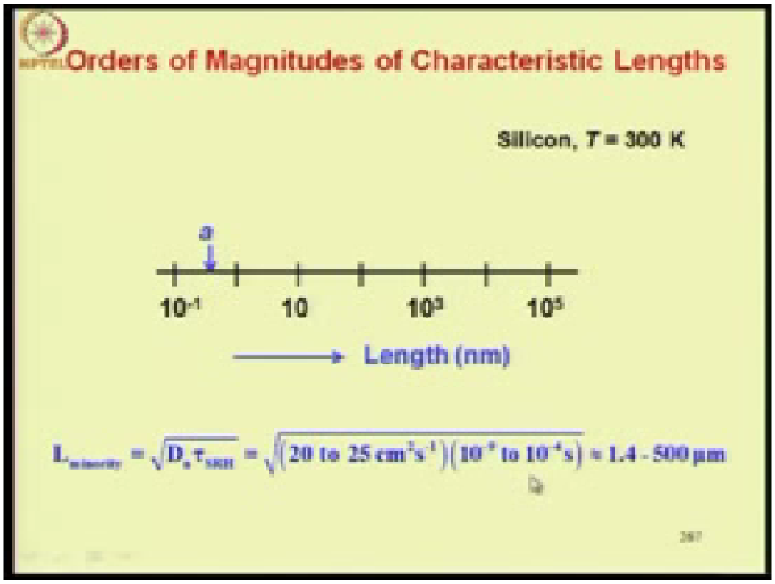

Orders of Magnitudes of Characteristic Lengths

Silicon, $T = 300$ K

$$L_{minority} = \sqrt{D_n \tau_{SRH}} = \sqrt{(20 \text{ to } 25\ cm^2 s^{-1})(10^{-9} \text{ to } 10^{-4} s)} \approx 1.4 - 500\ \mu m$$

Finally let us look at the characteristic lengths for silicon. Here we have placed the lattice constant a which is the smallest characteristic length that we will come across. For silicon, it is about 5.43 Angstroms or 0.543 nanometers. The minority carrier diffusion length is given by square root of the diffusion coefficient into the lifetime. Now the diffusion coefficient ranges from 20 to 25 centimeter square per second considering the various devices in which diffusion occurs and the lifetime ranges between 10 power -9 to 10+/-4 seconds.

So putting these quantities and finding out the minimum and maximum, you get range of 1.4 to 500 microns for the diffusion length, that is a very wide range as shown here.

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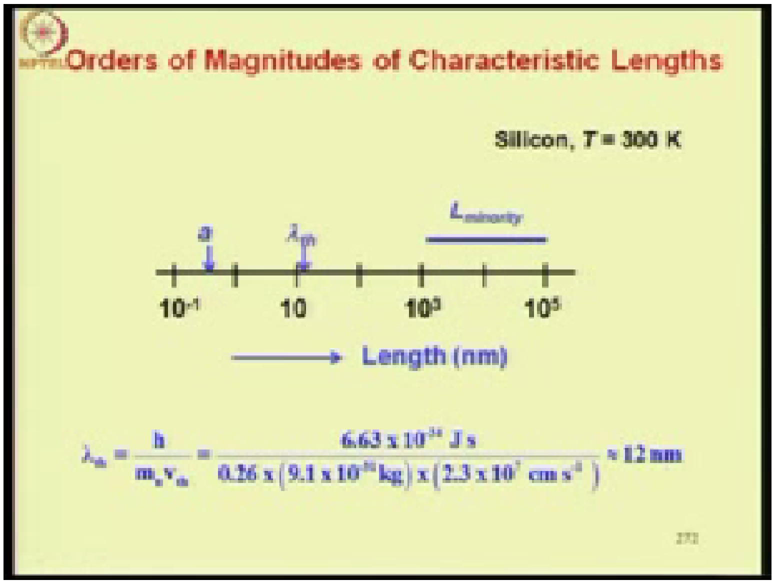

Orders of Magnitudes of Characteristic Lengths

Silicon, $T = 300$ K

$$\lambda_{th} = \frac{h}{m_c v_{th}} = \frac{6.63 \times 10^{-34}\ \mathrm{J\ s}}{0.26 \times \left(9.1 \times 10^{-31}\,\mathrm{kg}\right) \times \left(2.3 \times 10^{7}\ \mathrm{cm\ s^{-1}}\right)} \approx 12\ \mathrm{nm}$$

The De Broglie wavelength of thermal average electron is h/effective mass of electrons into thermal velocity. So putting down the various values using conductivity effective mass of electrons, you get a value of 12 nanometers at 300 K. That is placed here on the length axis.

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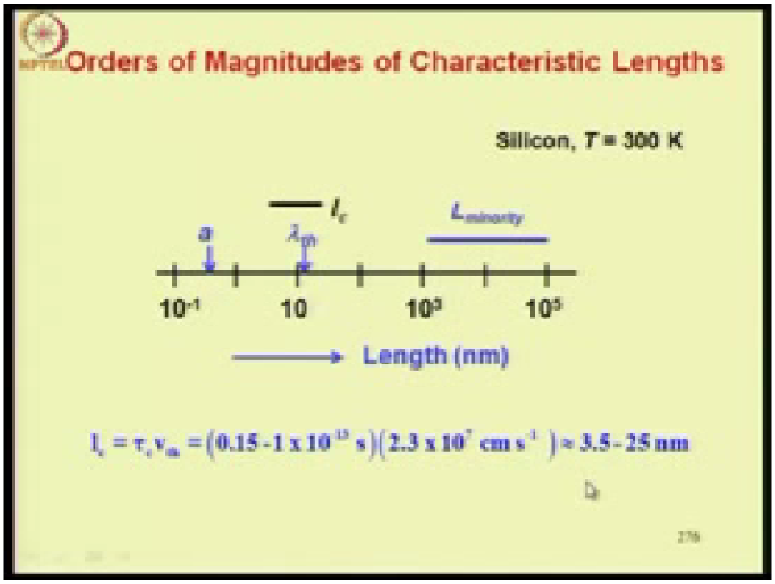

Orders of Magnitudes of Characteristic Lengths

Silicon, $T = 300$ K

$$l_c = \tau_c v_{th} = \left(0.15 - 1 \times 10^{-13}\ \mathrm{s}\right)\left(2.3 \times 10^{7}\ \mathrm{cm\ s^{-1}}\right) \approx 3.5 - 25\ \mathrm{nm}$$

Mean free length between collisions is equal to mean free time between collisions into thermal velocity. Now using the values of momentum relaxation time and the fact that mean free time between collisions are little less than the momentum relaxation times, we arrive at this range for tau C. The result is variation from 3.5 to 25 nanometers.

That is somewhere here. Now, this is really interesting which means that sometimes your mean free length between collisions can go below the de Broglie wavelength of thermal average electrons, which means the particle approximation will not be good, right, in some cases.

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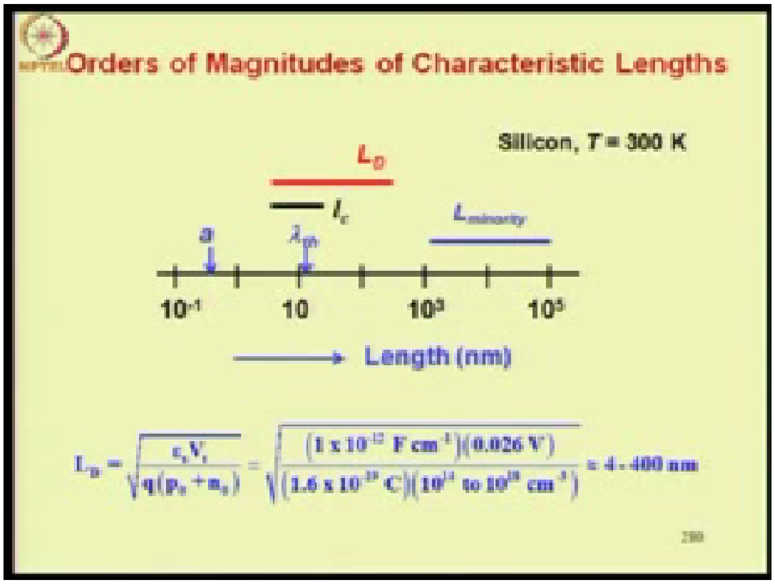

Orders of Magnitudes of Characteristic Lengths

Silicon, $T = 300$ K

$$L_D = \sqrt{\frac{\varepsilon_s V_t}{q(p_0+n_0)}} = \sqrt{\frac{(1\times10^{-12}\text{ F cm}^{-1})(0.026\text{ V})}{(1.6\times10^{-19}\text{ C})(10^{14}\text{ to }10^{18}\text{ cm}^{-3})}} \approx 4-400\text{ nm}$$

Let us look at the Debye length which is Epsilon S Vt/q times P0+n0 the square root over this expression. Putting down the values for silicon and using a doping range of 10 power 14 to 10 power 18, it really does not matter whether it is n type or p type because if it is n type, then n0 will be dominating over p0 and if it is p type, p0 will dominate over n0. So this results in a range of 4 to 400 nanometers.

And that is shown here. Now you see from here that generally the diffusion length is the largest characteristic length. Debye length is shorter, right, but generally this Debye length will be more in many devices than the mean free length between collisions which in turn will be more than the thermal average of de Broglie wavelength and all these lengths will be much more than the lattice constant.

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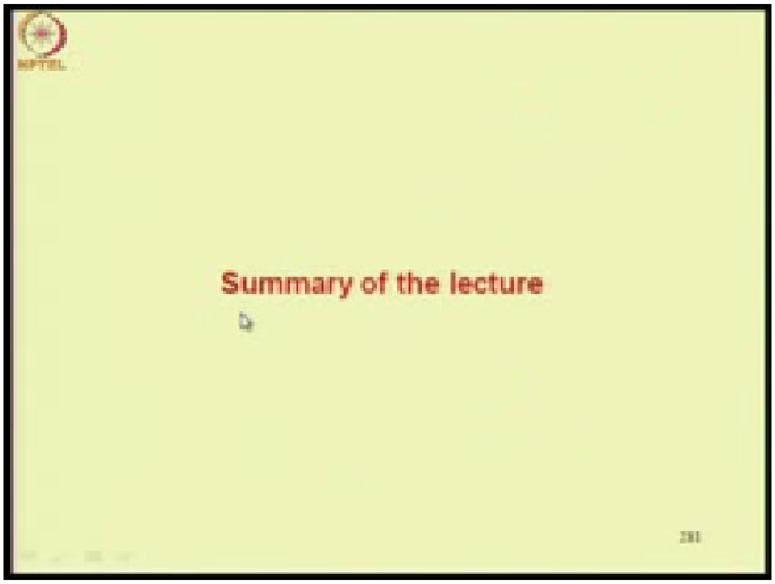

Summary of the lecture

Now with that we have come to the end of the lecture and hence a summary of the important points. So in this lecture, we introduced the Debye length which characterizes the decay of space charge when there is a small disturbance in the space charge and we said that the decay over distance occurs over 3 times a length constant called the Debye length.

The differential equation characterizing this length was the second order differential equation for space charge as a function of distance X and the formula for Debye length is square root of Epsilon S that is the Epsilon of the region in which you are finding the Debye length multiplied by the thermal voltage divided by charge on the electron q into the sum of the electron and hole concentrations, p0 and n0. So therefore the Debye length decreases as your doping increases and as thermal voltage increases or temperature increases, the Debye length increases, okay.

So in heavily doped regions or in regions where the mobile carrier concentration is high, the space charge region would be really very, very narrow and then be considered the orders of magnitude of various characteristic times and lengths, those can be shown on the slide as follows.

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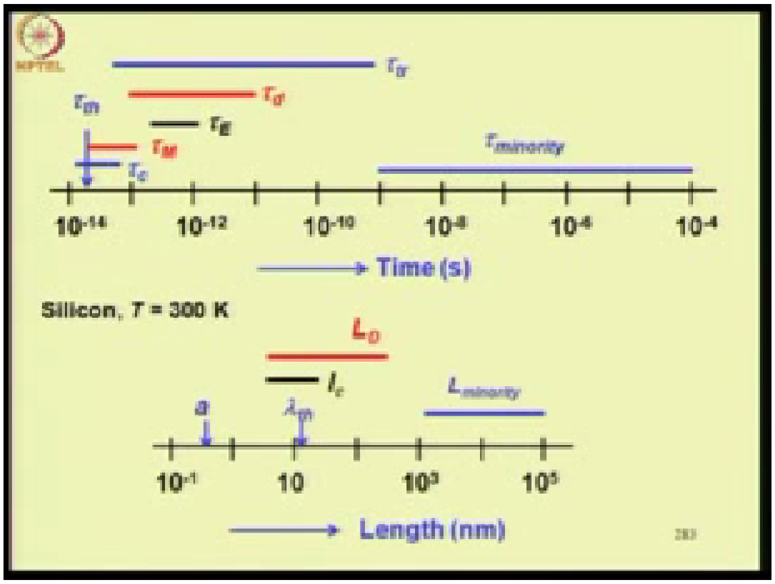

Where the minority carrier lifetime is the largest of the time constants and the time associated with the translation E = h nu, that is the time for which an electron should be alive without any action on the electron or the electron to be treated as a particle, that is the minimum. Other time constants fall in between. Generally the order would be, maximum time is minority carrier lifetime, then comes transit time, then comes dielectric relaxation time, then comes energy relaxation time, momentum relaxation time tau C, right in that order, maximum to minimum.

So this would mean, for instance that the dielectric relaxation time is smaller than the transit time and minority carrier lifetime in many devices so that space charge neutrality can be assumed even when you have time varying situations. More about this will be discussed later.

For length constants, we discussed the minority carrier lifetime, the Debye length, the mean free length between collisions, the thermal average wavelength of electrons and the lattice constant and we said that in many devices, the order of these characteristic lengths would be as follows: maximum would be minority carrier diffusion length then Debye length would be less than that, the mean free distance between collisions would be even less but which would be more than the thermal average wavelength so that we can treat the electron or hole as a particle in many situations.