Semiconductor Device Modeling
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Lecture – 23
Characteristic Times and Lengths

In the previous lecture, we have discussed the concepts of dilated relaxation time, momentum relaxation time and energy relaxation time. While discussing dilated relaxation time, we have discussed both injection of majority carriers and injection of minority carriers. In this lecture, we will discuss some more characteristic time and lengths.

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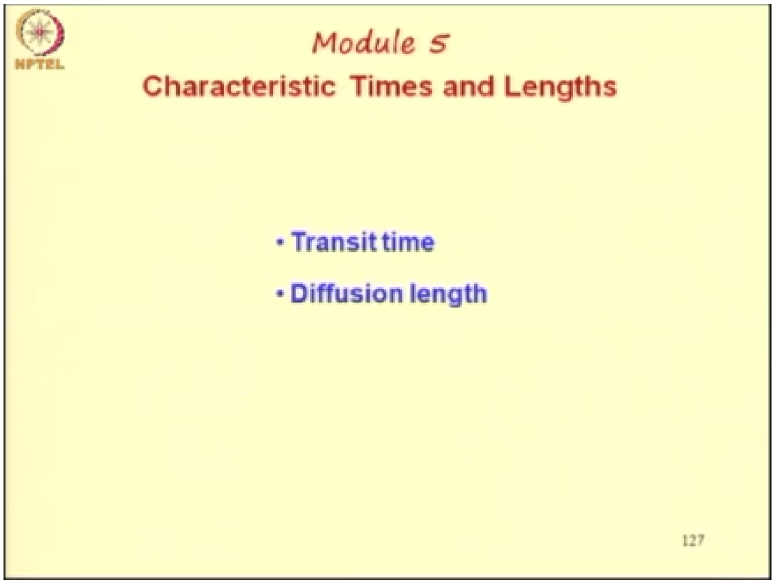

Module 5
Characteristic Times and Lengths

- Transit time
- Diffusion length

Let us look at transit time.

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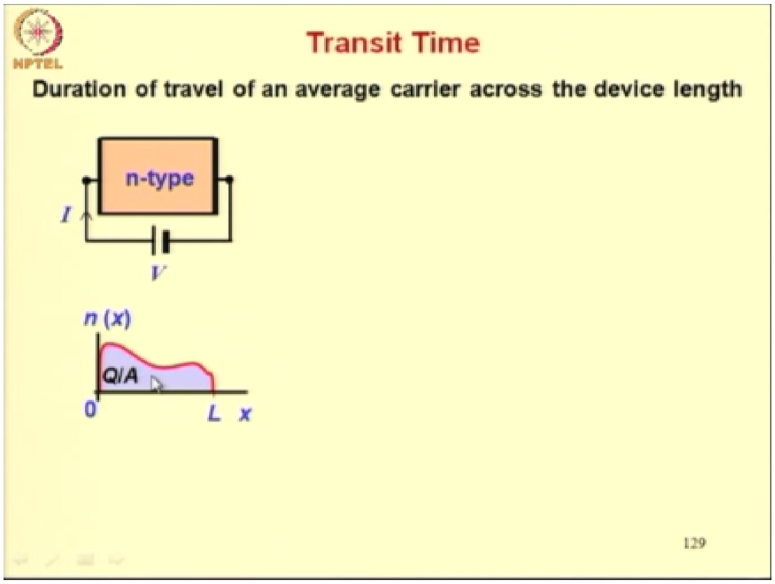

Transit Time

Duration of travel of an average carrier across the device length

This time is a duration of travel of an average carrier across the device length. The situation which introduces this time is shown here. So, you have an N type semiconductor across which a voltage is applied and a current setup. Now note the fact that the doping in the semiconductor need not be uniform. So, that is shown here using the fact that the electron concentration is arbitrarily varying with X. The length of the device is L.

We shall denote the area under this electron concentration distribution as charged by A. We are using charge per unit area because when you integrate N of X over X, you will get per. In the previous lecture, we discussed the concepts of dilated relaxation time, momentum relaxation time and energy relaxation time. For dilated relaxation time, we considered 2 situations, injection of majority carriers and injection of minority carriers.

In this lecture, we will discuss some more characteristic times and lengths. So, we shall discuss transit time, diffusion length and the Debye length.

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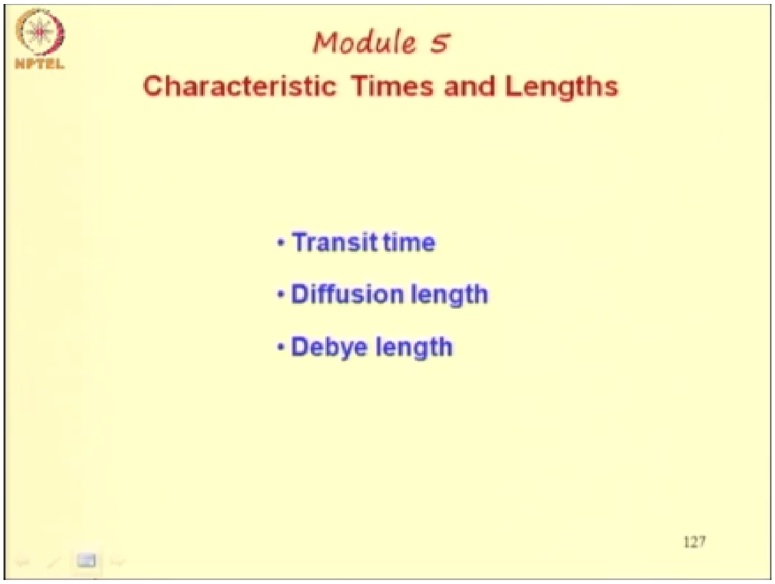

Module 5

Characteristic Times and Lengths

- Transit time
- Diffusion length
- Debye length

Consider transit time.

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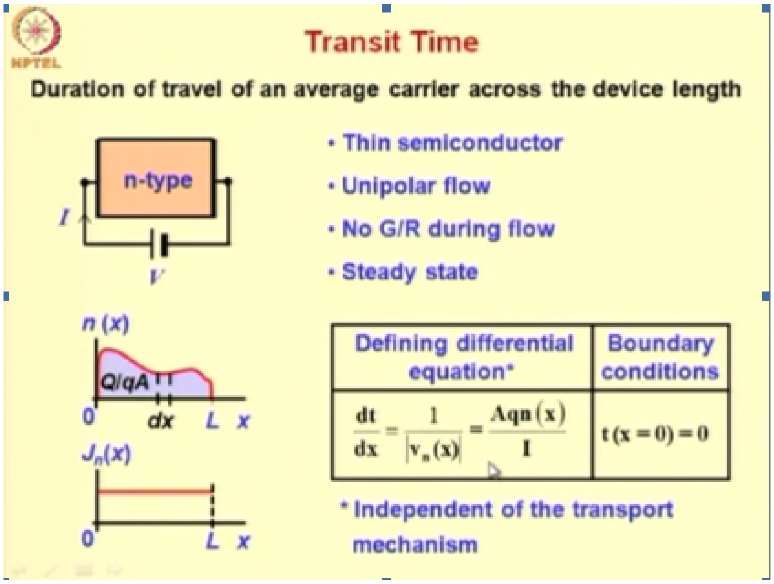

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Defining differential equation*	Boundary conditions
$\frac{dt}{dx} = \frac{1}{\lvert v_n(x) \rvert} = \frac{Aqn(x)}{I}$	$t(x=0)=0$

* Independent of the transport mechanism

This is a duration of travel of an average carrier across the device length. The situation introducing this time is shown here. You have an N type semiconductor across which a voltage has been applied and as a consequence a current is setup. Now, note that in contrast to situations we discussed so far, the N type semiconductor here need not be uniform. For example, it can have arbitrary variation in doping and as a consequence electron distribution.

The length of the device is L. The under this electron distribution, we shall denote as Q divided by small q into A. So, this is number of electrons per unit area. Now, this area is the area of cross-section of the device or area of the contacts. We shall consider a one-dimensional situation. Now, the one-dimensional assumption is justified because we shall consider a thin semiconductor. Then we will concentrate on the flow of any one polarity of electrons.

We will assume that there is no generation or recombination during flow and finally the situation will be regarded as steady state. Under these conditions, the electron current will remain constant with X and the defining differential equation for this situation is the time spent by the carrier in traversing a length dX, the time dT required to travel dX. So, dX is the length here = reciprocal of the electron velocity which can vary with X because the current is constant.

While in general the distribution of electrons changes as a function of X and this will be shown equal to the area of cross-section of the device into X into the electron concentration at X divided by the current I. The boundary condition in this case, the initial condition here would be the time for X = 0 is 0. So, we shall assume that the instant at which the electron starts from X = 0 is 0. Important to note is the fact that this differential equation is valid for any mechanism of carrier transport, so this is independent of transport mechanism.

For instance, in this case because the carrier concentration is varying with distance, there is a diffusion current and we have applied a voltage, therefore there is an electric field and drift current. So, in this situation you have both drift and diffusion currents. Now you can have situations where there is drift current only or diffusion current only. You can even have thermoelectric current.

So, the concept of transit time will apply in all cases and this equation that we are showing on the slide will apply independent of the transport mechanism. Now, let us know about deriving the equation.

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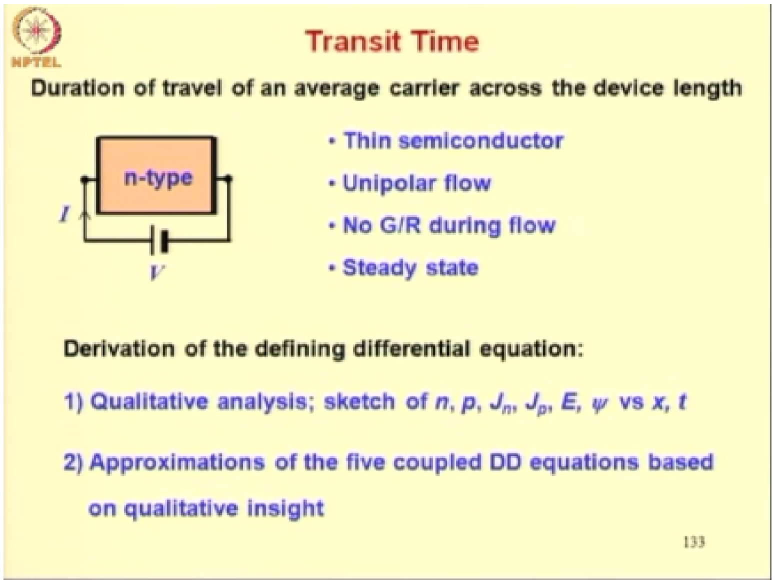

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

2) Approximations of the five coupled DD equations based on qualitative insight

So, the first set is qualitative analysis which involves sketching the electron hole concentrations, electron hole current densities, electric field and psi versus X. Now, we will not have any distribution of the function of time because it is a steady state situation. Further, since we are going to concentrate on one polarity of carriers only and we have chosen electrons here. So, therefore we will not be concerned with the concentration of holes and the hole current density.

So, now the picture would look something like this.

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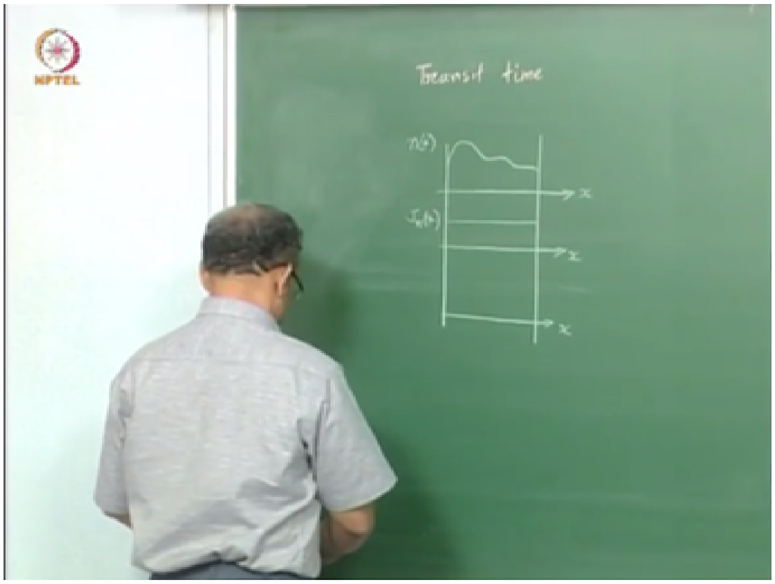

So, the electron concentration varies with X according to some arbitrary shape. We are not plotting the whole concentration because we are not concerned with it. The electron current density is, however, constant with X because it is a steady state situation. We shall show this fact also using equations. Now electric field and potential, we will find that will not be concerned with this information.

So, I will leave it you as an assignment to plot the electric field and potential. Take care to see the fact that both drift and diffusion currents are present because there is a slope here. So, using the fact that the electron current is constant with X, but the electron current is due to both diffusions because of the concentration gradient and drift. So, you will have to manipulate the equation to derive an equation for the electric field.

Then you will have to plot the electric field which will also very in some way with X depending on the concentration of electrons. What will be of interest to us is, however, the velocity of electrons. Now using a formula JnX = q times the electron concentration into the velocity of electrons where this q there will be a negative sign because the charge is negative. You can use this fact and then sketch the velocity.

Now we will be concerned with magnitude of the velocity. So, we are not concerned with the direction because if the electric field is from left to right which is what it is in our sample, then electrons will move from right to left. So, velocity will be negative, but we are plotting the magnitude here. So that velocity according to this formula will be JnX/- of QX. So, you manipulate this formula, this is what you get.

If you are interested in the magnitude, then it is simply the magnitude of this.

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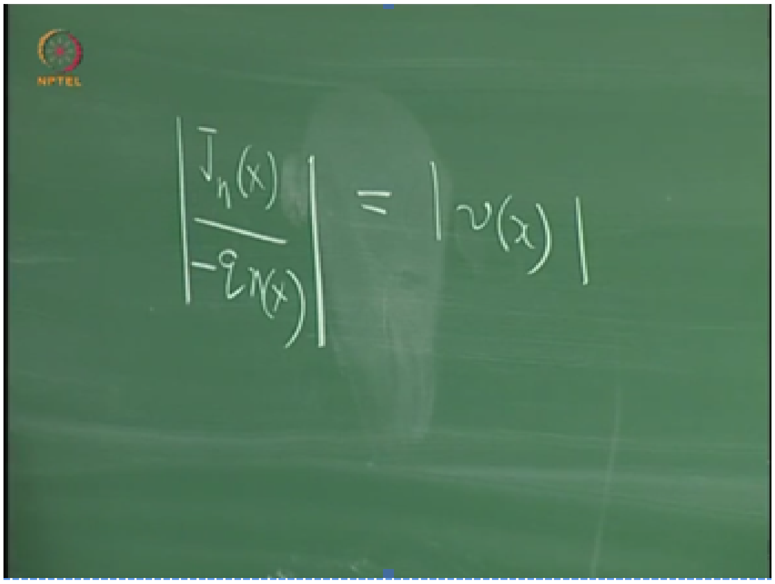

So, since JnX is constant and NX is varying, it is reciprocal of NX really proportional to reciprocal of NX.

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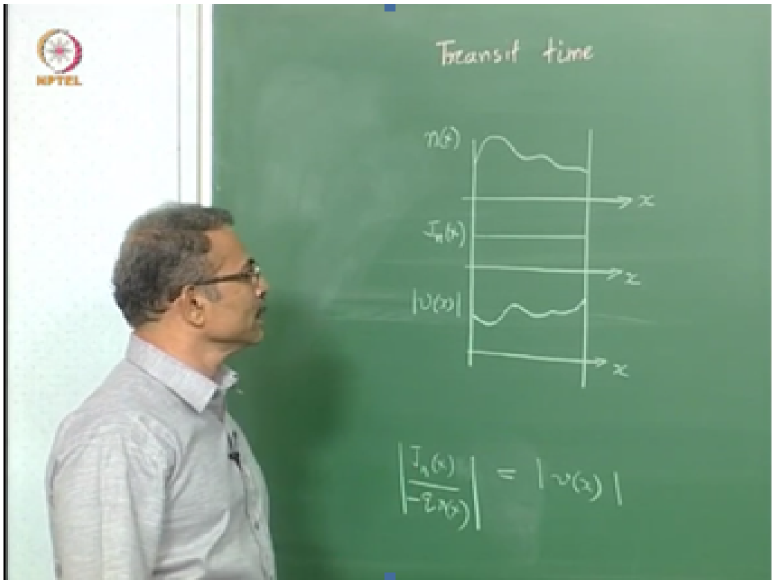

So, this could look something like this. Okay, so some distribution which is a mirror image. So, like this it goes and then it rises here okay. So, this is your velocity as a function of X. Now this information will be used to derive the transit time information.

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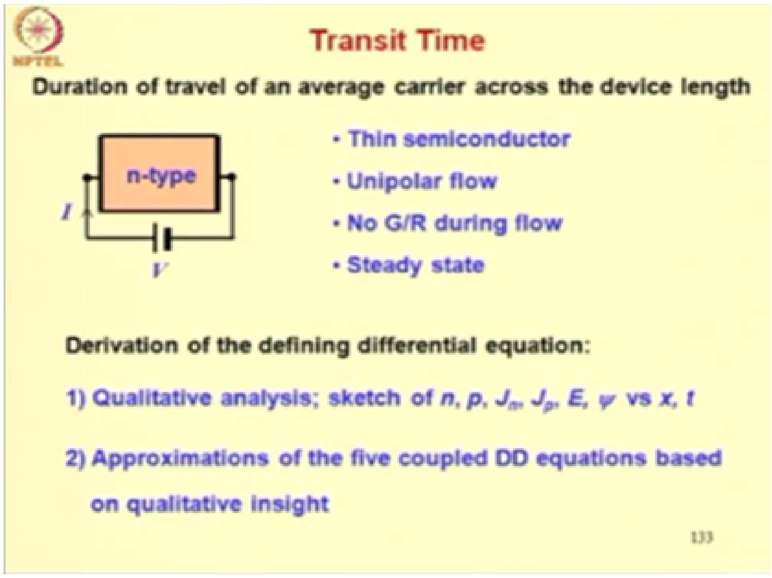

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

2) Approximations of the five coupled DD equations based on qualitative insight

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So next step is approximations of the 5 coupled drift diffusion equations based on qualitative insight.

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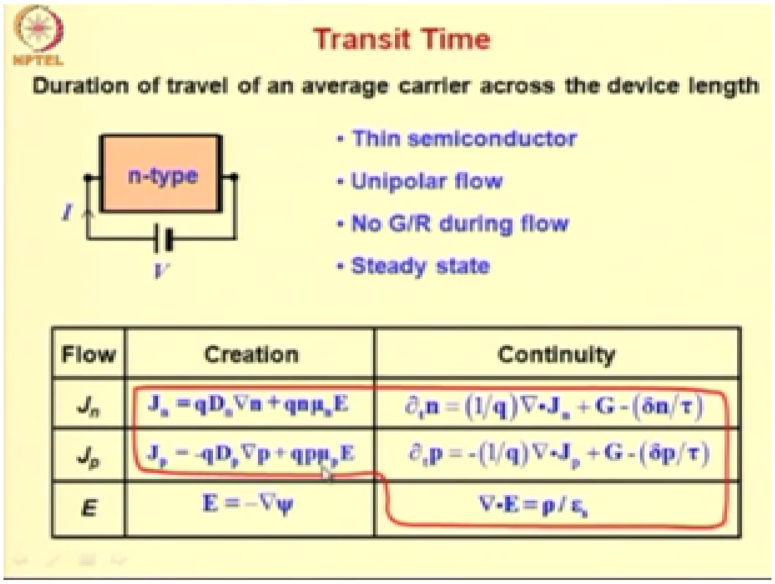

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Flow	Creation	Continuity
J_n	$\mathbf{J}_n = q D_n \nabla n + q n \mu_n \mathbf{E}$	$\partial_t n = (1/q)\nabla \cdot \mathbf{J}_n + G - (\delta n/\tau)$
J_p	$\mathbf{J}_p = -q D_p \nabla p + q p \mu_p \mathbf{E}$	$\partial_t p = -(1/q)\nabla \cdot \mathbf{J}_p + G - (\delta p/\tau)$
E	$\mathbf{E} = -\nabla \psi$	$\nabla \cdot \mathbf{E} = \rho / \varepsilon_s$

These are our 5 equations. Now the thin semiconductor approximation or condition leads to one-dimensional flow, unipolar flow. So, we are concerned with electrons alone. So, we strike out the equations for the holes. So, we are left with the 3 equations Jn, then the continued equation and the Gauss's law in the 5 coupled equations. No generation and recombination during flow helps us to strikeout the generation term G.

And the recombination term delta N/tau. Steady-state helps us to strike out the dou N/dou T term. So, this is 0.

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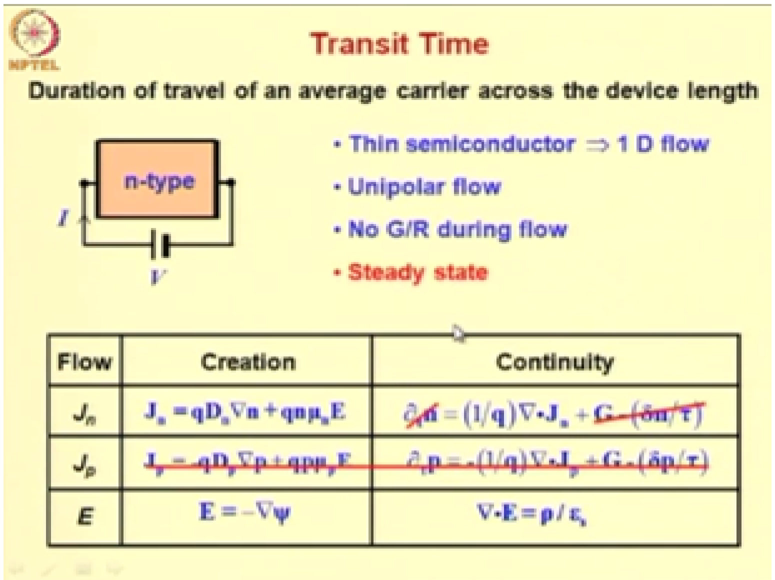

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor ⇒ 1 D flow
- Unipolar flow
- No G/R during flow
- Steady state

Flow	Creation	Continuity
J_n	$J_n = qD_n\nabla n + qn\mu_n E$	$\cancel{\partial_t n} = (1/q)\nabla\cdot J_n + \cancel{G - (\delta n/\tau)}$
J_p	~~$J_p = -qD_p\nabla p + qp\mu_p E$~~	~~$\partial_t p = -(1/q)\nabla\cdot J_p + G - (\delta p/\tau)$~~
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

Now, the next step is reduction of the 5 approximated equations and the 6th equation, the 6th equation is E = - Grad psi to an equation defining the characteristic time.

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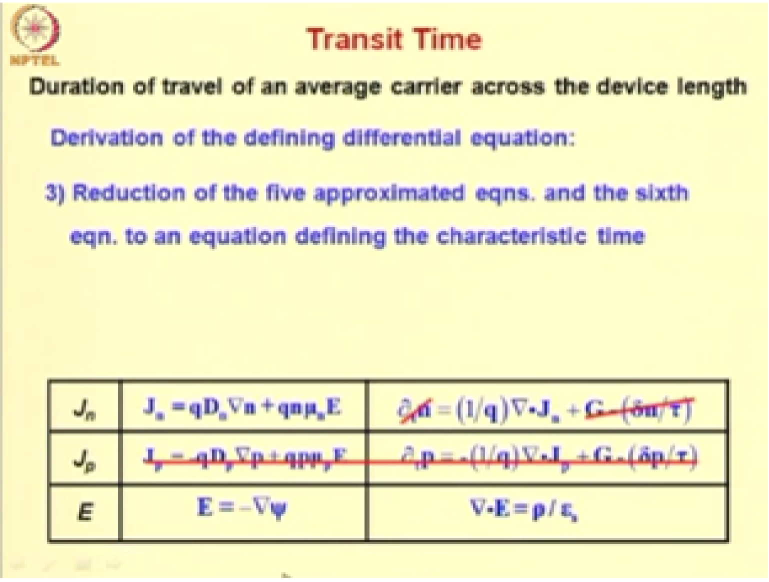

Transit Time

Duration of travel of an average carrier across the device length

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to an equation defining the characteristic time

J_n	$J_n = qD_n\nabla n + qn\mu_n E$	$\cancel{\partial_t n} = (1/q)\nabla\cdot J_n + \cancel{G - (\delta n/\tau)}$
J_p	~~$J_p = -qD_p\nabla p + qp\mu_p E$~~	~~$\partial_t p = -(1/q)\nabla\cdot J_p + G - (\delta p/\tau)$~~
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

Now, we will find that we will really not need these equations related to E. So, the current density equation, we will express in a compact form like this in terms of the velocity of electrons VX, because it is this velocity that is required for determining the transit time. So, we combine the diffusion and drift terms and write it as - of q times the electron concentration into the velocity.

So, this will be an effective velocity which includes effects of diffusion as well as drift. You know that the current density is expressed in more fundamental way in this formula, then the continued equation. So, you see here the equation is really diversions of Jn = 0, which means in one-dimensional case, DJn/dx is 0 or Jn is constant. Therefore, the modulus of Jn is expressed in terms of the current I and divided by the area of cross-section A.

So everywhere the current density is simply equal to the terminal current I divided by the area of cross-section. So, we combine these 2 and then we get an expression for reciprocal of the velocity as the function of X as A times Q times the electron concentration at X/I.

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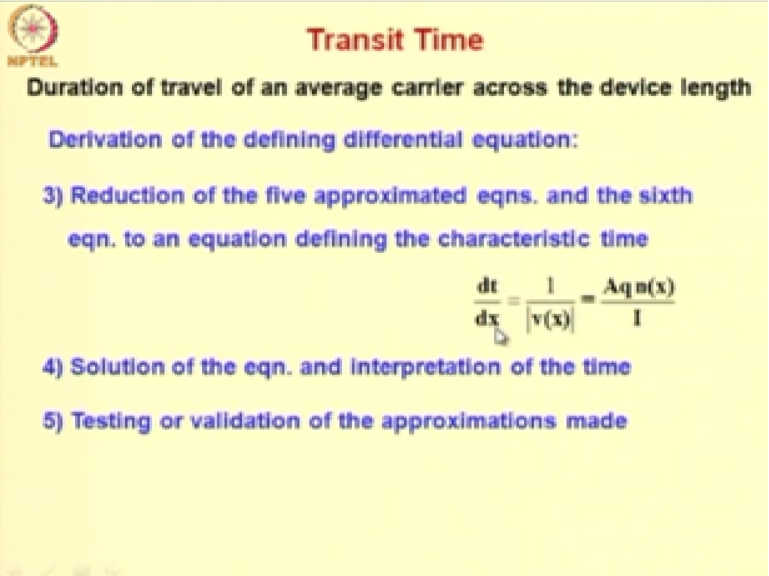

Transit Time

Duration of travel of an average carrier across the device length

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to an equation defining the characteristic time

$$\frac{dt}{dx} = \frac{1}{|v(x)|} = \frac{Aq\,n(x)}{I}$$

4) Solution of the eqn. and interpretation of the time

5) Testing or validation of the approximations made

Therefore, the equation defining the characteristic time is the dt/dx. So, time dt required to travel dx is 1/v(x) = A into q into n(x)/I. So, you know that dx is velocity. So, dt x dx is reciprocal of velocity and we are taking the modulus. The next couple of steps are solution of the equation and interpretation of the time and testing or validation of the approximations made.

Now, testing or validation approximations is not an important issue in our case because whatever simplifications we have done for the various equations, they follow directly from the conditions assumed related to the situation. For example, steady state, unipolar flow, one-dimensional flow, no generation recombination, and so on. So let us move on to the solution of the equations and interpretation of the time.

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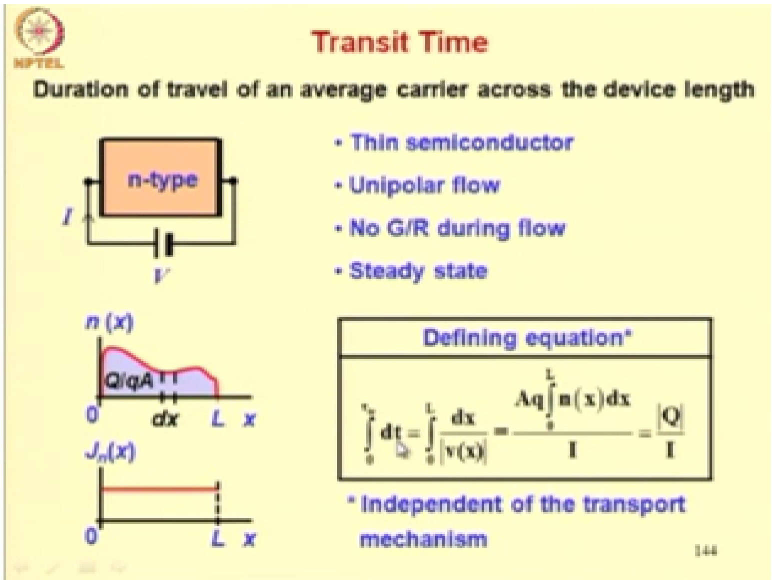

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Defining equation*

$$\int_0^{\tau_t} dt = \int_0^L \frac{dx}{|v(x)|} = \frac{Aq\int_0^L n(x)dx}{I} = \frac{|Q|}{I}$$

* Independent of the transport mechanism

So you can integrate the differential equation to get the solution. So, integrate dt when X = 0, the time instant is 0 that was initial condition and when the electron reaches L that is the end of the device, here this L and then the duration of electron travel is transit time. So, we are calculating the time required for the electron to traverse from X =0 or X = L, so that is our transit time. So, you integrate this.

This integration is nothing but integration of n(x) dx over 0 to L multiplied by A times q/I and this integral is nothing but the modulus of q, okay. Because in this case, this is electron charge and when you take the q into account, it will be the electron charge is negative and therefore we take the modulus because our transit time will be always positive. So, the result is transit time = modulus of q/I.

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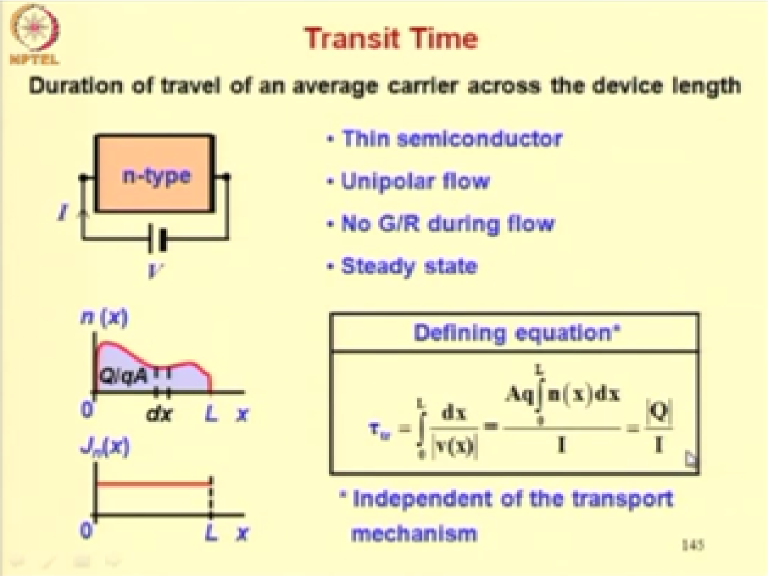

Transit Time

Duration of travel of an average carrier across the device length

- Thin semiconductor
- Unipolar flow
- No G/R during flow
- Steady state

Defining equation*

$$\tau_{tr} = \int_0^L \frac{dx}{|v(x)|} = \frac{Aq\int_0^L n(x)dx}{I} = \frac{|Q|}{I}$$

* Independent of the transport mechanism

So, this is our formula for transit time and this formula allows us to give an alternate interpretation of the transit time.

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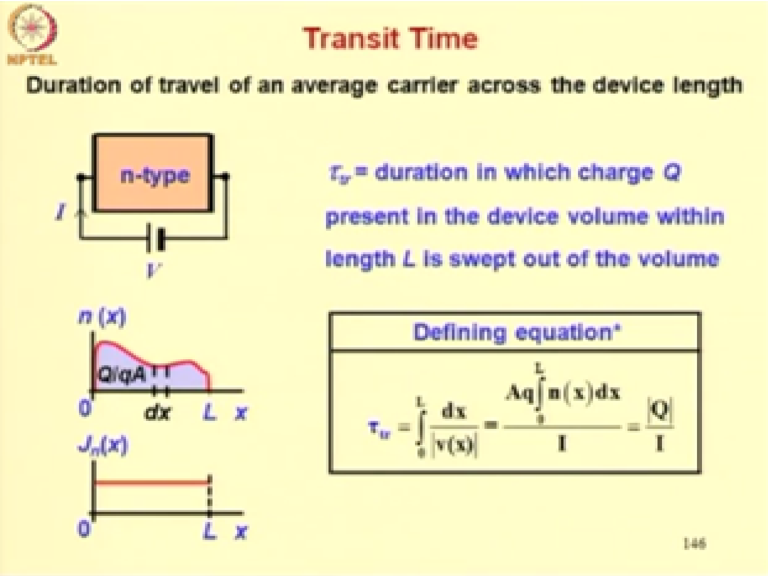

Transit Time

Duration of travel of an average carrier across the device length

τ_{tr} = duration in which charge Q present in the device volume within length L is swept out of the volume

Defining equation*

$$\tau_{tr} = \int_0^L \frac{dx}{|v(x)|} = \frac{Aq\int_0^L n(x)dx}{I} = \frac{|Q|}{I}$$

So the transit time is a duration in which the charge Q present in the device volume within length L is swept out of the volume. So, you can see that in this time supposing at any instant you have a number of electrons within the volume whatever way they may be distributed, you observe duration of transit time. The electron at this would have reached this end and therefore the entire charge within this volume would have been swept out.

So, this is how we are interpreting the transit time in an alternate manner. Now, you might wonder that when we were considering the characteristic times earlier such as momentum relaxation time, energy relaxation time, dilated relaxation time, minority carrier lifetime and so on, we consider transient situations, whereas here we are considering a steady state situation. Now can we not have a transient situation analogous to the situations we considered for transit times in the previous lectures.

Yes, we do have a transient situation where transit time will be used. For example, if you have a MOSFET and you switch the gate voltage, how long does it take for the drain current to be steady state. The time required for the drain current to reach steady estate is of the order of transit time. So, this situation and a similar situation for bipolar transistors when you switch the base of the bipolar transistor how long does the electric current take to stabilize.

That would also be transit time across the base. So, these situations will be discussed at the end of discussion of all the characteristic times and lengths.

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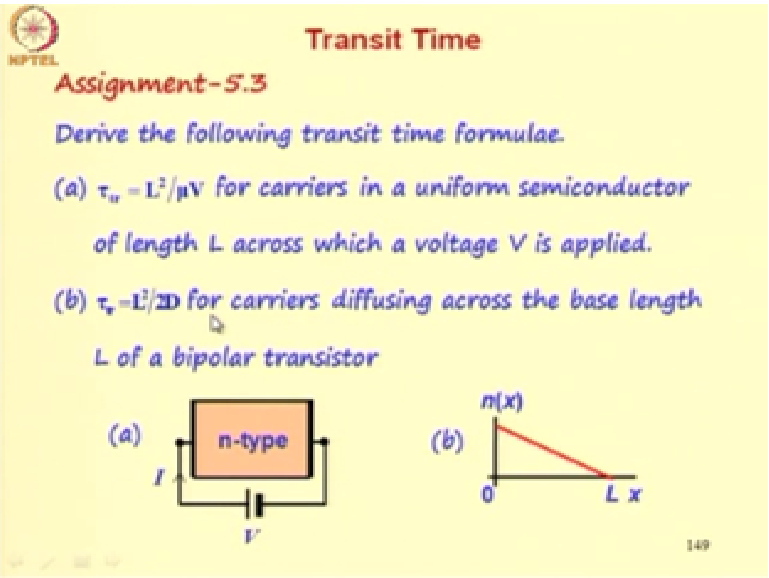

Transit Time

Assignment-5.3

Derive the following transit time formulae.

(a) $\tau_{tr} = L^2/\mu V$ for carriers in a uniform semiconductor of length L across which a voltage V is applied.

(b) $\tau_{tr} = L^2/2D$ for carriers diffusing across the base length L of a bipolar transistor

So for now, here is an assignment for you on transit time. So, derive the following transit time formulae. So, case A, the transit time for drift current is given by L square/mobility into applied voltage for carriers in uniform semiconductor of length L across which the voltage V is applied. So, this is your semiconductor and if you assume the electrons concentration to be uniform, there is no diffusion current there can only be drift current.

So, for the situation, this is the transit time. You have to show that. Similarly, in the case of diffusion, transit time is given by L square/2 times the diffusion coefficient for carriers diffusing across the base length L of a bipolar transistor. So, if you assume the L to be the length of the base of a bipolar transistor and you have a linear distribution of carriers. You take an NPN bipolar transistor for example, and you have only diffusion, recombination is negligible.

So, for this case you have to establish this particular formula for transit time.

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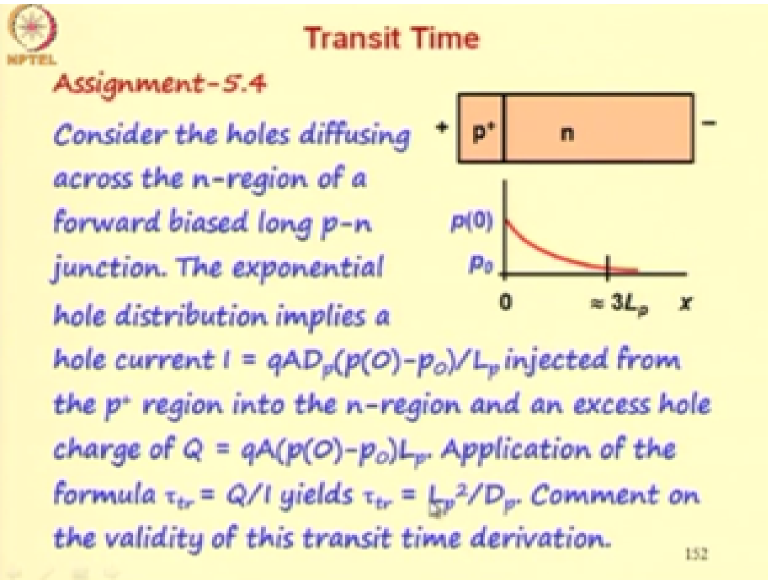

Transit Time

Assignment-5.4

Consider the holes diffusing across the n-region of a forward biased long p-n junction. The exponential hole distribution implies a hole current $I = qAD_p(p(0)-p_0)/L_p$ injected from the p^+ region into the n-region and an excess hole charge of $Q = qA(p(0)-p_0)L_p$. Application of the formula $\tau_{tr} = Q/I$ yields $\tau_{tr} = L_p^2/D_p$. Comment on the validity of this transit time derivation.

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Another assignment, consider the holes diffusing across the N-region of the forward biased long P-N junction. So, here this N region is assumed to be long, so that the excess carrier distribution here is exponential. The exponential hole distribution implies a hole current I = Q times area of the cross-section times the diffusion coefficient of holes into P(o)- P0 which is excess carrier concentration at X = 0 divided by LP injected from the P + region

Into the N region and an excess hole charge of Q = Q times A into excess carrier concentration at X = 0 into diffusion length. Now application of the formula; transit time = Q/I yields transit time = LP square/DP. So you substitute Q from here and I from here, you will get LP square/DP. Comment on the validity of this transit time derivation. So, is this correct. Can we use this LP square/DP result as a transit time of holes for travelling the long N region of P+N junction, that is the assignment.

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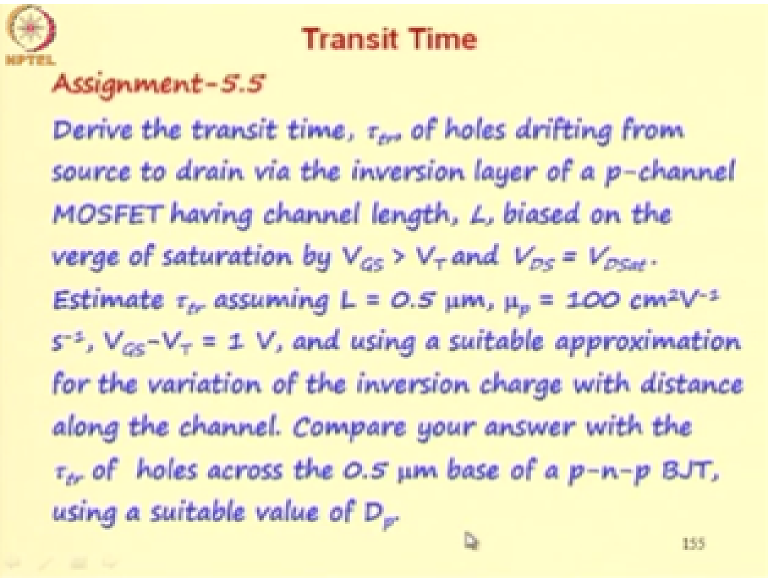

Transit Time

Assignment-5.5

Derive the transit time, τ_{tr}, of holes drifting from source to drain via the inversion layer of a p-channel MOSFET having channel length, L, biased on the verge of saturation by $V_{GS} > V_T$ and $V_{DS} = V_{DSat}$. Estimate τ_{tr} assuming $L = 0.5$ µm, $\mu_p = 100$ cm^{2}V^{-1}s^{-1}, $V_{GS}-V_T = 1$ V, and using a suitable approximation for the variation of the inversion charge with distance along the channel. Compare your answer with the τ_{tr} of holes across the 0.5 µm base of a p-n-p BJT, using a suitable value of D_p.

Third assignment derive a transit time tau TR of holes drifting from source to drain via the inversion layer of a P-channel MOSFET having channel length L biased on the verge of saturation by gates source voltage VGs more than VT and VDS = VDSat. Estimate the transit time assuming L equal 0.5 microns, UP = 100 cm square per V second and VGs - VT = 1 V and using a suitable approximation for the variation of the inversion charge with distance along the channel.

Compared your answer with the transit time of holes across the 0.5-micron base of a PNP BJT using a suitable value of diffusion coefficient of holes. So, here you will have to bank on your knowledge of MOSFET and bipolar transistors you have acquired in the level course on solid-state devices. So, if you do not have this knowledge, please refer to the lectures of the first level course on solid-state devices available in YouTube.

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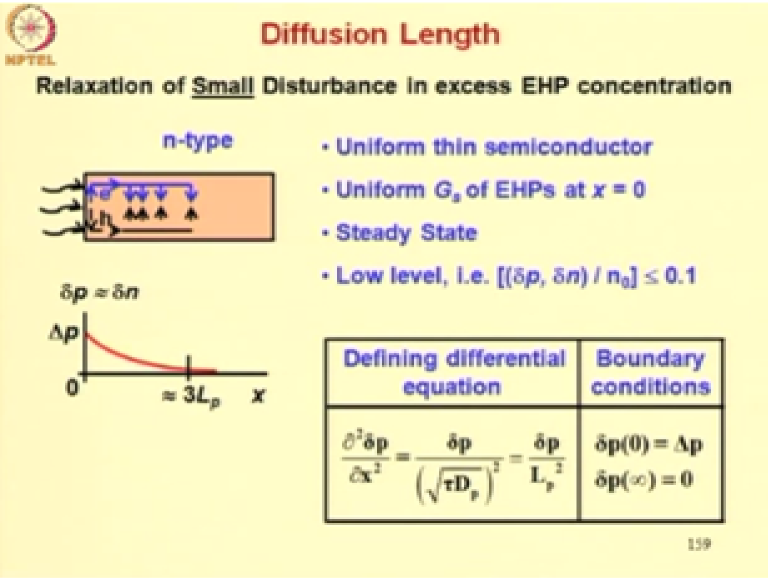

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_0] \leq 0.1$

Defining differential equation	Boundary conditions
$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^2}$	$\delta p(0) = \Delta p$ $\delta p(\infty) = 0$

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Let us move on to a characteristic length, namely the diffusion length. So far we were considering characteristic times. Now, this is the first characteristic length that we are discussing. The diffusion length is associated with relaxation of small disturbance in excess electron hole pair concentration. Here is a physical situation by way of example, so you have surface generation of carriers at one end.

So electron hole pairs are being generated in a thin volume near the surface and these electrons and holes move inside into the sample away from the surface because of diffusion. Now there is not just this diffusion as we will see. There can be drift also, but primarily the driving force for electrons and holes to move into the sample from the surface is the concentration gradient and that is a diffusion.

So putting down the conditions for this physical situation, a uniform thin semiconductor thin so that we can assume one-dimensional situation, uniform surface generation Gs of electron hole pairs at X = 0, so here the uniform word means uniform over the area of cross-section of the surface. Then, we assume steady-state conditions and we assume low-level conditions. Now, we will find that the excess holes and electrons will decay as a function of distance.

It will establish that the decay will happen over 3 times a length constant called the diffusion length of minority carriers here namely the holes because this is an N type semiconductor.

Further, the access electron and hole concentration would be approximately equal. Now, they would not be exactly equal, the reason for which will become clear now. However, they can be assumed to be very close to each other.

The defining differential equation for this case is a second order differential equation in X, dou square delta P/dou X square = delta P/square root of T times DP square which can be cast in the form delta P/LP square where LP = square root of the minority carrier lifetime tau into the diffusion coefficient DP and since this is a second order differential equation, you will have 2 boundary conditions.

Delta P at X = 0 is Delta P and faraway from the surface, there is delta P at X = infinity would be zero because the excess carriers would decay to equilibrium or relax to equilibrium. Now note that for characteristic times, except the transit time, the defining differential equation was the first-order equation in time, whereas for the characteristic length that we are considering now, it is a second-order differential equation in X.

However, as we will see the form of the equation is such that the solution would again be exponential as in the case of characteristic times. Now going on to derive the differential equation, the first step is sketching of NP, Jn, Jp, E, and psi versus X. Again we do not have anything as a function of T because there is a steady state situation. So, let us look at the results of the qualitative analysis.

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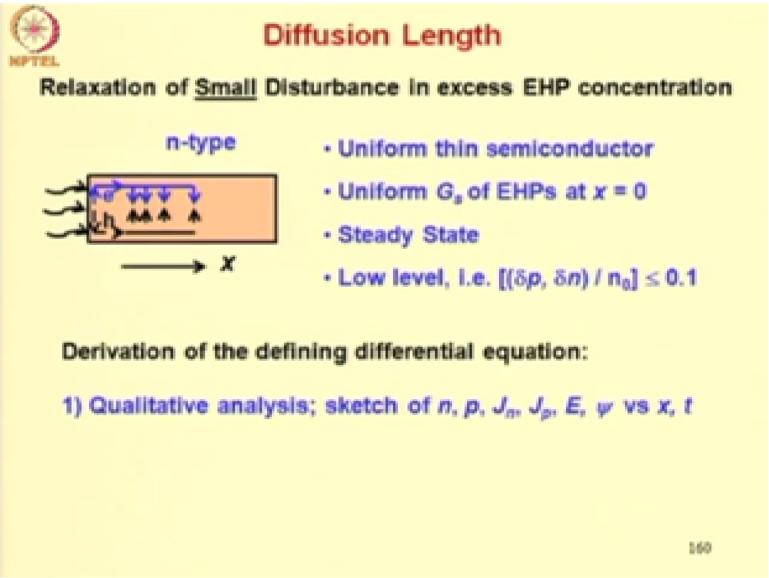

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

n-type

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_0] \leq 0.1$

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

So, this is your surface generation of an N type semiconductor. Your electron concentration will vary something like this and hole concentration also would vary something like this. Now we are showing both electron and hole concentrations on the same graph and on linear scale because if I use a log scale, since it is a low injection level condition, I will not be able to show any variation of the electron concentration. It would look just approximately constant.

Though I will be able to show the variation in the hole concentration. So, I will leave as an excise to you to plot the same graph on a semi-log plot where you use a log scale for concentration as a function of X, X will be a linear axis. Now, here these values are equilibrium values of electron and hole concentrations and these are the values of excess carrier concentrations at the surface where illumination is happening.

So delta N and P, we will find that delta N is approximately = delta P. Now, the concentration is decaying like this for electrons and holes. We show a cut here because both are on linear scale. The hole and electron currents, since the holes are moving from left to right, the current is positive and evidently the current will go on decaying because the carriers are recombining. Now that is what has been shown here by the flow diagram.

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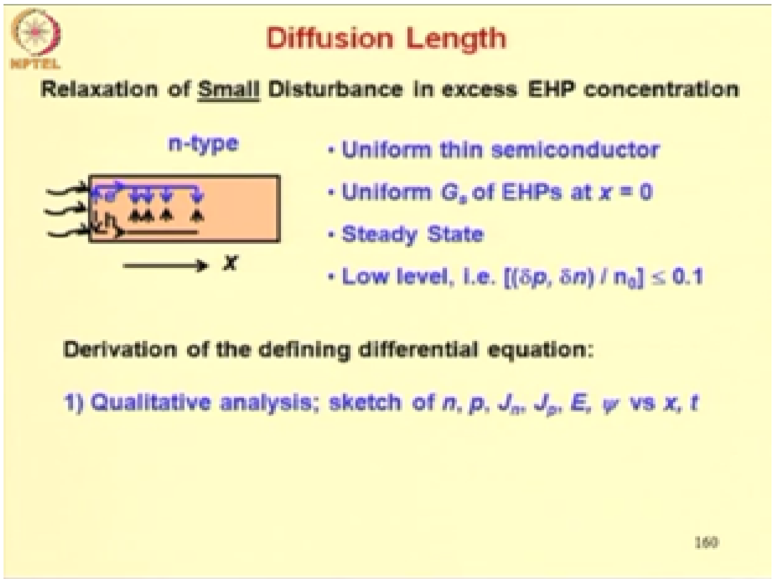

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_0] \leq 0.1$

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

So this arrow is here, they show recombination of carriers, okay and here the arrows are close together because recombination is more because excess carrier concentration is more and as you move away, the recombination goes on decreasing because the excess carrier concentration goes on decaying.

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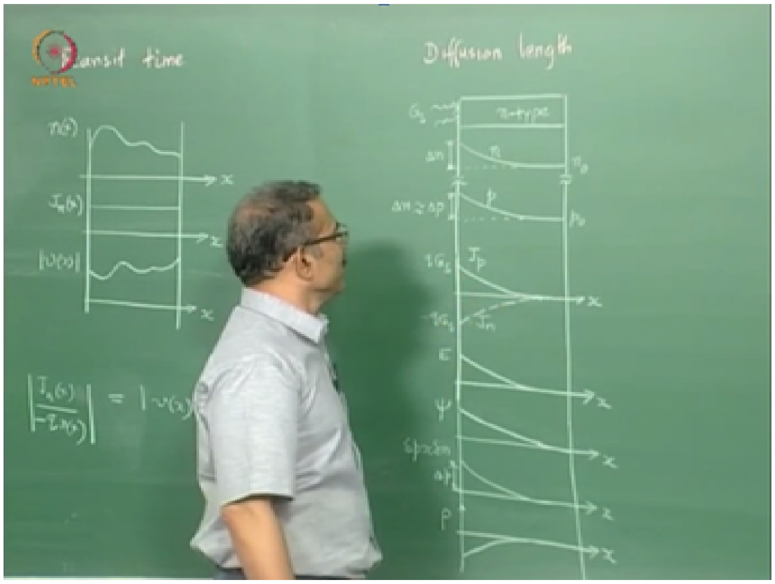

So, because of this reason, your current also will go on decaying and ultimately go to 0, faraway. The electron current, electrons are moving from left to right that implies the current from right to left and therefore the electron current density is shown on the negative axis. Now, since no current is injected into the sample, Jp+Jn should be = 0 and that is why the electron current distribution is a mirror image of the hole current distribution over this X axis.

Also, the Gs puts a boundary condition on Jp at X =0. The hole current is simply Q times Gs and electron current is simply - Q times Gs at this boundary. Coming to the electric field. Now you see that electrons move faster than holes. So, these electrons when they are diffusing, they will diffuse faster than holes why because diffusion coefficient of electrons is more than the diffusion coefficient of holes.

You know that DN/DP is 2.5 to 3 times and therefore if there was diffusion alone, then electrons will move faster than holes and electron current would be more than hole current. However, we want Jp+Jn to be 0, which means electron current magnitude should be equal to hole current magnitude. Now the only way you can achieve this is to have an electric field setup.

We have discussed this point in one of the earlier modules when we introduced the concept of mechanism of diffusion current. So, this electric field will slow down the electrons, but it will aid the holes and that is how it will bring Jp and Jn instep. So, at any X you will find the Jp will be equal in magnitude to Jn. So, evidently this field will be more wherever the concentration is more and this field will decay because faraway the carrier concentrations are becoming equal to equilibrium value and therefore there you do not need any field.

Potential, you can integrate the electric field and get the potential. Now potential variation, potential will be more at the X = zero point and then it will decrease. The electric field is from left to right which evidently means potential should be more on the left and less on the right that is what is shown here. We have assumed arbitrarily that the reference potential at X, very large, is 0 and with respect to this reference potential, we are plotting the entire potential.

Now, we can derive information about excess electron and hole concentration from N and P. So, because you think that neutrality should hold, you would expect delta P = delta N. However, you see that electric field is varying with X and Gauss's law therefore predicts that de/dx is non-zero. So, you know the Gauss’s law de x dx = rho/epsilon S. So, if D/dx is non-zero which is the case here to keep the hole current and electron current instep.

Then rho is non-zero which means delta N and delta P cannot be exactly equal. They will be only approximately equal. So long as we show that this rho is small which is what we will do later and this will be done by you as an assignment, so you will find that delta P is not exactly = delta N but approximately = delta N and therefore the excess carrier concentration can be shown as a single line here, initial value being delta P.

Now, we are writing these carrier concentrations in terms of minority carrier concentrations because as we will see that it is easy to solve for minority carrier concentration in this kind of a situation. The space charge rho can be derived. Now, how could you derive space charge. You cannot derive it from this information right because you do not know what is the difference between delta P and delta N.

However, using Gauss's law and this is electric field distribution, I can derive this space charge. So, D/dx is negative, therefore the space charge is negative and it decays because D/dx become 0, space charge become 0. Now how is a device charge neutral. You see though there can be space charge within a device, as a whole the device has to be charge neutral because no extra positive or negative charges can get into the device.

The positive and negative charges in the device have to be equal in magnitude. Now that overall neutrality is maintained by having a positive chargesheet at this end that is what is shown here, so this is a delta function. Now you can have a negative chargesheet also at the right end if there is a change in electric field here. However, you find that electric field is going to 0 and therefore there is no change in electric field when you move from within the device to outside.

Therefore, there is no surface charge here. So, this is the positive surface charge. Its value is equal to the area under this space charge current, okay, this negative part. So, that is our qualitative analysis of the situation. Now, we will use this qualitative analysis to get the equation.

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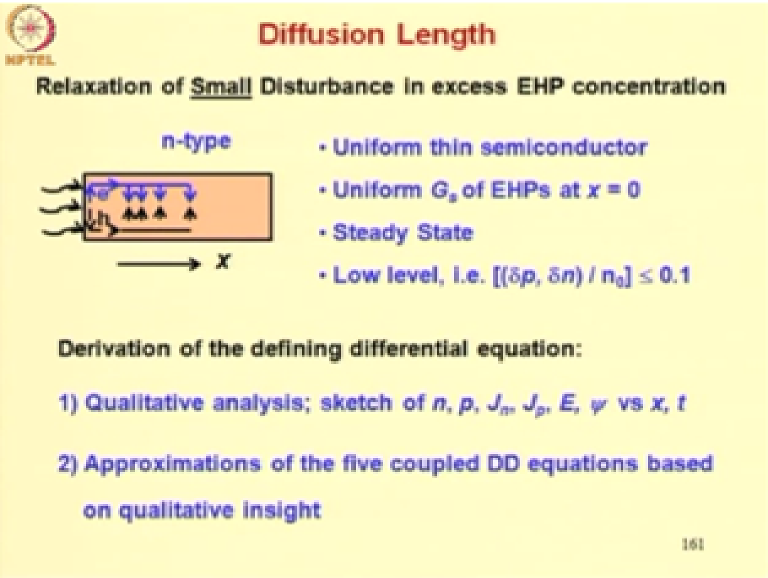

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_0] \leq 0.1$

Derivation of the defining differential equation:

1) Qualitative analysis; sketch of n, p, J_n, J_p, E, ψ vs x, t

2) Approximations of the five coupled DD equations based on qualitative insight

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So, the next step is approximations of the 5 coupled drift-diffusion equations based on qualitative insight. These are our 5 equations. Now uniform semiconductor. Because a semiconductor is uniform, n0 and P0 are constant with X, we can replace the gradient of N by gradient of delta N and gradient of P/gradient of delta P. Because gradient of delta N = gradient of N+gradient of n0 and gradient of n0 is 0 and similarly for holes.

Now, uniform surface generation. This means there is no volume generation and therefore we remove the excess generation terms from the continued equations. Since it is steady state, we can cancel out or remove the time varying concentration terms from the continued equation.

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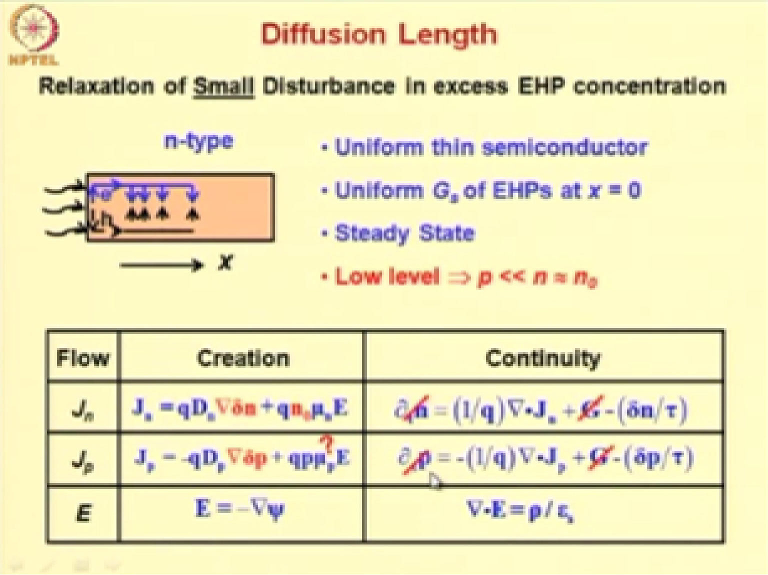

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level $\Rightarrow p \ll n \approx n_0$

Flow	Creation	Continuity
J_n	$J_n = qD_n \nabla\delta n + qn_0\mu_n E$	$\cancel{\partial n} = (1/q)\nabla\cdot J_n + \cancel{G} - (\delta n/\tau)$
J_p	$J_p = -qD_p \nabla\delta p + qp\overset{?}{\mu_p} E$	$\cancel{\partial p} = -(1/q)\nabla\cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

An important approximation finally or assumption is the low level which means the hole concentration is much less than the electron concentration which is approximately equal to the equilibrium electron concentration. This is the situation for low-level, now as a result you see the electron concentration here in the current density equation for electrons is replaced by n0. Now, there is a question mark put here what do we do with the drift current equation for minority carriers, okay.

Since P is much < normal, we anticipate that this drift current would be much less than the drift current of electrons. So, drift current of holes would be much less than drift current of electrons. Therefore, is a chance that we can remove the drift current. Well to know that, we must compare this drift current with the diffusion current of holes, right. It is not sufficient if I know the drift current of holes is much less than drift current of electrons because the term for drift current of holes appears along with the diffusion current of holes.

So, for now we will postpone this approximation of the drift current of holes and let us move further.

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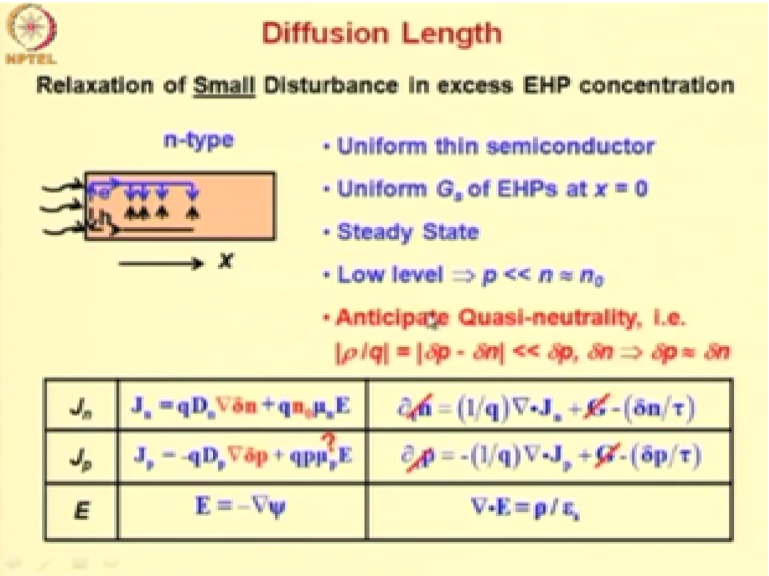

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level $\Rightarrow p << n \approx n_0$
- Anticipate Quasi-neutrality, i.e. $|\rho/q| = |\delta p - \delta n| << \delta p, \delta n \Rightarrow \delta p \approx \delta n$

J_n	$J_n = qD_n\nabla\delta n + qn_0\mu_n E$	$\cancel{\partial_t n} = (1/q)\nabla\cdot J_n + \cancel{G} - (\delta n/\tau)$
J_p	$J_p = -qD_p\nabla\delta p + qp\overset{?}{\mu_p}E$	$\cancel{\partial_t p} = -(1/q)\nabla\cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot E = \rho/\varepsilon_s$

Now, we anticipate a very important formation namely quasi-neutrality. What does this means. This means rho/queue the modulus which is = the modulus of delta P - delta N is much less than delta P or delta N okay. So, the difference between the excess carrier concentrations, the magnitude of that is much less than the individually the values of excess carrier concentration and that will allow us to make the approximation delta P approximately equal delta N.

So, please remember, so Delta P approximately = delta N does not mean delta P - delta N =0 but it means that this difference is much less than the excess carrier concentrations. Let us see the important consequences of this approximation.

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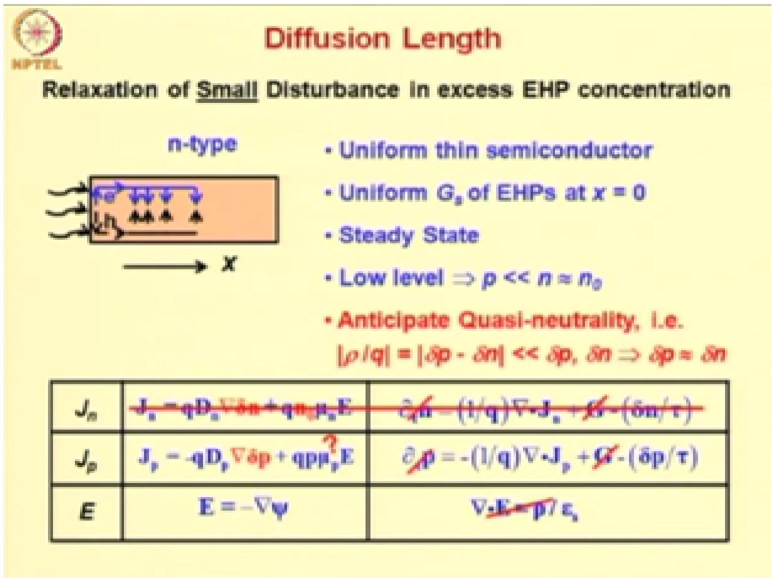

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level $\Rightarrow p << n \approx n_0$
- Anticipate Quasi-neutrality, i.e. $|\rho/q| = |\delta p - \delta n| << \delta p, \delta n \Rightarrow \delta p \approx \delta n$

J_n	~~$J_n = qD_n\nabla\delta n + qn_0\mu_n E$~~	~~$\partial \cancel{n} = (1/q)\nabla\cdot J_n + \cancel{G} - (\delta n/\tau)$~~
J_p	$J_p = -qD_p\nabla\delta p + q\overset{?}{p}\mu_p E$	$\partial \cancel{p} = -(1/q)\nabla\cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot \cancel{E = \rho}/\varepsilon_s$

So the first important consequence is that I do not have solve for the electron and hole concentrations both. I can solve for anyone of them and I can get the other one because of the approximate equality between the excess electron and whole concentration, so I strikeout the 2 equations corresponding to the electrons because electrons are majority carriers here and you see there appears to be a chance that I could neglect the diffusion current of holes.

Therefore, there is a possibly the simplification of the hole current equation. For electron current, I cannot make this simplification because I know the drift current is really large, okay, because the carrier concentration of electrons is large and therefore I will have the consider both diffusion and drift. There is also gradient of the excess electron concentration, so I cannot neglect diffusion.

So, since there appears to be a possibility of simplifying the current equation for minority carriers, I prefer to work with the minority carrier equations here. So, that is the logic in removing the equations for majority carriers if there is a choice between majority and minority carriers.

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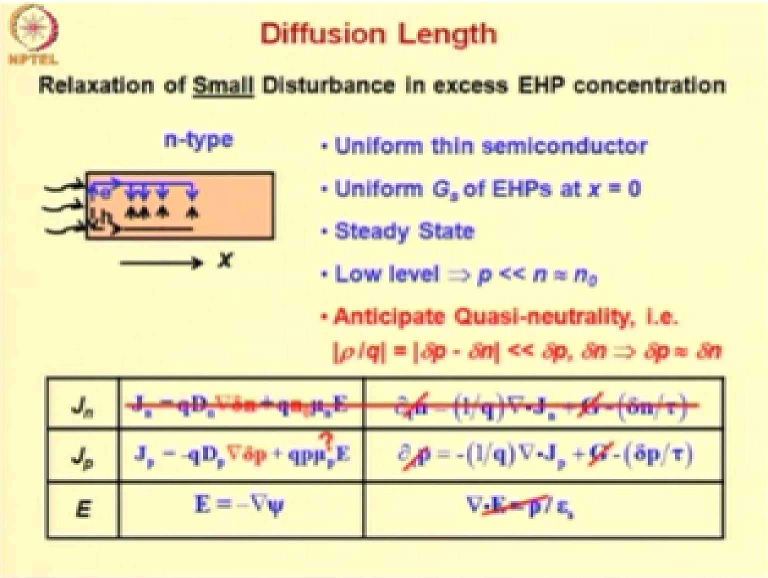

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level $\Rightarrow p << n \approx n_0$
- Anticipate Quasi-neutrality, i.e. $|\rho/q| = |\delta p - \delta n| << \delta p, \delta n \Rightarrow \delta p \approx \delta n$

J_n	~~$J_n = qD_n\nabla\delta n + qn_0\mu_n E$~~	~~$\cancel{\partial_t n} = (1/q)\nabla\cdot J_n + \cancel{G} - (\delta n/\tau)$~~
J_p	$J_p = -qD_p\nabla\delta p + \overset{?}{qp\mu_p E}$	$\cancel{\partial_t p} = -(1/q)\nabla\cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot \cancel{E = \rho}/\varepsilon_s$

Next, since I am going to assume quasi-neutrality, so rho is going to be small and therefore I may not have to use this Gauss's law in my ultimate solution, though while establishing the validity of quasi-neutrality, I may use this relation. Now you see that at this point, once I assume or anticipate quasi-neutrality, I can strike off this drift term for holes. Now let us see the logic in how quasi-neutrality assumption allows us to strikeout this particular term.

So, in other words now we shall be able to prove that this will be much less than the diffusion current of holes that is this term.

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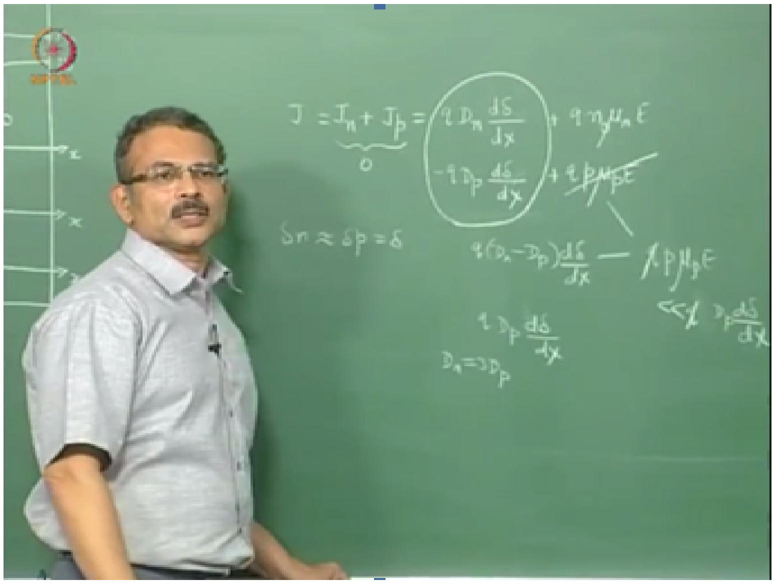

Now, you know that Jn+Jp which is the total current J is 0 here for our situation. Now I can write Jn+Jp as shown here. So, this is diffusion current of electrons, drift current of electrons + diffusion current of holes + drift current of holes. We are assuming a one-dimensional situation that is why the gradients have been expressed in terms of X direction alone.

Now quasi-neutrality means delta N = delta P approximately therefore let me use delta as a symbol for that approximate value of delta N and delta P. So, I am assuming delta N approximately = delta P = delta. So that I can simplify things and therefore I can club these 2 terms together and write this as Q times DN - DP into D delta/DX. Now you see some of this current + this current + this current should be 0.

Now, I know that this current is very small compared to this current why because P is much less than. In fact, we have put n0 for the electron concentration under low level and because of low-level P is much less than n0. So, now you see in this equation this current and this current has come together on the same side of the equation when I sum up electron and hole currents.

Therefore, now I know that when this appears along with this, I can neglect this. So, that is what I do. I strike off this. So, now you will appreciate why I am able to strike off the hole current. You see that we realize that because of the fact that Jn+Jp should be = 0, it turns out that the diffusion current of electrons and holes put together which is what of this is of the same order as the drift current of the electrons, okay.

So, Jn diffusion+Jp diffusion together is of the same order as this. Now DN - DP is of the same order as DP. So, for example, if I take the term Q times DP into D delta/dx and compare DN - DP. So, DN is supposed 3 times DP I assume, then this quantity will be 2 times DP into D delta/dx into Q and this quantity is DP into D delta/dx into Q.

So, now if I can strike off this current when it appears along with this term and this term, I can say that this quantity is much less than DN - DP into D delta - dx and that is this much less than 2 times DP into D delta/DX. So, I am deriving the relation from these 2 I get QP, UP into E is much less than 2 times DP, D delta/DX. I am assuming DN = 3 times DP just to give you a feel.

Now, if this is true, I can even assume it to be much less than DP into D delta/DX, right assuming that order is the same right and I will find in fact this is really very, very small compared to 2 times DP. So, it will turn out to be much less than even DP into D delta right here. So, there is a Q here that gets cancelled and that is how I will be able to make the approximation that drift current of holes is much less than diffusion current of holes.

This is the so-called diffusion approximation for minority carriers okay, which is often used to simplify analysis of situations involving excess carrier decay. So that is how we have stuck off this term. So, now we can reduce the 5 approximated equations and the 6th equation to a single equation defining the characteristic length as follows.

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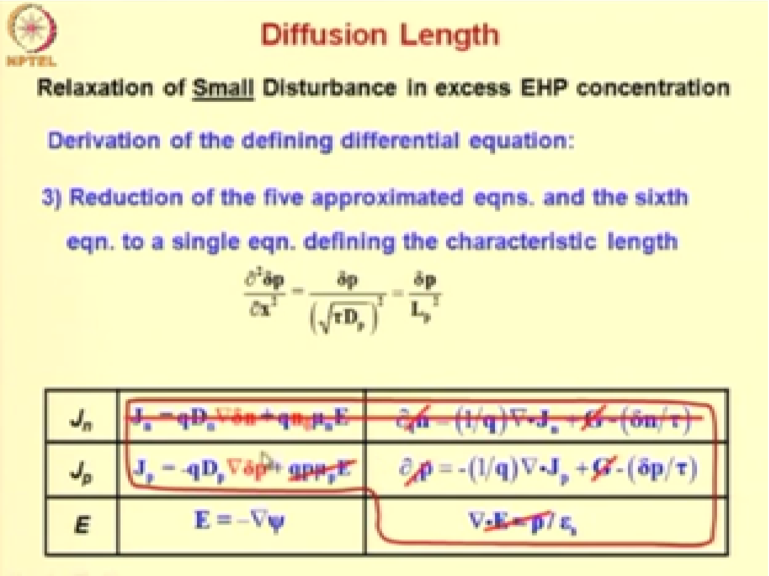

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^{\,2}}$$

J_n	~~$J_n = qD_n\nabla\delta n + qn_o\mu_n E$~~	~~$\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)$~~
J_p	$J_p = -qD_p\nabla\delta p + \cancel{qp\mu_p E}$	$\partial_t \cancel{p} = -(1/q)\nabla\cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot \cancel{E = \rho}/\varepsilon_s$

So, this is the equation you get it very easily as follows. So, for the hole current density you have only the diffusion current - QDP gradient of Delta P and here you have only 2 terms in a continued equation -1/Q queue diversions of Jp - delta P/Tau = 0. So, I substitute Jp into this. So, diversions of Jp becomes Q times delta P into Del square delta P when I substitute this here and negative sign will go away because there is a negative sign here and negative sign here.

Further, there is a Q here and Q here. so that will cancel each and therefore this term simplifies to simply dou square/dou X square of delta P. Now, there is a DP term here which will be clubbed with the minority carrier lifetime here. So, this term appears as delta P/Tau into DP that has been written as square root of Tau DP square. Now what is a logic in doing this. Now, you see that Tau into DP has units of length square because Tau is seconds, DP is centimeter square per second.

So, when you multiply this you get centimeter square and therefore square root of T into DP has units of length and that is why this is then cast as a length square.

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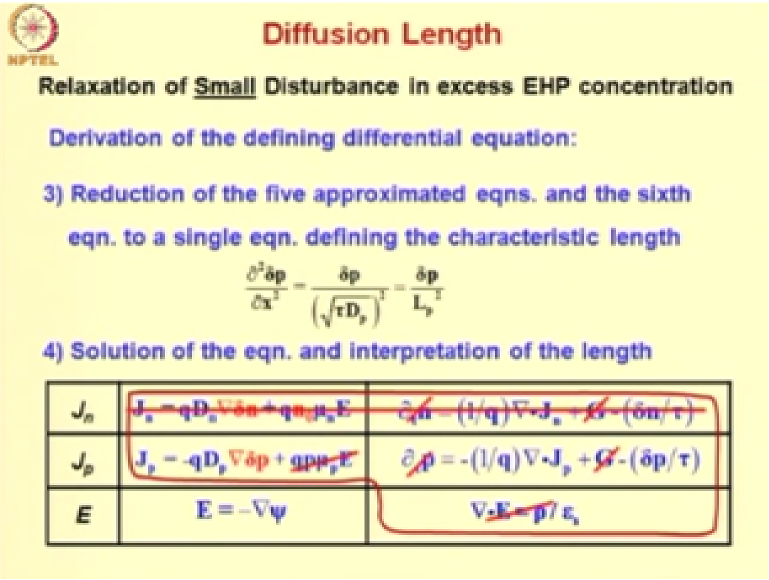

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

Derivation of the defining differential equation:

3) Reduction of the five approximated eqns. and the sixth eqn. to a single eqn. defining the characteristic length

$$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^2}$$

4) Solution of the eqn. and interpretation of the length

J_n	~~$J_n = qD_n \nabla \delta n + qn\mu_n E$~~	~~$\partial n = -(1/q)\nabla \cdot J_n + G - (\delta n/\tau)$~~
J_p	$J_p = -qD_p \nabla \delta p + \cancel{qp\mu_p E}$	$\partial \cancel{p} = -(1/q)\nabla \cdot J_p + \cancel{G} - (\delta p/\tau)$
E	$E = -\nabla \psi$	$\nabla \cdot \cancel{E = \rho} / \varepsilon_s$

Now comes a solution of the equation and interpretation of the length.

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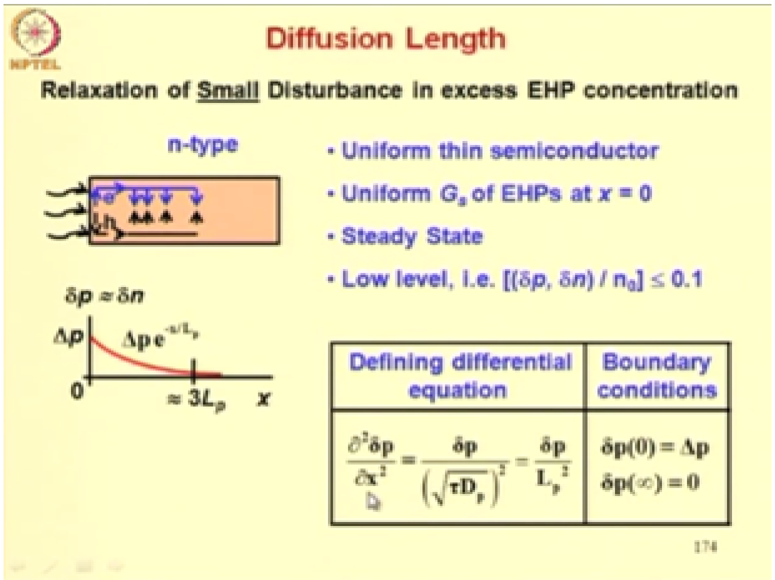

Diffusion Length

Relaxation of Small Disturbance in excess EHP concentration

- Uniform thin semiconductor
- Uniform G_s of EHPs at $x = 0$
- Steady State
- Low level, i.e. $[(\delta p, \delta n) / n_0] \leq 0.1$

Defining differential equation	Boundary conditions
$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{\left(\sqrt{\tau D_p}\right)^2} = \frac{\delta p}{L_p^2}$	$\delta p(0) = \Delta p$ $\delta p(\infty) = 0$

So, this is your differential equation and subjected to this boundary conditions it will have an exponential solution. You know this from your knowledge of differential equations very easily and therefore your solution is what is shown here. So delta P is capital delta P into E power -X/LP and therefore this LP can be interpreted as the average length over which the holes diffuse in the situation.

Now what is the situation. So, here you have holes recombining as they move, so therefore the diffusion length can be interpreted as the length over which an excess minority carrier moves before it recombines when there is a combination of diffusion and recombination, okay. So, either you can talk in terms of excess carriers or minority carriers. So, L suffix P is the length associated with holes which are minority carriers.

So, the average length over which a hole defuses before it tricks our minds. In this kind of situation where excess electrons and holes have been created at some point in the sample and thereafter they are diffusing and recombining, right.

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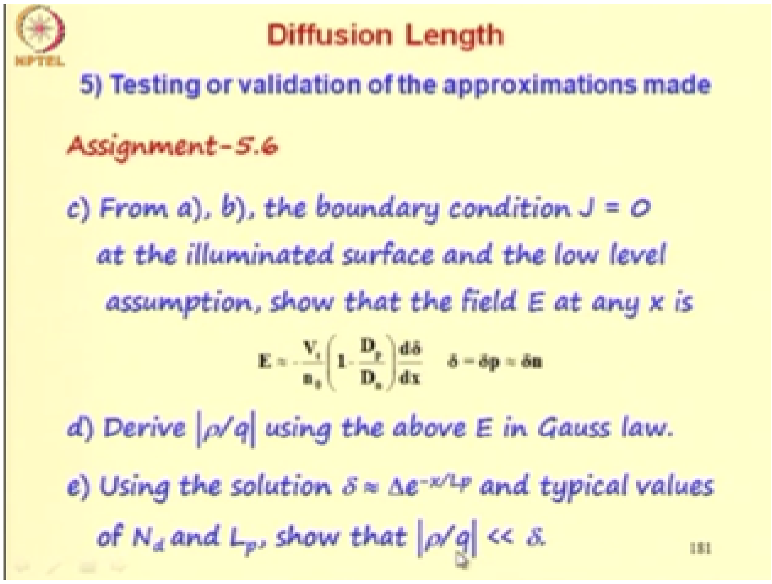

Diffusion Length

5) Testing or validation of the approximations made

Assignment-5.6

c) From a), b), the boundary condition $J = 0$ at the illuminated surface and the low level assumption, show that the field E at any x is

$$E \approx -\frac{V_t}{n_0}\left(1-\frac{D_p}{D_n}\right)\frac{d\delta}{dx} \qquad \delta = \delta p \approx \delta n$$

d) Derive $|\rho/q|$ using the above E in Gauss law.

e) Using the solution $\delta \approx \Delta e^{-x/L_p}$ and typical values of N_d and L_p, show that $|\rho/q| \ll \delta$.

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Now, finally it is very important to test or validate the approximation made. We have made significant approximations, namely the diffusion approximation for minority carriers and quasi-neutrality approximation. So, the testing or validation will be left to you as an assignment. So first the quasi-neutrality approximation. Establish the validity of the quasi-neutrality approximation in the situation introducing a minority carrier diffusion length as follows.

Assume the approximation to hold that is delta N is approximately = delta P. Now, this is a common method of testing approximation. We assume the approximation to hold, derive some result and see whether the result contradicts the assumption. If the result contradicts the assumption, then you know that the approximation does not hold. So, the next step show that under the conditions, delta N = delta P approximately and steady state the sum of the continued equations for N and P yield the result J = Jn+Jp is constant with X.

Next, from A and B the boundary condition J = 0 at the illuminated surface and the low-level assumption show sure that the field E at any X is given by this formula, that is E is approximately = - of the thermal voltage by equilibrium electron concentration into 1 - different quotient of holes by different coefficient of electrons into D delta/dx where delta = delta P approximately = delta N.

Now, derive modulus of rho/Q using the above. So, you have now an expression for the electric field, okay. Now this electric field is the same thing that we plotted here, so you see that the result that we just now showed on the slide amounts to relating the electric field to the excess carrier concentration, right. That is what we find, so electric field is related to gradient of excess carrier concentration.

Now, finally using the solution delta is approximately = the initial value of the excess carrier concentration into E power - X/LP and typical values of doping ND and diffusion length LP show that modulus of rho/Q is much less than delta. Now, that would be our quasi-neutrality approximation, right. So we have set that continuity of approximation is delta P - delta N is much < delta P or delta N.

Now delta P - delta N is here rho/Q and delta P or delta N is represented as delta here. Now, what would be typical value of ND, lets say 10 power 16 per centimeter cube. What would be a typical value of LP. Well you assume minority can lifetime of about say 0.1 m/sec okay and typical diffusion coefficient of say 10 cm square per second for holes and you can estimate this value.

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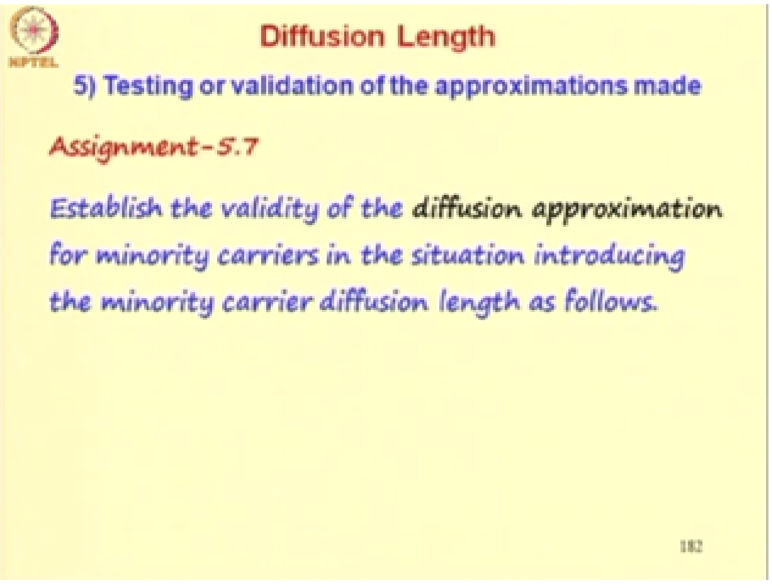

Diffusion Length

5) Testing or validation of the approximations made

Assignment-5.7

Establish the validity of the diffusion approximation for minority carriers in the situation introducing the minority carrier diffusion length as follows.

The next assignment 5.7 is establish the validity of the diffusion approximation for minority carriers in the situation introducing the minority carrier diffusion length as follows. Use the expression for electric field obtained in assignment 5.6 and the fact that P is approximately = delta and typical values of ND and LP that is a minority carrier diffusion length to show that the product of the diffusion coefficient of holes into the hole gradient DP/dx is much less than the product of hole concentration, hole mobility and electric field.

So this left hand side of this inequality represents the diffusion current and right hand side of inequality represents the drift current.

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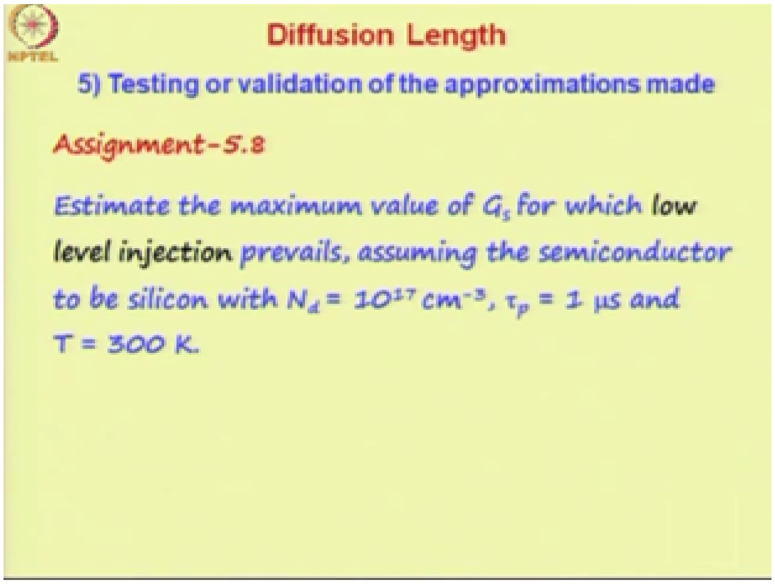

Diffusion Length

5) Testing or validation of the approximations made

Assignment-5.8

Estimate the maximum value of G_s for which low level injection prevails, assuming the semiconductor to be silicon with $N_d = 10^{17}$ cm^{-3}, $\tau_p = 1$ µs and T = 300 K.

Assignment 5.8. Estimate the maximum value of Gs for which low-level injection prevails assuming the semiconductor to be silicon with ND = 10 power 17 per centimeter cube, the lifetime of holes to be 1 msec and temperature to be 300K. So, the silicon is N type, therefore the lifetime is related to the holes.

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Summary of the lecture

So, we have come to the end of the lecture and so let us make a summary of the important points. So in this lecture we discussed the characteristic times namely transit time and characteristic length associated with minority carriers namely the diffusion length. For transit time, the defining differential equation was first-order as was the case with all other characteristic times that we discussed.

And the expression for transit time was the charge Q inside the device volume divided by the current I flowing into the device. So, the transit time was introduced considering a steady state situation. On the other hand, defining equation for diffusion length was second order and the formula for diffusion length was square root of diffusion coefficient into the lifetime associated with the minority carrier under consideration.

The transit time was the time taken by the carrier to travel from one point to another point within a device and diffusion length is the length over which are minority carrier defuses before it recombines in a situation where there is a flow of minority carriers due to a combination of diffusion and recombination. In the next lecture, we shall consider an important characteristic length, namely the Debye length.