Semiconductor Device Modeling

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Lecture – 22
Characteristic Times and Lengths

So in the previous lecture, we have begun a discussion of the characteristic lengths and times. In this module, we will discuss the derivation and utility of characteristic lengths and time. In the previous lecture, we have outlined how we would like to express the analysis of any situation in terms of graphs of electron concentration, hole concentration, electron current density, hole current density and electric field and potential and the function of distance and time.

So, any situation we analyze, we should express the various things that are happening in the situation in terms of these sketches. Now, all this is qualitative, we are not using any equations at this stage. The next step, we use the equations, we use the approximate forms of equations, so we also would like to learn in this module, how for various situations, the different equations are approximated to derive a simple relation for the characteristic time or length of interest.

We began the discussion on the dielectric relaxation time in the previous lecture, before that in the lecture, we also worked out, the various aspects of the minority carrier lifetime. So, we would like to continue with the discussion on dielectric relaxation lifetime in this lecture and also discuss other characteristic times.

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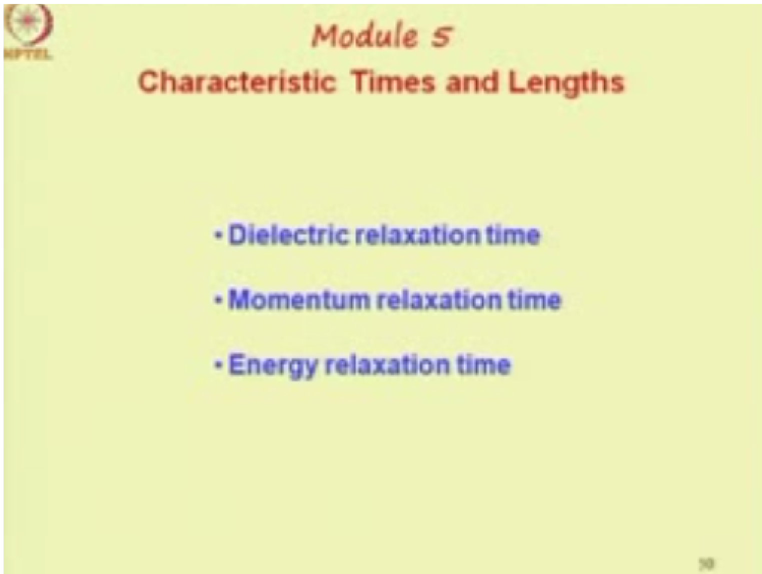

Such as the momentum relaxation time and the energy relaxation time.

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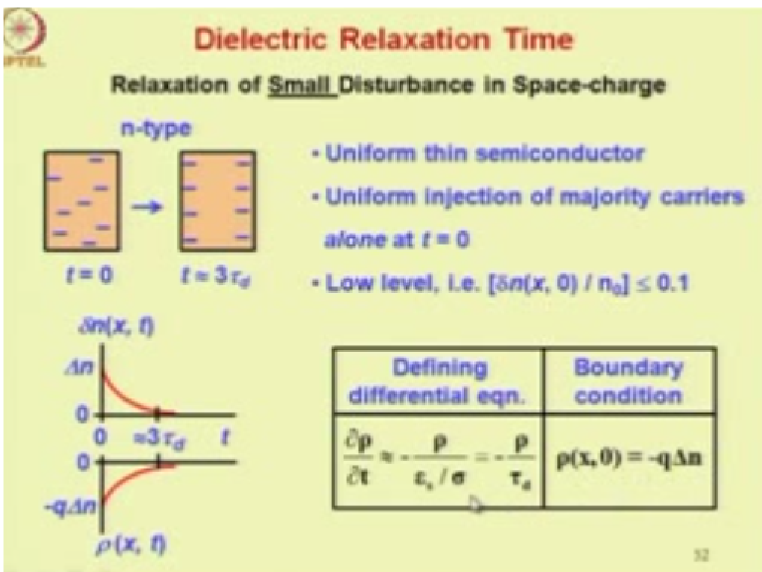

So this was the situation that we were considering in which majority carriers are injected uniformly throughout the volume at t = 0. The carrier concentration that is injected is very small compared to the majority carrier concentration, therefore, there is low injection level and we said that over a time, responding to the dielectric relaxation time the volume charge or space charge will decay to 0 and convert itself into charges at the surface.

So this is the process that we are analyzing, so we plotted the results of this process on graph like this, so here this shows the space charge as a function of time, for any x, since conditions are uniform for all x, the curve would be the same. Then, we also plotted the electron concentration, as a function of time, that is causing the space charge. The hole concentration is really not changing because we have not injected any extra holes, we are injecting only electrons.

Now, this was the defined differential equation, which we were trying to derive, and this is the boundary condition for solving this differential equation.

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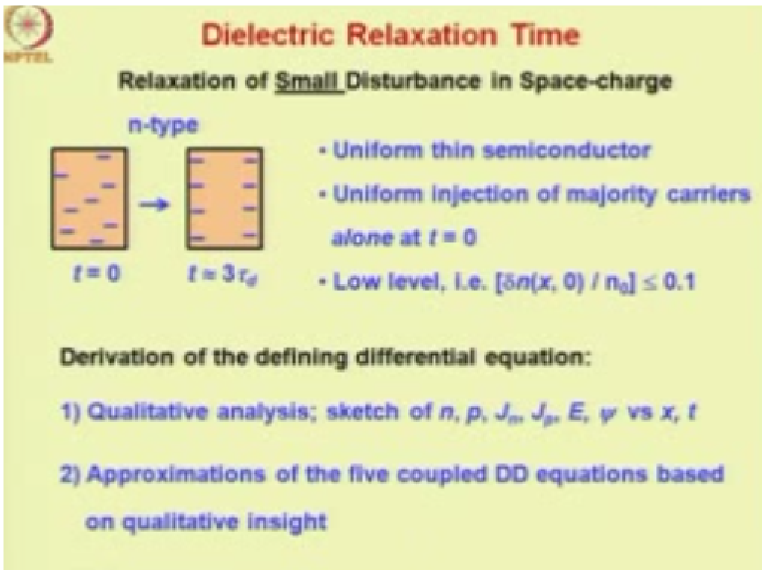

We completed the qualitative analysis in which we sketched n, p, Jn, Jp, E and psi versus x and t. We were going to approximate the coupled equations based on qualitative insight. So let us do that now.

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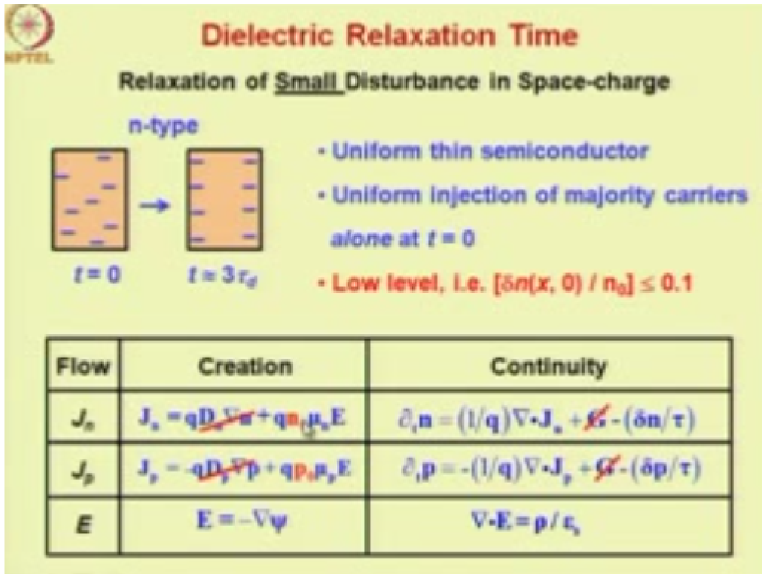

Now, these are the 5 coupled equations, as a first step, let us see the consequence of uniform in semiconductor and uniform injection of majority carriers alone at t equal 0. Now, let me remind you that we are considering a thin semiconductor because we would like to approximate the situation using a 1-dimensional analysis. So when we say thin semiconductor, it means this particular thickness of the semiconductor here is very, very small.

So you can approximate a situation as 1-dimensional, if the area of cross section is large, but the sample is very thin, right, then in the sample there is nothing moving to the sides, okay, so everything is moving from one end to the other end and that is why is the situation is really 1-dimensional. So what are the consequences of these uniform conditions. So the first important consequence is that there is no diffusion current of either electrons or holes.

So these 2 terms drop out because when np are uniform, gradient of n and gradient of p are 0. Now, also since there is the injection of carriers but no generation of access carriers. So we drop out this terms G from this continuity equation. Further since we are injection only electrons, the hole concentration remains equal to the equilibrium value, so the consequence of this namely injection of majority carriers alone the consequence of that condition is that p = p0.

And however will change as compared to the equilibrium, so it has been left just like that. Now, low level conditions, so, though n changes, the change in n is really very small as compared to n0, now this is important, please note this important point. So, the n is disturbed from equilibrium no doubt, for example, look at this.

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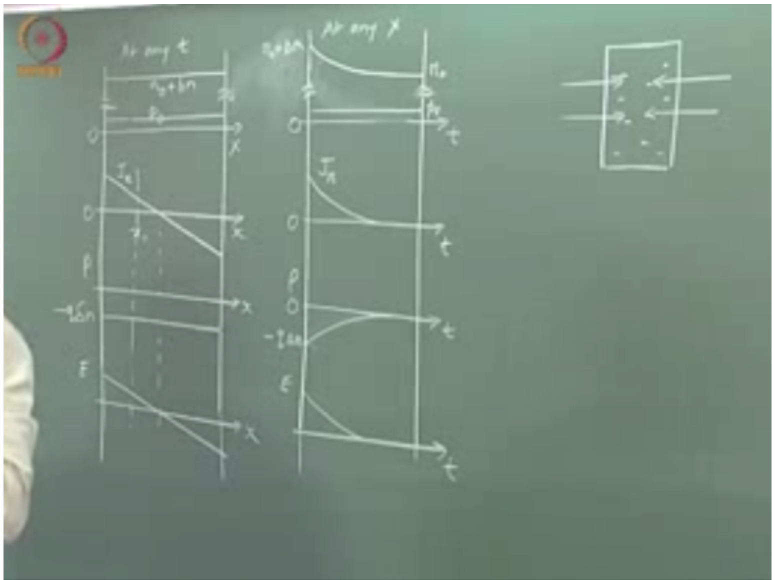

So your n at any x at as the function of time is different from n0, okay. So however, what we are saying is the disturbance is n is very, very small, low injection assumption and that is why in this equation here, where you had n that can be approximated by n0. The next step is reduction of the 5 approximated equations and the 6th equation to a single equation define the characteristic time. So these are the 5 approximated equations.

Now, how do we reduce them, to a single equation.

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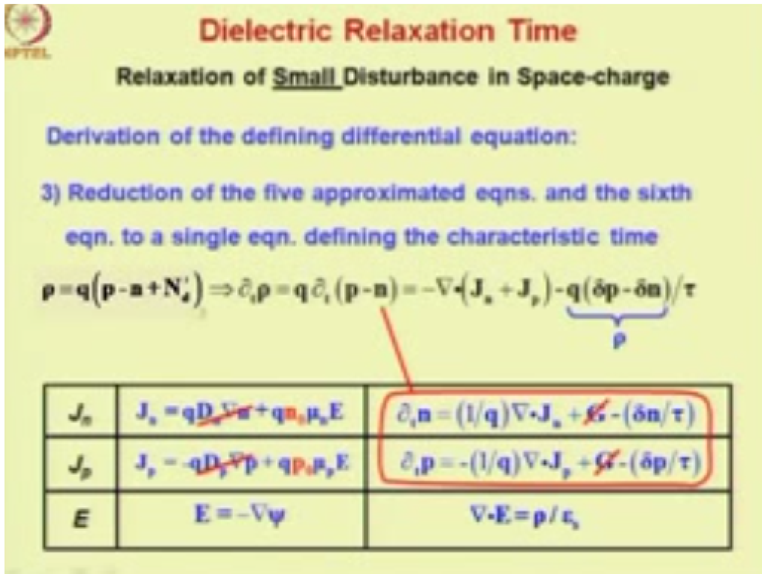

So, first note that we are interested in the space charge because that is really what we have disturbed and we want to know how the space charge is going to change with time. So, the equation for the space charge is p–n+nd+. Now as a consequence, if I take the time derivative gravity of the space charge, it would q times the time derivative of p–n, you see if I differentiate p–n+nd+, nd+ does not change with time, right.

So that is why, the differential of rho is differential of p-n. Now, we can write the expression for dou/dou t of p–n x q using these 2 continuity equations. So all that we do is we subtract the electron continuity equation from the hole continuity equation and consequence is shown here, so we have negative divergence of jn+jp–q times the delta p–delta n/tau. Now, delta p–delta n x q is nothing but rho, now this can be understood very easily from this equation.

Because under equilibrium conditions, there is no space charge, p0–n0+nd+ is really 0, okay.

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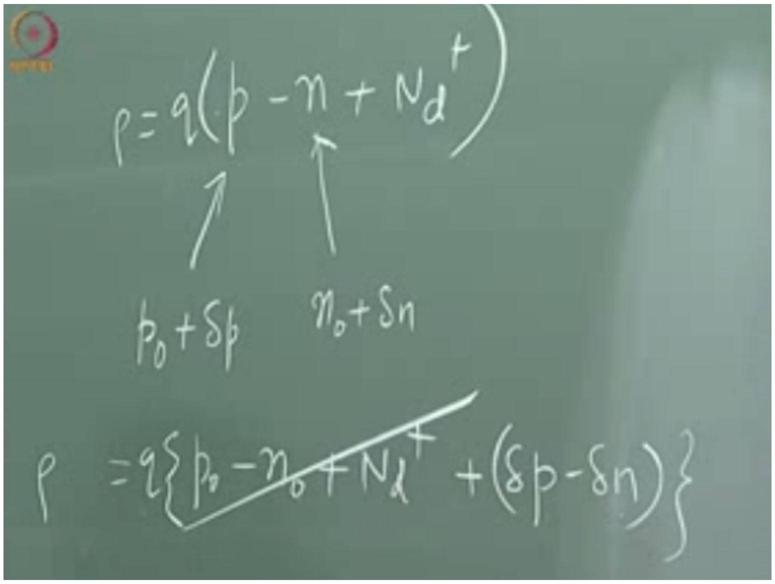

So consider this, this is p0+delta p, this is n0+delta n. Now, p0–n0+nd+, so I can write this quantity as p0–n0+nd+, +delta p–delta n. Now, this is the quantity we are saying is 0, because under equilibrium, the semiconductor is neutral, so therefore if rho is q times this, then we can write the same thing as q times this quantity is which is nothing but q times delta p–delta n. So that is what we are saying here.

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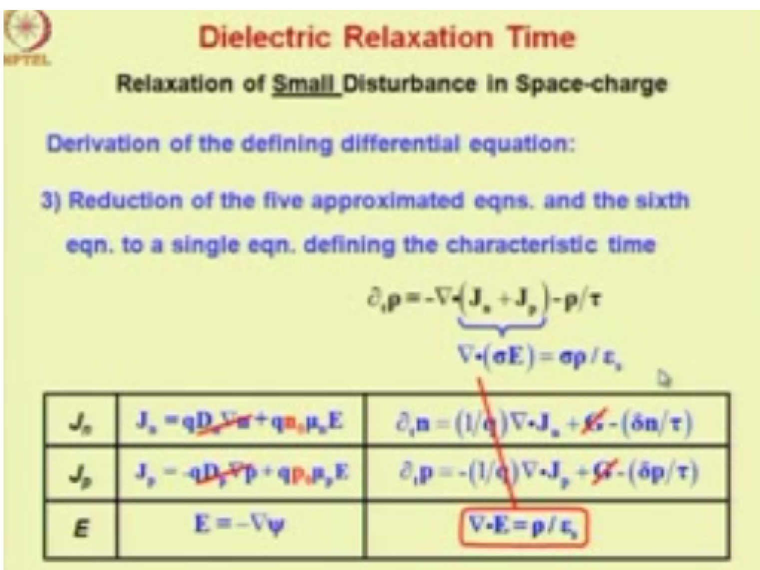

So consequently we have simplified the equation for time derivative of rho to this form using the se to continuity equations. Next, look at the sum jn+jp, that is = sigma times e, this is obtained by summing of j and jp here, where diffusion currents have been crossed out, they do not exist, so when I sum up the drift current of electrons and drift current of holes, I get this formula. So sigma = q times n0 mu n+q times p0 mu p.

Now I use the Gauss law, when I take the diversions of sigma e, because there is a diversion from here, I am divergence here. So divergence of sigma e is nothing but sigma into divergence of e because sigma is constant with x. And divergence of e is rho/epsilon s and that is how you get this tau here.

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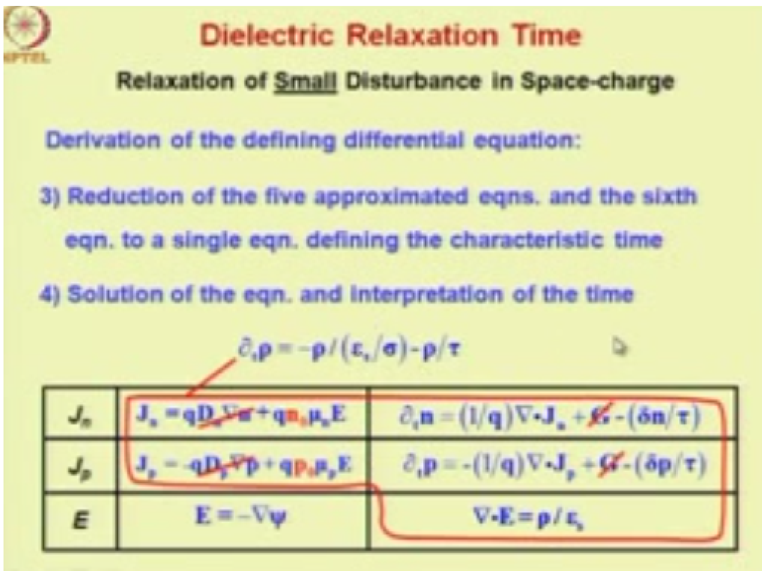

So consequently, when I club all the 5 equations, I have reduced these equations to this form, dou rho/dou rho = –rho/epsilon s/sigma–rho/tau. Now, we need to solve the equation and interpret the time constant over there.

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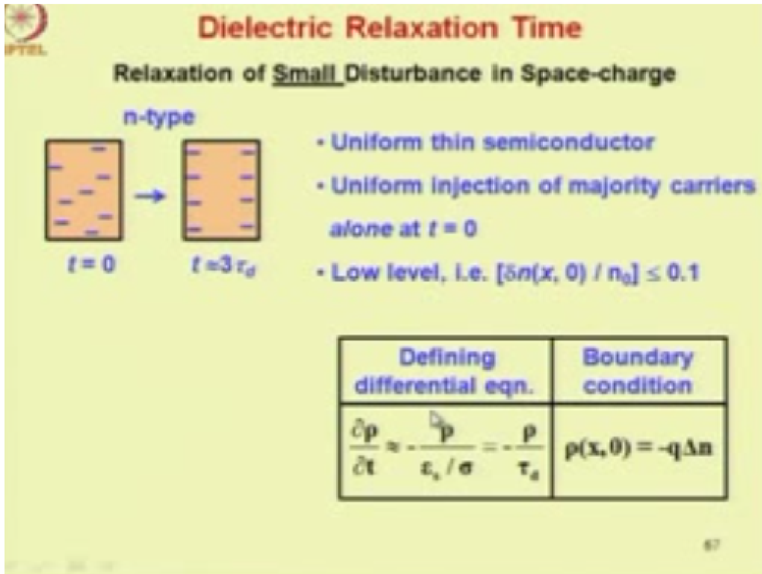

For that purpose, we write the equation here in this form, note that when I have come to this form of the equation, I have neglected the term rho/tau, where tau is a minority carrier lifetime, okay. Now this is because their dielectric relaxation time as we will see will turn out to be very, very small compared to minority carrier life time.

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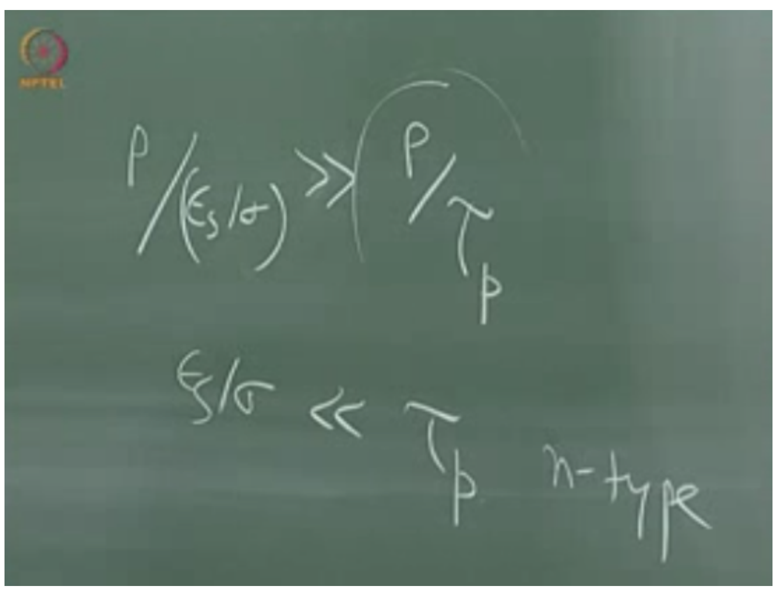

So as a result, rho/epsilon s/sigma -, so what we are saying is this quantity will be much greater than rho/tau, that is why neglect this quantity and this because epsilon s/sigma would be much less than tau, this is minority carrier lifetime, so in this case since we have n type sample, this is tau p. So, because of this fact, we are able to write the equation in this form. Now, if you want to solve this equation, you must use this boundary condition.

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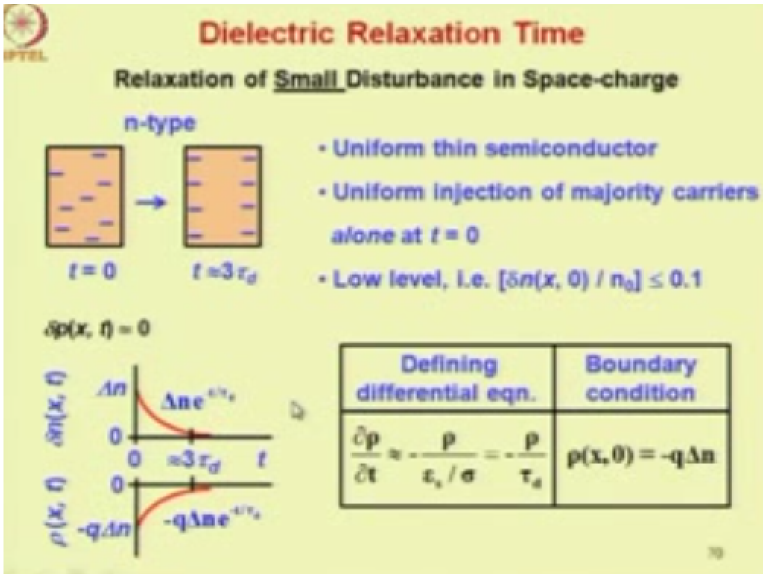

So when you do that, you will end up with an exponential solution, it is straight forward to see, we have solved equations like this, so this equation is of the form that is analogous to the equation that we discussed related to decay of the excess carrier concentration with time, under low injection level. So rho of x, t is – q x times delta n that is the initial value of the injected electron into e power –t/tau d, where tau d is dielectric relaxation time that is = epsilon s/sigma.

So from here we can interpret that this the time required for the space charge to decay, in fact more specifically since it is a simple exponential law like this, when you go 3 times tau d, the charge will decay to 5% of the initial value. Now, what are the components of the space charge, so we would like also to know how the hole concentration and electron concentration changes with time.

Now, you recall, we have said already that the hole concentration does not really change, so delta p, excess hole concentration is really 0 everywhere, as the function of time and distance, because we have not injected any holes, so if you make this approximation and use the fact that rho is nothing but delta p–delta n into q, you can derive the relation for delta n. So, delta n is nothing but – of rho/q, because delta p is 0, so this is your form of decay of delta n, is also exponential.

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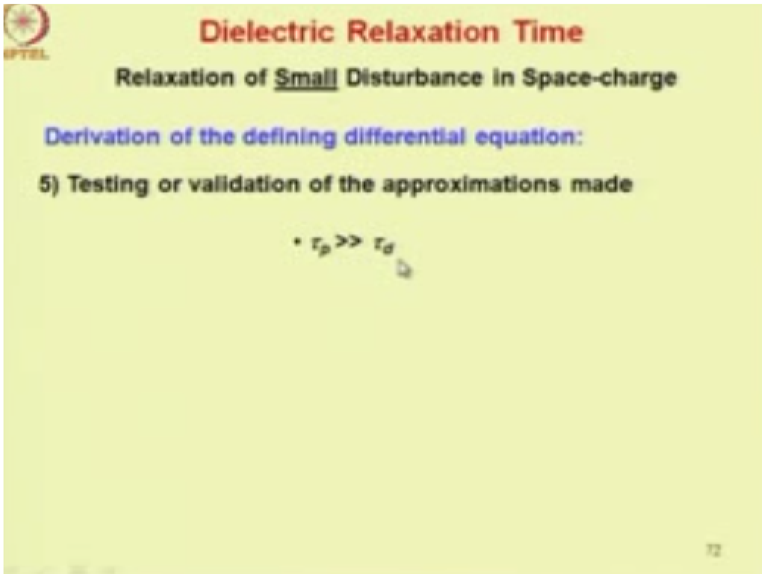

Now, finally it is very important to test or validate the approximations made. So, what are the approximations we have made, so first approximation is tau p much greater than tau d. Now, let us check this approximation using some typical values.

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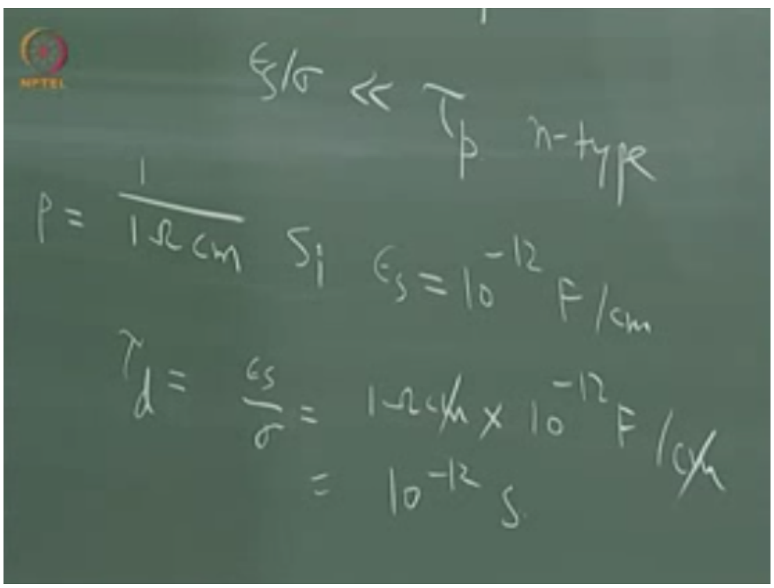

Supposing, I take a semiconductor, which is 1-ohm centimeter of resistivity, that means, sigma = 1/1-ohm cm. Let us say, this is silicon and so your epsilon s will be 10 power–12 farads per cm. Now for this case, you will get tau d, which is = epsilon s/sigma will be = 1-ohm cm into 10 power–12 farads per cm, I cross the centimeter out, this is = 10 power–12 seconds or 1 picoseconds.

Now you know the order of lifetime in silicone, right, it would be 10s of nanoseconds to microseconds, several microseconds, right, it can be even tens of microseconds, so you really see that this approximation, tau p is much greater than tau d is valid.

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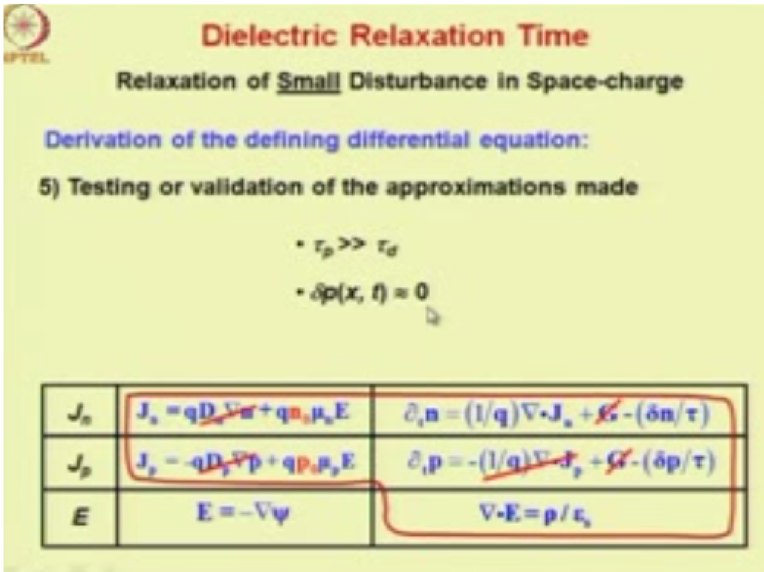

Let us look at the next approximation, delta p of x, t is 0, how we justified in making this approximation, now this is related to the fact that JP is really very small. So, that is why we are crossing out the JP here, so if you want to check this approximation, we should refer to the hole continuity equations here. Now, JP, it depends on 9 not, equilibrium hole concentration and you know that equilibrium hole concentration is extremely small, that is why we have cross this out, so if you cross this out, then you equation becomes dou p/dou t is –delta p/tau.

Now p is nothing but p0+delta p, so dou p x dou t is dou delta p/dou t. So, in other words, this gives rise to an exponential kind of solution for delta p, however, since the initial value of delta p is 0, we are not injected any access holes in the beginning. So initial value of delta p is 0, final value of delta p is also 0, because ultimately even if there was any holes, they should have decayed to the equilibrium.

So what is an exponential, whose initial value is 0 and final value is 0, exponential of the function of time, well evidently it is 0 everywhere, right. So that is why, our approximation delta p, x, t is approximately 0 is valid.

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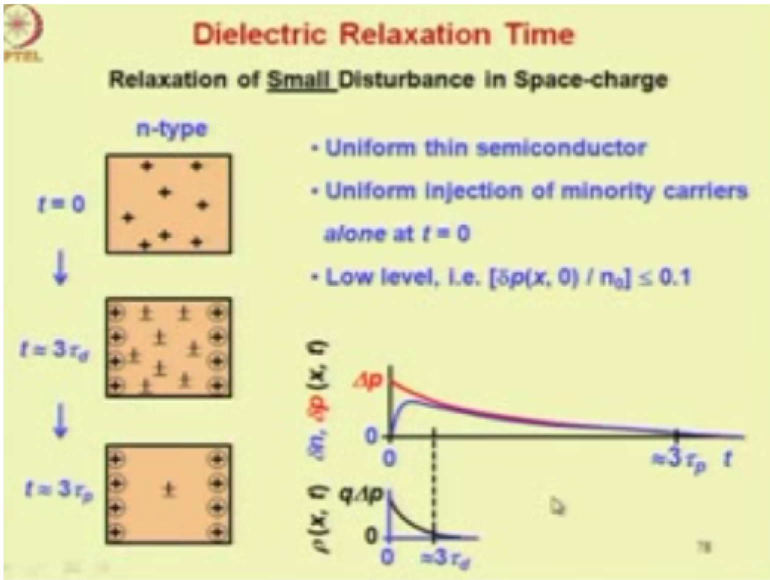

Now let us look at another form of dielectric relaxation and that is injection of holes instead of electrons. So, minority carrier injection, so except for the fact that we are injecting minority carriers, all other conditions are same. So, you have uniform thin semiconductor, you have uniform injection of minority carrier alone at t = 0 and you have low level conditions. So, delta p that is injected is very small compared to the equilibrium value of majority carrier electron concentration.

What happens in this case, this is a more interesting case, right? As we will see here, there will be 2 time constants coming in, one associated with decay of holes that would be of the order of lifetime of holes and other one associated with decay of space charge caused by the holes, that too be much shorter that would be the order of dielectric relaxation time. So let us how, this is the case, what is going to happen.

The moment you put holes in the semi conductor, immediately electric field will be developed, just the way we discussed for the case of electrons. So, for exam, if I were to sketch here, for the case for holes, instead of the electric field going in, it would go out from the sample, that is all. So that is the difference, so this x direction. So, this electric field is going to act on holes as well as electrons.

Now, please note, this is an n type sample, so it really contains a large number of electrons, like this sample. So, though the charge is created by holes, the electric field that results from the charge is going to have a strong effect on the electrons and not so much effect on the hole themselves which are create the electric field, because the hole concentration is really small, okay, this is a low injection level and the majority carrier are electrons.

So the electric field caused by the holes, will have a strong impact on the electrons and it will quickly try to draw electrons in from the surface. So, each hole will get paired with the electron. No doubt as soon as the holes are injection they will start recombining, why because the holes are surrounded by such a large number of electrons. But so quickly will the electrons come in and neutralize the charge, that during this period, the amount of holes that recombine is very small.

Now, moment the electrons have come in, what is left behind at the surface well you have positively charge donors, because they are ones we are giving these electrons. Since, ours is a 1-dimensional case, we are not showing any donors on this side, right. So that is what is shown here. So after about 3 times, the time that we call as dielectric relaxation time, we will find the charge would have almost got neutralized.

So you have holes as well as electrons, so you see there is an interesting situation where you have excess electron hole concentration within the volume, compare it with the situation of dielectric relaxation when excess majority carriers were injected. By the end of the 3 times tau d period, no electrons were left in the volume, right they had all moved to the surface, now however, you have excess electron hole pairs, in the volume.

And you do have a charge on the surface, however, it is not because of mobile carriers but because of fixed donors. Now, what is going to happen, these access carriers are going to recombine. So in about the 3 times, the hole lifetime, which is the minority carrier lifetime, all these access carriers would have recombined, okay. Note that holes have started recombining, right after $t = 0$ when you injected them.

Therefore, from t = 0 to about 3 times the hole lifetime the entire process of return to equilibrium will be going on. Now, plotting the results in the form of graph, the space charge rho will decay as shown here, as a function of time. The hole concentration will decay much more slowly because decay of the hole is happening over the period 3 times the hole lifetime. The electron concentration is increasing first because the electrons are drawn in.

You see, there are no electrons here, so electron concentration is 0 and then the electrons are drawn in to the volume, so their concentration is increasing the volume that is what is shown here. However, after they are get paired with holes, they start recombining, so that is why they are falling here, so that is how you get this kind of a behavior.

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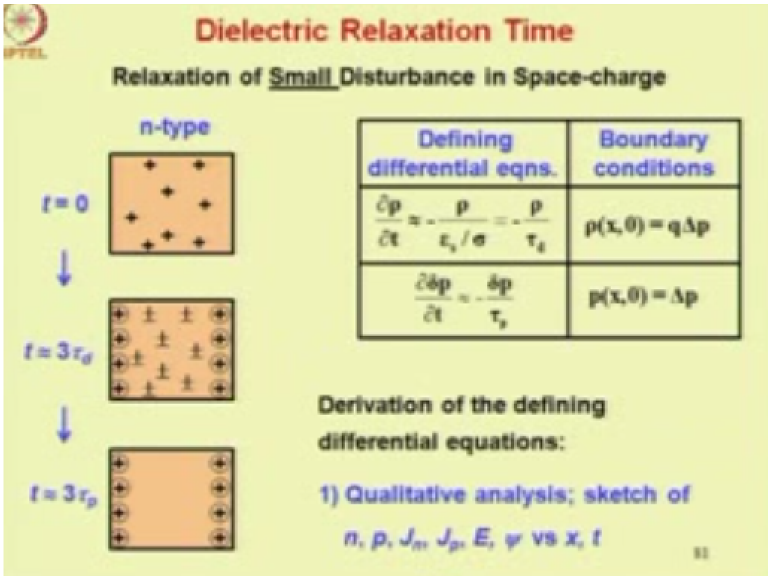

These are defining equations, so you have 2 equations, you do not have a single equation here, right, you have 2 equations. One for the space charge, that is a blue curve here, so this is a equation for the space charge, dou rho/dou t. We are going to derive this, I am just stating the equation here, so dou rho/dou t is approximately = rho/tau d.

And you have an equation related to the decay of hole concentration that is dou delta p/dou t is approximately = –delta p/tau p and these are the boundary conditions, rho. Initial value of rho is q times delta p, that is the hole concentration that is injected at t = 0. Now, it is uniform condition therefore for all x the value is the same okay. Now the same information is put here, hole concentration, here in terms of the space charge due to holes.

Here it is in terms of the hole concentration, so this boundary condition will be used for the red curve, we derive the red curve, this boundary condition, will be used to derive the blue or the space charge curve. Now, let us go about deriving the equation, so first is the qualitative analysis, now this qualitative analysis will be exactly identical, to what we have done for dielectric relaxation due to majority carrier injection, therefore, I will leave it to as you as an assignment.

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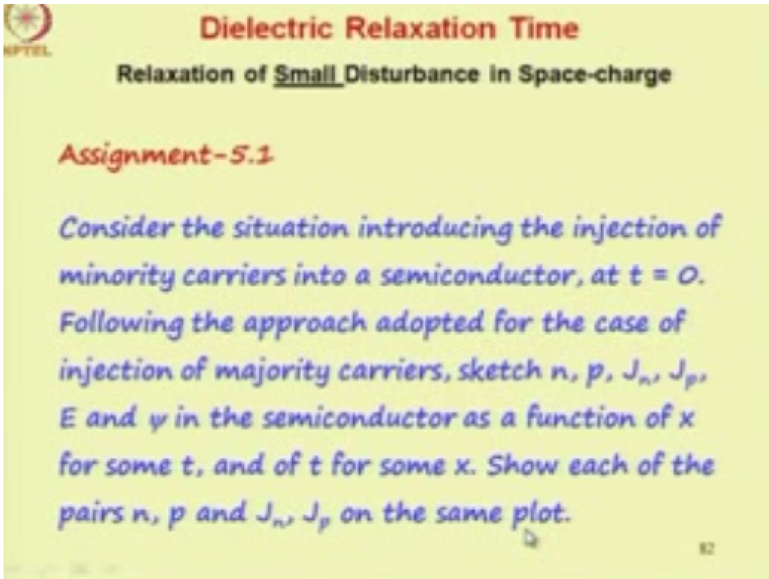

The only thing you must consider is that the space charge here is position, instead of negative, so you must take that into account and draw np, j and jp, e, psi etc. So this is the assignment, consider the situation introducing the injection of minority carriers into a semiconductor at t = 0. Following the approach, adopted for the case of injection of majority carriers, sketch n, p, jn, jp, e and psi in the semiconductor as a function of x for some t.

And as the function of t for some x, show each of the pairs n, p and jn, jp on the same plot, okay. So this is what you should do in your assignment. Let us proceed further, now, to approximate the equations, we will derive the formula.

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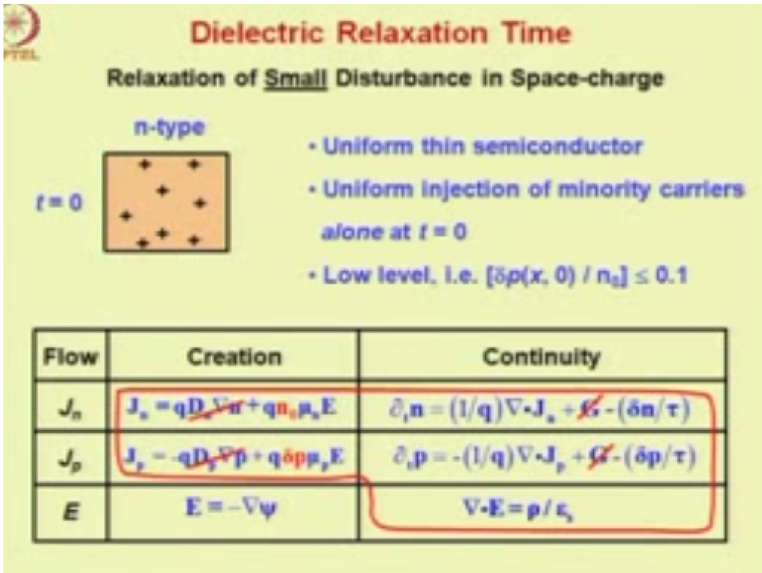

Note, to approximate the equations, we must aware of the conditions, right, so these are your conditions. So, these are 5 coupled equations, now these 3 conditions, uniform thin semiconductor, uniform injection of minority carriers at t = 0 and low level, make you strike off the terms indicated here, following the same approach that we employed for majority carrier injection.

So, diffusion currents are absent because conditions are uniform within the sample. There is no excess volume generations, because we have injected only minority carriers and there is no generation of electron hole pairs. N is replaced by n0 because injection level is low, however, there is one deviation, from the previous case of majority carrier injection and that is since the hole concentration has been disturbed, the term p here has been replaced by delta p.

This term was p0, when we considered majority carrier injection. However, here, it is delta p, now this is because p = delta p+p0, and you know that p0 is so small, that delta p normally dominates over p0, so therefore p is approximately delta p itself.

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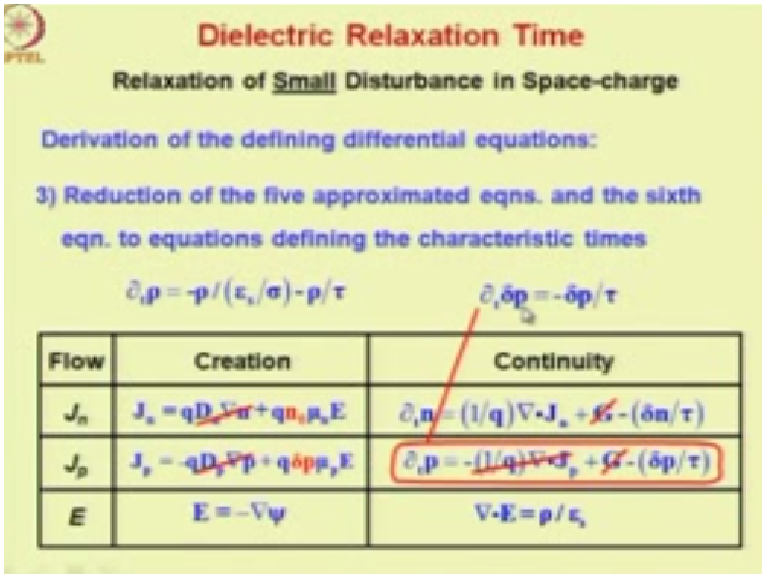

The next step is reduction of the 5 approximated equations and the 6th equation to defining the characteristic times. So, following the approach of majority carrier injection, you can easily show that the 2 continuity equations, 2 current density equations and the Gauss law, together reduced to this equation for space charge, dou rho/dou t=– f rho divided by epsilon s/sigma where sigma is the conductivity of the sample – rho/tau, where tau is the minority carrier lifetime.

Now, there is one point, we noted here, when you sum up the drift currents of electrons and holes, here you have delta p in place of p0, which enters into the expression for sigma. So, strictly speaking, the conductivity will be slightly enhanced, because delta p is more than p0, however, we do not bother about this enhancement because, it is low injection level and delta p is much less than a not.

And it is really this n0, that decides your conductivity. Now, we need an equation for the decay of holes and that is obtained from the continuity equation for holes which is circled here. In this equation, we shall neglect the divergence of jp, and that is because jp itself very small, so the jp is q delta p mu p into e but since it is low injection level the delta p is very, very small compared to n0.

So, unlike in the electron continuity equation, where we cannot really neglect this divergence of jn, in hole continuity equation, we can neglect this in comparison to the other term here, that is the recombination term. Now consequently, your equation for hole concentration will become dou/dou t of delta p = -delta p/tau, where, this dou x dou t of p has been set = dou/dou t of delta p. So, the reason for this is that p = p0+delta p and dou/dou t of p0 is 0, so dou/dou t of p is dou/dou t of delta p.

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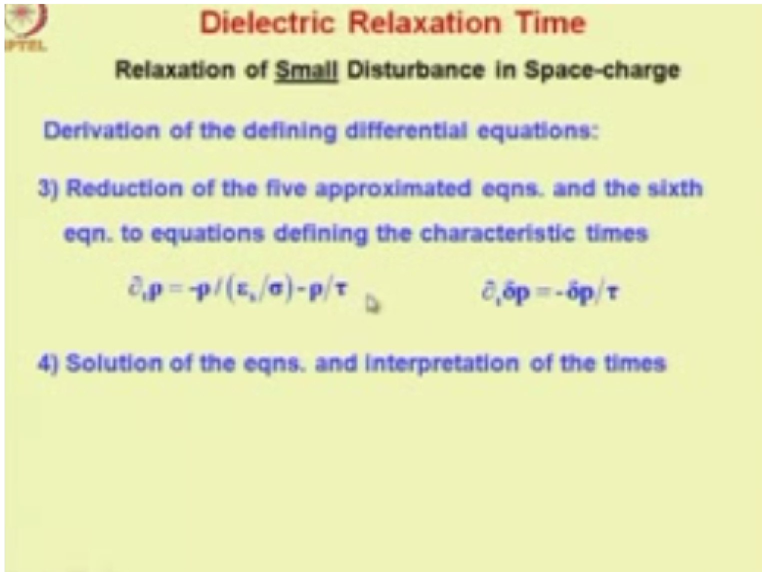

Now comes the solution of the equations and interpretation of the times. So, these are the 2 equations whose solutions we need to consider and interpret the time constants over there.

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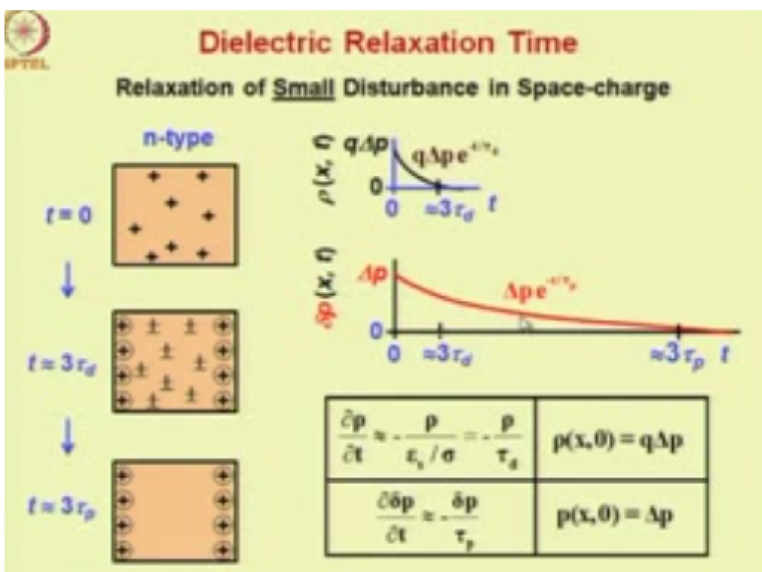

So let us take the first equation, dou rho/dou t is rho/tau d for the space charge, following the approach of majority carrier injection, we have neglected the term responding to rho/tau in the equation, this is because the dielectric relaxation time tau d will turn out to be much less than the minority carrier lifetime. So subject to this boundary condition, if you solve this for shorter differential equation in time, you will get an exponential solution and that will be given by the initial value q times delta p into exponential of -p/tau d.

So by about 3 times tau d, your space charge would have fallen to 5% of the initial value. The equation for holes is also a first order differential equation in time. The only difference is that the time constant here is minority carrier life time, unlike in this case, where it was dielectric relaxation time. So subject to the initial condition here, your solution will again be an exponential.

So, this is interesting, what we find is the characteristic times, that we have considered so far, all emerge from a first order differential equation of the quantities involved with respect to time. Now what remains is delta n, the electron concentration, now we do not need a separate differential equation for this, because, delta n is given by delta p-space charge rho/q, so by taking the difference between this red curve associated with delta p and this black curve here, associated with rho, so you divide this by q.

So taking the difference of these 2 curves, you get this particular curve for electron, excess electron concentration which rises and then falls. You can see the 2 limits, why delta and 0 here, because this initial value and the initial value of the space charge divided by q are the same. Therefore, when you subtract one from the other, you get 0, finally for a very large T also, space charge is 0 and delta p also goes to 0.

Therefore, the difference of the 2 also goes to 0, so that is how this decay is 2 0. So in between you will reach a peak. The location of the peak and the value of the peak is an assignment for you.

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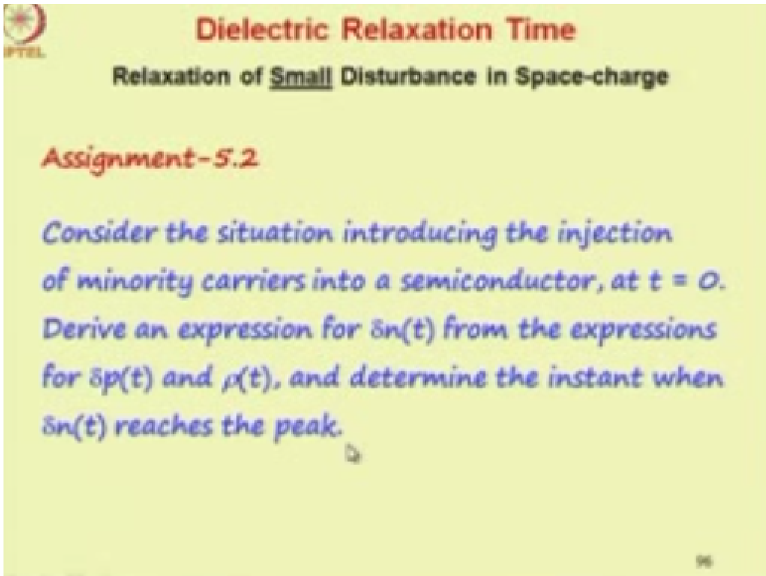

So this assignment is considered the situation, introducing the injection of the minority carriers into a semiconductor at t =, derive an expression for delta n as a function of t from the expression for delta p as a function of t and a rho has a function of t and determine the instant when delta n as a function of t reached the peak. Now you should also find out what is value of the peak, though it is not exactly written down in the assignment, you must do that also.

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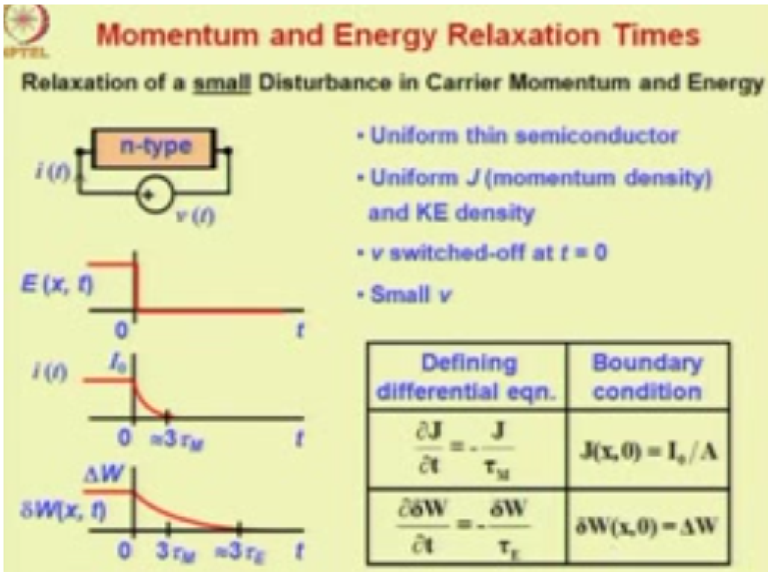

Let you move on the anther characteristic time or another pair of characteristic times namely the momentum and energy relaxation times, now these pertain to relaxation of a small disturbance in carrier momentum and energy. Now, we are discussing these 2 characteristic times together, because when you disturb the momentum, you disturb the energy also, okay. So how you disturb the momentum.

Evidently, all the carrier velocities should be aligned that is one thing and the velocities or speed increase, right and therefore, there is always the increase in energy also, the situation under which you will see the affects of these times, is shown here, a very simple situation. Take a uniform semiconductor and apply a time varying voltage, so it is really voltage that switches at t = 0 from a non 0 value to 0.

So suddenly the voltage is switched and you are monitoring the effect of this switching on the current. So to analyze this situation, you look at the electric field picture inside the device. The switching of the voltage at the terminals will cause the electric field picture also to switch inside the device. So, the electric field at any x as a function of t is given by this particular step. Now, since condition are uniform, for all x your electric field is the same.

So let us put down the conditions in words, the first important condition is a uniform thin semiconductor so which means doping is uniform, hole and electron concentrations are uniform. Then, uniform J or the carrier momentum density. So when you say carrier momentum density, it includes both electron and hole momentum densities. So, you will recall from our discussions in earlier modules that the current density actually is reflection of the momentum density.

So, uniform J and uniform kinetic energy density, so this means these quantities do not change with distance, so specially uniform. Now v is switched off at t = 0 and very importantly, we use a small v, so this electric field is really small. Now, this is important because, we have earlier seen the effect of switching of the electric field when the amount of electric field is very, very high, so we have discussed phenomena such as velocity overshoot, when you suddenly switch the electric field from a low value to a high value.

So, those kind of complications, where you have velocity overshoot, in fact, one can talk about a velocity undershoot, when you switch the electric field from a high value to suddenly 0 value. So, we do not want to get into those kind of complications, so we are assuming that the electric field is small. Now the complications that we talked about, that is undershoot and overshoot and so on, of the velocity, they arise because the relaxation time, such as momentum relaxation time, changes with the energy of carriers and energy of carriers itself is changing with time.

So momentum relaxation time changes with time. So, the complications that we see arise from variation of the characteristic time with time as the entire system is returning to equilibrium. So we do not want these kind of variations to set in, in our case, the characteristic times involved would be constant as a function of time. Now one more thing, the sample here appears to be long, so it does not really seem to correspond to the description here, it is a thin semi conductor.

This is because, this diagram is not drawn to scale, so you must always visualize this sample to be thin sample like this and which has a very large are of cross section, okay. So the results would be something like this, the current as a function of time would decay. I not is the initial value of the current, when the electric field was switched and within about 3 times the momentum relaxation time, the current would decay to 0.

So, this decay of the current really indicates the decay of the momentum. We shall find that the define in differential equation, would be a first order equation for J in time, where the time constant is the momentum relaxation time and this would be the boundary condition, so the current density is uniform over x, at t = 0, it is given by the initial current divided by the area of the sample, so that is cross sectional area of this n or this end of the sample.

The energy inside the sample will decay much more slowly and in about 3 times the energy relaxation time, this decay would be completed. Now, energy relaxation time, is more than the momentum relaxation time. This is something that we have explained in our earlier module. Now, please also recall when we are talking about the energy of the carriers, this energy normally pertains to the random thermal energy in addition to the directed energy.

So, when the momentum is decaying, the directed energy component is definitely decaying. However, there is a random component also which was also enhanced because of the presence of electric field, so when we are talking of decay of the kinetic energy density, we are talking about decay of both the random component and the directed component right and that is why this decay is different.

The decay of the energy occurs over a different time or duration than the decay of the current, because this includes the random component of the kinetic energy also, apart from the direct component. So, we shall find that the corresponding equation is the first order equation in time, given by this formula and this is the initial condition for this equation.

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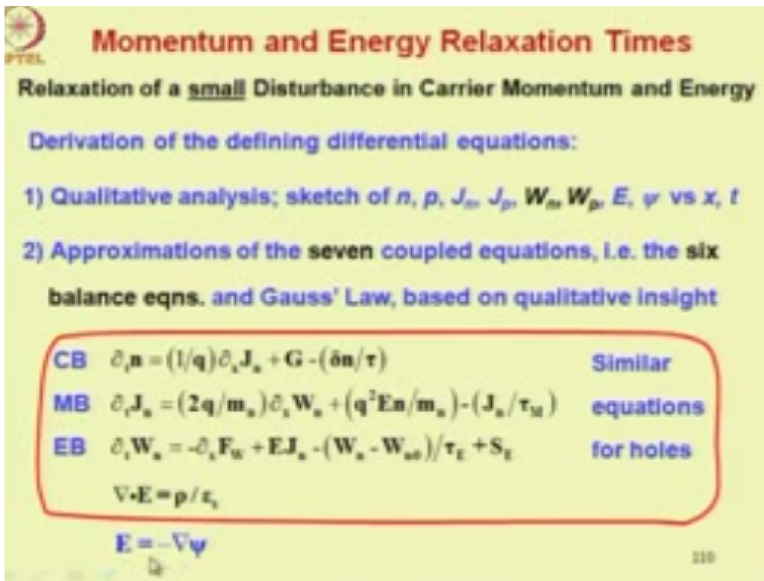

Now let us proceed through the steps for deriving the defining differential equation. The first step is the qualitative analysis, now results of this analysis are sketched in terms of these quantities, either function of x and t. Notice one difference between the situation here and situations we considered so far, where he had used the drift diffusion model. Now, in this case, we will have to use the energy and momentum balance equations as well.

So, we will have to use the balance equations, that form of the transport equations, okay. So, therefore, the energy densities Wn and Wp of electrons and holes are additional quantities that enter into the picture, so this has been shown in black. In addition to np, jn, jp, e and psi, you have this pair of quantities which have to be considered for analysis of this situation.

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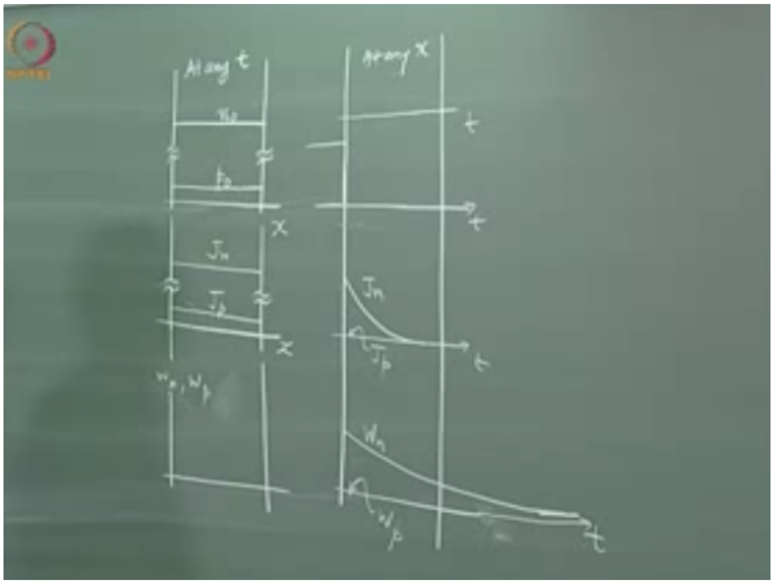

Now what would this distribution look like, well it is very simple, at any t as the function of x if you plot, electron concentration would be constant, hole concentration if I want to show on the same graph I have to show a break because hole concentration is really very small, so this is p and really this concentration = n0 and p = p0. Now if you sketch other quantities, like jn and jp, as a function of x at any time.

These would also be uniform, so your jn would be let us say like this and jp will be very, very small compared to jn because n0 is very small and it is drift current, they would be uniform. Now, similarly, one scan sketch Wn and Wp, they would also be uniform like this. The electric field, would also be uniform as a function of x, so I am not sketching them. Psi, however, at a instant of time, what would happen to psi.

Well, if the e, now e is actually 0, we do not have to plot e as a function of x, because we have switched when I show as a function of time. So at any x, as a function of time, n0, p0, there is no difference, so I am not plotting. It remains the same as the function time, jn and jp, let me plot that here, so jn and jp, they are decaying, so I will show decay of jn for example, on the same scale if I try to show jp, unless I show a cut like this, I cannot show, right.

But I want to show jn and jp relative to each here, so jp I will not show, so it is almost it will be just this line actually, jp almost 0 as a function of time, and electric field, say if sketch here, the electric field it has suddenly gone to 0, so really here I do not have to sketch electric field, it is 0 as a function of x. Wn and Wp, well they would decay as a function of time but decay of Wn and Wp would be rather slow.

So in fact if I want to show on the same scale, I should really show it like this, slowly, right decaying slowly as compared to this. So, this is Wn, the Wp will also be really very small as compared to Wn. It would be something like this. Now, in this case, I will leave it to you to sketch the potential that is straight forward thing and does not really matter as far as our analysis is concerned.

So, these are you distributions of the various quantities as a function of x and t. The next step is approximations of the 7 coupled equations. Now really speaking there are 8 equations out of which 7 are coupled. The 8 equations are because you have added 2 equations corresponding to the electron and hole energy densities. The 7 coupled equations including the 6 balance equations, namely the equation shown here.

The carrier balance equation, the momentum balance equation and the energy balance equation, these are shown for electrons and you have similar 3 equations for holes. The momentum balance equation converts to the current density equation of the drift diffusion model, if you consider steady state situation that is this term goes away and you also break up the kinetic energy term, right to get the diffusion and thermoelectric currents.

In addition to the 6 balance equations, you have the Gauss law, so these are the 7 equation. So you have to approximate the 7 coupled equations based on qualitative insight, which we have gained from plot or sketch of these quantities as the function of x and t. So these are 7 coupled equations but total number of equation is 8, this is the additional equation.

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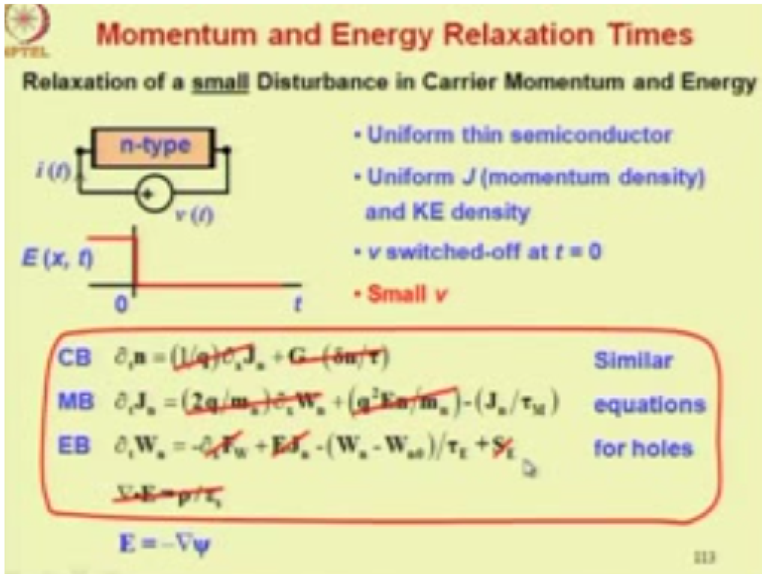

Now the approximation of these equations can be done very easily as follows, since conditions in the semiconductor sample are uniform, all the special derivatives with respect to x followup, so they go to zero, so this terms are struck off. Then, since we switched off a t = 0 in the duration in which the phenomenon is being observed, the electric field is 0, so what is why the electric field related terms are stuck off and the Gauss law also is not necessary to be solved.

Now, since the v is small, the electric field here was small, there was really no question of any excess generation of recombination, so these terms are not there and this term also which is related to generation and recombination, source term in the energy, balance equation is also struck off.

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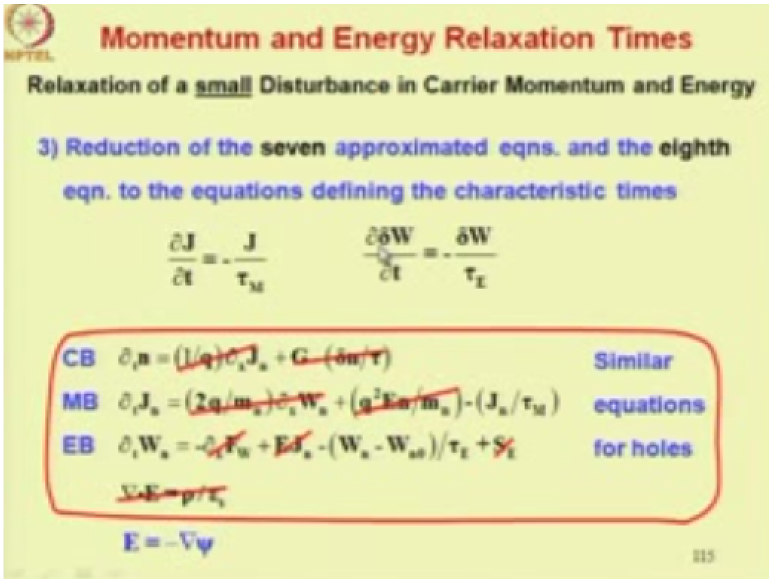

The next step is reduction of the 7 approximated equations and the 8th equations to the equations defining the characteristic times. So, clearly, these are 2 equations that we are left with, so dou j/dou t=-j/tau m. This equation arises from this momentum balance equation and a corresponding equation for holes, so you add up the momentum balance equation for electrons and holes to get the equation for total current.

Similarly, for the kinetic energy density, so this equation, is derived from the energy balance equation here after these terms are all removed. Wn-Wn0 is really re-presented as delta Wn and we sum up the electrons and holes, the energies for these 2 so you get W. So delta W is the difference between the energy of the carriers and the energy of the carrier under equilibrium.

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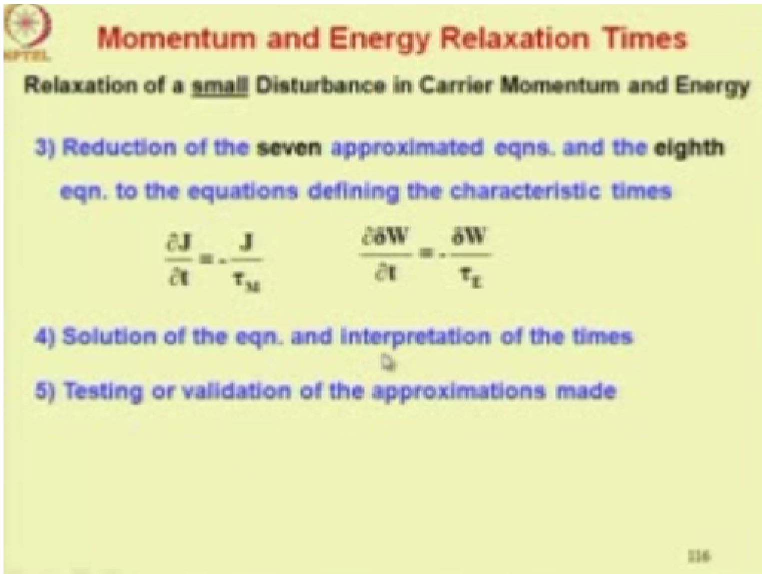

So the next step is solution of the equation and interpretation of the times and testing or validation of the approximations made. Now testing or validations of the approximations is not really an issue in this case because our conditions in the sample are such that they allowed us to delete so many of the term, so we really did not do any significant approximations as such. Whatever terms we removed from the equations, they followed from the conditions given.

So, let us look at the solution of the equations and interpretation of the times.

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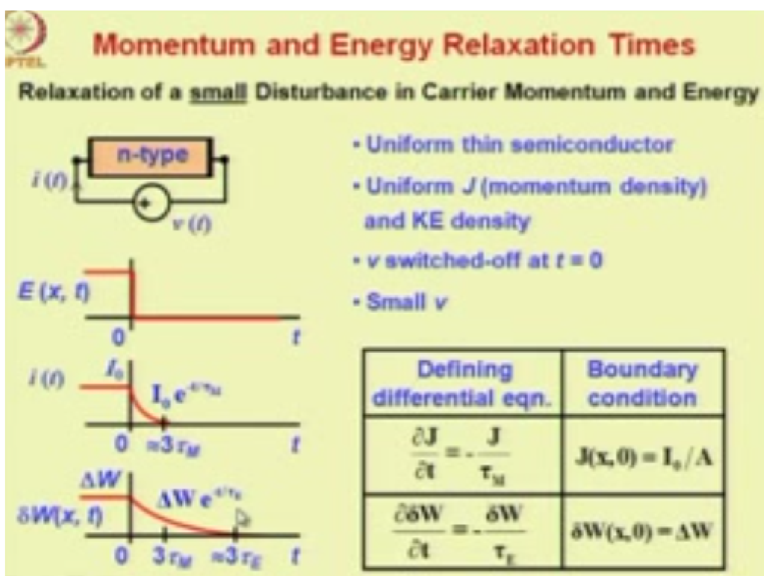

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Consider the equation for j, this is the first order differential equation, subject to this boundary condition, so your solution would be an exponential of this form, so this gives you really the eye of t, how this current changes with time. So, this current is reflection of the momentum of the carriers and similarly the first order differential equation for the kinetic energy here will give you this solution, subject to this boundary condition.

This is again, another exponential, okay with respect to energy relation time.

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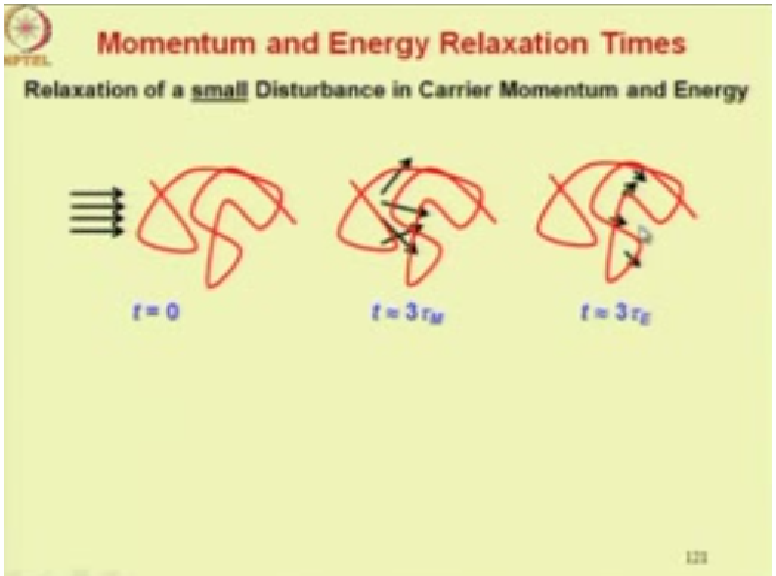

Now before we close this lecture, we just want to remind you that of the discussion related to energy and momentum relation times, how the energy relaxation time is more that the momentum relaxation time, so you recall that we have considered a hypothetical beam of carriers, let us electrons, assuming no interaction between these electrons, assuming no interactions between these electrons.

So repulsive forces, so the beam is parallel and it somehow imparted a velocity in this direction as it enters a carrier population in thermal equilibrium which is all moving around randomly, within a time equal to the momentum relaxation time, multiplied by 3 of course, the momentum of the carriers changes though it is energy does not change. So you see the length of the arrows here is the same as the length of the arrows here because the speed of the carrier has not changed.

So, because of scattering the direction has changed but the speed has not changed, right that is why the energy has not changed, though the momentum has changed because whenever direction changes, the momentum is affected. And after a much longer time, the carriers also lose, speed as they collide more and more with other carriers in random motion and their energy becomes equal to that of the carrier energy right or energy of the numerous carriers which are randomly moving and in thermal equilibrium.

So, this is the time when energy relaxes completely. Now, with that we have come to the end the lecture. To summarize, in this lecture, we have considered situations involving space charge relaxation and relaxation of carrier momentum and energy, so space charge relaxation, we consider 2 possibilities, in one case, we created a space charge at the instant t = 0 by injecting majority carriers in a semi conductor and we found that the space charge relaxes in dielectric relaxation time.

The next situation we considered was we injected minority carriers into a semi conductor and found that even the space charge due to minority carriers also relaxes in the same dielectric relaxation time as that of charge due to majority carriers, however, the important difference is that the minority carrier themselves take a very long time as compared to the dielectric relaxation time, namely the minority carrier lifetime to relax to equilibrium.

So space charge relaxes quickly when you inject minority carriers, but the minority carriers themselves right, they take much longer time to return to equilibrium, so the relaxation of the space charge happens by drawing in of majority carriers which pair up with the minority carrier to neutralize the space charge. Then we considered the energy and momentum relaxation times, and showed how the characteristic equations defining all these various characteristic times are all first ordered equations in time.

So in the next lecture, we shall consider the transit time and a few characteristic lengths like the diffusion length and the divided length.