

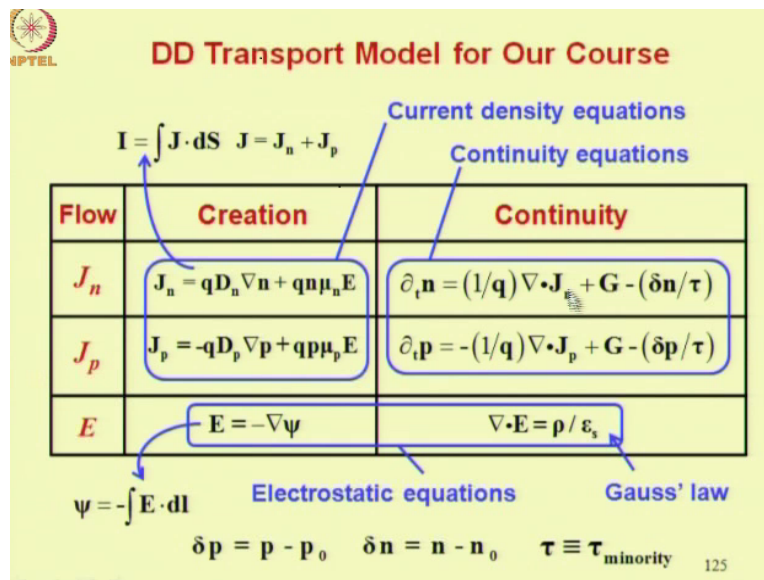
Semiconductor Device Modeling
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Lecture - 19

Drift-Diffusion Transport Model: Equations, Boundary Conditions, Mobility and Generation / Recombination

In the previous lecture, we have completed modelling of the mobility we took into account the scattering mechanisms that occur in the bulk as well as near the surface, now what remains to be done is to develop expressions for the generation and recombination mechanisms.

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So let us look at the drift diffusion model, so you find the generation term coming here both in electron continuity equation as well as hole continuity equation and the $\delta n / \tau$ is the recombination term in the electron continuity equation and the $\delta p / \tau$ is the recombination term in the hole continuity equation.

You will recall we have said that in most cases quasi neutrality will hold in the semiconductor and in that case δn and δp would almost be the same, therefore very often we only use the same δ as excess carrier concentration in both electron as well as hole continuity equations, so we have maintained δn and δp as 2 separate symbols because we would

like to just take care of even though very rare cases when there may not be space charge neutrality.

So we are looking at expressions for these 2 quantities, now with that our drift diffusion model would be complete because the mobility terms appeared in the current density equations and the diffusion coefficients can be expressed in terms of mobility, so with that the drift diffusion model would be completed.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$
$$= \delta (\tau_{\text{SRH}}^{-1} + \tau_{\text{Auger}}^{-1} + \tau_{\text{Radiative}}^{-1})$$

Let us begin with the Recombination rates, now you would recall when we discussed the qualitative model for carrier transport we had pointed out the various interactions that are possible of electrons, holes, phonons and photons at that point we are pointed out that there are 3 possible mechanisms of generations and there are corresponding 3 mechanisms of recombination.

So connecting to the discussion that we had earlier we can write the 3 mechanisms of recombination as Thermal recombination, Auger recombination and Radiative recombination. Now it will be our endeavor to show that the sum total of these 3 mechanisms of recombination the rates can be summed up as excess carrier concentration divided by lifetime.

So we will show that the recombination rate are so what we are talking about is the volume recombination rate so per centimeter cube per second, so that can be shown to be proportional to the excess carrier concentration and the denominator that comes here is the so-called lifetime, now we will find that at high injection level for auger and radiative recombination's the lifetime is not a constant.

Therefore, it is not correct to say that the recombination rates are proportional to the excess carrier concentration under high injection level conditions for radiative and auger recombination rates, however for thermal recombination we will find that this is a very good method of expressing the recombination rate both at low and high injection levels.

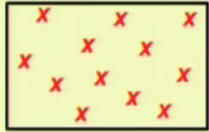
So we can expand the recombination lifetime into the mechanisms associated with each of these lifetime associated with each of these 3 mechanism and therefore we can write δ/τ as δ into 1 by the recombination rate due to thermal mechanism which is modelled using SRH theory that is the Shockley Read Hall theory then +1 by the auger recombination lifetime +1 by the radiative recombination lifetime.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

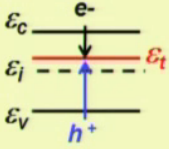
$$R_{\text{Thermal}} \equiv R_{\text{SRH}}$$



Trap concentration
 $N_t \text{ (cm}^{-3}\text{)}$



Carrier capture by a trap



Let us begin with the thermal recombination which is synonymous with the recombination rate determined from Shockley Read Hall theory, so Shockley Read and Hall gave a theory of a recombination process which essentially involves recombination via traps, so this is the picture

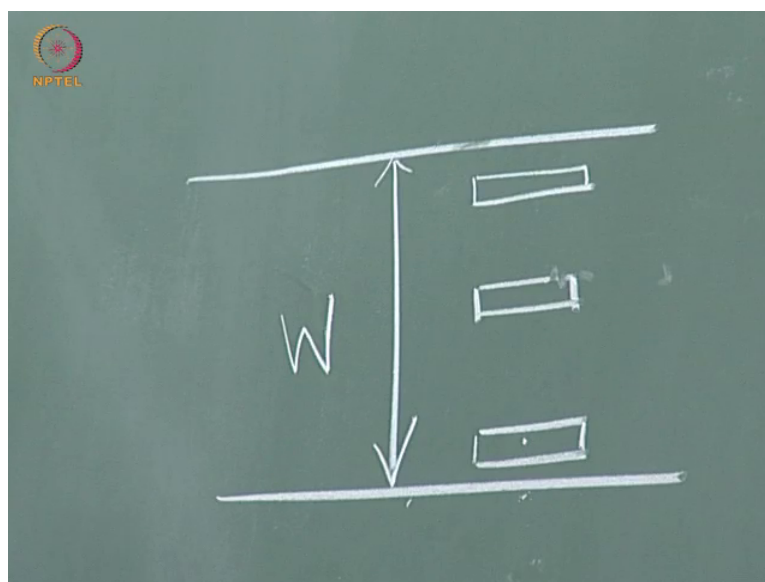
of the semiconductor which contains at select places traps, these can be either impurities or defects the concentration of these traps is N_t per centimeter cube.

Now in this theory the recombination only happens at these sides, so electrons and holes they may meet elsewhere they will not recombine okay they will recombined only in the places where there are traps, now this kind of a recombination dominates in silicon, now the traps need to have a particular characteristic in order to be a efficient recombination sides and that property is that the energy level corresponding to them should be close to the middle of the energy gap.

Here we are gaming tacitly that all the traps have the same energy level, now in practice you can have the traps spanning an energy spectrum, however what is important is as we understand from this theory the trap levels which are close to intrinsic level alone are very efficient in recombination in causing recombination the traps which have energies further away from the intrinsic level are less and less efficient okay.

So even in such a case we can use this SRH theory based on a single trap level to get an effective recombination rate, now why should the trap level be near the middle of the energy gap well a simple explanation is with the help of an analogy in which suppose you are trying to cross or jump across a canal okay or stream.

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Now let us say this is the width of the stream and you want to jump across, now evidently the probability that you can jump across will reduce as the width of the stream increases, now let us say the width is such that it is quite difficult for you to jump across in one jump right, now assume that to ease your problem a step is provided in between so that you can jump onto this step and then from this step you can take another jump and then go across to the other end.

Now intuitively it is obvious that its best situation for you would when this step is right in the middle because if this step were very close to this end then this jump to the step becomes difficult though the other jump is easy, on the other hand if this step was very close to this end then jumping onto the step is easy but from the step jumping on to the other side is difficult, so somewhat similar situation prevails for the traps.

So in this mechanism what is happening is electrons is captured at the trap followed by a hole capture or a hole may be captured first and then electrons will be captured by the trap and then they will recombine now the energy and momentum associated with electron and hole would be released as a phonon, so the impurity atom will vibrate and that is how a phonon is generated, so that is why it is called a thermal recombination mechanism right.

Because it is phonons that are involved that are released at the consequence of this recombination, so the process of capture is illustrated here, now in the theory it is assume that an electron for example will be captured if it comes when within the certain area around the impurity, so you can imagine a certain volume around the impurity if a carrier enters into that volume then it is said to be captured.

So evidently if you take a cross section of the volume perpendicular to the direction of flow of the carrier direction of motion of the carrier then that cross section would be an area and that area is normally called the capture cross section, so in this case for example we have shown a capture cross section corresponding to an electron which is incident in this direction and similarly a capture cross section for a hole.

Evidently if this capture cross section of the impurity is larger than the possibility that the carrier will be captured will be more, so that is why this recombination rate depends on the capture cross section of the impurity apart from the concentration of the impurities.

Now another thing that will affect recombination rate is the rate at which the carriers are captured that depends on with what velocity the carrier approaches this cross section and that velocity is nothing but the thermal velocity, therefore in this model the thermal velocity, the capture cross section of the trap and the trap concentration would enter into the formula for recombination rate.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Thermal}} \equiv R_{\text{SRH}} = \frac{pn - n_i^2}{\tau_p (n + n_i) + \tau_n (p + p_i)} \quad pn \geq n_i^2$$

$$\tau_p = \frac{1}{\sigma_p v_{th} N_t} \quad \tau_n = \frac{1}{\sigma_n v_{th} N_t}$$

Trap concentration
 $N_t \text{ (cm}^{-3}\text{)}$

So the formula is given by this expression we are not deriving the formula okay, we are just explaining the various terms in this formula based on the model that we just discussed, so evidently if pn product is more than ni square then you have excess recombination. Therefore, the recombination depends on pn-ni square, then you have 2 time constants so called time constant associated with the hole capture and time constant associated with electron capture.

Now these time constants are given by tau p is 1/sigma p into thermal velocity into Nt, Nt is a trap concentration, sigma p is a capture cross section for holes and v-th is the thermal velocity an analogous expression is therefore electrons.

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Recombination Rates

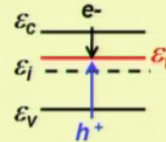
$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Thermal}} \equiv R_{\text{SRH}} = \frac{pn - n_i^2}{\tau_p(n + n_t) + \tau_n(p + p_t)} \quad pn \geq n_i^2$$

$$\tau_p = \frac{1}{\sigma_p v_{\text{th}} N_t} \quad \tau_n = \frac{1}{\sigma_n v_{\text{th}} N_t}$$

$$n_t = n_i \exp\left(\frac{\epsilon_t - \epsilon_i}{kT}\right) \quad p_t = n_i \exp\left(\frac{\epsilon_i - \epsilon_t}{kT}\right)$$

$$\epsilon_t = \epsilon_i \Rightarrow n_t = p_t = n_i$$



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The terms n_t and p_t try to incorporate the efficiency of the trap as determined from its energy level, so we just said a little earlier that the trap should be somewhere near the middle of the energy gap for it to be an efficient recombination site, so this information is contained in these terms n suffix t and p suffix t , so n suffix t is n_i into exponential of $E_t - E_i/kT$ and p_t , p suffix t is n_i into exponential of $E_i - E_t/kT$.

So n suffix t represents electrons concentration assuming that the electrons were occupying this trap this was like the conduction band edge and similarly p_t is hole concentration assuming that the trap location was like a valence band edge, now in practice the maximum recombination rate occurs for $E_t = E_i$. And since there is a spectrum of energy over which this trap exists we try to find out the recombination rate for the maximum situation and use that as index of recombination rate.

Now for that condition n_t and p_t become $= n_i$ and let us see what the formulae or they would look quite simple.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}) = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Thermal}} \equiv R_{\text{SRH}} = \frac{pn - n_i^2}{\tau_p (n + n_t) + \tau_n (p + p_t)} \quad pn \geq n_i^2$$

$$\tau_p = \frac{1}{\sigma_p v_{\text{th}} N_t} \quad \tau_n = \frac{1}{\sigma_n v_{\text{th}} N_t} \quad \epsilon_t = \epsilon_i \Rightarrow n_t = p_t = n_i$$

Low level (n-type) : $\tau_{\text{SRH}} \approx \tau_p$ High level : $\tau_{\text{SRH}} \approx \tau_p + \tau_n$

$$\sigma_p, \sigma_n \sim 5 \times 10^{-16} \text{ cm}^2 \quad N_t \sim 1 \times 10^{14-17} \text{ cm}^{-3} \quad \tau_p, \tau_n \sim 2 \text{ ns} - 2 \mu\text{s} \text{ at } 300 \text{ K}$$

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So if you take an n-type silicon then the SRH lifetime is approximately = tau suffix p, so the hole lifetime right because we always talk about lifetime of minority carriers, on the other hand high level lifetime is sum total of a time constant associated with hole capture and time constant associated with the electron capture, so high level life time in SRH theory is more than the low level lifetime for tau p = tau n, the high-level lifetime is twice that of low level lifetime.

Now let us see how we can get these expressions in terms of from starting from this relation $pn - n_i^2 / \tau_p (n + n_t) + \tau_n (p + p_t)$

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The handwritten derivation on the chalkboard shows the following steps:

$$R_{\text{SRH}} = \frac{pn - n_i^2}{\tau_p (n + n_t) + \tau_n (p + p_t)}$$

For n-type low-level, $n \approx n_0$ and $p = \delta n_0$. The condition $p, n \gg n_i$ and δ, n_0 is noted. The equation simplifies to:

$$R_{\text{SRH}} = \frac{\delta n_0^2}{\tau_p n_0 + \tau_n \delta n_0}$$

For high level, the equation simplifies to:

$$R_{\text{SRH}} \approx \frac{\delta^2}{\tau_p + \tau_n}$$

Now, in our case, p_t is p_i and n_t is n_i . Now let us resume n type semiconductor and low level this means the electron concentration n is approximately = the majority carrier concentration of equilibrium, the minority carrier concentration p however can be approximately assumed to be = δ itself excess carrier concentration of holes, because this $+p_0$ is actual value but this is going to be very small.

This p_i is nothing but n_i actually we normally use that symbol n_i even for intrinsic concentration of holes, now recognize the fact that this n is n_0 much more than a n_i therefore this is really not important, similarly the excess carrier concentration will generally be much more than n_i , so p will be δ but and this n_i would not be important, so let us remove this let us make this n_0 , let us remove this and let us put δ instead of this here.

Now let us look at the numerator $pn - n_i^2$, so this would be p is nothing but δ n is nothing but n_0 and if p and n are each much less than n_i because p is δ and n is n_0 , then this n_i^2 can be neglected, so numerator is simply δn_0 , so your result is this above thing is = δ into n_0 upon τ_p into $n_0 + \tau_n$ into δ , however this n_0 is again much more than δ or δ is much less than n_0 because we are resuming low level.

Therefore, this term drops out and the n_0 , n_0 cancels and you get δ/τ_p , do a similar exercise for high injection level, for high injection level each of p and n is replaced by δ because δ is much more than the equilibrium concentration of electrons and holes and you can neglect this n_i^2 . So for high level R_{SRH} is approximately = numerator is simply δ^2 , denominator.

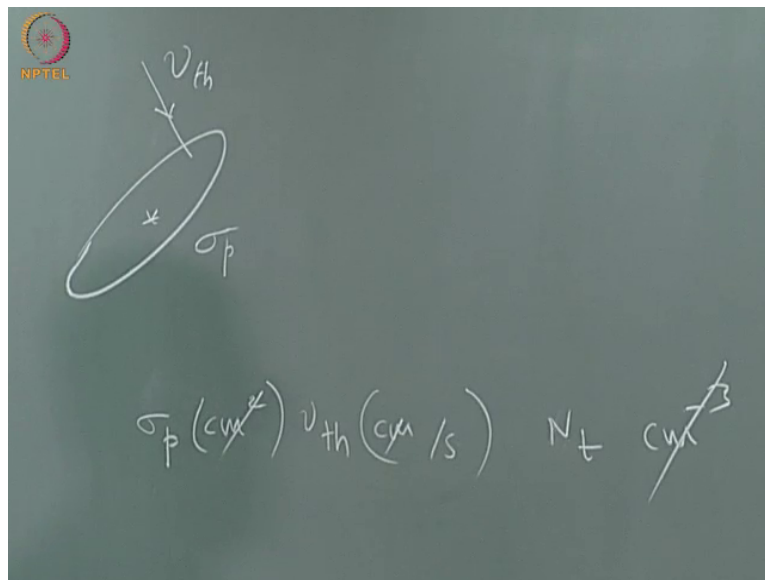
So n_0 is replaced by δ and here n and p also replaced by δ n_i , n_i very small so you have δ into $\tau_p + \tau_n$, so you cancel one δ out. So you get this as your recombination lifetime okay at high level and this τ_p is recombination time constant or lifetime for n type semiconductor for p type low level lifetime would be τ_n . But for both p and n type the high level lifetime would be the same $\tau_p + \tau_n$.

Let us look at some typical values of the various parameters, the capture cross section 5×10^{-16} per centimeter square, now recall that 10^{-16} per centimeter square is 10^{-8} centimeters into 10^{-8} centimeters that is about an angstrom into an angstrom right, so 1 angstrom square, so the capture cross section is 5 angstroms square. The size of the atom is very close to this right.

So it makes sense that the capture cross section would be approximately the sphere of influence of the atom right, the size of the atom it should be related to that, the trap concentration in practice varies from 10^{14} to 10^{17} per centimeter cube and the constants τ_p and τ_n vary from 2 nanoseconds to 2 microseconds, so because of this range of N_t . So time constants are shown here.

The physical interpretation of the time constants would not be difficult at all, because $\sigma_p v_{th}$ into N_t is a rate at which the carriers are captured and therefore the time constant is nothing but 1 by that, so let us look at this part.

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So suppose this is your capture cross section for a hole and carriers are coming at thermal velocity, then the rate of capture would be depending on dimensionally σ_p is centimeter square, thermal velocity is centimeter per second, so this tells you how many electrons will be

captured so you have to multiply this by Nt whose dimensions is per centimeter cube, so this, this cancels and therefore you get the rate at which the carriers are captured.

Because like that you have Nt impurities each capturing holes and similarly capturing electrons, so that is how the product of σ_p into $v_{thermal}$ into Nt gives you the rate and therefore 1 by this rate will gives you the time okay between 2 consecutive captures.

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The slide, titled "Recombination Rates", features the IPTTEL logo in the top left corner. It displays the equation $R \text{ (cm}^{-3} \text{ s}^{-1}) = R_{Thermal} + R_{Auger} + R_{Radiative} = \delta / \tau$. Below the equation, the term R_{Auger} is highlighted. To the left, a diagram shows an electron (e^-) and a hole (h^+) interacting with another electron (e^-) in a box labeled "n-type". To the right, an energy level diagram shows the conduction band (ϵ_c), an intermediate energy level (ϵ_i), and the valence band (ϵ_v).

Let us look at the auger recombination here, the recombination can only happen near an electron or a hole now in n type semiconductor electrons are in majority and holes are in minority therefore it will happen near the presence of an electron, so what happens in this process an electron and hole when they come near an electron the chance of recombination increases because this electron can take away the energy and momentum.

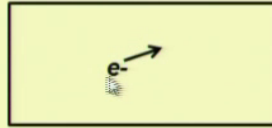
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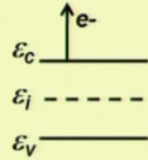
Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

R_{Auger}



n-type



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So at the end of the process your electrons and holes have disappeared and what you have is an energetic electron okay, same thing is shown here on the energy band picture.

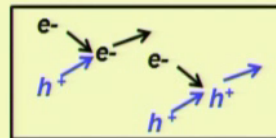
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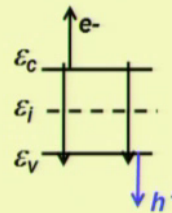
Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Auger}} = (pn - n_i^2)(nC_n + pC_p) \quad pn \geq n_i^2$$



High Level



High level what we have shown is okay for low level, for high level whether n type or p type you will have high concentration of electrons and holes and therefore the recombination rate increases, so both possibilities exist where the energy is released to an electron or to a hole, same thing is shown here on the energy band picture, so this electron its energy state is what is shown here.

And this hole that is taken away the energy and momentum its state is what is shown by this arrow here, this arrow represents the recombination of these electrons and holes, so the formula for all auger recombination is given by this relation, let us see how this relation make sense if pn is more than n_i^2 then only you have the recombination, however excess carriers will recombine only near an electron or a hole.

Therefore, the recombination rate should be proportional to the concentration of electrons and holes, now you use different constants of proportionality for electron and the holes, so that is how you get this formula.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Auger}} = (pn - n_i^2)(nC_n + pC_p) \quad pn \geq n_i^2$$

Low level (n-type): $\tau_{\text{Auger}} \approx \frac{1}{C_n N_d^2}$

High level: $\tau_{\text{Auger}} \approx \frac{1}{(C_n + C_p)\delta^2}$

$C_n \sim 3 \times 10^{-31} \text{ cm}^6 \text{ s}^{-1}$ $C_p \sim 1 \times 10^{-31} \text{ cm}^6 \text{ s}^{-1}$
at 300 K

Under low level starting from the formula following the same approach that we used for Shockley Read Hall recombination you can substitute the various concentrations here in terms of the excess carrier concentration, the equilibrium carrier concentration and so on and you will arrive at this results which are shown here, so this is given as an assignment because this is state forward manipulation.

What you find here is that the auger lifetime is inversely proportional under low level conditions to the doping level square of the doping level, so it rapidly falls as doping increases, now therefore auger recombination becomes important at high doping's only, but at high doping's the

lifetime will change rapidly with doping the low level lifetime, the high-level lifetime on the other hand depends on excess carrier concentration and square of this concentration okay.

The fact that you are getting square of concentrations either at high level or low level can be seen from simple dimensional analysis of this formula, therefore at high-level conditions you really cannot say that you can write the recombination rate as excess carrier concentration by a constant lifetime, so you cannot say the excess carrier recombination rate is proportional to the excess carrier concentration right that would not be correct.

because life time would be inversely related to excess carrier concentration square, so really speaking you should say that recombination rate varies as cube of the excess combination if I put this formula into this solution I will get cube of excess carrier concentration, to give you a field for number C suffix n is about 3 into 10 power - 31 centimeter power 6 per second okay and C p is also of the same order except that the constant proportional is little smaller.

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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$R_{\text{Radiative}}$

e^- →

h^+ →

ϵ_C ———

ϵ_i - - -

ϵ_V ———

Now let us look at the radiative recombination in this case an electron and hole can recombine anywhere and release a photon, so electron and hole before recombination this is the state the recombination is represented on the energy picture like this.

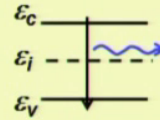
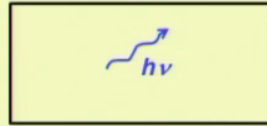
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Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Radiative}} = \frac{G_i}{n_i^2} (pn - n_i^2) \quad pn \geq n_i^2$$



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At the end of the recombination process you have a photon released that is why it is called radiative, evidently this form of recombination will be prevalent in those semiconductors where the electrons and holes when they recombined they do not have to release much of momentum, so they have to release energy equal to energy gap, so that particle which has a very small momentum for a given energy alone can participate in this process and such a particle is the photon.

So the formula for radiative recombination since it is a direct recombination between conduction and valence band and it does not depend on where the electron is therefore you have only a proportionality constant coming here, so $pn - n_i^2$ square into a constant of proportionality this is G_i/n_i^2 square, so this constant of proportionality has been cast in this form because under equilibrium the generation rate of direct recombination should be equal to recombination rate right, so it is from that consideration you get this kind of relation.

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Radiative recombination

$$G_i = A n_i^2$$

$$G = A pn$$

$$G = \left(\frac{G_i pn}{n_i^2} \right)$$

So supposing you consider under equilibrium G_0 for radiative recombination, so G_0 should be equal to some constant into n_i square, so that we will call as G_i actually intrinsic generation rate that is the under equilibrium so is equal to some constant into n_i square, so let us say the constant is A , now the generation rate when it is not an equilibrium it will be proportional to p into n and it has the same constant of proportionality A .

So you can clearly see from here that I get $G/G_i = pn/n_i$ square and therefore the G comes out as G_i/n_i square into p into n , so that is how that formula is arrived at, now but we have to consider the recombination excess recombination this is G but what we are interested is $G - G_i$ okay, so when you take $G - G_i$ you get $pn - n_i$ square, the constant of proportionality A is G_i/n_i square.

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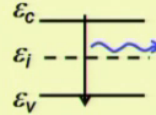
Recombination Rates

$$R \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = R_{\text{Thermal}} + R_{\text{Auger}} + R_{\text{Radiative}} = \delta / \tau$$

$$R_{\text{Radiative}} = \frac{G_i}{n_i^2} (pn - n_i^2) \quad pn \geq n_i^2$$

$$\text{Low level (n-type): } \tau_{\text{Radiative}} \approx \frac{n_i^2}{G_i N_d}$$

$$\text{High level: } \tau_{\text{Radiative}} = \frac{n_i^2}{G_i \delta}$$



$$G_i / n_i^2 = 1.8 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} \text{ (Si)} \quad 7.2 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1} \text{ (GaAs)}$$

at 300 K

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Again it is the assignment to you just show that radiative recombination under the low level gives a lifetime = $n_i^2 / G_i N_d$, so here the radiative recombination lifetime falls with doping level, Auger recombination lifetime fell as square of the doping level, high level lifetime on the other hand decreases as excess carrier concentration for Auger this was $1/\delta^2$ for radiative it is $1/\delta$.

Now this will explain why in semiconductors where the radiative recombination dominates like gallium arsenide if your doping level is high your lifetimes will be poor, so you cannot control the resistivity and lifetime independently whereas in semiconductor like silicon where the thermal recombination rate dominates because of the energy gap structure and this recombination rate the thermal recombination depends on the trap concentration and not on the majority carrier concentration.

Therefore, in silicon the lifetime and resistivity can be controlled independently the values of G_i/n_i^2 for silicon and gallium arsenide are shown here, clearly showing that the recombination rates this particular recombination rate in gallium arsenide is almost 5 to 6 orders of magnitude higher right you can see from this time constant, this constant of the recombination rate constant of proportionality.

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Recombination Rates

Assignment-4.6

Derive the following relations for lifetime

$$\text{Low level (n-type): } \tau_{\text{Auger}} \approx \frac{1}{C_n N_d^2}$$

$$\text{High level: } \tau_{\text{Auger}} \approx \frac{1}{(C_n + C_p) \delta^2}$$

$$\text{Low level (n-type): } \tau_{\text{Radiative}} \approx \frac{n_i^2}{G_i N_d}$$

$$\text{High level: } \tau_{\text{Radiative}} = \frac{n_i^2}{G_i \delta}$$

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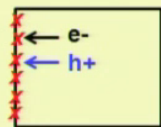
So this is the wording of the assignment derive the following relations for lifetime, so high level and low level lifetimes for auger and radiative recombination process.

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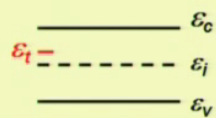


Recombination Rates

$$R_{\text{surface}} (\text{cm}^{-2} \text{ s}^{-1}) = s \delta|_{\text{surface}}$$



Trap concentration
 $N_{it} (\text{cm}^{-2})$



$$R_{\text{surface}} = \frac{pn - n_i^2}{s_p^{-1}(n + n_t) + s_n^{-1}(p + p_t)} \quad pn \geq n_i^2$$

$$s_p = \sigma_p v_{th} N_{it} \quad s_n = \sigma_n v_{th} N_{it}$$

$$n_t = n_i \exp\left(\frac{\epsilon_t - \epsilon_i}{kT}\right) \quad p_t = n_i \exp\left(\frac{\epsilon_i - \epsilon_t}{kT}\right)$$

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Now let us look at the surface recombination rate, now like you can have recombination in the volume when you move near the surface the carriers may combined at the surface, so recombination rate near a surface can be more than that in the bulk and therefore there will be tendency for carriers to move towards the surface, now why should the recombination rate near the surface be more because you know any surface is disruption the lattice.

And therefore lot of defects are naturally present wherever the disruption of the periodic lattice okay lot of states with in the energy gap are created whenever you have defects, so surface is the defects therefore lot of states are there in addition there can be some impurities and so on sticking on to it, so the surface is really a source of impurity also apart from defects, therefore recombination rate there can be higher.

Now that is the recombination rate that we are talking about and its unit is per centimeter per second and we would like to show as in the case of the volume recombination that we would like to write this recombination rate as proportional to the excess carrier concentration at the surface, so if you use a simple dimensional analysis you can see that the constant of proportionality will turn out to be a velocity.

So I can leave it to you as an assignment simple assignment the left hand side units are per centimeter square per second and delta is per centimeter cube, so the constant of proportionality should be centimeters per second and therefore it is called a velocity more specifically it is called surface recombination velocity, so let us see a model for this surface recombination velocity, this model would be analogous to that for lifetime.

Now since the surface recombination mainly happens because of defects or impurity levels are defect levels within the energy gap we can use the SRH theory to develop a model for this case as well, so here E_t is the effective energy level of the traps near the surface and the concentration of the traps would be per centimeter square and we have represented this concentration using suffix i as against the volume concentration of trap so N_{it} interface traps so i stands for interface.

After this the model is just analogous to the volume recombination rate by SRH theory, so R_{surface} is given by this formula SRH formula, where the time constant τ_p is replaced by $1/S_p$ by surface recombination velocity constant for holes and similarly here for electrons you can just check dimensionally does that this approach works when you replace $\tau_p/1/S_p$ and $\tau_n/1/S_n$.

So the surface recombination velocity would be given by $\sigma_p \cdot v_{\text{thermal}} \cdot N_{it}$, so here again the traps are capturing the electrons and holes and the rate at which the capture happens

depends on the velocity with which the carriers approach traps. I leave it to you as an exercise to show that when you replace this n suffix t volume concentration of traps by N_{it}, the left hand side the whole thing becomes a velocity right.

Because 1 centimeter unit is coming out and therefore instead of seconds you get a velocity okay, so I leave it to you to show that.

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Recombination Rates

$R_{\text{surface}} (\text{cm}^{-2} \text{ s}^{-1}) = s \delta|_{\text{surface}}$

← e⁻

← h⁺

$$R_{\text{surface}} = \frac{pn - n_i^2}{s_p^{-1}(n + n_t) + s_n^{-1}(p + p_t)} \quad pn \geq n_i^2$$

Trap concentration $N_{it} (\text{cm}^{-2})$

$s_p = \sigma_p v_{th} N_{it} \quad s_n = \sigma_n v_{th} N_{it}$

$\epsilon_t = \epsilon_i \Rightarrow n_t = p_t = n_i$

Low level (n - type): $s_{\text{SRH}} \approx s_p$ High level: $s_{\text{SRH}} \approx (s_p^{-1} + s_n^{-1})^{-1}$

$\sigma_p, \sigma_n \sim 5 \times 10^{-16} \text{ cm}^2 \quad N_{it} \sim 1 \times 10^{9-13} \text{ cm}^{-2} \quad s_p, s_n \sim 10^{-10^5} \text{ cm s}^{-1}$
at 300 K

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Here again we bother about the maximum surface recombination rate that happens for $E_t = E_i$ in that case n_t and p_t is n_i and these are your formulae for low level surface recombination rate that is in n type it is S_p , in a p type it is S_n . And high level however whether it is n or p type it is simply given by $1/S_p + 1/S_n$ whole power - 1, so you are simply replacing $\tau_p/1/S_p$, $\tau_n/1/S_n$ and these are the values.

So the capture cross sections are of the same order as in the case of volume recombination however n_t is replaced by the interface trap concentration and here per centimeter square this varies between 10^9 to 10^{13} and the surface recombination velocities vary from about 10 to 10^5 per centimeters per second, so 10 is the kind of limit for the ultra-clean surfaces where defects are less and no impurities and so on okay.

Now this kind of velocities you will see at silicon, silicon dioxide interfaces grown on ultra-clean environment okay in that case the silicon, silicon dioxide interface surface recombination velocity will be small.

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Generation Rates

$$G \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$$

$$G_{\text{Thermal}} = -R_{\text{SRH}} = \frac{n_i^2 - pn}{\tau_p(n + n_t) + \tau_n(p + p_t)} \quad pn \leq n_i^2$$

$$\tau_p = \frac{1}{\sigma_p v_{th} N_t} \quad \tau_n = \frac{1}{\sigma_n v_{th} N_t}$$

$$n_t = n_i \exp\left(\frac{E_t - E_i}{kT}\right) \quad p_t = n_i \exp\left(\frac{E_i - E_t}{kT}\right)$$

• $p, n \ll n_i$, as in depletion region
 • $E_t = E_i$

$$G_{\text{Thermal}} = \frac{n_i}{\tau_g} \quad \tau_g \approx \tau_p + \tau_n$$

The diagram shows energy levels ϵ_c , ϵ_i , and ϵ_v with an electron e^- moving from ϵ_v to ϵ_c and a dashed line at ϵ_i .

Now let us look at generation rates in analogy to the recombination rates the generation rates are dependent on 3 processes Thermal generation, generation by Impact or Impact ionization and generation by Light okay or Photoionization, so the process of thermal recombination is balanced by the in equilibrium the process of thermal recombination is balanced by the process of thermal generation, the process of auger recombination is balanced by impact ionization.

And the process of radiative recombination is balanced by the process of photoionization right, so these are inverse processes of the recombination rates, recombination process that we discussed, while under equilibrium the recombination and generation are in balance, under excess carrier condition the recombination balances the generation, on the other hand you can have situations where you can create excess carriers by this generation processes.

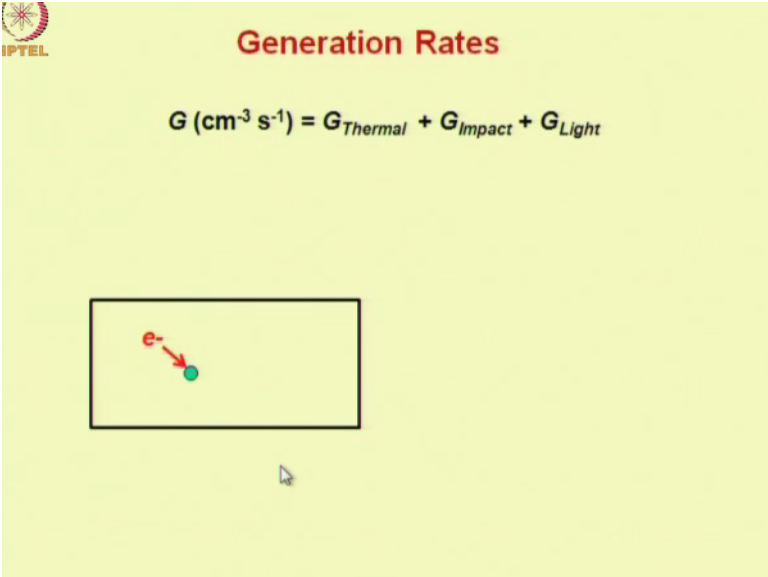
So the thermal generation process the same as SRH given by SRH recombination theory except that you are talking about the situation when pn product is less than n_i square, so that is why G_{thermal} is nothing but negative of R_{SRH} , so if pn is more than a n_i square you would use

the recombination you will assume that there is excess recombination if pn is less than n_i^2 you would assume that there is excess generation by the SRH approach.

So everything is the same as that we discussed earlier except that p and n are much less than n_i and for $E_t = E_i$ your thermal generation rate can be shown to be n_i/τ_g , so this is again left you as an assignment it is a very straight forward situation since p into n , p and n both are much less than n_i in the numerator you have just a n_i^2 and denominator this n and p drop off and you put p_t and $n_t = n_i$, n_i cancels, 1 numerator n_i cancels and you get this.

So the generation time τ_g is called regeneration time, so the generation time or generation lifetime is $\tau_p + \tau_n$, so you see that the generation time is more than the recombination time under low levels, it can be twice if τ_p and τ_n are same twice that of the low level lifetime recombination lifetime.

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The slide features a yellow background. At the top left is the IPTEL logo. The title "Generation Rates" is centered at the top in red. Below the title is the equation $G (\text{cm}^{-3} \text{s}^{-1}) = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$. In the center, there is a diagram showing a red electron (e^-) with a red arrow pointing towards a green circle representing an atom. A mouse cursor is visible below the diagram.

Let us look at the next process namely the impact generation now here an energetic electron is colliding with an atom this circle here represents an atom, the electron is scattered its direction of motion is changed but it has given a lot of energy and that energy is sufficient to create an electron hole pair, so valence electron bonding to the silicon atom has become free giving rise to a hole and an electron so this is the process of impact ionization.

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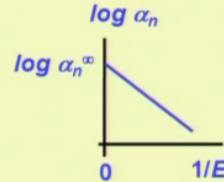
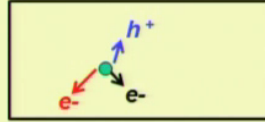


Generation Rates

$$G \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$$

$$G_{\text{Impact}} = \frac{\alpha_n |J_n| + \alpha_p |J_p|}{q}$$

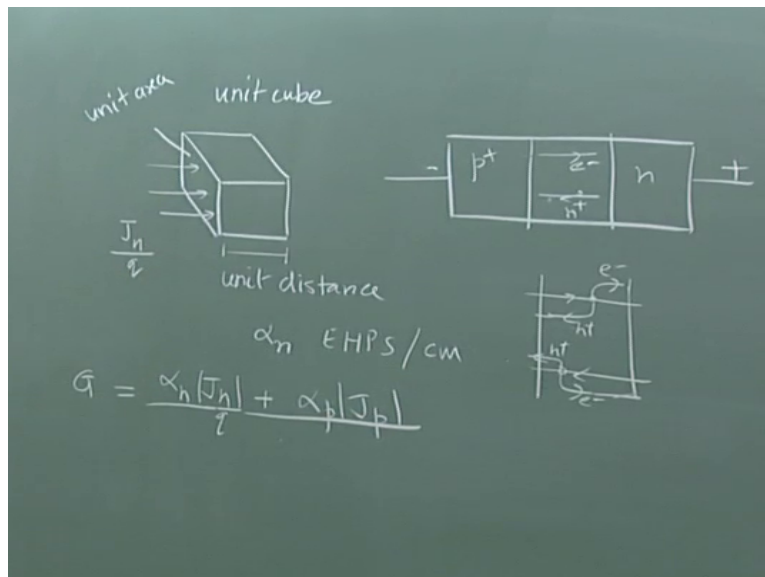
$$\alpha_{n,p} = \alpha_{n,p}^{\infty} \exp(-E_{n,p}^{\text{crit}}/E)$$



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So the formula for impact ionization can be derived easily, let me read out and then show how you can get it, so the impact ionization rate per centimeter cube per second is = the current density of electrons multiplied by a constant proportionality alpha n which is called the ionization constant which represents the number of electron hole pairs created by a moving electron over 1-unit distance, so per centimeter unit of alpha n is per centimeter.

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Similarly, you have ionization because of hole current as well, so let us see how you can get this formula suppose I concentrate on a unit cube in a semiconductor, this is a unit area and therefore the flux of carriers into this area let us say for electrons if I am talking about would be J_n/q , now

each electron while moving over a unit distance since this is a unit cube, this is unit area and this is unit distance.

So over a unit distance each carrier which is moving create alpha electron hole pairs that is our definition of alpha n, so alpha n EHPs per centimeter, since it is a unit distance so 1 centimeter therefore alpha n EHPs are created, now if each electron creates alpha n EHPs then there are how many electrons J_n/q entering at this rate therefore its stands to reason that alpha n J_n/q is number of electron hole pairs that would be created in this volume okay per unit time.

Because so many electrons are entering per unit time and then leaving this volume right, so that is why this is the generation rate, now we put a modulus because you know in practice let us take a situation such as reverse bias PN junction where this happens, so suppose this is the depletion layer p+n reverse bias, now the electrons will move from negative to positive whereas holes will move in this direction.

Now the contributions of electrons and holes independent of the direction or flow should add up okay, therefore you at alpha p J_p to alpha n J_n and you are dividing by q and you put a modulus sign so that you take care of any sign changes right J_n could be positive in one direction and negative in other direction, so this is your total generation rate, so if you try to show with the flow diagram what is happening inside.

So each electron is generating an electron hole pair, so let us say this is an electron, this is a hole and each hole also generating an electron hole pair, so the hole crosses in this direction electron goes in this direction, here the hole goes in this direction electron goes like that right, so this is the process that we are talking about impact ionization. So this is the formula now an empirical relation has been found for the ionization coefficient evidently.

If the field is more this ionization coefficient will increase, an exponential relation has been found so that can be that appears like a straight line over a log coefficient versus $1/E$ graph, so experimentally these things have been determined and it has been found that when you plot the experimental data on a log alpha n on y axis and $1/E$ on the x axis you get a straight line. That is

how this formula is arrived at, so clearly the ionization coefficient increases rapidly as electric field increases.

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Generation Rates

$$G \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$$

$$G_{\text{Impact}} = \frac{\alpha_n |J_n| + \alpha_p |J_p|}{q} \quad \alpha_{n,p} = \alpha_{n,p}^{\infty} \exp(-E_{n,p}^{\text{crit}}/E)$$

For silicon, at 300 K

$\alpha_n^{\infty} = 1 \times 10^6 \text{ cm}^{-1}$	$E_n^{\text{crit}} = 166 \text{ kV cm}^{-1}$
$\alpha_p^{\infty} = 2 \times 10^6 \text{ cm}^{-1}$	$E_p^{\text{crit}} = 2 \text{ MV cm}^{-1}$

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For silicon at 300 k these are the values of the alpha's and the critical electric field, so you find that critical electric fields are of the order of megavolt per centimeter per holes and an order less for electrons right, about an order less and the number of electron hole pair is generated per centimeter is about 10 power 6. So if you convert this to for micron to give you some field for the lens that are used in devices modern devices are of the order of microns lens, so that is 100 electron hole pairs per micron.

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Generation Rates

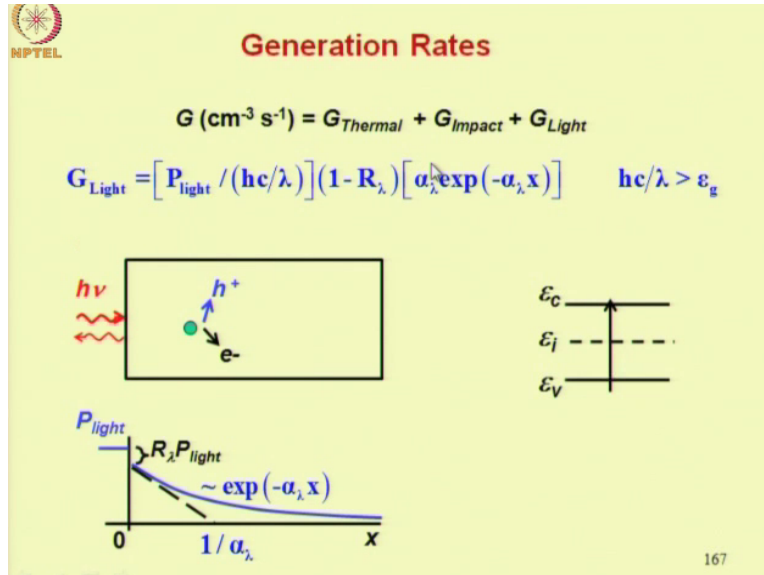
$$G \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$$

G_{Light}

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Now let us look at generation because of light that is a final form of generation, so here light is falling on a semiconductor and what is happening is that part of this light is reflected remaining goes in and gets absorbed, so what happens in the process of absorption so one of the same process that we are showing physically here is being shown in energy diagram.

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So when the light is absorbed by an atom the photon is absorbed it creates an electron hole pair, so the photon has been removed and instead electron hole pair has appeared this is a process of generation because of light, to some extent it has an analogy to impact ionization in the sense there also electron hole pair have been generated as a carrier moves through here instead of a carrier the source of electron hole pair is photon.

So the photon is ionizing the atoms as photons are getting absorbed into the sample, so if you show the details of how there incident intensity is changing as the function of distance, so P_{light} here stands for the power incident per unit area light power incident per unit area, now as you move inside there is a sudden drop in this power because a part of it is reflected. That is why is quantity is reflection coefficient into the incident P_{light} .

The reflection coefficient shown as the function of lambda wavelength of the light because that is how it is in practice, now as the light is absorbed its intensity falls it has been found that the intensity of light or power if you want to call it in terms of watts per centimeter square it goes on


falling exponentially, so this constant of proportionality is like the ionization constant of impact ionization.

Alpha suffix lambda because it depends on the lambda, now $1/\alpha_{\lambda}$ has the units of distance so α_{λ} is number of electron hole pairs created by photons per unit distance, so $1/\alpha_{\lambda}$ has units of length and therefore this distance here represents approximately the distance over which the light is absorbed in fact 3 times this distance is actually the correct distance over which light is absorbed most of the light would be absorbed.

Now putting this information together one can derive this formula write down this formula very easily, so here $P_{\text{light}}/hc/\lambda$ is the flux of photons per unit area, hc/λ is the energy of photon, so you divide power per unit area by energy you will get the number of photons per unit area, then $1-R_{\lambda}$ is the amount of light that gets through into the semiconductor because R is the reflection coefficient.

So whatever gets through is absorbed at this rate exponential of $-\alpha_{\lambda}x$, now each unit distance will create α_{λ} electron hole pairs, so that is how you get this formula generation as a function of x.

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Generation Rates

$$G \text{ (cm}^{-3} \text{ s}^{-1}\text{)} = G_{\text{Thermal}} + G_{\text{Impact}} + G_{\text{Light}}$$

$$G_{\text{Light}} = \left[\frac{P_{\text{light}}}{hc/\lambda} \right] (1 - R_{\lambda}) \left[\alpha_{\lambda} \exp(-\alpha_{\lambda} x) \right] \quad hc/\lambda > \epsilon_g$$

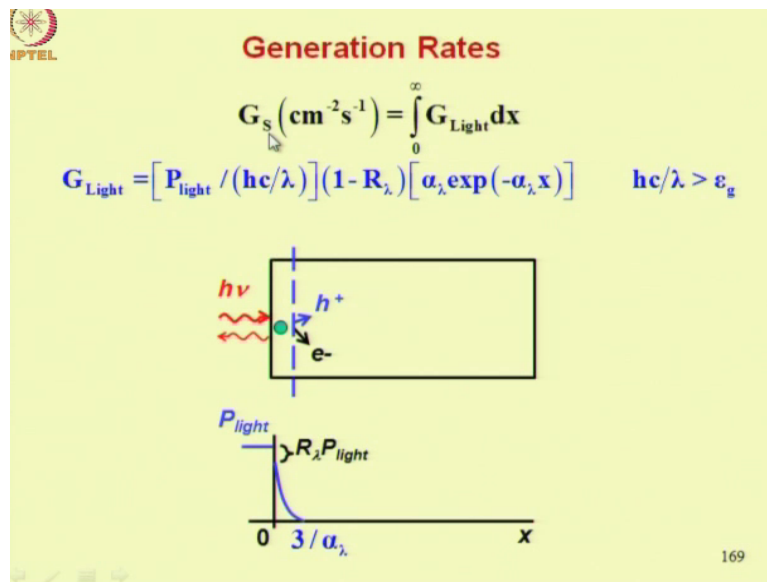
Symbol	Description	Typical value for Silicon
P_{light}	Incident light power	$\sim 100 \text{ mW cm}^{-2}$
λ	Incident light wavelength	$\lambda < 1 \text{ } \mu\text{m}$ for $hc/\lambda > \epsilon_g$
R_{λ}	Reflection coefficient	0.2-0.3
α_{λ}	Absorption coefficient = No. of EHPs generated cm^{-1}	$10^2, 10^4$ for $\lambda = 1, 0.5 \text{ } \mu\text{m}$
x	Depth of absorption	1 - 500 μm

Now what we said in words has been put down in this table, the meanings of various symbols
 P_{light} is incident light power typically its value is about 100 milliwatts per centimeter square if you take for example the solar spectrum of course it is a function of λ the amount of power density over the spectrum we are giving here some typical values. The λ is incident light wavelength generally since only light which has energy a photons which have energy more than energy gap will create electron hole pairs.

Therefore, for silicon that λ corresponds to 1 micron so λ less than 1 micron will create a electron hole pairs. Reflection coefficient is of the order of 0.2 to 0.3 that means 20 to 30% light is reflected by an anti-reflection coating methods and so on, you can reduce this reflection. Alpha λ is the absorption coefficient number of electron hole pairs generated per centimeter it is about 100 per 1 micron wavelength and 10 power 4 for 0.5 micron wavelength.

It is very sensitive function of λ , you expect that as a λ falls the absorption will increase evidently because the energy of the photon is increasing and x is depth of absorption.

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Now we shall close the lecture with formula for surface generation G suffix s per centimeter square per second, now you if you integrate the light generation rate volume generation rate due to light over distance you will get the surface generation, so here we are assuming that for some wavelength of light the alpha λ is so high that $1/\alpha$ λ is small.

So light is getting absorbed in very short distance from the surface, $3/\alpha \lambda$ is approximately the distance over which the light is absorbed, therefore if the semiconductor is very long compared to this distance we can treat the generation that has happened in this volume thin volume as surface generation and we can just talk about how many electron hole pairs are emanating from this per centimeter square per unit time right that is the surface generation.

So since this thickness is very small a very small amount of the electron hole pairs will really recombined and so all of them will be coming out, so you can give this whole thing as a surface phenomenon since the volume is very thin, so here all that you have done is you take this expression for G light and integrate so when you integrate the exponential term disappears and $\alpha \lambda$ goes away and this is your surface generation rate.

So since we have come to the end of this lecture, let us make a quick summary of the important points, this summary will really very short because we are now going to discuss module summary, summary of this whole module on drift diffusion module and that we will do in the beginning of the next lecture, so therefore a very short summary of this lecture here. So in this lecture, what we have discussed is the various expressions for generation and recombination processes, various generation, recombination processes.

So the generation, recombination process that we discussed are the thermal generation, the impact ionization generation and the generation because of light or photoionization and the corresponding counterparts for recombination are the thermal recombination, the auger recombination and the radiative recombination, we discussed both volume recombination as well as surface recombination.

Because at the surface the recombination rates are higher because of presence of defects and impurities, so similarly we discussed both volume generation and surface generation. So we said that if light is absorbed in a very thin region near the surface, then the volume recombination in this thin region can be treated effectively as the volume generation in this thin region can be

effectively treated as the surface generation as electron hole pairs coming out of this volume per unit area per unit time, so we derived the expressions for all these cases.