Semiconductor Device Modeling
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Lecture - 14
Semi-classical Bulk Transport: EM field and Transport Equations

In the previous lecture, we have discussed the balance equation approach in which we converted the Boltzmann's Transport Equation into a set of balance equation for carrier concentration, for current density, for kinetic energy density and so on. This we achieved by multiplying the Boltzmann's Transport Equation with a function of momentum P and then integrating the result over various allowed states.

We worked out what will be the various terms of this balance equation. Now in this lecture we would like to develop the balance equations further, see how the equations can be used to model velocity saturation, velocity overshoot, velocity field curve and so on. The balance equations are the ones which are most often used for device modeling and one rarely uses the Boltzmann's Transport Equation itself.

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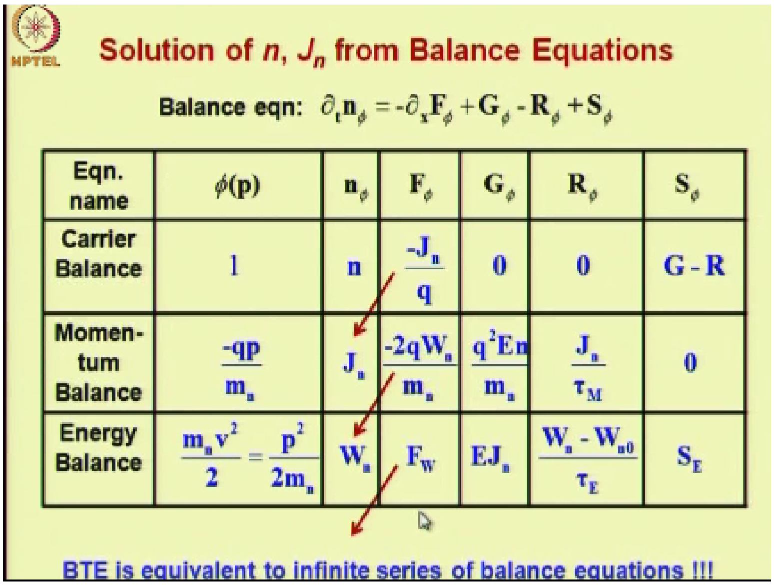

Solution of n, J_n from Balance Equations

Balance eqn: $\partial_t n_\phi = -\partial_x F_\phi + G_\phi - R_\phi + S_\phi$

Eqn. name	$\phi(p)$	n_ϕ	F_ϕ	G_ϕ	R_ϕ	S_ϕ
Carrier Balance	1	n	$\frac{-J_n}{q}$	0	0	$G - R$
Momentum Balance	$\frac{-qp}{m_n}$	J_n	$\frac{-2qW_n}{m_n}$	$\frac{q^2En}{m_n}$	$\frac{J_n}{\tau_M}$	0
Energy Balance	$\frac{m_n v^2}{2} = \frac{p^2}{2m_n}$	W_n	F_W	EJ_n	$\frac{W_n - W_{n0}}{\tau_E}$	S_E

BTE is equivalent to infinite series of balance equations !!!

Now let us see the arrangement of the balance equation. To solve for the carrier concentration N we need the information about the carrier current density Jn. This is obtained from the momentum balance equation. So here this equation expresses the current density Jn. However, what you find is that you have got a new variable namely the kinetic energy density.

So you need to know Wn if you want to solve for Jn. Now where do you get the Wn from, you get Wn from the energy balance equation, but then this seems to be going on and on because now you have introduced the kinetic energy flux FW. So how do we get FW? Well, the way out would be to write another equation. Now there seems to be a problem that you seem to have infinite number balance equation.

So the equation for carrier concentration introduces the carrier flux Jn. The equation for Jn introduces the kinetic energy Wn. The equation for kinetic energy introduces the kinetic energy flux FW and it goes on and on. So how do we deal with this situation?

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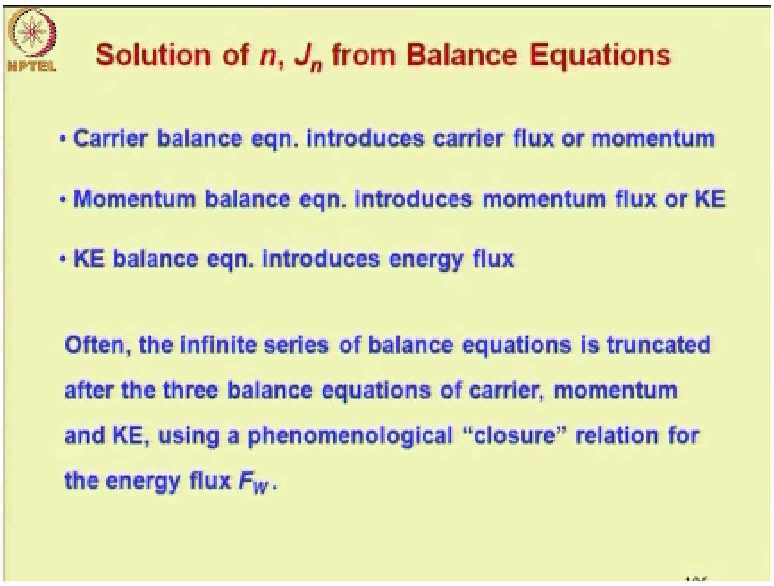

Solution of n, J_n from Balance Equations

- Carrier balance eqn. introduces carrier flux or momentum
- Momentum balance eqn. introduces momentum flux or KE
- KE balance eqn. introduces energy flux

Often, the infinite series of balance equations is truncated after the three balance equations of carrier, momentum and KE, using a phenomenological "closure" relation for the energy flux F_W.

The situation is carrier balance equation introduces carrier flux or momentum. Momentum balance equation introduces momentum flux or kinetic energy and kinetic energy balance equation introduces energy flux. The way out is the following. Often the infinite series of balance equations is truncated after 3 balanced equations of carrier, momentum and kinetic energy using a phenomenological closure relation for the energy flux FW.

So what you do is you pause or you stop after the 3 equations. Now you do not have a relation for FW solved from other fundamental equation. So what do you do you make an assumption about FW. This assumption is based on your observation of how kinetic energy flux varies with various quantities. For example, consider the heat flux you can write the heat flux as proportional to the temperature gradient.

So you write heat flux=a constant into the temperature gradient. Now how do you write this? It is not as though you have derived this relation from fundamental principles, but you observe an Ohm’s law kind of relations or linear behavior between cause and effect. So for a spring for example you write the force = the constant into the displacement. So the force exerted by the spring is proportional to the amount by which you elongated.

Now using these kinds of observations you are writing an approximate relation for FW. So this approach of writing a relation based on some intuitive understanding, but not necessarily from fundamentals is called a phenomenological approach. So you are using a phenomenological relation for FW to terminate the series of balance equations and that is why the relation that you use for FW is called a closure relation.

Because now the form is getting closed, the form of equations is getting closed. It is not going on and on.

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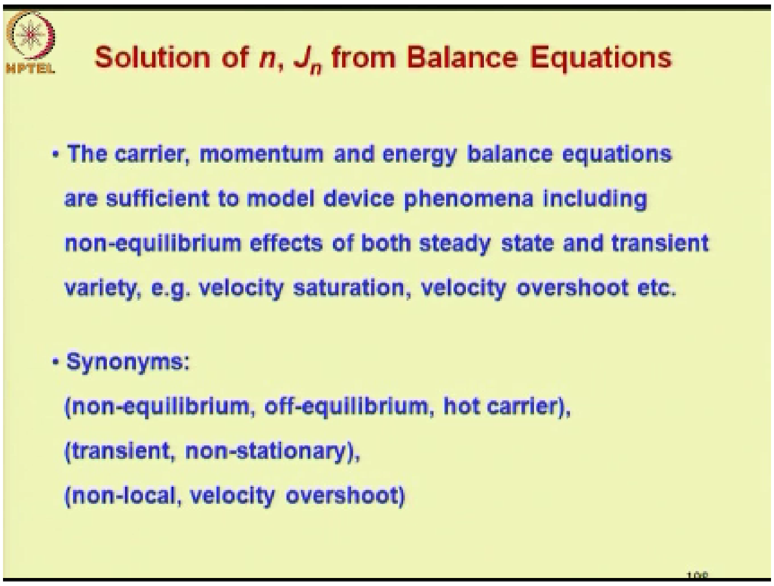

Solution of n, J_n from Balance Equations

- The carrier, momentum and energy balance equations are sufficient to model device phenomena including non-equilibrium effects of both steady state and transient variety, e.g. velocity saturation, velocity overshoot etc.
- Synonyms:
  (non-equilibrium, off-equilibrium, hot carrier),
  (transient, non-stationary),
  (non-local, velocity overshoot)

It turns that the carrier momentum and energy balance equations are sufficient to model device phenomena including non-equilibrium effects of both steady state and transient variety such as the velocity saturation, velocity overshoot etcetera. These effects we have already introduced in the context of the qualitative model of the various transport phenomena. Let me just alert you to some of the synonyms.

Many times some words are used interchangeably to mean the same thing. For instance, the terms non-equilibrium, off equilibrium, hot carrier is often used to mean the same. Similarly, the word transient means the same as non-stationary and the word non-local effects and velocity overshoot effects are also used interchangeably.

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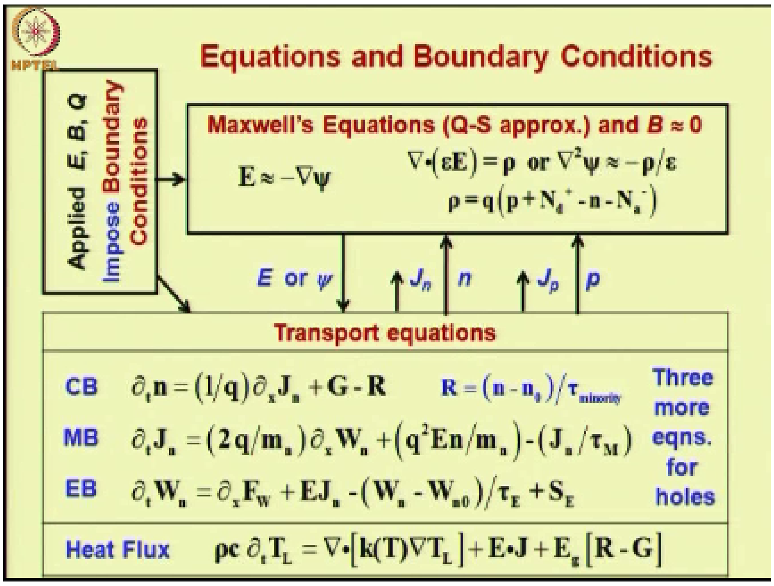

Equations and Boundary Conditions

Applied E, B, Q

Impose Boundary Conditions

Maxwell's Equations (Q-S approx.) and $B \approx 0$

$\mathbf{E} \approx -\nabla\psi$

$\nabla\cdot(\varepsilon\mathbf{E}) = \rho$ or $\nabla^2\psi \approx -\rho/\varepsilon$

$\rho = q\left(p + N_d^+ - n - N_a^-\right)$

Transport equations

CB $\partial_t \mathbf{n} = (1/\mathbf{q})\partial_x \mathbf{J}_n + \mathbf{G} - \mathbf{R}$ $\quad$ $\mathbf{R} = (\mathbf{n} - \mathbf{n}_0)/\tau_{\text{minority}}$

MB $\partial_t \mathbf{J}_n = (2\mathbf{q}/\mathbf{m}_n)\partial_x \mathbf{W}_n + \left(\mathbf{q}^2\mathbf{En}/\mathbf{m}_n\right) - (\mathbf{J}_n/\tau_M)$

EB $\partial_t \mathbf{W}_n = \partial_x \mathbf{F}_W + \mathbf{EJ}_n - (\mathbf{W}_n - \mathbf{W}_{n0})/\tau_E + \mathbf{S}_E$

Three more eqns. for holes

Heat Flux $\rho c\, \partial_t \mathbf{T}_L = \nabla\cdot\left[\mathbf{k(T)}\nabla\mathbf{T}_L\right] + \mathbf{E}\cdot\mathbf{J} + \mathbf{E}_g\left[\mathbf{R} - \mathbf{G}\right]$

So let us summarize our picture of the equations and boundary conditions. So at this point of time our picture looks as shown in this slide. Let us focus on the transport equations because we already are familiar with the Maxwell equations under quasi-static approximation and neglect of the magnetic flux B. So you have the carrier balance equation that is dou N/dou T = 1/Q into dou/dou X of Jn+ the generation rate of electrons and recombination rate of electron hole pair.

Often derive the recombination rate in this form of carrier concentration - the equilibrium value divided by the minority carrier lifetime. The momentum balance equation on the other hand is given by dou Jn/dou T = 2 Q divided by the effective mass electrons the whole term multiplying the spatial derivative dou/dou X of Wn the energy density +Q square into electric field into the carrier concentration N divided by the effective mass MN - the current density Jn divided by the momentum relaxation time.

The energy balance equation is given by a time derivative of Wn = dou/dou X of the energy flux FW+ the heat generated so called ohmic losses electric field into the current density at that point - Wn - the equilibrium value of Wn that is Wn0 divided by the energy relaxation time + a source term which signifies the contribution of electron hole pair generation recombination processes to the carrier energy.

Now, you have 3 more equations for holes. We have written down this equation for electrons. You have 3 more equations for holes. So you have 6 equations here + one must not forget the equation for heat flux and that is given by the mass density rho into the heat capacity C multiplied by the time derivative of the lattice temperature equal to the spatial derivative or diversions of the thermal conductivity times the gradient of the temperature, lattice temperature + the electric field into J the ohmic losses.

So this term is analogous here so this term here E times Jn except that the J here includes the electron current density as well as the hole current density + the heat that is generated by excess recombination processes of electron hole pairs. So R - G into the energy gap. So the Maxwell equations give you the information about the electric field and they in turn require the input as concentration of electrons and holes because you need to use the space charge here to get the electric field.

Now these are supplied by the transport equations which also give you the current density Jn and the hole current density JP and this transport equation require the electric field information as the input which is obtained from the Maxwell equation. You impose suitable boundary conditions to solve the differential equation so that is our picture and this is the transport model that will be used to analyze various devices.

So in fact, these are the equations which are used very often. We will also see some further approximations of this equation to derive what is called the drift diffusion transport model which is much more common than the models based on all the balance equations.

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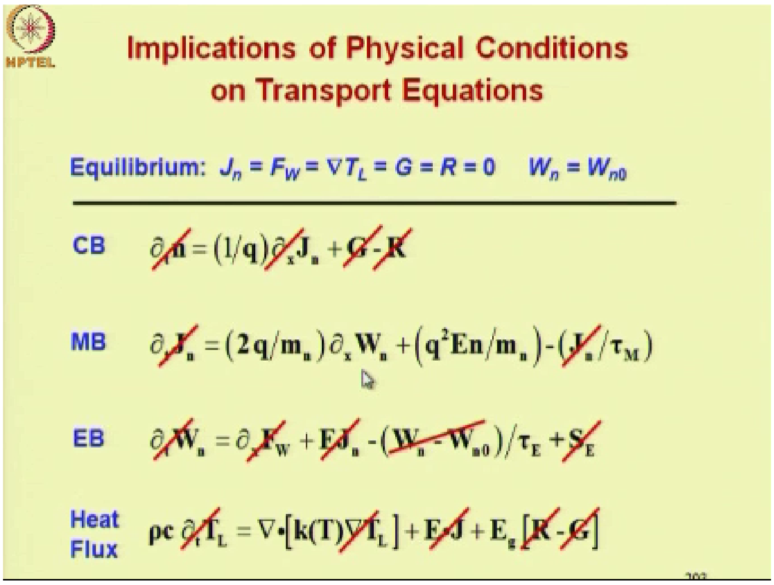

Implications of Physical Conditions on Transport Equations

Equilibrium: $J_n = F_W = \nabla T_L = G = R = 0 \quad W_n = W_{n0}$

CB $\cancel{\partial_t n} = (1/q)\cancel{\partial_x} J_n + \cancel{G} - \cancel{R}$

MB $\cancel{\partial_t J_n} = (2q/m_n)\partial_x W_n + (q^2 En/m_n) - (\cancel{J_n}/\tau_M)$

EB $\cancel{\partial_t W_n} = \cancel{\partial_x F_W} + \cancel{EJ_n} - (\cancel{W_n - W_{n0}})/\tau_E + \cancel{S_E}$

Heat Flux $\rho c\, \cancel{\partial_t T_L} = \nabla \bullet [k(T)\cancel{\nabla T_L}] + \cancel{E \cdot J} + E_g[\cancel{R} - \cancel{G}]$

Now let us look at implications of some physical conditions on transport equations. So very often we do not have to deal with all the terms of these differential equations. They look fairly formidable. However, I want to remind you that we have said that all these equations have a common form because these are all talking about conservation balance for continuity of some physical quantity.

And they can be related to the familiar whole continuity equation so this you must always bear in mind. Let us start with the physical condition of equilibrium. So under equilibrium current density Jn the kinetic energy flux FW both arE0 because the condition of equilibrium is no net flow of charge or energy so that is what is captured here when you say Jn and FW arE0.

Further another condition of equilibrium is that the temperature is uniform throughout the device. Now this is expressed in the form the gradient of TL lattice temperature is 0. Further you do not have generation or recombination processes, excess generation of recombination processes. G and R represent excess generation and recombination so these arE0. Similarly, the kinetic energy density of electrons Wn = the equilibrium value.

Now as a consequence of this many of the terms in this equations will drop out. So the equilibrium picture of these equations is as follows. So you see most of the terms are getting dropped out. The carrier balance equations really become trivial. So things are not changing with time under equilibrium therefore all the time derivatives arE0 here in this column the left hand side of all equations arE0.

Further, since Jn itself is 0, this term does not exist. G and R also arE0. So carrier balance equation really become trivial all terms = 0. Similarly, you find energy balance equation also all terms are becoming 0 and same thing about the heat flux. However, it is interesting to know for the momentum balance equation there are 2 terms which are noN0 here.

While Jn/tau M goes to 0 because Jn is 0 and left hand side is also 0 because time derivative are absent. Now what does this mean? Now you see that there can be conditions of non uniform doping in a semiconductor though the semiconductor is under equilibrium and you would recall from discussions in the first level course that under non-informally dope conditions.

You do have an electric field to counterbalance the diffusion that is created by non uniform doping. So non uniform doping causes the majority carrier concentration to be non uniform and the minority carrier concentration also to be non uniform and therefore there are diffusion currents of mobile carrier electrons are holes. However, since under equilibrium there should be no Jn or JP the diffusion current should be balanced by drift current.

And that is how an electric field is set up. Now that is kind of situation that is described by these 2 terms. So you see for non-uniformly dope semiconductor E is noN0. However, since other terms arE0 these 2 terms should sum to 0. So therefore it shows that there is a noN0 spatial gradient of kinetic energy density. Now as we will see this is due to the presence of the carrier concentration gradient and that is causing the diffusion current.

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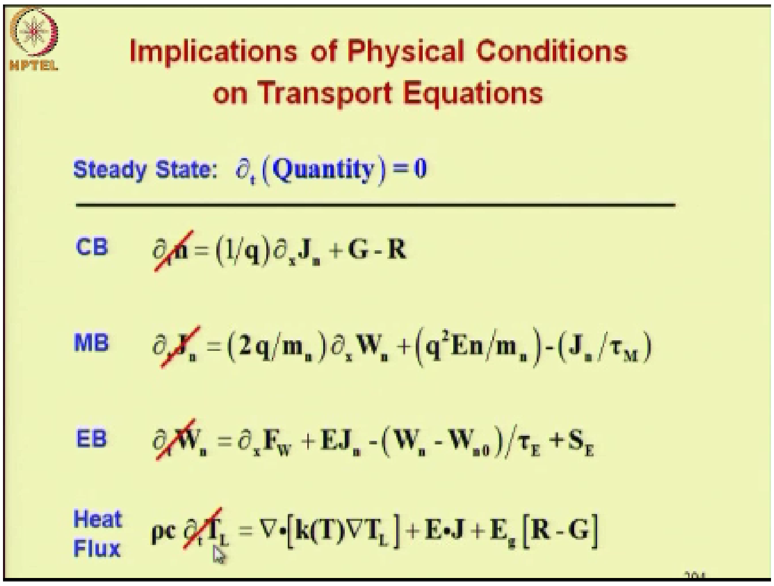

Implications of Physical Conditions on Transport Equations

Steady State: $\partial_t(\text{Quantity}) = 0$

CB $\cancel{\partial_t n} = (1/q)\partial_x J_n + G - R$

MB $\cancel{\partial_t J_n} = (2q/m_n)\partial_x W_n + (q^2 E n/m_n) - (J_n/\tau_M)$

EB $\cancel{\partial_t W_n} = \partial_x F_W + E J_n - (W_n - W_{n0})/\tau_E + S_E$

Heat Flux $\rho c\,\cancel{\partial_t T_L} = \nabla\cdot[k(T)\nabla T_L] + E\cdot J + E_g[R - G]$

Let us take the next important condition namely the steady state. In this case all time derivatives go to 0-time derivative of all quantities dou/dou T of any quantity is 0. So what is happening is the left hand side of all these equations is being said to 0.

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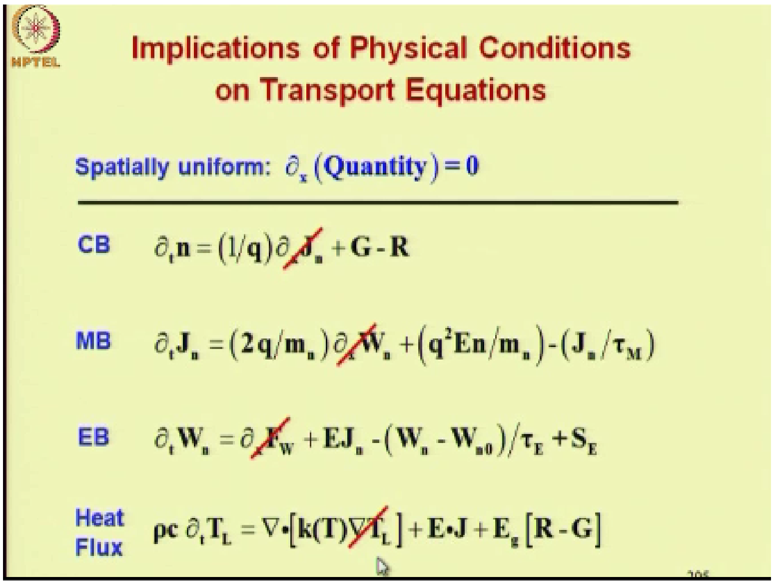

Implications of Physical Conditions on Transport Equations

Spatially uniform: $\partial_x(\text{Quantity}) = 0$

CB $\partial_t n = (1/q)\cancel{\partial_x J_n} + G - R$

MB $\partial_t J_n = (2q/m_n)\cancel{\partial_x W_n} + (q^2 E n/m_n) - (J_n/\tau_M)$

EB $\partial_t W_n = \cancel{\partial_x F_W} + E J_n - (W_n - W_{n0})/\tau_E + S_E$

Heat Flux $\rho c\,\partial_t T_L = \nabla\cdot[k(T)\cancel{\nabla T_L}] + E\cdot J + E_g[R - G]$

Let us take another important situation spatially uniform condition. In this case the spatial derivatives of quantity go to 0. For example, in carrier balance equation dou Jn/dou X is 0. In momentum balance equation dou Wn/dou X is 0. In energy balance equation the dou/dou X of FW is 0. In fact, this is the reason why in many situations you do not really need information about FW.

Because they are spatially uniform and finally in the heat flux also the gradient of the lattice temperature is 0.

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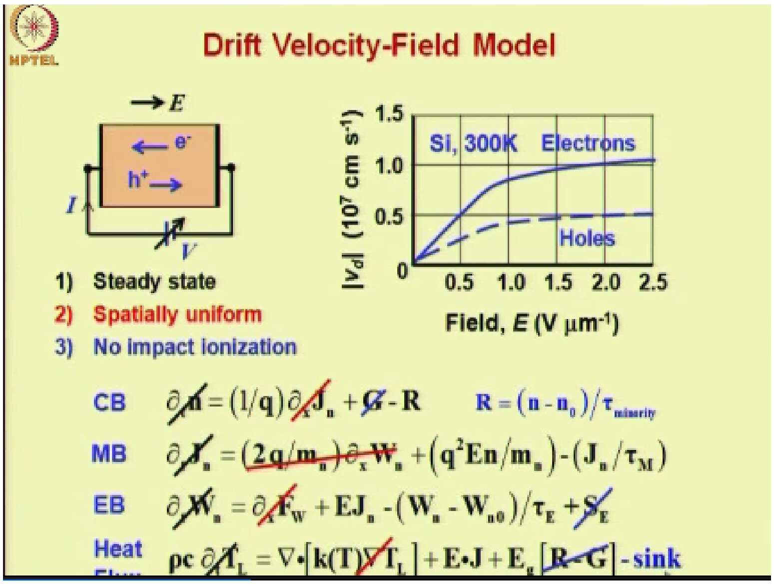

Drift Velocity-Field Model

1) Steady state
2) Spatially uniform
3) No impact ionization

CB $\cancel{\partial_t n} = (1/q)\cancel{\partial_x} J_n + \cancel{G} - R$ $R = (n - n_0)/\tau_{minority}$

MB $\cancel{\partial_t J_n} = \cancel{(2q/m_n)\partial_x W_n} + (q^2 E n/m_n) - (J_n/\tau_M)$

EB $\cancel{\partial_t W_n} = \cancel{\partial_x} F_W + E J_n - (W_n - W_{n0})/\tau_E + \cancel{S_E}$

Heat $\rho c\, \cancel{\partial_t T_L} = \nabla \bullet [k(T)\cancel{\nabla} T_L] + E \bullet J + E_g [\cancel{R - G}] - sink$

Now let us go on to model the velocity field curve based on this information about balance equation. Now this curve is shown here for silicon at 300 K for electrons and holes. So basically what we want to model is a linear increase for small electric field followed by a saturating tendency for large electric fields.

As we have remarked whenever we want to develop a model for any characteristics, we must also have a diagram of the biasing arrangement of the device in mind which tells us what is a device geometry and what is the applied signal. So this is the situation that is depicted here. So you have a uniform semi conductor a voltage is applied DC voltage is applied and there is a current and this DC voltage is very slowly varied.

This is very important that we are not varying it rapidly. So the variation is so slow that corresponding to any voltage the conditions in the device can be regarded as steady state. This voltage sets up an electric field E that is causing the electrons to move from the right contact to left contact but holes moves from left to right. So first we should write down the conditions before writing the equations.

So the conditions are steady state then spatially uniform and no impact ionization. Now why are we saying this no impact ionization because we are dealing with high electric fields here when the velocity tends to saturate and we want to be sure that our fields are not so high that there is excess generation because of impact ionization. So we will not analyze that situation where impact ionization may be present. Now what would our equations look like.

So let us put down the equations here. We are doing this modeling for electrons and now let us simplify the equations using the conditions so which are the terms which is drop out. So since situation is steady state all the left hand side terms here have dropped out. Since it is spatially uniform the spatial derivatives have dropped out. This is shown by the red crosses here because spatially uniform is shown in red color.

The steady state is shown in black color and then no impact ionization. This means there is no generation excess generation of carriers. Now evidently this means that the recombination also is 0 because excess recombination is 0 because you see this term is 0 in carrier balance equation this term is 0. So evidently R has to bE0. This means the electron concentration would be equal to the equilibrium value of the concentration.

Now, look at the term SE in energy balance equation. Now we are neglecting a contribution of generation recombination processes to the kinetic energy. Now this is because we have just now remarked that there are no excess generation or recombination. So therefore there is no energy contribution from these processes. Also note that because the G and R have becomE0 as obtained from the carrier balance equation the R - G term here in the heat flux equation drops out.

Now you see there is a problem we have to introduce a sink term here otherwise the heat flux equation becomes inconsistent why suppose we do not have the sink term. Now, you see what is happening because of steady state condition this term is 0, time derivative is 0 because of uniform conditions that is what we have claimed the spatial derivative is 0 and because there is no excess generation or recombination this term is 0.

Now, this seems to lead to the condition that E dot J is 0 that is no heat generation or ohmic loss which is evidently wrong because E is noN0 and J is noN0. So therefore there has to be another term non-0 term here. Now what has happened is we have not taken into account in this equation the rate at which the heat is lost by the semi conductor to the surroundings.

The equation that we have written down here talks about the sources of heat within or sources or sinks of heat within the semiconductor so that is not sufficient. We need to have a sink term here otherwise there will be inconsistency so that is the sink term. However, in practice we are not going to use this heat flux equation. So we will not concern ourselves with the sink term here.

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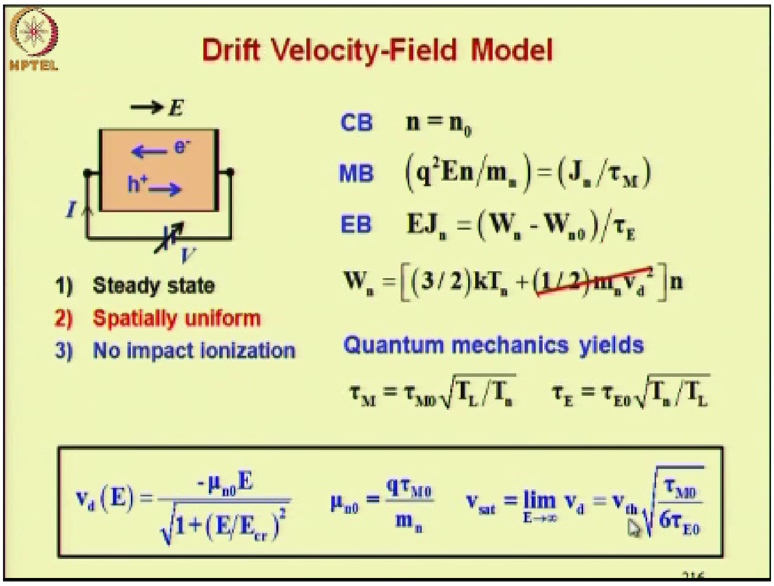

Drift Velocity-Field Model

1) Steady state
2) Spatially uniform
3) No impact ionization

CB $n = n_0$

MB $\left(q^2 E n / m_n\right) = \left(J_n / \tau_M\right)$

EB $E J_n = \left(W_n - W_{n0}\right) / \tau_E$

$W_n = \left[(3/2) k T_n + \cancel{(1/2) m_n v_d^2}\right] n$

Quantum mechanics yields

$\tau_M = \tau_{M0}\sqrt{T_L / T_n} \qquad \tau_E = \tau_{E0}\sqrt{T_n / T_L}$

$$v_d(E) = \frac{-\mu_{n0} E}{\sqrt{1 + \left(E/E_{cr}\right)^2}} \qquad \mu_{n0} = \frac{q\tau_{M0}}{m_n} \qquad v_{sat} = \lim_{E \to \infty} v_d = v_{th}\sqrt{\frac{\tau_{M0}}{6\tau_{E0}}}$$

So we will work with the carrier balance, the momentum balance and energy balance equation. So the simplified forms of this equation are given here after all the other terms have been removed. So carrier balance equation gives the electron concentration = equilibrium value. The momentum balance equation gives equality between these 2 terms and energy balance equation will give you this.

Now what about tau M and tau E? The relation for tau M and tau E have to be obtained from quantum mechanics because they are related to the scattering phenomena and we have discussed that the scattering itself has to be treated quantum mechanically. So these relations are obtained from quantum mechanics. We have not done this derivation we are accepting the result.

Now that being the case we need to express the energy balance equation in terms of the carrier temperature because you see the equations derived for tau M and tau E from quantum mechanics consists of the electron temperature here. To convert the energy balance equation into equation for electron temperature we use this relation for kinetic energy density Wn in terms of the carrier temperature Tn.

So, recall that this kinetic energy density can be split into 2 parts random component called the thermal component and the direct heat component called the drift component. Since it is energy density you have the electron concentration terms coming there. Now usually we make the approximation that we neglect the direct heat component. We assume that the random component is very large.

Now these approximations is definitely quite good for very high electric fields where the direct heat component saturates. It is also good for very low electric field. When the direct heat component is small, you know, that even under equilibrium you have a large kinetic energy density because carriers are moving about randomly. So near equilibrium also the direct heat component is very small compared to the random component and for very high electric fields when the velocity saturates the random component goes on increasing.

So even there this approximation holds good somewhere in between the approximation may not be valid to that extent. However, we shall continue to use this approximation over the entire range to get a simple equation. Now I leave it as an assignment to you to show that simple algebraic manipulations with these equations here. In fact, you do not have to use the carrier balance equation this is independent.

You basically have to work with these 2 equations because the Jn here enters in both these equations really these are 2 equations to be solved simultaneously and in this you substitute Wn in terms of Tn from here and also use this formula for tau M and tau E. The result turns out to be in the following form.

The drift velocity is the function of electric field given by -mu N0 into electric filed by square root of 1 + electric field by a critical electric field square in which the mu is 0 is Q times the momentum relaxation time under equilibrium by the effective mass of electron. And the saturation velocity is obtained by taking the limit E tending to infinity in this equation and what you find is that when you take E tending to infinity.

You can see that this term will be very large and so square root of a square term is E itself and therefore this E will cancel with this E and therefore you will get a saturation value and that saturation value should be shown as thermal velocity into square root of tau M0/6 times tau is 0. So this tells you that the saturation velocity is of the order of thermal velocity. Now you will have to use the relation for thermal velocity.

So you know that the kinetic energy density or kinetic energy for carrier under equilibrium you have the relation half M into V thermal square = 3/2 times the Boltzmann constant into the lattice temperature so you will have to use that relation.

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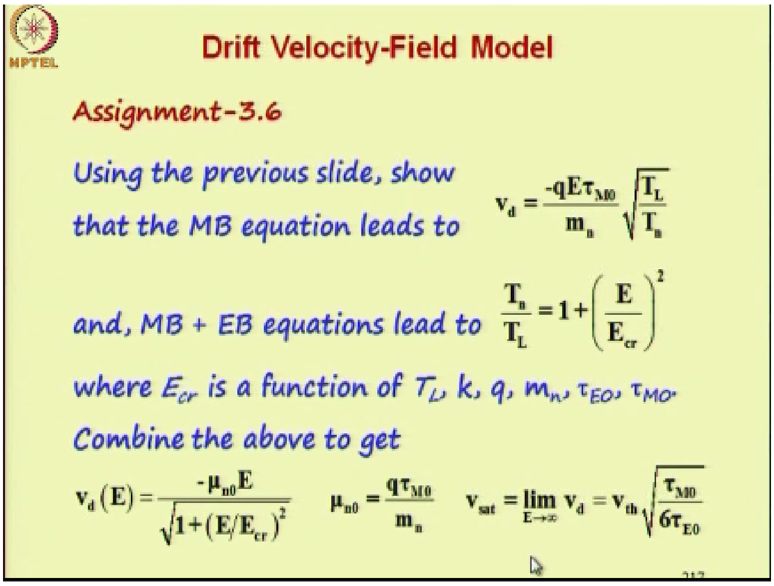

Drift Velocity-Field Model

Assignment-3.6

Using the previous slide, show that the MB equation leads to $v_d = \frac{-qE\tau_{M0}}{m_n}\sqrt{\frac{T_L}{T_n}}$

and, MB + EB equations lead to $\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$

where E_{cr} is a function of T_L, k, q, m_n, τ_{E0}, τ_{M0}.

Combine the above to get

$$v_d(E) = \frac{-\mu_{n0}E}{\sqrt{1+(E/E_{cr})^2}} \qquad \mu_{n0} = \frac{q\tau_{M0}}{m_n} \qquad v_{sat} = \lim_{E\to\infty} v_d = v_{th}\sqrt{\frac{\tau_{M0}}{6\tau_{E0}}}$$

Let me give you some hints for this assignment. So using the previous slide show that the momentum balance equation leads to this formula for Vd then if you combine the momentum balance and energy balance equation you will obtain this. So the algebraic manipulation is your responsibility. I am telling you some hints for the individual steps.

Now where E critical is a function of the lattice temperature TL Boltzmann constant K the electronic charge Q effective mass of electrons MN. The energy relaxation time close to equilibrium and momentum relaxation time close to equilibrium and then you can combine these 2 equations to show this model.

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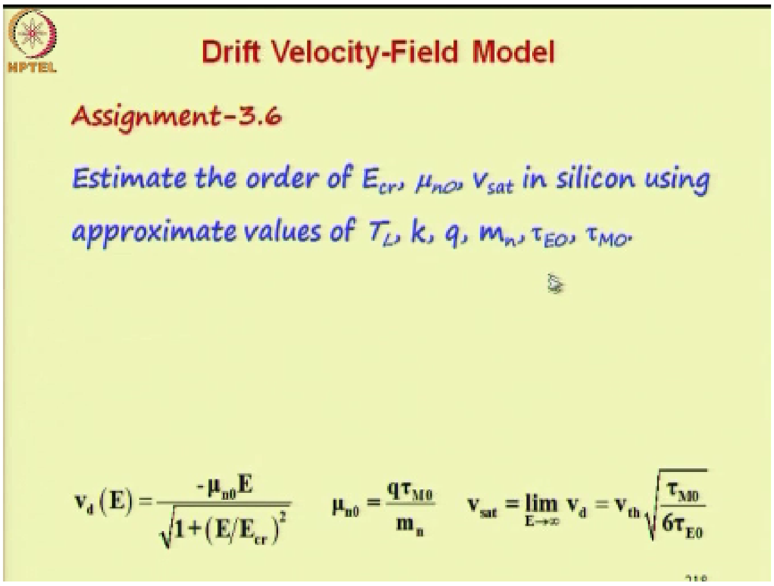

Drift Velocity-Field Model

Assignment-3.6

Estimate the order of E_{cr}, μ_{n0}, v_{sat} in silicon using approximate values of T_L, k, q, m_n, τ_{E0}, τ_{M0}.

$$v_d(E) = \frac{-\mu_{n0}E}{\sqrt{1+(E/E_{cr})^2}} \qquad \mu_{n0} = \frac{q\tau_{M0}}{m_n} \qquad v_{sat} = \lim_{E\to\infty} v_d = v_{th}\sqrt{\frac{\tau_{M0}}{6\tau_{E0}}}$$

I would also like you to estimate the order of critical electric field, the mobility of electrons for low electric fields mu N0, saturation velocity in silicon using approximate values of these quantities. Just to give you some feel tau M0 is of the order of a few 10s of a picoseconds and tau E0 is of the order of picoseconds. You can get the other quantities from books or internet.

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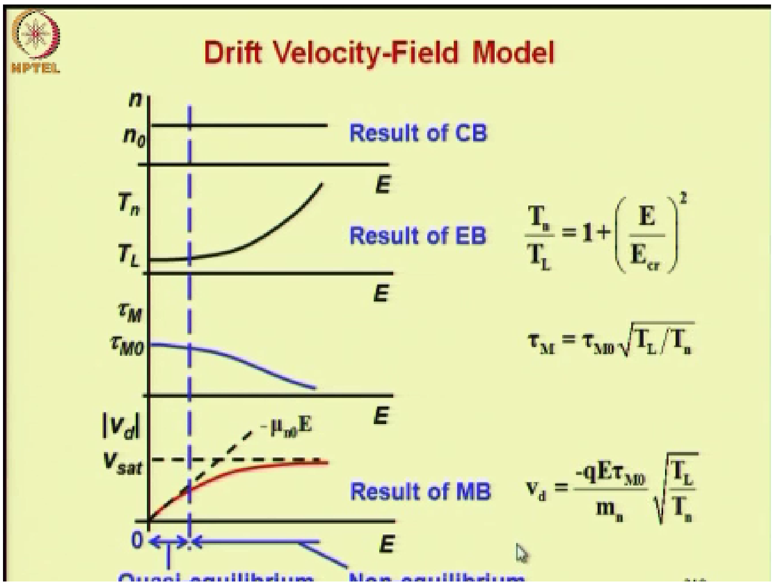

Drift Velocity-Field Model

Result of CB

Result of EB

$$\frac{T_n}{T_L} = 1 + \left(\frac{E}{E_{cr}}\right)^2$$

$$\tau_M = \tau_{M0}\sqrt{T_L/T_n}$$

Result of MB

$$v_d = \frac{-qE\tau_{M0}}{m_n}\sqrt{\frac{T_L}{T_n}}$$

Now here is the graphical representation of the results. This slide has actually been pulled up from the qualitative analysis, qualitative modeling. So why does the velocity saturate. Now this is your velocity field curve. Now why does velocity saturate it is because the momentum relaxation time is failing down and at this end it is inversely proportion to electric fields. So why does the momentum relaxation time fall down it is because of the electron temperature rising rapidly here.

So this is the graph representing the result of energy balance which is Tn/TL = 1+E/E critical square. The result of carrier balance is N remains = N0. Now this shape of tau M as a function of electric field is obtained from quantum mechanical relation tau M = tau M 0 into square root of TL/Tn where the Tn is expressed in terms of electric field from our energy balance equation.

So the momentum balance equation on the other hand gives you this expression for the drift velocity. Note that under quasi equilibrium conditions very close to the origin here. You have a straight line segment which can be cast in the form of minus of mu N0 into E. So here is a summary of our model. The conditions in our semiconductor or steady state, spatially uniform and no impact ionization.

Under such conditions, if you vary the voltage across the semiconductor slowly you increase voltage slowly. The current versus voltage relation will be linear in the beginning and then it will approach saturation. So since mu N0 is the constant slope in this linear segment close to the origin or for low fields this is called low field mobility.

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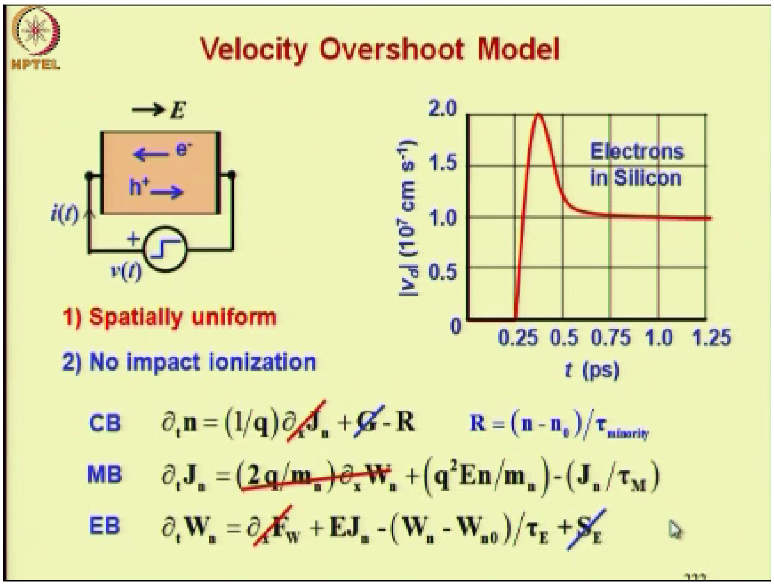

Velocity Overshoot Model

1) Spatially uniform

2) No impact ionization

CB $\partial_t \mathbf{n} = (1/\mathbf{q})\cancel{\partial_x \mathbf{J}_n} + \cancel{G} - \mathbf{R}$ $\mathbf{R} = (\mathbf{n} - \mathbf{n}_0)/\tau_{minority}$

MB $\partial_t \mathbf{J}_n = \cancel{(2\mathbf{q}/\mathbf{m}_n)\partial_x \mathbf{W}_n} + (\mathbf{q}^2\mathbf{E}\mathbf{n}/\mathbf{m}_n) - (\mathbf{J}_n/\tau_M)$

EB $\partial_t \mathbf{W}_n = \cancel{\partial_x \mathbf{F}_W} + \mathbf{E}\mathbf{J}_n - (\mathbf{W}_n - \mathbf{W}_{n0})/\tau_E + \cancel{S_E}$

Now similarly let us look at the velocity overshoot, modeling of the velocity overshoot which is a little bit more complex issue as compared to the velocity saturation. However, we will find that the same set of energy balance equations will be used here and we will be approximating the terms depending on the conditions. So our conditions are that you have a semi conductor uniform and you are suddenly stepping the voltage to a high value.

As a result, there is a transient current and this transient current which is reflected in the drift velocity varies as shown here. So it goes up and it overshoots the saturation value. This is the electric field step because of the applied voltage. Now we will assume that the conditions are spatially uniform and there is no impact ionization. Again because this is a high field effect we are going to ignore impact ionization so that our analysis is simple.

So let us begin with the equations again. Now because of spatial uniformity the spatial derivatives have all been neglected. Now look at this carrier balance equation. This term is 0 because no impact ionization the term G is 0. Now we can show that the consequence of this relation dou N dou T = -R is that the R should also bE0. Now why now this is because you see when at the instant when the electric field is just stepped so you take the condition initial condition.

At this point there is no excess generation. So there is no excess recombination either so your N = N0. Now under steady state conditions when dou N/dou T is 0. R has to bE0 because other terms arE0. So again under steady state condition also your N = N0. So initial condition at T = 0 when the electric field is stepped N = N0 finally also N = N0. And if you now substitute this form of the recombination process here you find that dou N/dou T versus N should you an exponential relation.

Now an exponential relation whose initial value is N0 and final value is N0. So you know that this segment is nothing, but a constant segment. Now that is why the carrier balance equation gives you N = N0. Now again we have removed the contribution of the generation recombination processes to the energy balance equation SE because there is no generation or recombination over and above the equilibrium value.

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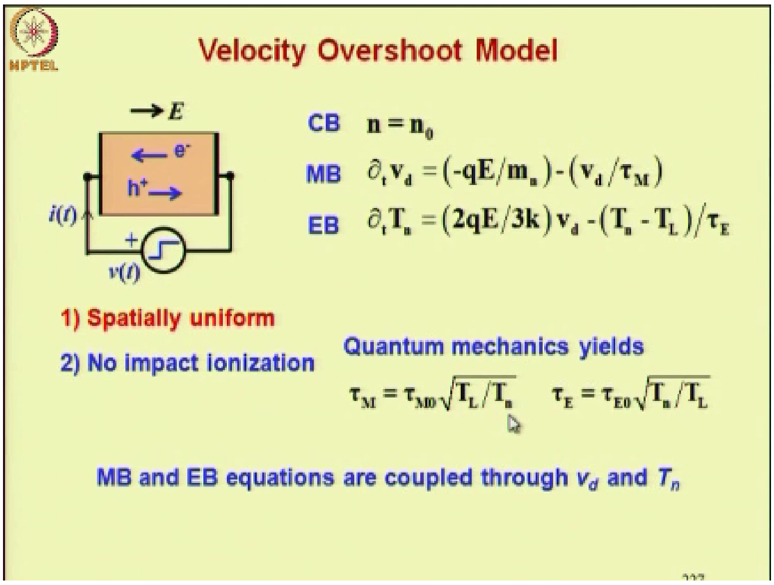

Velocity Overshoot Model

CB $n = n_0$

MB $\partial_t v_d = (-qE/m_n) - (v_d/\tau_M)$

EB $\partial_t T_n = (2qE/3k) v_d - (T_n - T_L)/\tau_E$

1) Spatially uniform

2) No impact ionization

Quantum mechanics yields

$\tau_M = \tau_{M0}\sqrt{T_L/T_n}$ $\tau_E = \tau_{E0}\sqrt{T_n/T_L}$

MB and EB equations are coupled through v_d and T_n

The simplified equations are put here we are following exactly the same approach that we followed for velocity saturation okay velocity field curve. The only thing is the equations are little bit more complex because here time derivative have to be considered. So tau M and tau E are obtained from quantum mechanics and since they are given in terms of the electron temperature we convert the kinetic energy balance equation to an equation in terms of electron temperature for which we use this expression.

And we neglect the direct heat component regarding the thermal component to dominate. Now the result of that would be the equation shown here okay. Now what we have done is we have also removed the term Jn and instead we have converted the equation into an equation of the drift velocity. This is done readily because you know that Jn is nothing, but - Q times the electron concentration into the drift velocity.

So if you divide the Jn equation/- Q times N you get the drift velocity because ultimately we want to model the drift velocity and not the Jn, we want the velocity overshoot characteristics. So now you find that the momentum balance and energy balance equations are coupled through drift velocity and the electron temperature. So you find the drift velocity entering here.

And you have the electron temperature entering in momentum balance equation why because tau M depends on electron temperature as shown by this formula.

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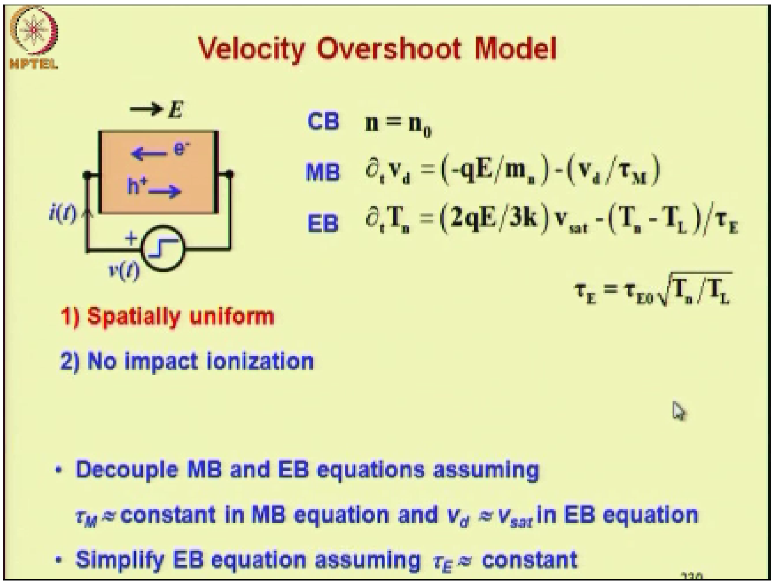

Velocity Overshoot Model

CB $\mathbf{n} = \mathbf{n}_0$

MB $\partial_t \mathbf{v}_d = \left(-q\mathbf{E}/m_n\right) - \left(\mathbf{v}_d/\tau_M\right)$

EB $\partial_t \mathbf{T}_n = \left(2q\mathbf{E}/3k\right)\mathbf{v}_{sat} - \left(\mathbf{T}_n - \mathbf{T}_L\right)/\tau_E$

$\tau_E = \tau_{E0}\sqrt{T_n/T_L}$

1) Spatially uniform

2) No impact ionization

- Decouple MB and EB equations assuming $\tau_M \approx$ constant in MB equation and $v_d \approx v_{sat}$ in EB equation
- Simplify EB equation assuming $\tau_E \approx$ constant

So the first step will be to simplify or decouple this equation. So these are approximation that we are doing for analytical solution. Otherwise numerically you can always solve 2 couple differential equation, but we want analytical solution so how do you decouple. So you decouple momentum balance and energy balance equations assuming tau M is constant in the momentum balance equation.

So here we will assume tau M to be constant and Vd is approximate is = V sat in the energy balance equation. So moment you said Vd as a constant = V sat you do not need this equation for this velocity and moment you said tau M as constant you do not need the equation of the electron temperature to estimate this value. So the results are shown here so we have replaced Vd/V sat constant and we have removed tau M because it is a constant, but we still have tau E as a function of electron temperature with a square root sign here.

Now that is going to complicate the solution of this differential equation. So if you put tau E as a function of square root tau M. So what we will do is next simplification we do is that we simplify energy balance equation assuming tau E is constant. So here we will set this to be constant. So we remove the tau E expression also. Now we are dealing with considerably simplified equation.

You can see that this is a simple differential equation first order and its solution will be an exponential for Vd as a function of time. Similarly, this is an equation which is also a first order equation in Tn that is electron temperature and it will also give you an exponential solution as a function of time.

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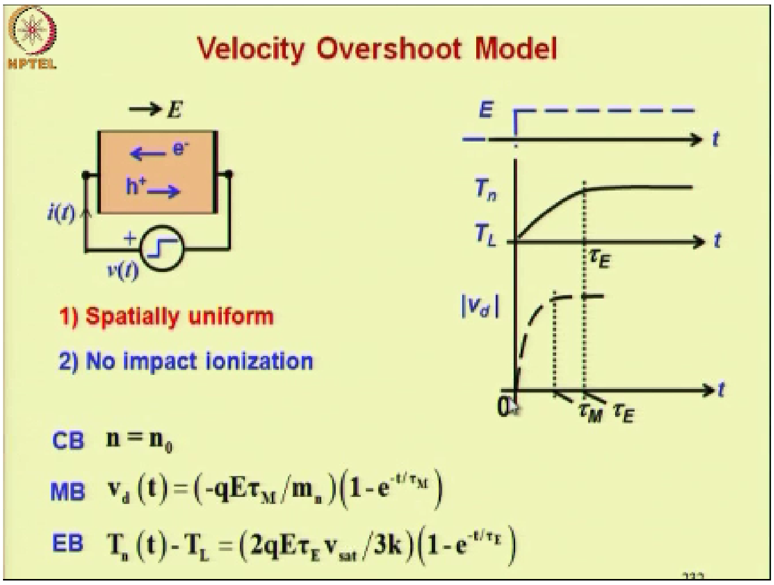

Velocity Overshoot Model

1) Spatially uniform

2) No impact ionization

CB $\mathbf{n} = \mathbf{n}_0$

MB $v_d(t) = \left(-qE\tau_M/m_n\right)\left(1 - e^{-t/\tau_M}\right)$

EB $T_n(t) - T_L = \left(2qE\tau_E v_{sat}/3k\right)\left(1 - e^{-t/\tau_E}\right)$

Now that is what is shown here. So the momentum balance equation gives you an exponential relation for the drift velocity as a function of time assuming that tau M is constant and the energy balance equation gives you equation for electron temperature as a function of time. Now these 2 solutions have been plotted here.

So Tn is exponentially rising from TL under equilibrium to some saturation value under steady state corresponding to the high electric field or high voltage that you apply so this is electric field step and similarly the momentum balance equation gives you an exponentially rising drift velocity as a function of time. Now here we have shown approximately to scale the fact that the momentum relaxation time tau M is less than the energy relaxation time tau E.

So this variation in Tn happens over a much longer period than the variation in VD. You will recall from the qualitative modeling that we had said that the fact that tau M happens to be less than tau E or much less than tau E is actually very important for the velocity overshoot to occur.

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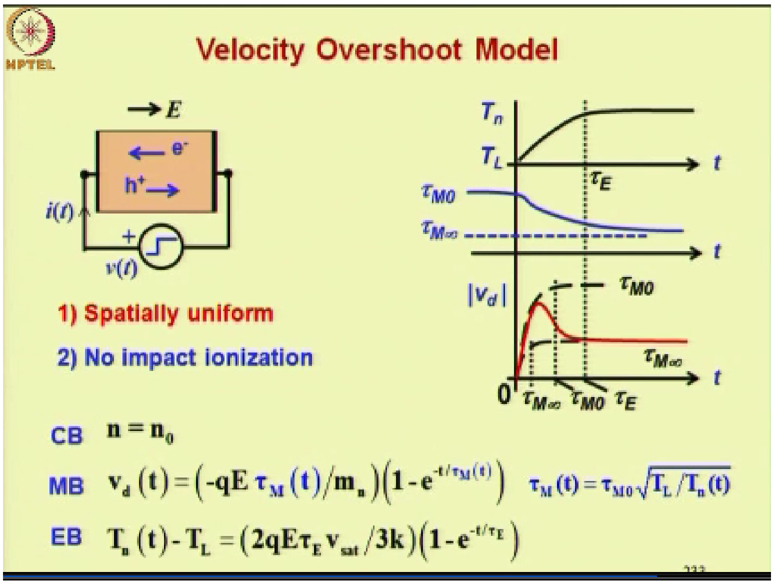

Velocity Overshoot Model

1) Spatially uniform

2) No impact ionization

CB $n = n_0$

MB $v_d(t) = \left(-qE\,\tau_M(t)/m_n\right)\left(1 - e^{-t/\tau_M(t)}\right)$ $\tau_M(t) = \tau_{M0}\sqrt{T_L/T_n(t)}$

EB $T_n(t) - T_L = \left(2qE\tau_E v_{sat}/3k\right)\left(1 - e^{-t/\tau_E}\right)$

Now we do not seem to have got the velocity overshoot yet. Now that is just one step ahead now. So what we do is we now set the tau M as a function of the electron temperature Tn. So we are solving for electron temperature from this equation and that value will substitute here to get that tau M and we will make that tau M as a function of time. Now notice that this is an approximate approach.

First we assume tau M to be constant to get exponential solutions, but now we are introducing the time dependent tau M to get the velocity overshoot. So note that the tau M is temperature dependent quantity in 2 places of the solution of the momentum balance equation here this place as well as this place. Unless you assume the tau M to be time dependent in both places.

You will not get the overshoot and how does it give you the overshoot that is represented here. So your tau M varies it decreases from high value to a low value. 2 L is what tau M is 0 and tau M infinity. So the exponential solutions for 2 extremes assuming constant tau M are given here this tau M 0 and this tau M infinity. So evidently when you put the tau M to be time varying between 2 limits in this equation.

Your curve is going to follow the curve for tau M 0 up to a point and then it will drop and then come to tau M infinity. In fact, this is what I leave it to you as an assignment to show that when you assume tau M as a function of time here according to this formula you will actually get a velocity overshoot.

So you will have a region where the velocity versus time will have a negative value or you can also analytically approach the problem as finding out the derivative DVd/DT and show that it goes to 0 at some non-zero times. And when it goes to 0 the D square Vd/DT square has appropriate sign to show that it is a maximum.

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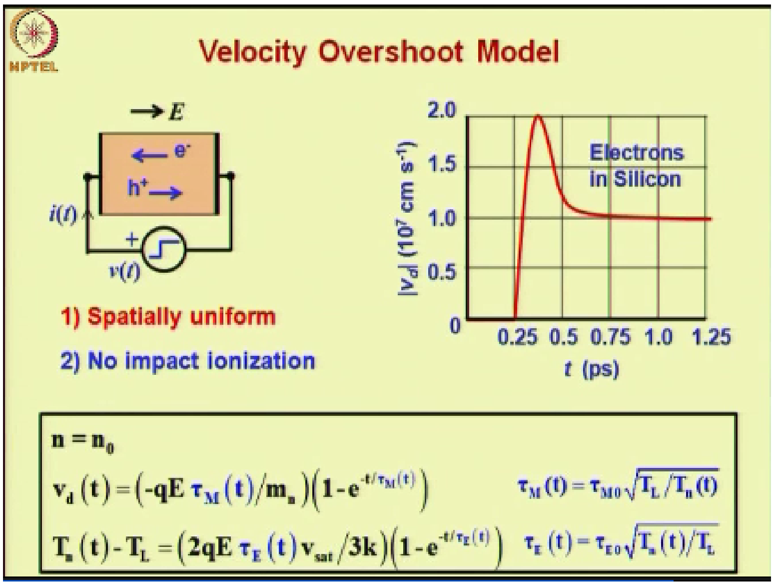

Velocity Overshoot Model

1) Spatially uniform

2) No impact ionization

$n = n_0$

$v_d(t) = \left(-qE\,\tau_M(t)/m_n\right)\left(1 - e^{-t/\tau_M(t)}\right) \qquad \tau_M(t) = \tau_{M0}\sqrt{T_L/T_n(t)}$

$T_n(t) - T_L = \left(2qE\,\tau_E(t)\,v_{sat}/3k\right)\left(1 - e^{-t/\tau_E(t)}\right) \qquad \tau_E(t) = \tau_{E0}\sqrt{T_n(t)/T_L}$

So this is our model of velocity overshoot of this characteristics. Now you can improve the model, you can also include the variation of energy relaxation time with time because energy relaxation time also will change in fact, it increases because the electron temperature is increasing with time. So I can feed this time dependence at this location and this location and then I can solve this equation. My computation would be more, but a model will be more accurate.

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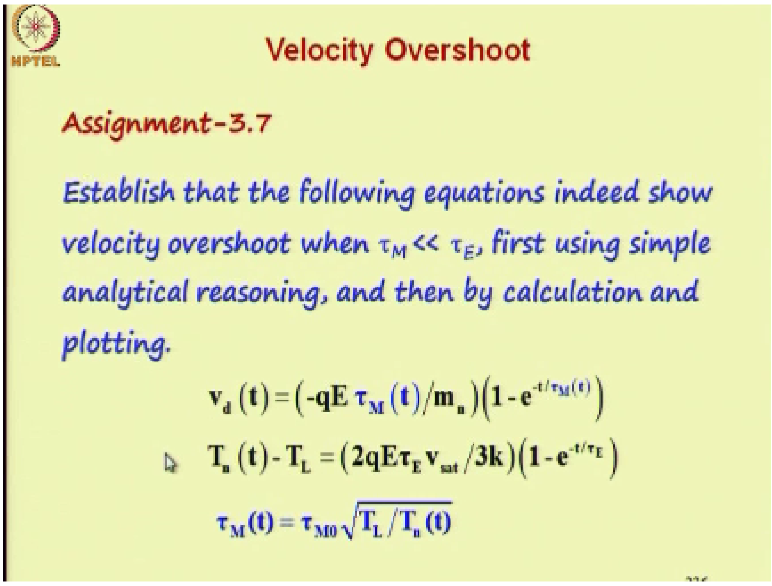

Velocity Overshoot

Assignment-3.7

Establish that the following equations indeed show velocity overshoot when $\tau_M << \tau_E$, first using simple analytical reasoning, and then by calculation and plotting.

$$v_d(t) = \left(-qE\,\tau_M(t)/m_n\right)\left(1 - e^{-t/\tau_M(t)}\right)$$

$$T_n(t) - T_L = \left(2qE\tau_E v_{sat}/3k\right)\left(1 - e^{-t/\tau_E}\right)$$

$$\tau_M(t) = \tau_{M0}\sqrt{T_L/T_n(t)}$$

So this is assignment established that the following equations indeed show velocity overshoot when tau M is much < tau E. First using simple analytical reasoning so I told you take derivative and find out the sign of the derivatives and so on. And then by calculation and plotting you can actually calculate and plot. You can use MATLAB or any other calculator.

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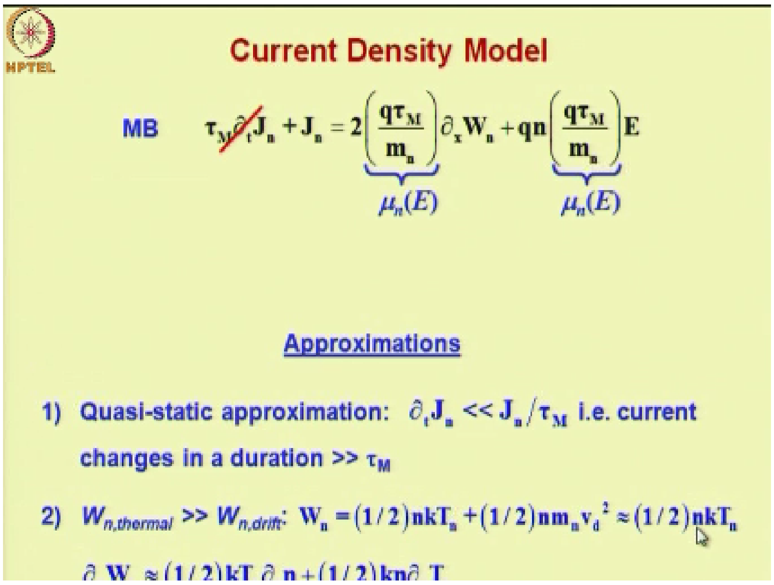

Current Density Model

MB $$\tau_M \cancel{\partial_t} J_n + J_n = 2\underbrace{\left(\frac{q\tau_M}{m_n}\right)}_{\mu_n(E)}\partial_x W_n + qn\underbrace{\left(\frac{q\tau_M}{m_n}\right)}_{\mu_n(E)}E$$

Approximations

1) Quasi-static approximation: $\partial_t J_n << J_n/\tau_M$ i.e. current changes in a duration >> τ_M
2) $W_{n,thermal} >> W_{n,drift}$: $W_n = (1/2)nkT_n + (1/2)nm_n v_d^2 \approx (1/2)nkT_n$

$\partial W \approx (1/2)kT\,\partial n + (1/2)kn\partial T$

Now finally let us discuss the model for current density. You will recall that when we discuss the various transport phenomena semi classical transport phenomena we said that current can be because of drift diffusion or thermoelectric current so temperature gradient. So we want expressions for the current density for this driving forces. Now this is what we would like to get from the balance equation.

The relevant balance equation here is the momentum balance equation because it is this equation which contains the Jn terms. Let us show how some approximations of this equation will lead us to a simple model for current density Jn in terms of the various driving forces namely the electric field, the concentration gradient and the temperature gradient. So we have reproduced the momentum balance equation.

And note that this quantity here is called the mobility which is the function of electric field because tau M varies with electric field. The first approximation we make in this equation is a quasi-static approximation. We assume that the time varying term is small. Now this approximation works fairly well because momentum relaxation times are very, very small and the quasi-static approximation amounts to assuming that dou Jn/dou T is much < Jn/tau M.

So you see we have said that whenever we neglected a term we also do so with respect to or with reference to another term. So it is not as though some terms becomE0 and that is why we neglected. So whether a term is small or large depends on its relative value compared to other terms in the equation. So the simplest term that you can compare with to get a feel for what this approximation means is Jn.

So when we said tau M dou Jn/dou T is much < Jn then it means the current changes in a duration much > tau M. So long as your changes in current density are in a time that is much more than fractions of picoseconds this will work. So this is going to fairly high frequencies. The next approximation we make is that thermal energy is much greater than the drift energy. So this approximation related to Wn.

You see we have to find out this spatial derivative of Wn. So we are doing the approximation for that purpose. Now writing the equation in terms of electron temperature this amounts to the approximation Wn thermal much greater than Wn drift amounts to assuming that this half nkTn+ half nm N Vd square is approximately equal to the thermal component itself. So we have neglected the direct heat component and we are using half here.

Because we are using one dimensional version of the equation. Now if you use 3 dimensional versions, you know, that this could be 3/2 nkTm. Now when you differentiate this term with respect to X to get dou X/dou Wn. So you will get 2 terms here one related to the derivative of N with respect to X and other related to derivative of Tn with respect to X because you have the product of N and Tn.

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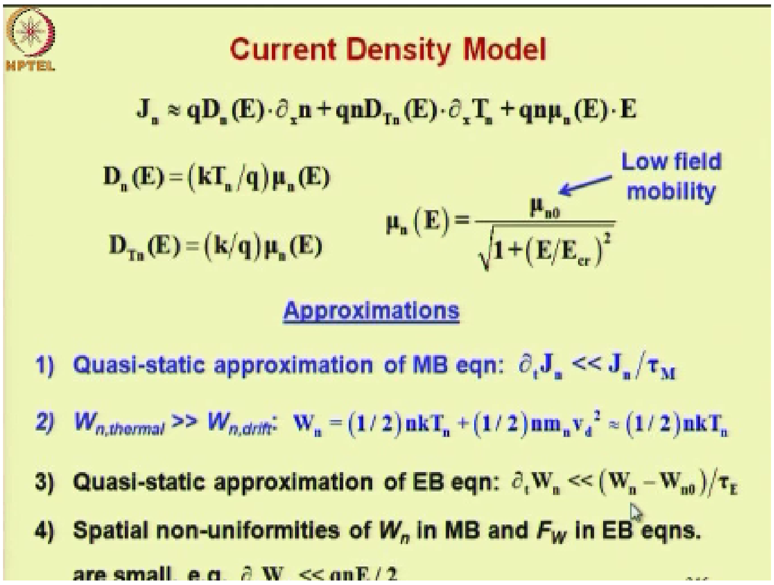

Current Density Model

$$J_n \approx qD_n(E)\cdot\partial_x n + qnD_{T_n}(E)\cdot\partial_x T_n + qn\mu_n(E)\cdot E$$

$$D_n(E) = (kT_n/q)\mu_n(E)$$

$$D_{T_n}(E) = (k/q)\mu_n(E)$$

$$\mu_n(E) = \frac{\mu_{n0}}{\sqrt{1+(E/E_{cr})^2}}$$

Low field mobility

Approximations

1) Quasi-static approximation of MB eqn: $\partial_t J_n << J_n/\tau_M$
2) $W_{n,thermal} >> W_{n,drift}$: $W_n = (1/2)nkT_n + (1/2)nm_n v_d^2 \approx (1/2)nkT_n$
3) Quasi-static approximation of EB eqn: $\partial_t W_n << (W_n - W_{n0})/\tau_E$
4) Spatial non-uniformities of W_n in MB and F_W in EB eqns. are small, e.g. $\partial W << qnE/2$

Now let us substitute the result of differentiating Wn with respect to X into the current density equation and see how it look like or rather the momentum balance equation and see how the equation looks like. So now under these 2 approximations namely the quasi-static approximation where we have dropped out dou Jn/dou T term.

And using this derivative of the current density with respect to X the equation becomes Jn is approximately equal to few times the diffusion coefficient of electrons multiplied by the concentration gradient +Q into carrier concentration N into D suffix Tn this is called thermal diffusivity in analog 2 the diffusivity associated with the concentration gradient. So Q into carrier concentration into thermal diffusivity into the temperature gradient.

So here we are looking at the carrier temperature +Q times the carrier concentration N into the mobility mu N into E. Now the diffusivity DN thermal diffusivity D suffix Tn and mobility mu suffix N are all functions of electric fields. So this term represents diffusion this is related to carrier concentration gradient.

The term dependent on the temperature gradient represents the thermoelectric current and the term dependent on electric fields represents the drift current. Let us look at the formulae which relate the diffusion coefficient and thermal diffusivity to the mobility. So this is the so called the Einstein relation. You have come across already in the first level course what we see. However, is that the Einstein relation you had discussed in the first level course was related to the equilibrium conditions.

Whereas here we are using it for high electric field regions also. So that is the difference. Similarly, the thermal diffusivity D suffix Tn depends on the mobility why are this formulae K there is a Boltzmann constant by Q into the mobility. Now this is the reason why one talks only of the mobility of carriers in a semi conductor and one tries to model mobility, measure mobility and so on.

So you will find in books the data for mobility given, but many times the data for diffusion coefficient or thermal diffusivity is rarely given. The reason is you can always derive this data from the mobility data because of these relations. So these relations are therefore very, very important. Now the 2 approximations have been put in abbreviated form here because we are going to encounter more approximations as we consider formulae for mobility.

So the mobility mu N is a function of E and a function is given by this formula. This we have just derived when we discuss the velocity field relation. We model the velocity field relation from the balance equations. Therefore, you will recall from that modeling exercise that there are some approximations associated with this result also. Now those are the quasi-static approximations of the energy balance equation in which we neglect the dou Wn/dou T term.

Now when you neglect this term evidently this term is small however we have been emphasizing that a term is small not in that it is absolutely 0, but it is small compared to the other terms. So which term do you compare with? So as we have shown here you compare it with the term Wn - Wn0/tau E. The reason for that is look at the energy balance equation.

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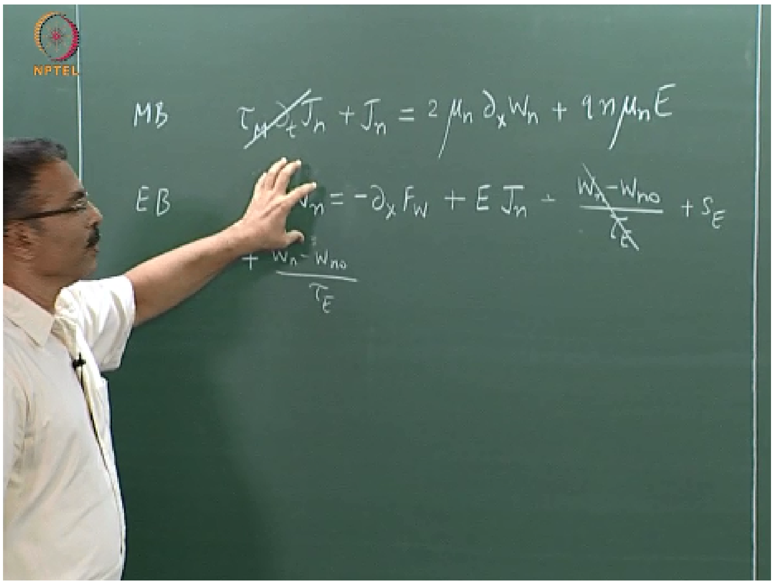

Now here before I proceed further I wish to point out an error in this equation the way it is written there should be a negative sign here. And this negative sign had actually been missed in the tables that I have shown earlier in this lecture so please make that correction if you have taken down notes and note these corrections. However, this does not affect any of the results that we have discussed.

Because we have already shown that we do not use this term at all in modeling at least the kind of modeling that we have done. Now returning to this energy balance equation we want to neglect this term so we would like to compare this term with any of the terms here. So one term that you can compare it with is this. So just like for momentum balance equation we move this Jn term from the right hand side to the left hand side.

And they are allowed us to compare these 2 terms. We can move this term to the left hand side and then it will appear as +Wn-Wn0/tau E. So we remove it from here. Now we can compare these 2 and if this is very small compared to this then you say that the quasi-static approximation or steady state approximation for kinetic energy is applicable. So that is what we have done here.

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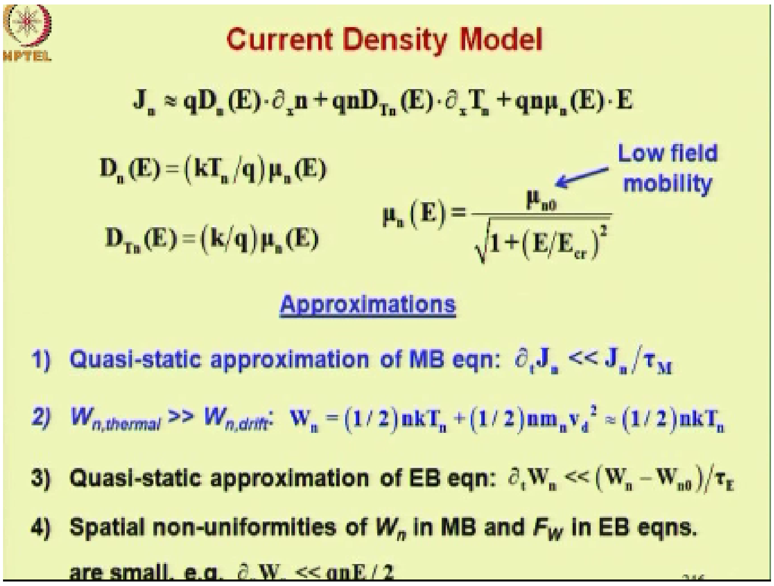

Current Density Model

$$J_n \approx qD_n(E)\cdot\partial_x n + qnD_{T_n}(E)\cdot\partial_x T_n + qn\mu_n(E)\cdot E$$

$$D_n(E) = (kT_n/q)\mu_n(E)$$

$$D_{T_n}(E) = (k/q)\mu_n(E)$$

$$\mu_n(E) = \frac{\mu_{n0}}{\sqrt{1+(E/E_{cr})^2}}$$

Low field mobility

Approximations

1) Quasi-static approximation of MB eqn: $\partial_t J_n \ll J_n/\tau_M$
2) $W_{n,thermal} \gg W_{n,drift}$: $W_n = (1/2)nkT_n + (1/2)nm_n v_d^2 \approx (1/2)nkT_n$
3) Quasi-static approximation of EB eqn: $\partial_t W_n \ll (W_n - W_{n0})/\tau_E$
4) Spatial non-uniformities of W_n in MB and F_W in EB eqns. are small, e.g. $\partial\ W\ \ll qnE/2$

Similarly, we made another approximation namely that we assume the conditions to be uniform when we derive this formula for mobility. So you will recall that this formula for mobility involved a relation for the electron temperature in terms of the electric field and that is where we have done this approximation that we have been talking about when we derive that equation for electron temperature.

So spatial non-uniformity of Wn in the momentum balance and FW that is a flux of kinetic energy in the energy balance equations are small. So for example dou Wn/dou X is much less than QNE divided by 2. So here again we are comparing the term that we are neglecting with other terms in the equation. Look at this so dou Wn/dou X in a momentum balance equation comes here.

Now we compare it with this term you can say that mobility can be removed and so if this term is much less than this term then you can neglect this term that is what we are saying. One can similarly write down in expression for neglecting FW. So if you want to do that you should look at the terms here. You can compare it for example with this term for neglecting and you can write down a similar relation.

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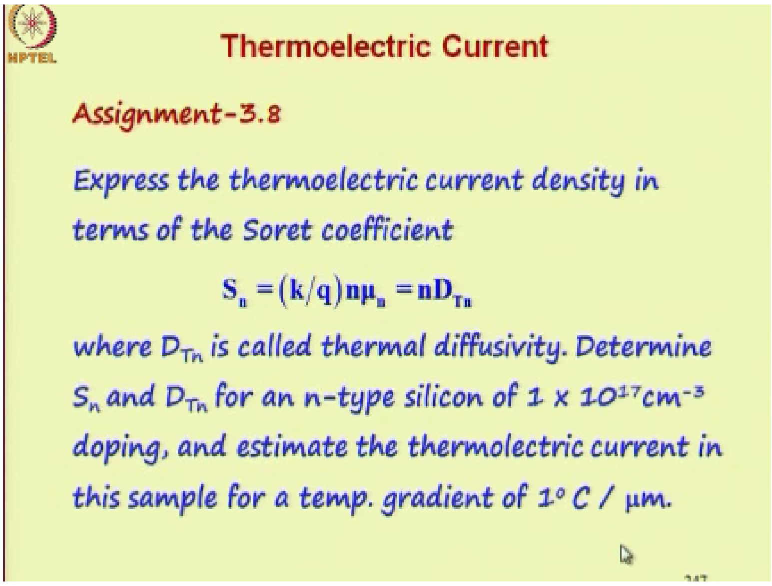

Thermoelectric Current

Assignment-3.8

Express the thermoelectric current density in terms of the Soret coefficient

$$S_n = (k/q)n\mu_n = nD_{Tn}$$

where D_{Tn} is called thermal diffusivity. Determine S_n and D_{Tn} for an n-type silicon of 1 x 10^{17}cm^{-3} doping, and estimate the thermolectric current in this sample for a temp. gradient of 1° C / μm.

Here is an assignment before we close the lecture. Express the thermoelectric current density in terms of the Soret coefficient Sn = k/q into carrier concentration N into mobility = carrier concentration into the thermal diffusivity. So DTn is called the thermal diffusivity. Determine Sn and DTn for an N-type silicon of 1 x 10 power 17 per centimeter cube doping and estimate the thermoelectric current in this sample for a temperate gradient of 1-degree centigrade per micron.

Now since we are at the end of the lecture let us make a summary of the important points. So in this lecture, we developed the energy balance equations approach for modeling devices. We showed that the Boltzmann transport equation is equivalent to an infinite series of balance equations. Now this series is terminated after 3 balance equations namely those of the carrier, concentration, the momentum density and energy density.

And this termination is achieved by a closure relation for the energy flux that enters into the energy balance equation. Then we showed how using these 3 balance equations you can model velocity overshoot, velocity saturation and the current density. The current density involving the drift current, the diffusion current and thermoelectric current.

And we showed that you need to do a number of approximations of the momentum balance equation to achieve these simple relations for current density in which the current density is shown as sum of 3 components. One due to concentration gradient, one due to temperature gradient, carrier temperature gradient and the other due to electric field. We will continue the discussion in the next lecture.