Semiconductor Device Modeling

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Lecture - 11

Semi-classical Bulk Transport: EM field and Transport Equations

In the previous lecture 3 lectures we have been discussing about the equations for Semi-classical carrier transport. So, we have introduced for different levels of description of carrier transport. Starting from the microscopic level to the macroscopic level. The most fundamental microscopic level is the one in which you treat the carrier individually and regard the carrier as a wave.

The next approximate level is where you treat the carrier individually here also but you regard the carrier as a particle. The next approximate level you treat the carrier in terms of their population and you look at how in a sufficiently large population of carriers the carriers are distributed over momentum. You talk in terms of a function that talks about this distribution and this distribution function then enables you to derive current and carrier concentration and other quantities.

And the most approximate level which is used very, very often is the ensemble approach like the approach of the distribution function but considering only the average values of carrier concentration, current density and so on in a local volume. So, we even out or average out the momentum distribution of carriers because that detailed information about distribution is not required for many practical situations of modeling.

Then we have discussed in detail how the individual carrier view point is followed to get the device current and carrier concentration. You can treat the electron as a wave and you can find out using Schrodinger equation the wave function psi for each of the electrons. In terms of this wave function psi you can then determine the carrier concentration and current density. These are most fundamental level.

When you treat the carrier as a particle and treat the carriers individually you can find out the state of the carriers which are nothing but the position and momentum values at any instant of time using Newton’s second law. And then in terms of the position and momentum of different carriers you can determine the current density and carrier concentration. Now, in this lecture we want to begin a discussion of the ensemble approach.

The fundamental ensemble approach in which you talk in terms of the distribution function or distribution of carriers over momentum.

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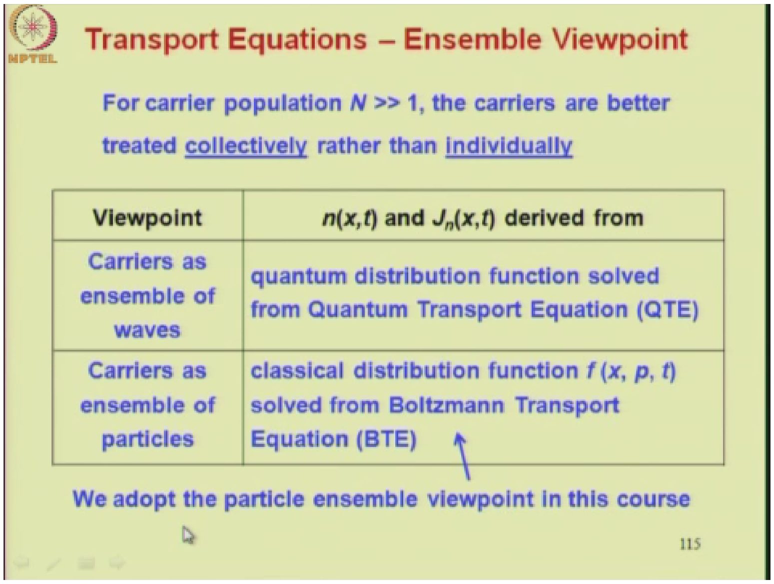

Transport Equations – Ensemble Viewpoint

For carrier population $N >> 1$, the carriers are better treated collectively rather than individually

Viewpoint	$n(x,t)$ and $J_n(x,t)$ derived from
Carriers as ensemble of waves	quantum distribution function solved from Quantum Transport Equation (QTE)
Carriers as ensemble of particles	classical distribution function $f(x, p, t)$ solved from Boltzmann Transport Equation (BTE)

We adopt the particle ensemble viewpoint in this course

For carrier population much greater than one the carriers are better treated collectively rather than individually. Now, though we would be discussing in detail the approach where you treat the carrier as a particle and then consider ensemble of carriers. We would just like to mention that there is an analogous approach in which you can treat the carriers as wave and yet talk about ensemble of carriers.

So look at the population of carriers in terms of group of waves and then analyze the motion so such an approach is the view point where carriers are regarded as ensemble of waves. In this approach, the carrier concentration n and the current density Jn as a function of position and time are derived from the quantum distribution function solved from what is called the quantum transport equation.

On the other hand, the approach that is of interest to us in this course where carriers are regarded as ensemble of particles. The n and Jn are derived from classical distribution function f of x, p, t. So, this function is a distribution function depends on the position the momentum and the time instant and this distribution function is solved from what is called Boltzmann Transport Equation. So, we adopt the particle ensemble view point in this course.

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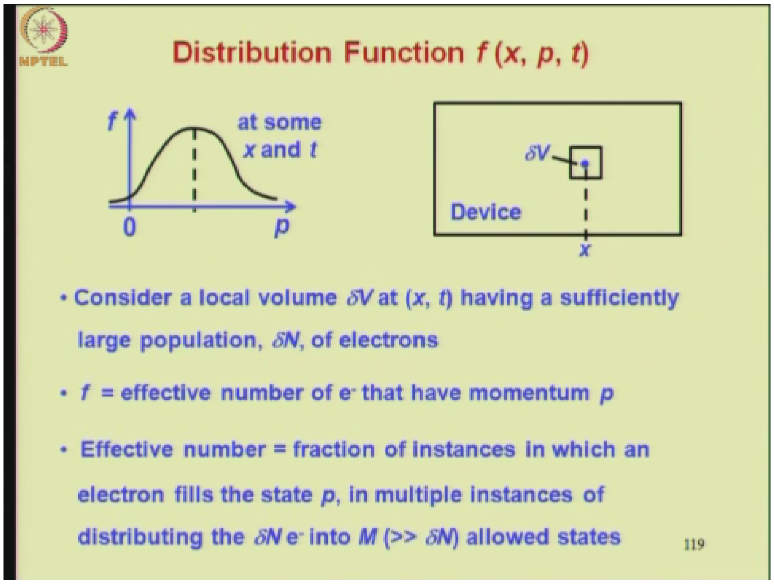

Distribution Function $f(x, p, t)$

- Consider a local volume δV at (x, t) having a sufficiently large population, δN, of electrons
- f = effective number of e^- that have momentum p
- Effective number = fraction of instances in which an electron fills the state p, in multiple instances of distributing the δN e^- into M ($>> \delta N$) allowed states

Let us try to understand the distribution function. So the shape of this function is show here for any instant of time t and at some position x in the device. Now we shall be mostly using the 1-dimensional form of the equation. That is why we are talking of position in terms of x. However, at some places it will be necessary to consider the 3 dimensional picture in which case x should be replaced by r.

So, the carriers are distributed over momentum in somewhat shape like this. So what does the shape indicate it indicates that there are a large number of carriers having momentum about this dotted line here and there are smaller and smaller number of carriers having momenta much higher than this value and momenta much less then this value. So, that is the meaning of the function dropping off.

Now, in terms of an abstract definition you can say that f is the probability that an electron at position x and time instant t has the momentum p. Now, this is an abstract definition for example talking about thermodynamic equilibrium we give an abstract definition that the state in which for every process there is an inverse process going on at the same rate is called an equilibrium state.

However, we also gave a more practical definition which enables us to apply the definition of equilibrium in practice and we said that an equilibrium state is one in which a semiconductor has uniform temperature throughout its volume but no net motion of charge or energy from any point to any other point. Now similarly we want to convert this abstract definition of f, namely the probability that an electron at x, t has momentum p into a more practical definition.

For this purpose, what we do is we construct a device in which we take a local volume delta v, a small volume delta v at position x. Now this volume delta v is sufficiently large so that it has a sufficiently large population delta N of electrons. Now, though we talk in terms of electrons we have already mentioned this several times that a similar discussion would apply for holes. Now if this is the picture that we are looking at delta N electron in delta V then f denotes the effective numbers of electrons that have momentum p.

Now let us see what the effective number of electrons means? The effective number is the fraction of instances in which an electron fills the state p, in multiple instances of distributing the delta N electrons into M allowed states. Where in practice M always happen to be much, much greater then delta N. Now this effective number is in fact, < 1 because it is a fraction. So now you may have a doubt.

How can you have fractional numbers of electrons in a state? Because you would think that there has to be a minimum of 1 electron and if < 1, the number of electrons that only be 0. So, how is that we are using an effective number of electrons in a state that is fractional. Now, this point has to be understood in detail. So please note that is why we have used the term effective so we are not saying it is the number of electrons. It is an effective number.

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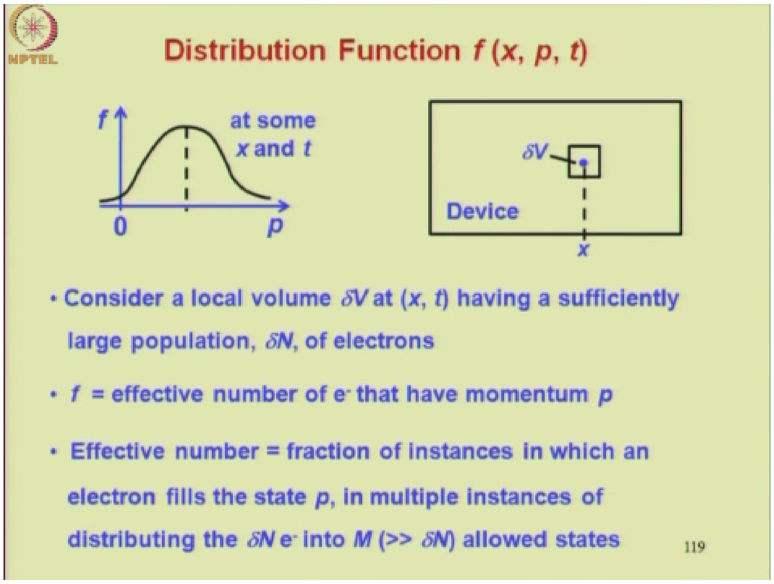

Distribution Function $f(x, p, t)$

- Consider a local volume δV at (x, t) having a sufficiently large population, δN, of electrons
- f = effective number of e^- that have momentum p
- Effective number = fraction of instances in which an electron fills the state p, in multiple instances of distributing the δN e^- into M (>> δN) allowed states

So, please look at the definition carefully. So, we are looking at the population of delta N electrons and delta N is sufficiently large and there are M allowed states which this electron can occupy and we are looking at how these delta N electrons are distributed. So, let me explain this point in detail.

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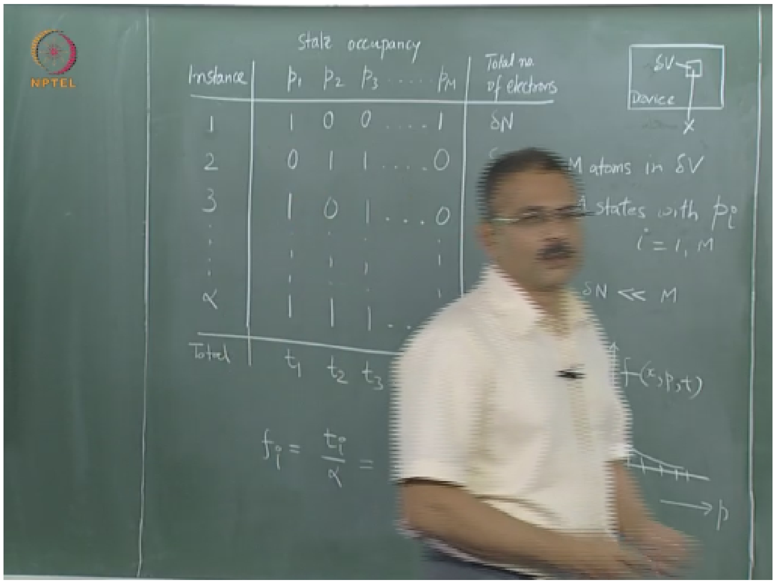

Now let us assume that the atoms in the delta V volume each of them contribute a state an allowed state evidently the number of allowed states will increase proportionately with delta V. So, if you delta V is more the number of states would be more. Let us say there are M atoms in delta V and each atom contributes a state therefor there are M states. And each state has a momentum Pi where i goes from 1 to M.

Now delta V contains delta N electrons we want to distribute this delta n electron into these M states. Now, the picture in the device is such that the group of electrons rapidly change their distribution in another words if I look at one instant of time I may have one distribution of delta N electrons into M states and then the electrons can rapidly change state. They may occupy some other delta N states out of M. So, let us set out the possible distributions.

So here we have a table where we have a column which tells the instance when we are looking at the volume delta V and then you have the momenta p1, p2, p3 up to Pm and we are talking of occupancy of the state. So at one instance I look at this delta V volume and I found let us say the following. This is instance number 1, p1 state was occupied I show it by number 1. If a state is not occupied I show it as zero.

Say p2 was not occupied then may be p3 was also not occupied and so on and may be Pm was occupied. Now at this point please note that the number delta N of electrons in delta V is always going to be much less than the number of atoms M. Let me give you an example to illustrate this point you take silicon in 1 cm cube of silicon you have 5 into 10 to the power 22 silicon atoms. Now, if you are dou is about say 10 to the power 18 then you would have 10 to the power 18 electrons per centimeter cube in the same volume.

Now if I scale down the volume the ratio of electrons to the number of atoms would always remain 10 to the power 18/5 into 10 to the power 22 which is really a very small number. 10 to the power 18 is very small compared to 5 into 10 power 22. So, that is how this delta N in practice is going to be always much less then M. In another words the number of 1’s that will appear here would be very few and most of the states would be empty.

Now let us make here a column for total number of electron. So if I add up these 1’s then I will get delta N because actually I am distributing delta N electrons. Now, I looked at the volume delta V at another instance, closely following this instance. So because the situation is dynamic the electrons are rapidly changing their states. So I would probably find at the second instance some distribution like this may be and again if is sum up these I will get delta N.

At a third instance I may find something like this again this sum up to delta N. Now I should take a very large number of instances. But you might think that if I take so many instances I would be observing the system over a long period of time. Now, you are right that we must observe the system over a long period of time because the distribution function f is actually a long time average.

However, please note that as we have remarked earlier in the course whenever we say something is long, something short or something large, something is small it is always relative. So the time that we are looking at is sufficiently long so that I can look at may be 1,000 or 10, 000 or million instances but this time will be sufficiently small as compared to the time in which the distribution function changes significantly.

Suppose there is a change in as a function of time. So, there is not much change in the distribution function over this time interval when you have these 1000s of instances. So that is our assumption. So let us say the number of instances is alpha where as we said alpha can be very large and let us say the alpha instance you had something like this. So, here also you have delta N.

Now, if I sum up this column I will get the number of instances in which this state p1 was occupied or electron has momentum p1. So let me do that. Let me total. So let us say this totals up to some t1. Similarly, I can total this up I may get t2 instances. Here t3 and so on and here t m. Now, if I take the fraction of instances in which electron was in state t1 or electron has momentum p1.

So that fraction is f = t upon alpha because I have observed this whole thing over alpha instances out which t1 instances, the electron was in state t1. So similarly I can write a general formula in the ith instance the ratio was ti/alpha and that I call as fi so this is the rally of the distribution function corresponding to the state i which has a momentum pi. So, this fi is nothing but f of x, pi, t.

So, this is the fraction of t instances in which the electron had momentum pi now that is really the meaning of this effective number. So you see ti/alpha it is a fraction of instances in which the electron occupied the state pi. Now you can also interpret the instances in another way. Suppose instead of looking at different instance of time in a given device I could look at a large number of devices at any instant.

So, I am looking at let us say alpha devices at any instant and in each device, all devices are identical. So in each device we are looking at the same volume delta V at some location x and we are observing how the delta N electron are distributed and we are finding this kind of different distributions and then we arrive at the same number. So, there are 2 ways of interpreting these instances.

One is I look at one device at several instance of time but the duration of your time in which I am looking at all this is sufficiently small as compared to the time in which the distribution function changes significantly. But it is sufficiently large so that I have many instances to look at because only then this statistical average will have some meaning. So, if the number of instance is large I will find ti/alpha will approach a constant value.

So that is the meaning of the distribution function. Now, since there are a large number of states I am getting fi at different states. So the picture is something like this. So you have multiple states like this. This is the p axis, say p1, p2 and so on and in each thing you are getting a value fi so that value is plotted here say something like this. And then when you have a large number of them you can plot an envelope for this something like this and then this is your distribution function f.

Now, let us re-read what we have said about the distribution function as a summery. So, f represents the effective number of electrons that have momentum p. That effective number is nothing but the fraction of instances in which an electron fills the state p, in multiple instances of distributing the delta N electrons of the volume delta V into M allowed states and M always happens to be much greater then delta N.

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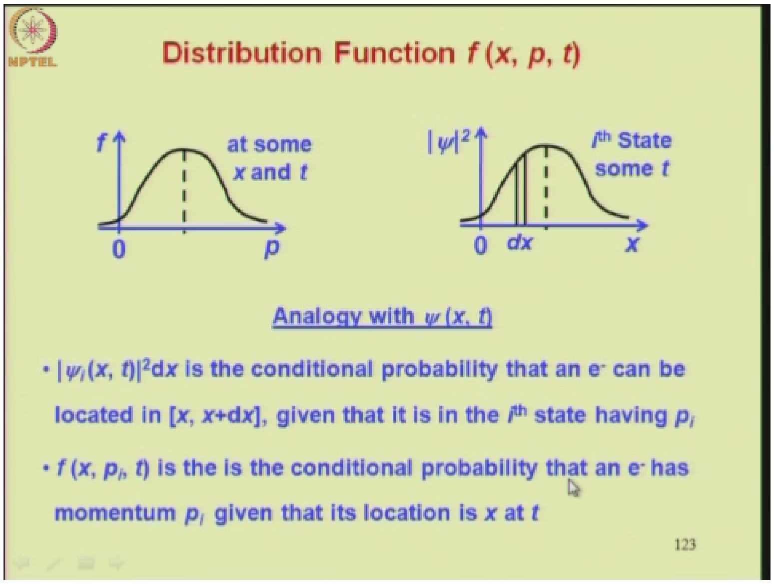

Distribution Function $f(x, p, t)$

Analogy with $\psi(x, t)$

- $|\psi_i(x, t)|^2 dx$ is the conditional probability that an e^- can be located in $[x, x+dx]$, given that it is in the i^{th} state having p_i
- $f(x, p_i, t)$ is the is the conditional probability that an e^- has momentum p_i given that its location is x at t

So we would like to emphasis that f is a classical concept, since it defines position and momentum x, p simultaneously. Let us gain a much deeper appreciation of this distribution function by pointing out its analogy to wave function. Both the functions are plotted here now f has not been plotted as a function of p whereas the modulus psi square, psi is the wave function, has been plotted as a function of x.

Now, as you know modulus of psi square is the probability, density function solved from Schrodinger equation and f is the function that is solved from Boltzmann's Transport Equation that equation we are going to consider shortly. Now here this psi square corresponds to the ith state because different electrons will have different states and the ith electron the way function corresponds to that at some instance time whereas this distribution function is for some location x of the device at instance t.

Now psi i square dx is a conditional probability that an electron can be located in the interval x to x+dx given that it is in the ith state having momentum pi. So, this is what is indicated in the diagram. So dx is this interval at x and area under this curve within this interval is the conditional probability that you can find an electron in this location given that this is the wave function for that because this is the wave corresponding the ith state which has momentum pi.

In analogous manner, let us write the definition of the distribution function that we already know.

So f of x, pi, t is the conditional probability that an electron has momentum pi given that its location is at x at instance t. So, while modulus of psi square dx is a conditional probability of locating an electron in a spacial interval given its momentum the classical distribution function f is the conditional probability that the electron has a momentum given its location.

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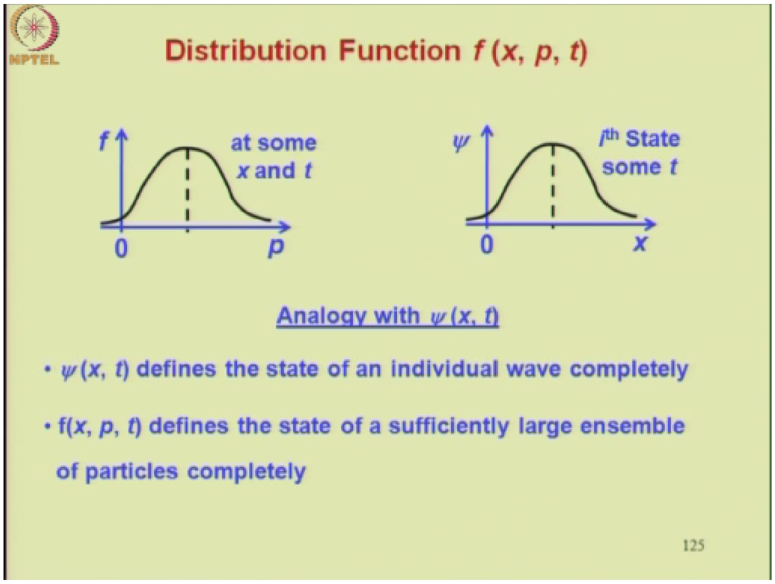

Distribution Function $f(x, p, t)$

Analogy with $\psi(x, t)$

- $\psi(x, t)$ defines the state of an individual wave completely
- $f(x, p, t)$ defines the state of a sufficiently large ensemble of particles completely

Another aspect of the analogy is that psi is a wave function or a function describing the state of an individual electron. So psi (x, t) defines the state of an individual wave completely in an exactly analogous manner. f of x, p, t defines the state of a sufficiently large ensemble of particles completely. So both are definitions of state and both definitions are such are that from these definitions you can get any quantity of interest.

However, the difference is that psi corresponds to regarding the electron as a wave and the psi is different for different electrons. So it is a individual carrier view point. Whereas the f corresponds to treatment of carriers as particles and it is for a group of particles. It describes the situation for a group of particles rather than individual particles.

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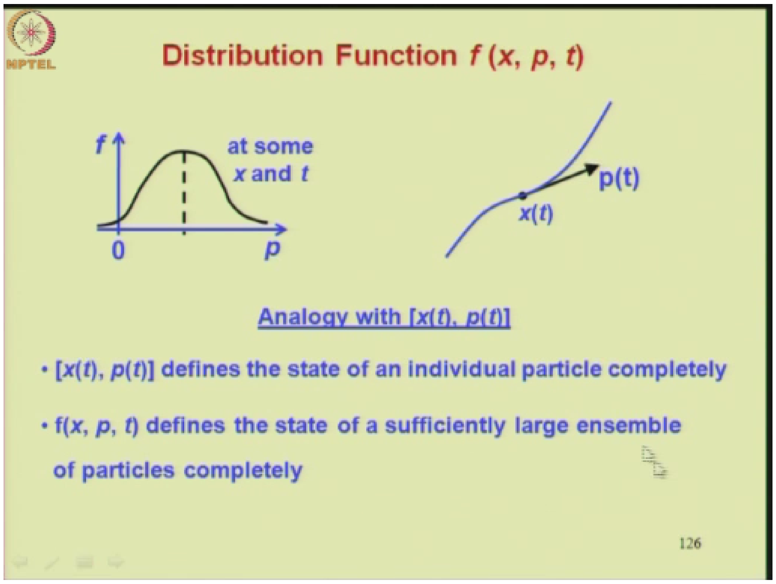

Distribution Function $f(x, p, t)$

Analogy with $[x(t), p(t)]$

- $[x(t), p(t)]$ defines the state of an individual particle completely
- $f(x, p, t)$ defines the state of a sufficiently large ensemble of particles completely

Let us also look at the analogy with the definition of state of an individual carrier from the classical point of view. Particle point of view so in this analogy you know that the state of an individual particle is described in terms of 2 parameters, positon and momentum at t. Now, x of t, p of t defines the state of individual particle completely. So if you know this value x and p at any instance of time and you know the force that is applied.

Then you can predict the trajectory of the particle if you know the initial position and momentum. Similarly, f of x, p, t defines the state of a sufficiently large ensemble of particles completely thus if you know this function for a group of particle this function is shown here and you know the force applied to the group of population and if you know the initial values corresponding to this distribution function.

Then you can find out all quantities of interest for this particular group how it evolves with time? How the various quantities change with time and positon all these things can be calculated starting from the distribution function.

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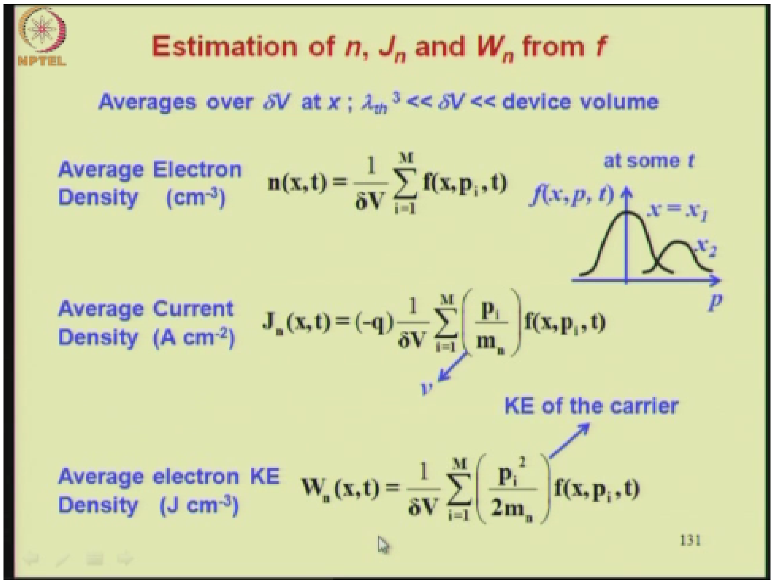

Estimation of n, J_n and W_n from f

Averages over δV at x ; $\lambda_{th}^3 << \delta V <<$ device volume

Average Electron Density (cm^{-3}) $$n(x,t) = \frac{1}{\delta V}\sum_{i=1}^{M} f(x,p_i,t)$$

Average Current Density $(A\ cm^{-2})$ $$J_n(x,t) = (-q)\frac{1}{\delta V}\sum_{i=1}^{M}\underbrace{\left(\frac{p_i}{m_n}\right)}_{v} f(x,p_i,t)$$

Average electron KE Density $(J\ cm^{-3})$ $$W_n(x,t) = \frac{1}{\delta V}\sum_{i=1}^{M}\overbrace{\left(\frac{p_i^2}{2m_n}\right)}^{\text{KE of the carrier}} f(x,p_i,t)$$

Let us see how we can estimate the carrier concentration, the current density and kinetic energy density W n from f. Let me just spend a couple of sentences on what does W n mean? So, W n is kinetic energy density. So what is the unit of this? Unit of energy Joules and density per unit volume. So that is its inert, that is its dimension. Now the formula that we are showing here are with reference with the diagram for the distribution function is shown here.

So, the function has been shown for 2 locations in the device x = x1 and x = x2. The average electron density within a volume n of x, t. It is given by this formula 1/delta v which is the local volume at x into sigma, i = 1 to M the distribution function f corresponding to the momentum states pi so i here corresponds to the momentum states. So there are n momentum states p1, p2, p3 up to pm.

So what this formula says that if you simply sum up the occupancy of the allowed states, M allowed states as given by the distribution function and divided by the local volume which are contributing to the states then you get the concentration. Now here a point that we would like to repeat is that the average works over delta V the local volume at x which is chosen such that it is much greater than the volume obtained from the thermal average wavelength of the electron.

But much less than the device volume so that you can capture the variations in n, in the device property. At the same time, you can treat the carrier as a particle. Because the volume is much more than the de Broglie wavelength of average thermal electron, the size that you obtain from this wavelength. Now let us spent a few minutes on understanding this formula, you can understand this based on the same table.

You see if sum up these delta N here in this column I will get alpha times delta N. So, what this means is if I sum these t i together I will get the sum as alpha into delta N. So, I can write sigma t i =alpha into delta n where i goes from 1 to M.

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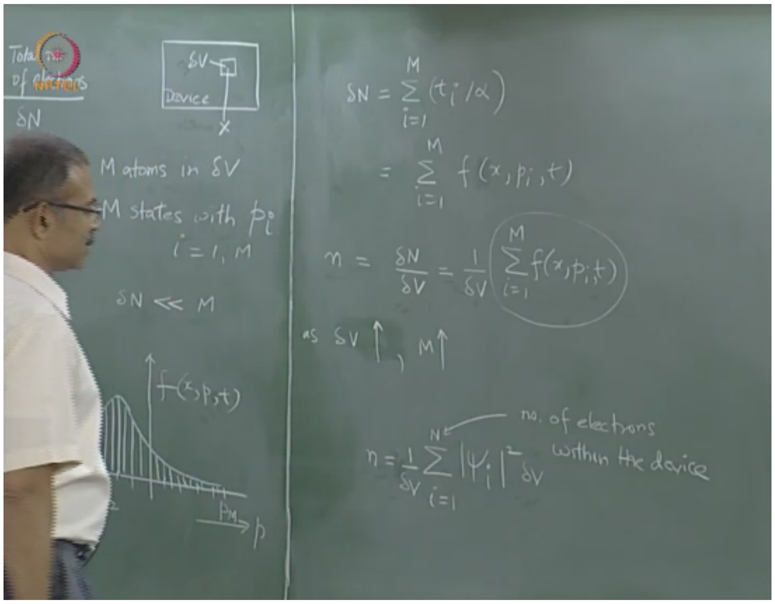

Total of electrons

δN

M atoms in δV

M states with p_i

$i = 1, M$

$\delta N \ll M$

$$\delta N = \sum_{i=1}^{M} (t_i/\alpha)$$

$$= \sum_{i=1}^{M} f(x, p_i, t)$$

$$n = \frac{\delta N}{\delta V} = \frac{1}{\delta V} \sum_{i=1}^{M} f(x, p_i, t)$$

as $\delta V \uparrow$, $M \uparrow$

$$n = \frac{1}{\delta V} \sum_{i=1}^{N} |\psi_i|^2 \delta V$$

no. of electrons within the device

Now if I divided this ti/alpha by moving this alpha to the right hand side I get this formula which is nothing but fi. So, the fraction of instances in which the electrons occupy the state ti. So, this is what we have written here this is fi so which is nothing but f of x, pi, t. Thus n is nothing but delta n electrons of volume delta V. So, this is the approach which enables me to find out that the delta V volume which contributes M states from the M atoms within the volume.

How many of the states are occupied and how are they occupied? So, that information enables me to find out the number of electrons in the states and if I take this ratio I get n. So, that is why your formula is sigma f x, p i, t. i = 1 to M, 1/delta V. So, this quantity is nothing but the delta n electrons. You can understand this point even graphically. So you see that this is your function and these are the values of the function at corresponding to different ti.

So, this is let us say somewhere here you have Pm. I am adding these heights. Now this M is very large please remember though this height is fraction, very small fraction but the number is large. So when I add up all these some of all these gives me this number delta N. So, you see an important point that this M depends on delta V this we have said already. So, as delta V increases the numbers of atoms also increases and so the number of states also increases.

So, what is going to happen is suppose I draw a picture like this for an increased delta V then my number M will more. This means I will see many more lines here because many more states, the M has increased and p2 which was this now is the state. Now I will have to sum up these lines so definitely my sum will increase the heights. These heights I am summing up but then you see the denominator has also increased.

So that is how the numerator/denominator this ratio will remain N. So, long as you take the delta V sufficiently large the value of N that you get from this approach by dividing this by this will approach a constant value. That is why we impose the condition that delta V should be sufficiently large. Now, before we leave this discuss of how this formula for N is derived based on the distribution function let us show the analogy of this formula?

To the formula for carrier concentration in the individual carrier view point regarding the carrier as a wave so there you will recall we had written a formula like this. The electron concentration n = the modulus of the wave function psi i for the ith state square. And summing up the i's for all the electrons within the device. M is the number of electrons within the device. So, this is the device and you are looking at all the N electrons.

And we said this is because electron is regarded as a wave so it pervades allover it moves everywhere so it has some presence so an electron here will also contribute to something in delta v. So, that is why you have this formula. Now let us see the correspondence between this formula and this formula. So, you can see that there is delta V here so what I could do is? I could multiply this by delta V and divide it by delta V.

Now, you know that this quantity modulus of psi i square delta V is the probability of finding an electron within the volume delta V assuming that electron is in the ith state given that the electron is in the ith state are having momentum pi what is the probability that I will find it within the volume V at some location? This is what it gives. So like that if I find out for all the electrons, for all the states.

And then if I divide it by the volume delta V in which I am trying to observe the presence of electrons I will get the number n. So, that is how you will see the correspondence between this formula and this formula. So psi i square delta V is analogous to the f. This we have already pointed out earlier while point out analogy between f and psi I square. Please note that here you have the square of the function and multiplied by delta V.

So, dimensionally this and this is same. So product of psi i square into delta V has the same dimension as f and f is dimensionless. So, this together is dimensionless. Also note difference however that here you are considering all the electrons so the state corresponds to all the electrons which are there. However here i = 1 to M these are the states contributed by the atoms in volume delta V.

Let us look at the formula for current density. So Jn this is given by -q into the average velocity. Now how are you getting the average velocity when you multiply the distribution function by the velocity which is nothing but the momentum by the mass. So momentum by the mass the velocity so if I want to find the average velocity then I have to multiply this distribution function by the v and then I have to sum up for all the states.

Now note one thing carefully that when I do this exercise I am getting the product electron concentration into the velocity and not the average velocity alone. So, I am getting the average of the product of the electron concentration and velocity when I do this exercise because we have said f is the effective number of electrons in state pi and vi is there velocity. So, when I multiply this effective number by velocity and do a summation I am getting the overall product.

And since V is the source of these electrons so we have divided by delta V you can check the dimensionally this whole thing turns out to be ampere per centimeter square. Now it is not very difficult to visualize how I could get the average kinetic energy density of electrons. Now that is given by –you replace this v here by the kinetic energy to get the average of the kinetic energy.

So, Pi square/2mn is the kinetic energy. Now multiply it by the effective number of electrons with this energy and then you submit up over all the allowed energies or momentum and divided by the volume to get the density.

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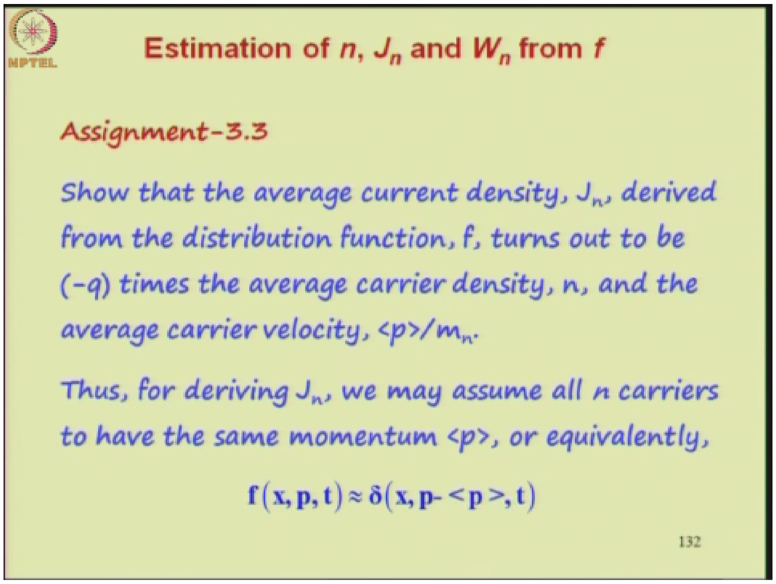

Estimation of n, J_n and W_n from f

Assignment-3.3

Show that the average current density, J_n, derived from the distribution function, f, turns out to be (-q) times the average carrier density, n, and the average carrier velocity, <p>/m_n.

Thus, for deriving J_n, we may assume all n carriers to have the same momentum <p>, or equivalently,

$$f(x, p, t) \approx \delta(x, p - \langle p \rangle, t)$$

Now here is an assignment for you show that the average current density J n, derived from the distribution function f, turns out to be -q times the average carrier density, n and the average carrier velocity average p, this is average p divided by mn. Thus, for deriving Jn, we may assume all n carriers to have the same momentum average p, or equivalently f of x, p, t can be approximated as delta function located at average momentum p. So, what we are saying is the following.

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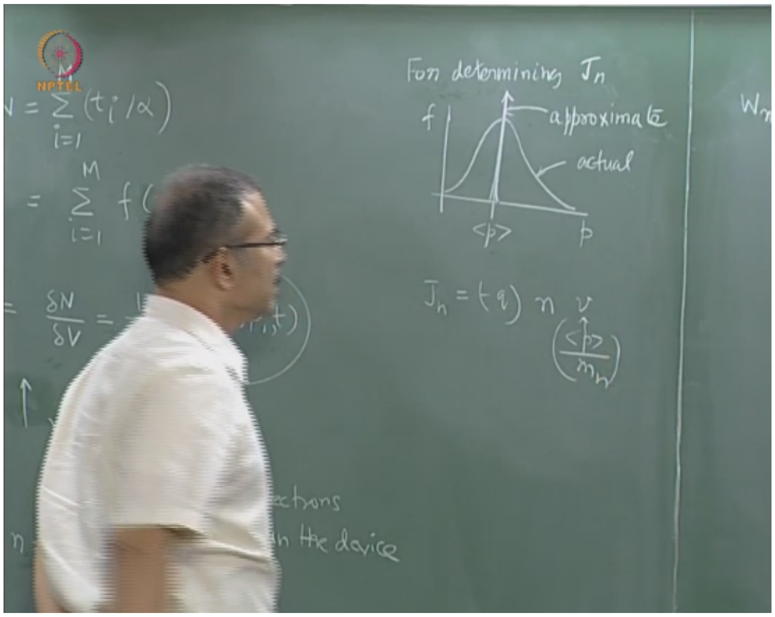

That for purpose of determining Jn this approximation works. So, this is your distribution function actual. This is your average value of momentum what we are saying is you can assume it to be delta function at this point is approximate. So which is a considerable approximation because you are assuming that all the n electrons have the same momentum or velocity and even then you are getting the same result as you would get if you take into account the detailed distribution of momentum.

However, the caution is that this is useful only for determining Jn. Now you cannot extend it for example to determining the average kinetic energy. So in effect we are saying the complicated formula for Jn that we showed amounts to simply writing Jn equal -q into this average carrier concentration into average velocity. This average velocity is written as p/mn. That is what we want to show.

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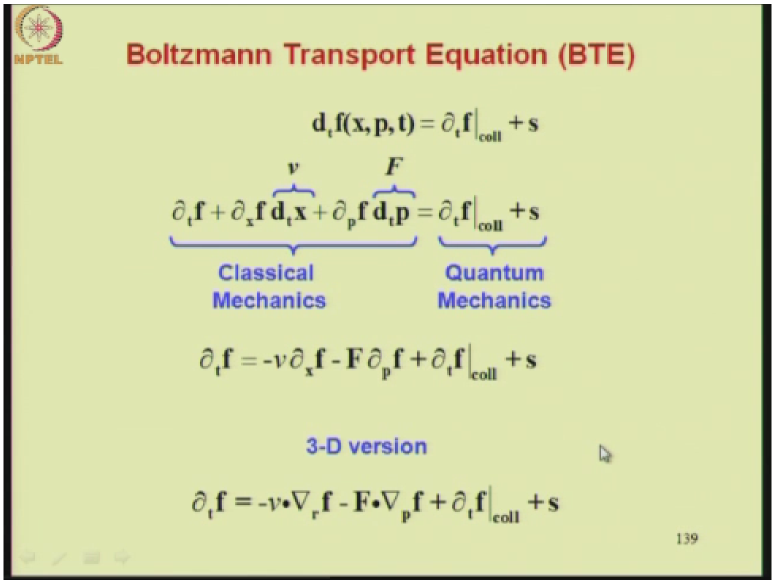

Boltzmann Transport Equation (BTE)

$$d_t f(x,p,t) = \partial_t f\big|_{coll} + s$$

$$\underbrace{\partial_t f + \partial_x f\, \overbrace{d_t x}^{v} + \partial_p f\, \overbrace{d_t p}^{F}}_{\text{Classical Mechanics}} = \underbrace{\partial_t f\big|_{coll} + s}_{\text{Quantum Mechanics}}$$

$$\partial_t f = -v\,\partial_x f - F\,\partial_p f + \partial_t f\big|_{coll} + s$$

3-D version

$$\partial_t f = -v\bullet\nabla_r f - F\bullet\nabla_p f + \partial_t f\big|_{coll} + s$$

Now let us look at the Boltzmann transport equation. This is the very, very important equation which tells you how to derive the distribution function f in a given situation. So like Schrodinger equation gives you the wave function psi. The Boltzmann transport equation gives you the distribution function f. The compact form of this equation is d f/dt. The total derivative of f with respect to time is caused by is = dou f/dou t collisions the change in f.

Because of collisions plus the change in the concentration because of generation or recombination or net generation. That is generation minus the combination. So this is s a source term for concentration and this is a source term for momentum. So because the state of the group of electrons will depend on or rather description of group of electrons is in terms of both momentum, concentration and so on.

Therefor we have to look at how the distribution function will change because of collusion. So when electron encounter collision their momentum will change as a result of collision so distribution function changes. Similarly, somewhere there is excess generation or recombination then again you are losing carriers and therefor again the distribution function will change because loosely speaking area under the distribution function is number of carriers.

Now you can expand this term df/dt considering the fact that f is the function of 3 variables x, p and t. So, using partial derivatives you can write this as dou f/dou t+dou f/dou x into dx/dt+dou f/dou p into dp/dt. Now, dx/dt is nothing but rate of change of the position of electrons with time that is the velocity v of the carriers. And rate of change of momentum with time dp/dt is nothing but the force which is applied and which is changing the distribution function.

Now, these 3 terms together represent classical mechanics because you are talking about the velocity of the particles. We are talking about the momentum and the force which is changing the momentum. So all these are parts of a classical picture. On the other hand, the right hand side has to be determined from quantum mechanics. So, you will recall we have said that scattering has to be treated quantum mechanically.

Scattering rate and so on if you want to find out you have to do analysis based on quantum mechanics to explain why scattering has to be treated quantum mechanically. Similarly, the source term s also has to be derived from quantum mechanics. So this is how the Boltzmann's transport equation combines elements of classical and quantum mechanics. And this is not surprising because we have said the carrier transport is semi classical.

So you cannot use only classical laws or Newton’s laws to derive the information about the carrier motion in a semiconductor. So this is a statement of semi classical nature of transport. Now, I can reorganize this function, this equation in this form where I keep the partial derivative with respect to time on the left hand side and push the other terms on the right. Now each form of arrangement of equation as positive feature.

So this form that we discussed now has advantage that it can be derived from the compact form and it has classical mechanics on the left hand side and quantum mechanics on the right hand side. So it clearly separates the 2 form of analysis on the 2 sides of the equations. The great advantage of the form that we have written down now here where dou f/dou t is put on the left hand side.

And all of the terms on the right hand side is that you can now see a correspondence between the Boltzmann's transport equation and for example the whole continuous equation and recognize that this transport equation is nothing but a statement of continuity, balance or conservation of a quantity namely the distribution function. Now, let us do that.

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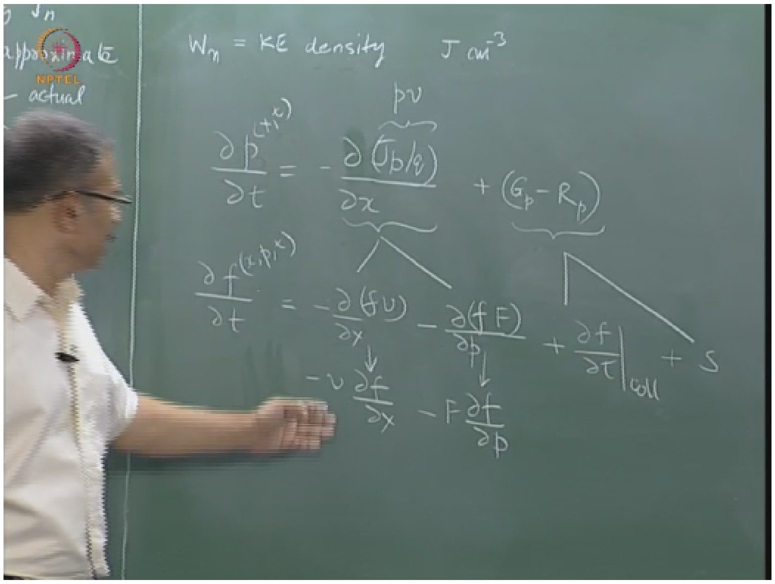

So whole continuous equation is dou p/dou t = -dou/dou x of Jp/q. This is nothing but the flux which can be written as whole concentration into velocity. So +g -r or Gp-Rp. This is net generation. Now let us write similarly the Boltzmann's transport equation. Now p is replaced by f so dou f/dou t now this is =-dou/dou x of the flux because of f that is like p into v I can write here f into v.

However, while p is a function of x, t. f is a function of x, p, t therefor like a derivative with respect to x I should also have a derivative with respect to p, in analogous manner. Now, if you write that, that would look like this so -dou/dou p of a flux in the p dimension. So flux in next dimension consists of f into dx/dt, that is the velocity. So, flux in the p dimension would be f into dp/dt that is nothing but the force.

Now plus you have a source term because of momentum dou f/dou t due to collisions because this is the function of momentum. Now you have a source term because of carrier concentration as well. Because it is the function of x and p both so you have 2 terms. So now what you find is each of this term has 2 corresponding terms and similarly this term also has 2 corresponding terms. Because here you have only x where as you have x, p.

So that is how a great advantage you see of writing equation in this form is that you can recognize the Boltzmann's transport equation to be a continued equation and by showing its analogy to the whole continued equation we have made ourselves very comfortable with this equation and we can therefor remember it very easily. Now still this is not in the form in which we wrote down there has be a small rearrangement to be done here.

The v has to be moved out of this derivative and this f has to moved out of this derivative. Now, I will not discuss what are the reasons we can do it there are reasons for this. Right now our aim is to familiarize ourselves with the form of the equation. So, if you move the v out it become -v dou f/dou x and here it becomes -F dou f/dou p and of course this becomes like this. Now you can recognize the form that we wrote down in 1- dimension.

We will write the 3 dimensional form also before we close the discussion on this equation. So, the 3-D version is dou f/dou t= minus of v. the gradient of f in the r direction special gradient. Now since v and gradients are vectors I must clearly tell whether it is a cross or a dot product. It is a dot product. In 1-dimensional version this problem does not arise because v and the gradient in x direct are all in the same direction.

Similarly, here you have f dot the gradient with respect to p momentum. So gradient of f with respect to p+dou f/dou t collision +s. So, this is the 3-D version of the Boltzmann's transport equation. Now with that we have come to the end of this lecture.

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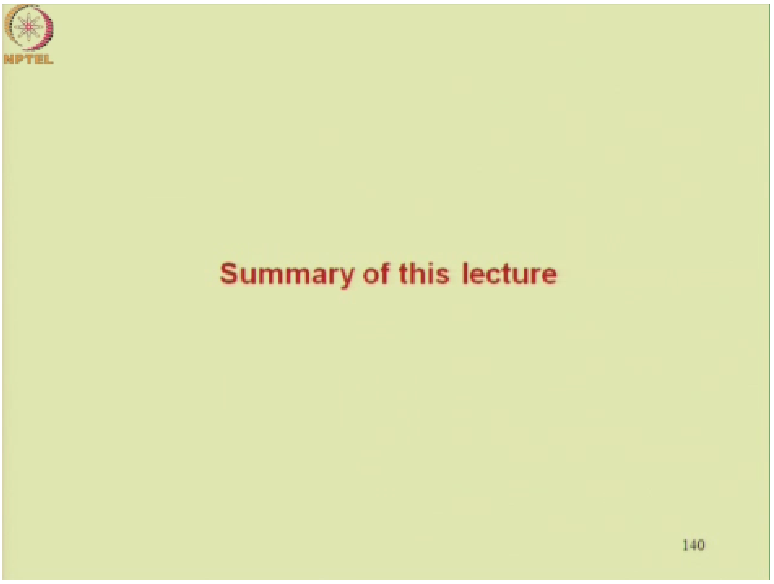

Summary of this lecture

Let us make a summer of the important points. So, in this lecture we begun a discuss of the fundamental ensemble point of view in which carriers are treated in terms of their group and carrier is regarded as a particle. So as a group of particles and therefore we want to apply laws of classical mechanics to treat this. However, we also need quantum mechanics because catering itself is quantum mechanical in nature.

And also generation recombination processes formula for that has to be derived from quantum mechanics. So, we have discussed the meaning of the distribution function that describes a collection of electrons. How the distribution function tells you the distribution of the carriers over the various momentum. Then we have given the formulae for n, Jn and the kinetic energy density Wn in terms of the distribution function.

So, how the distribution function helps you to find out the quantity of interest in a device namely n, Jn and Wn. And then finally we have explained the equation that should be used to derive the distribution function f that equation is called the Boltzmann's transport equation. We have shown the correspondence of this equation to the whole continued equation and established that this equation is also a conservation equation, a balanced equation or a continuity equation.