Digital Image Processing

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Module 02 Lecture Number 09

Application of Distance Measures

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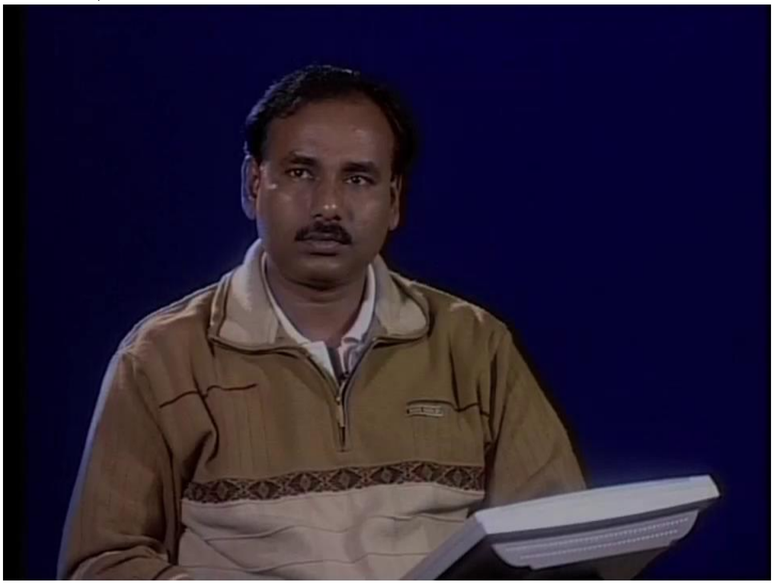

Hello, welcome to the video lecture series on Digital Image Processing.

Let us

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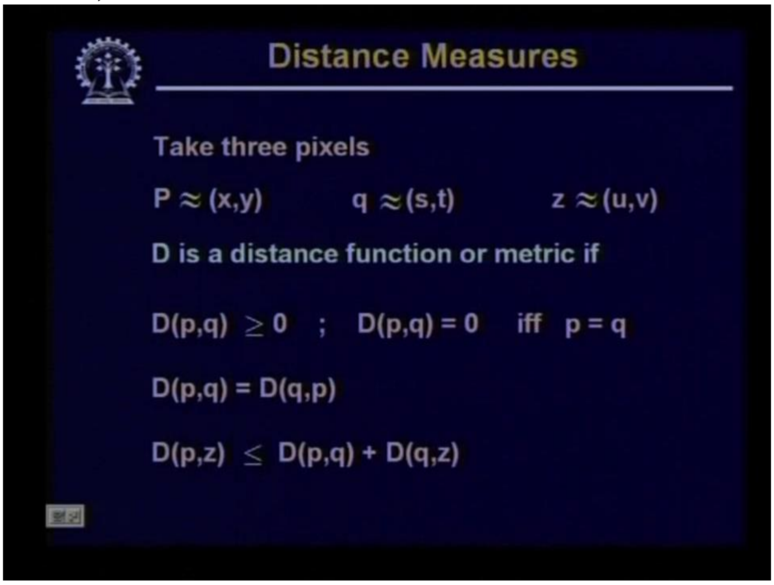

move to another operation, another concept that is distance measures.

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Now finding out the distance between two points, we are all familiar that if I know the coordinate or the location of two different points I can find out what is the distance between the two points. Say for example what I can do is if I have two points, say one point p and other

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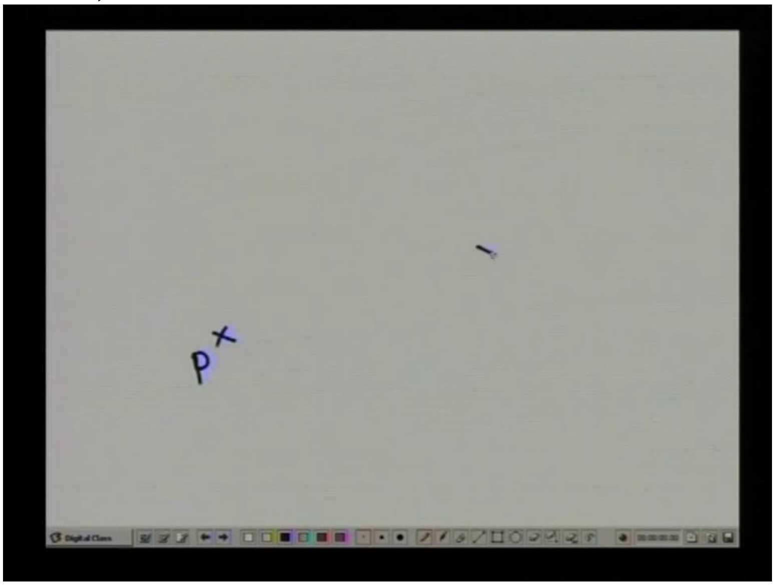

point q and I know that the coordinate of point p is given by x y and coordinate of point p q is given by s and t then we all know from our school level mathematics that the distance between

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the points p and q is given by the relation that

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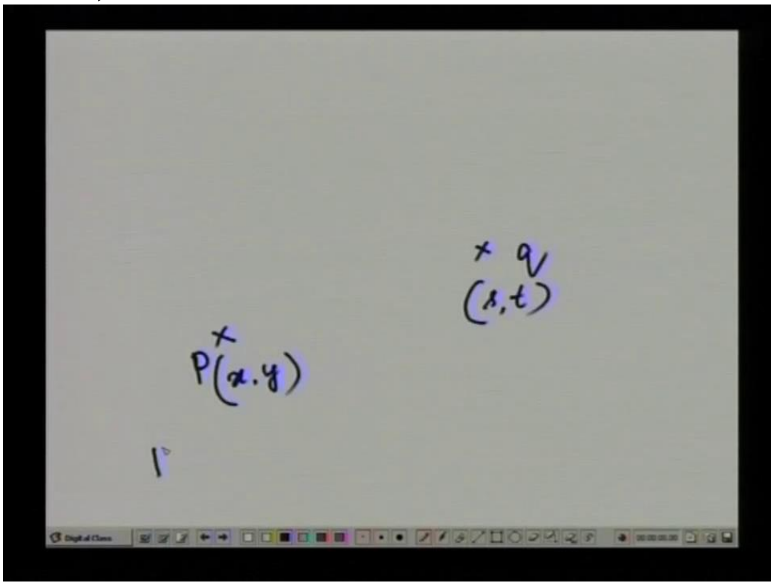

I represent this as d p q that is distance between p and q that is given by x minus s square plus y minus t square and square root of this term

So this is what all of us know

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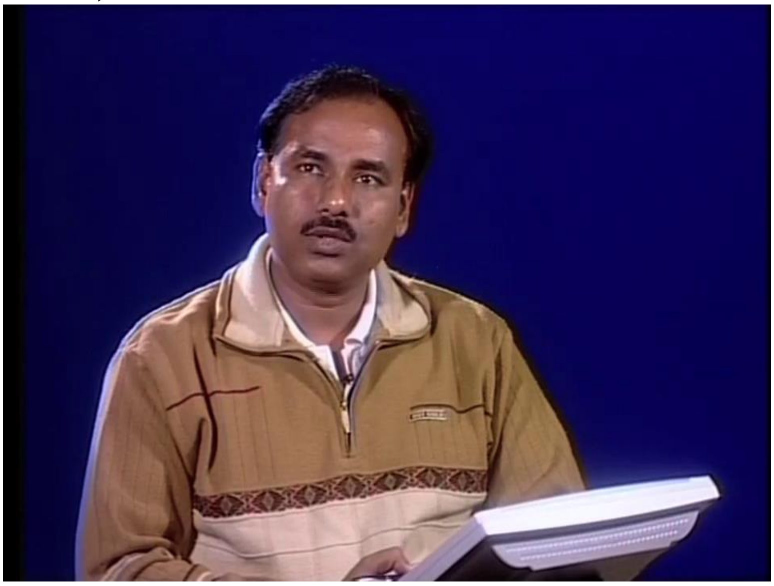

from our school level mathematics Now when i come to digital domain then this is not only this is not the only distance measure that can be used There are various other distance measures which can be used in digital domain those distance measures are say city block distance, chess board distance and so on. So to see that

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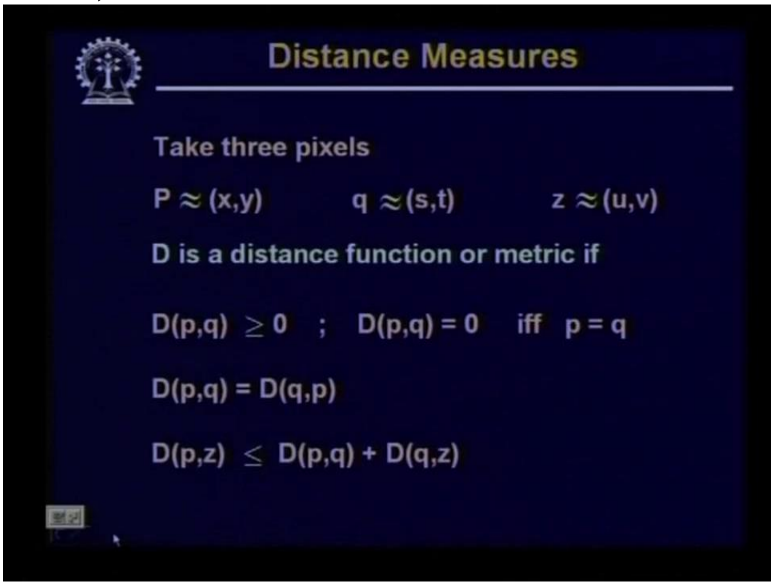

if d is a distance function or a distance metric then what

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is the property that should be followed by this distance function d; so for this let us take 3 points we take 3 points here p

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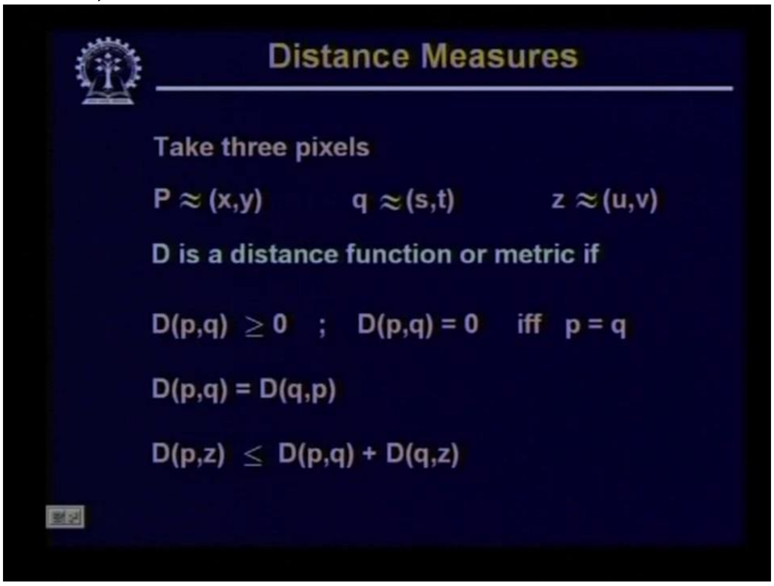

having a coordinate x y q having a coordinate s t and I take another point z having the coordinate u v Then d is called a distance measure, is a valid distance measure or a valid distance metric if d p q is greater than or equal to 0 for any p an q any two points p and q d p q

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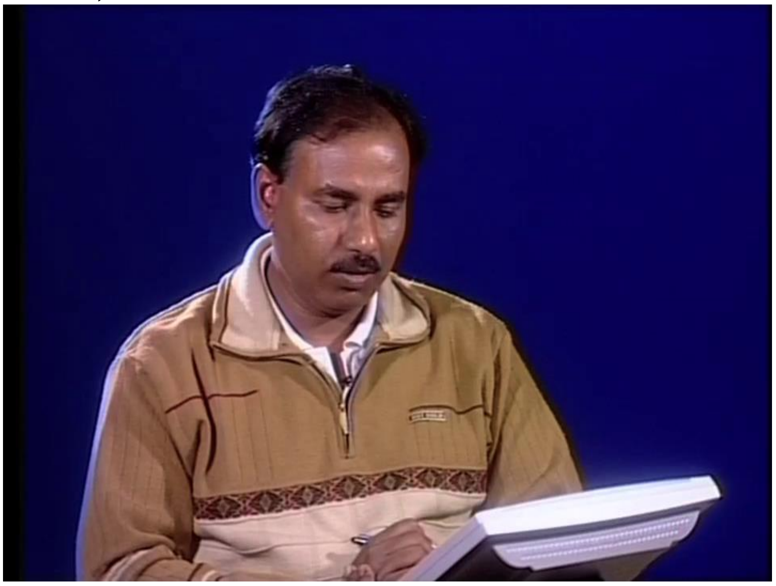

must be greater than or equal to 0 and d p q will be 0 only if p is equal to q. So that is quite obvious because the distance of the point from the point itself has to be equal to 0.

Then the distance metric

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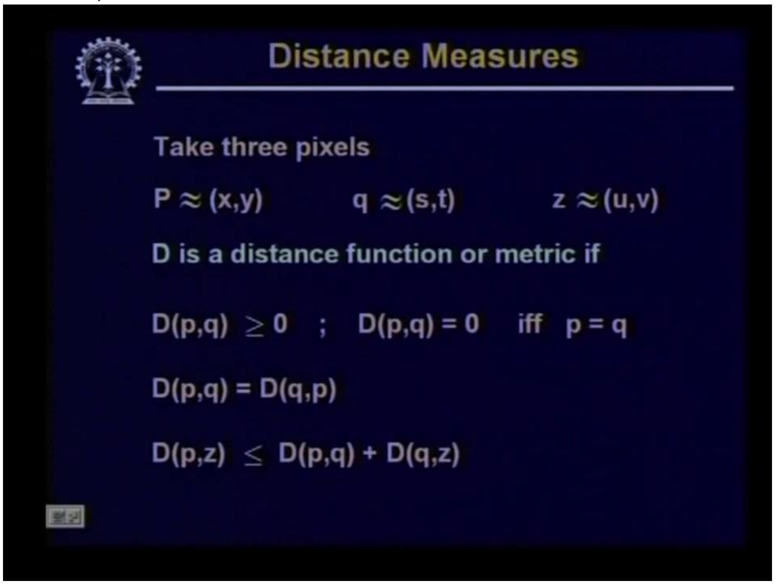

distance function should be symmetric. That is, if I measure the distance from p to q that should be same

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distance function should be symmetric. That is, if I measure the distance from p to q that should be same as the distance if I measure from q to p that is the second property that must hold true that is

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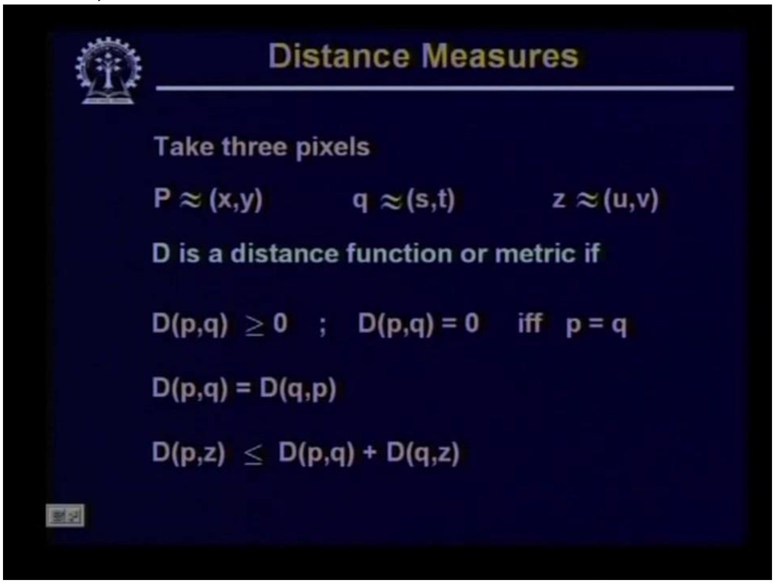

d p q should be equal to d q p. And there is a third property which is an inequality; that is if I take a third point z then the distance between p and z that is d p z must be less than or equal to the distance between p and q plus the distance between q and j, z.

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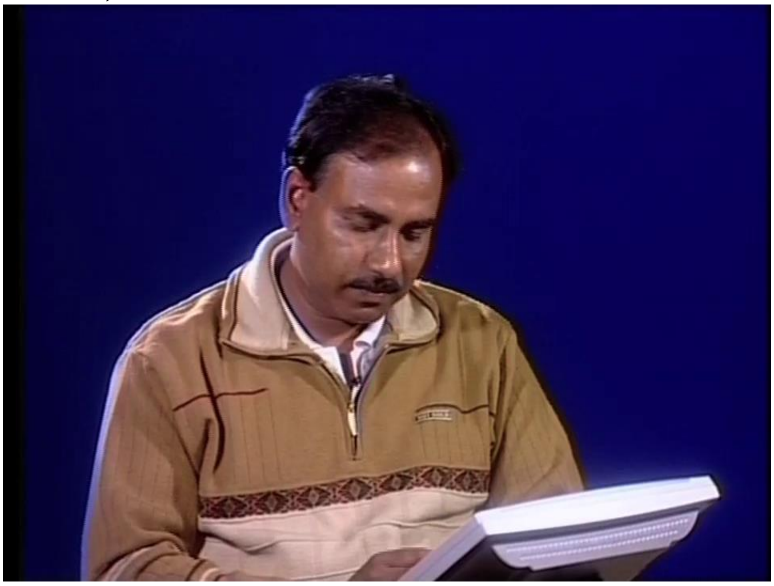

And this is quite obvious, again from

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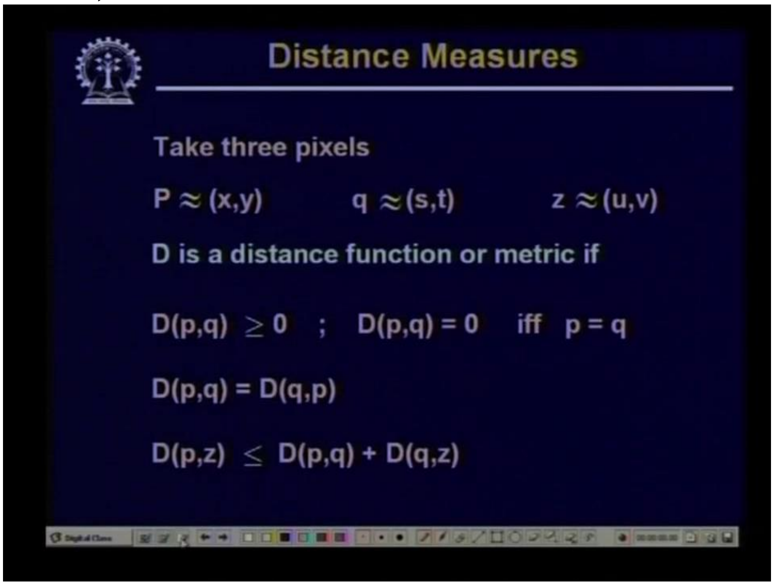

our school level mathematics you know that if I have say 3 points

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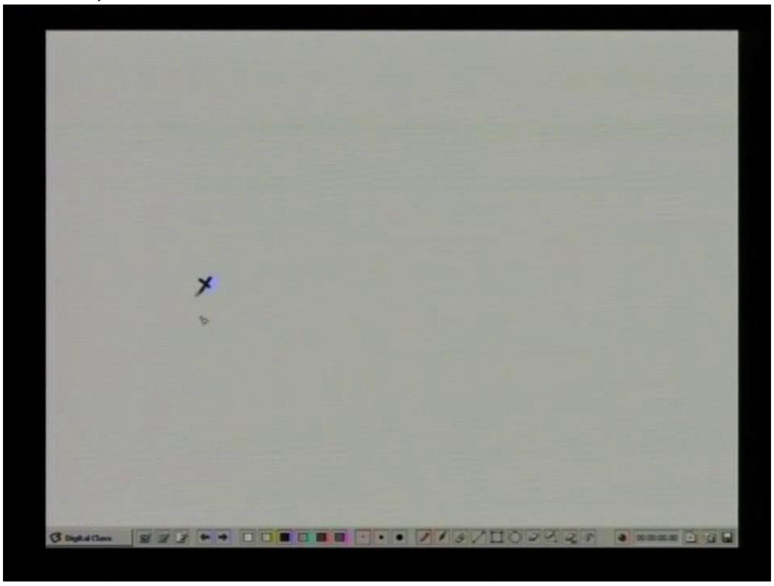

p, q and I have another point z and if I measure the distance between p and z this must be less than the distance between p q plus the distance between p z. So this is what we all have done

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in our school level mathematics and the same property must

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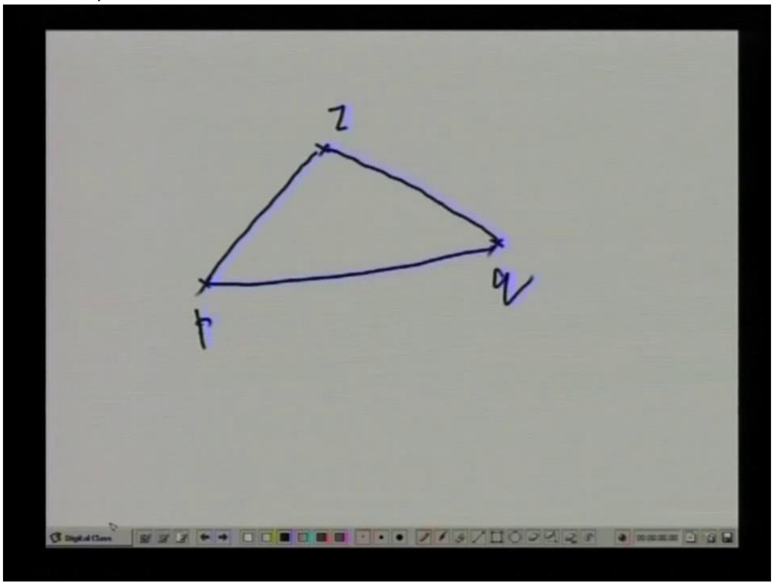

hold true even in case

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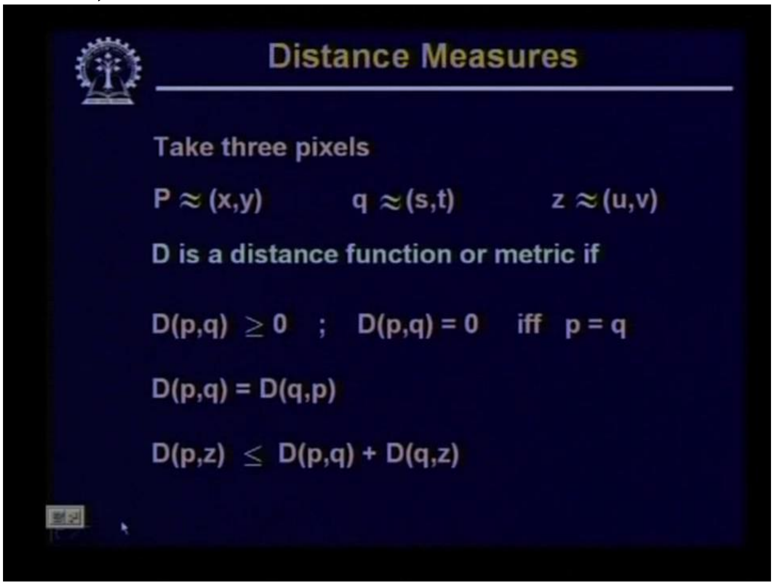

digital domain where we talk about different

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other distance functions So these are the 3 properties which must hold true for a function if a function is to be considered as a distance function or a distance metric.

Now the first of this, that is the distance between p and q which we have already seen that if p has a

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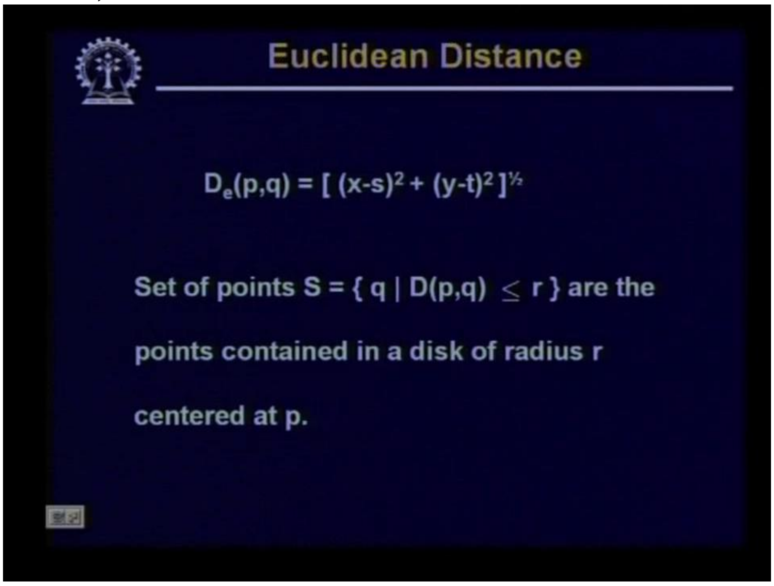

coordinate x y and q has a coordinate s t then d p q the distance between p and q is equal to x minus s square plus y minus t square and square root of this whole term. This is a distance measure which is called an Euclidean distance. So in case of Euclidean distance you will find that set of points q

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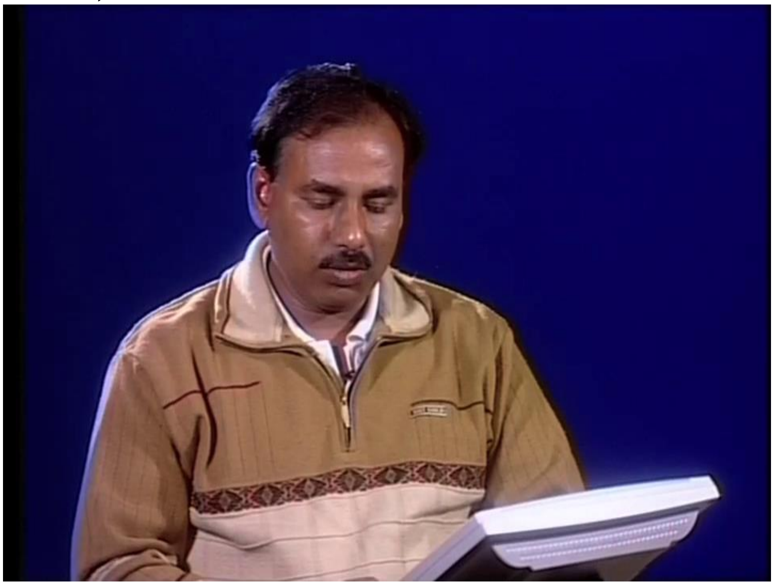

that is d p q the distance between p and q, obviously we are talking about the Euclidean distance is less than

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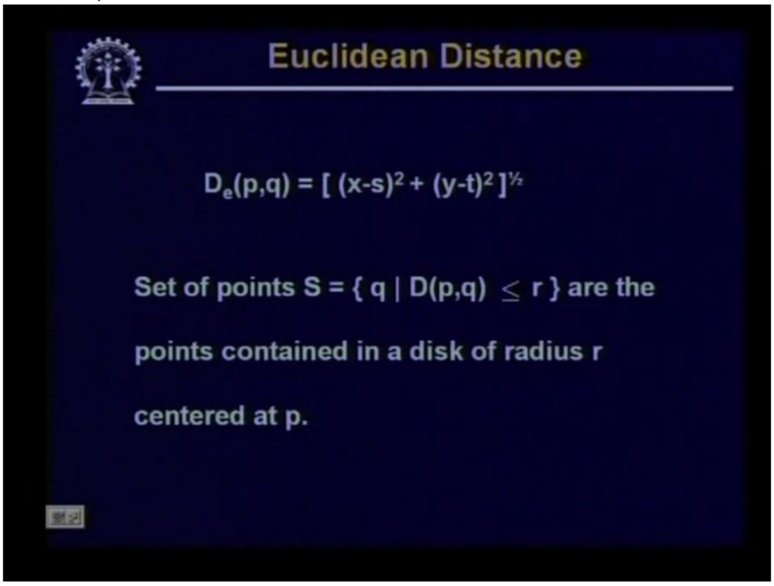

or equal to some value r so set of all these points are the points contained within a disk of radius r where the center of the disk is located

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at location p And again this is quite

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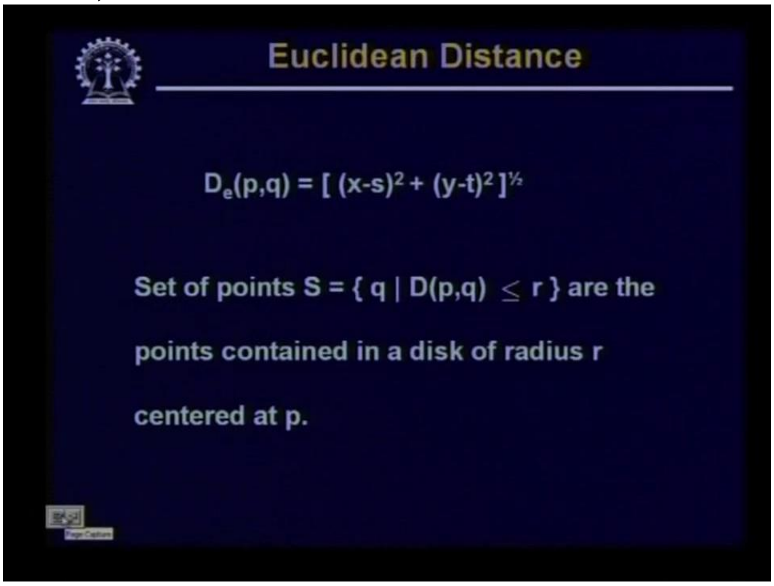

obvious you will find that suppose I have a point p here

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and I take a point q and I say that the distance between p and q is r. So if I take set of all these points where the distance is equal to r that forms a circle like this, so all other points having a distance less than r from the point p will be the points within this circle. So set of all these points where the distance value is less than or equal to r, obviously we are talking about the Euclidean distance

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in that case the set of all these points forms a disk of radius r where the center of the disk

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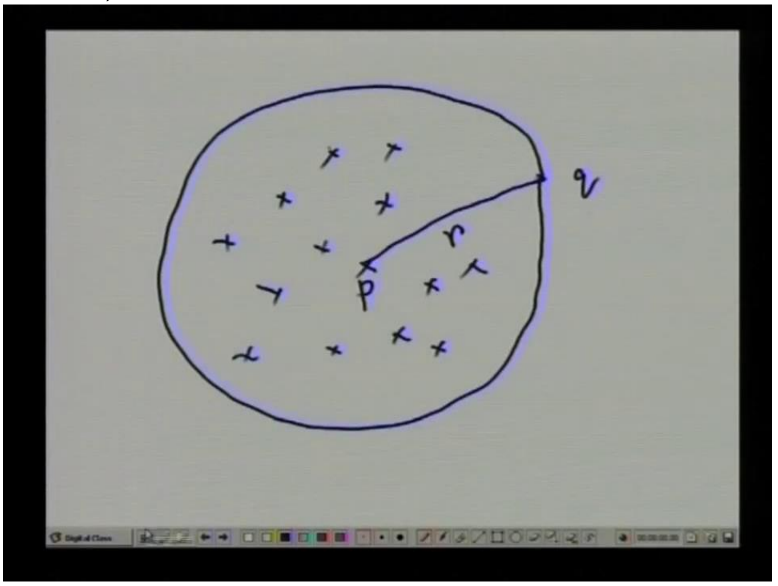

is at location p

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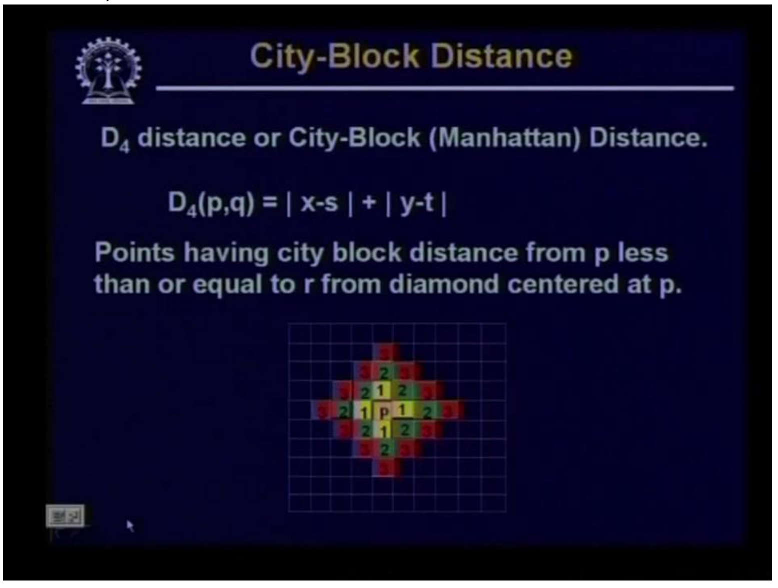

Now coming to the second distance measure which is also called D 4 distance or city block distance or this is also known as Manhattan distance. So this is defined as D 4 p q is equal to x minus s absolute value plus y minus t absolute value.

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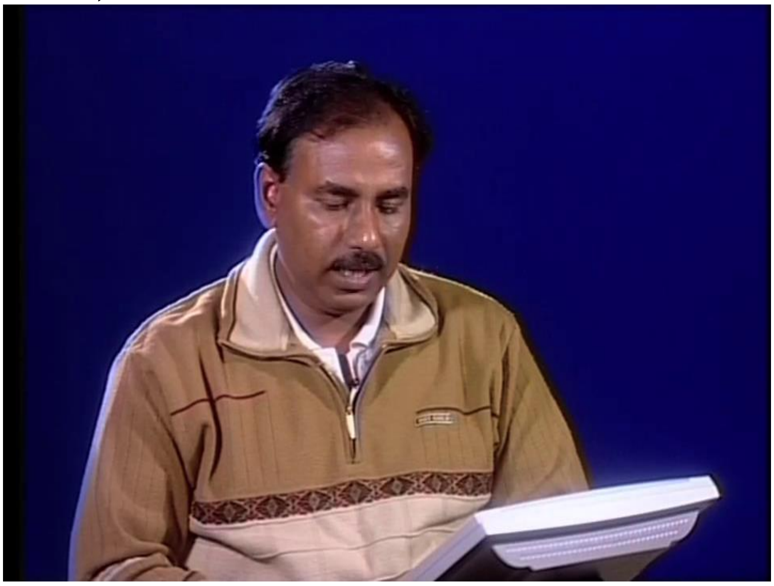

So in this case you will find it is something like this,

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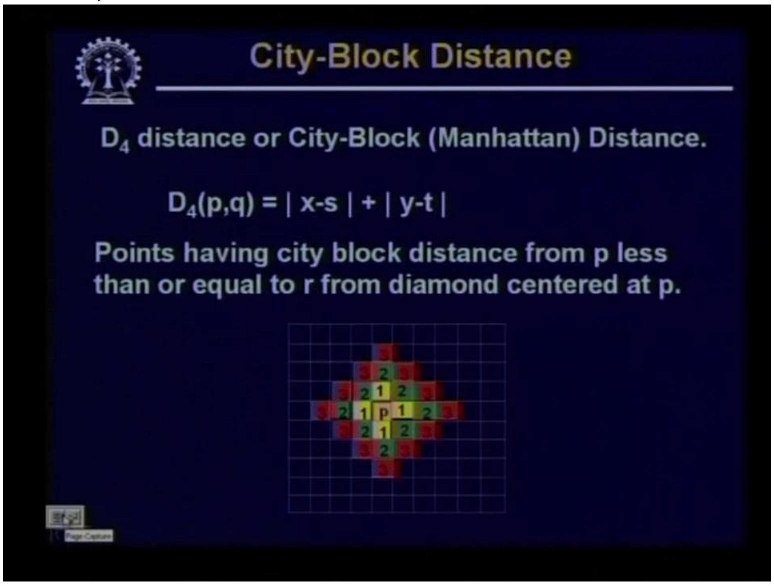

that if I have point p

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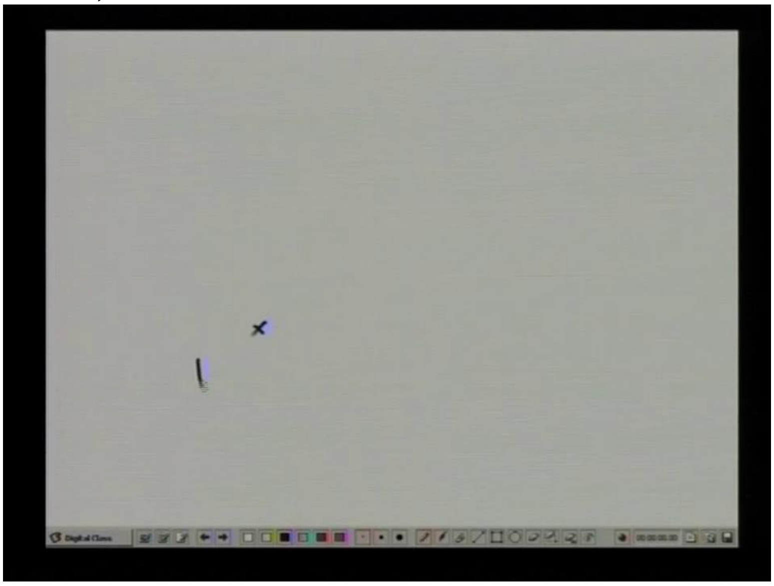

with coordinate x y and I have point q with coordinate s t so this D 4 distance as it is defined that it is equal to x minus s, take the absolute value plus y minus t, again you take the absolute value. So this clearly indicates that if I want to move from point p to point q then how much distance I have to move along the x direction and how much distance I have to move along the y direction. Because x minus s the absolute value of this is the distance travelled along x direction and y minus t absolute value of this is the travel, is the distance travelled along the y direction. So the sum of these distances along x direction and y direction gives you the city block distance that

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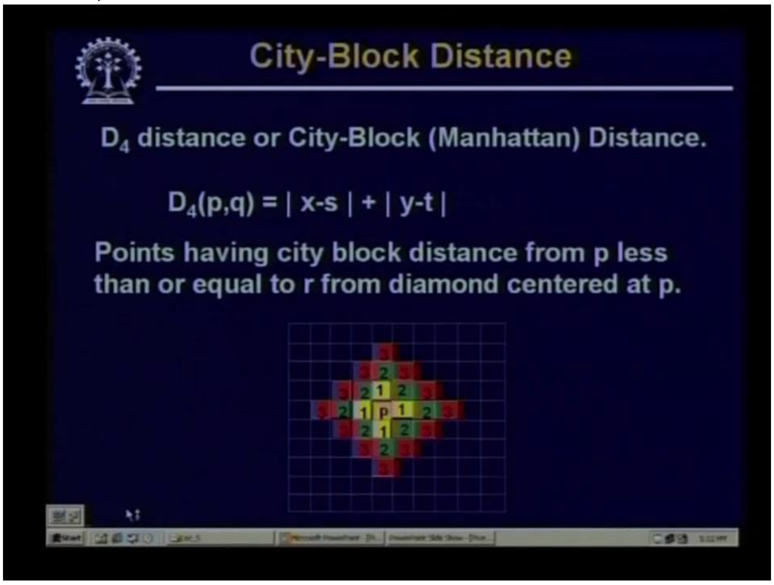

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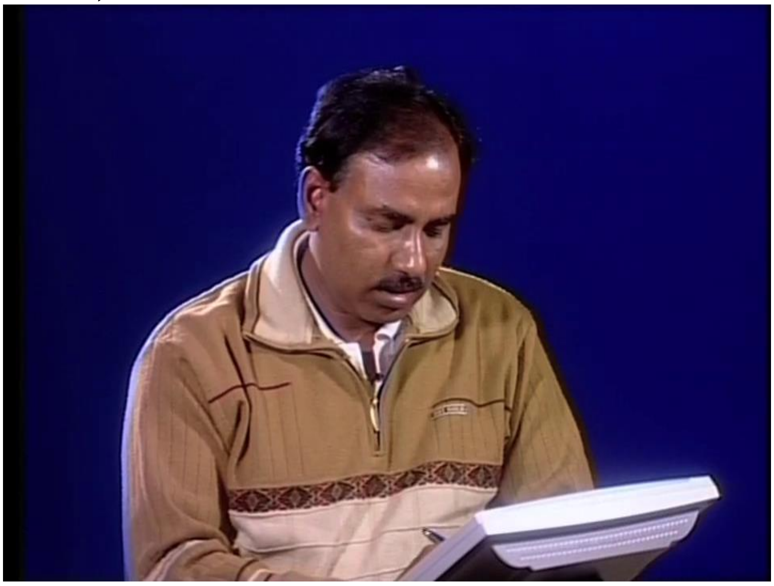

D 4 And here you find that the points having

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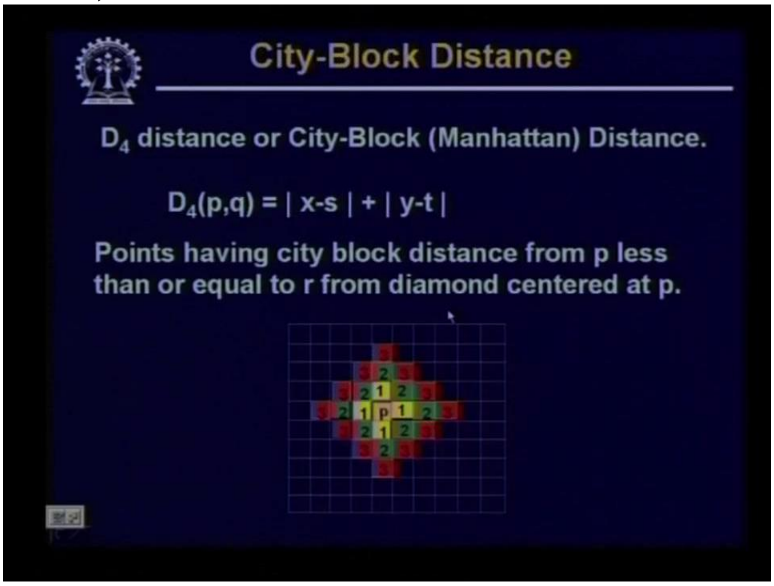

the city block distance from point p less than or equal to some value r will form a diamond centered at point p, so which is quite obvious from here

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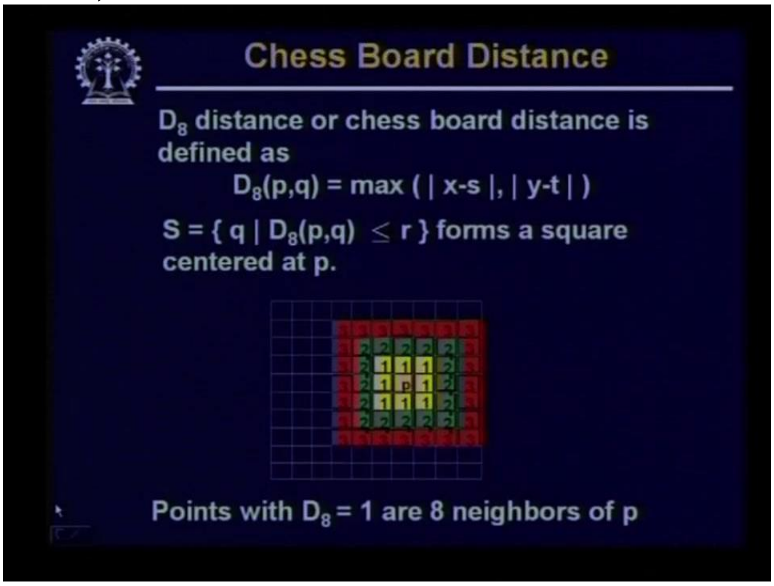

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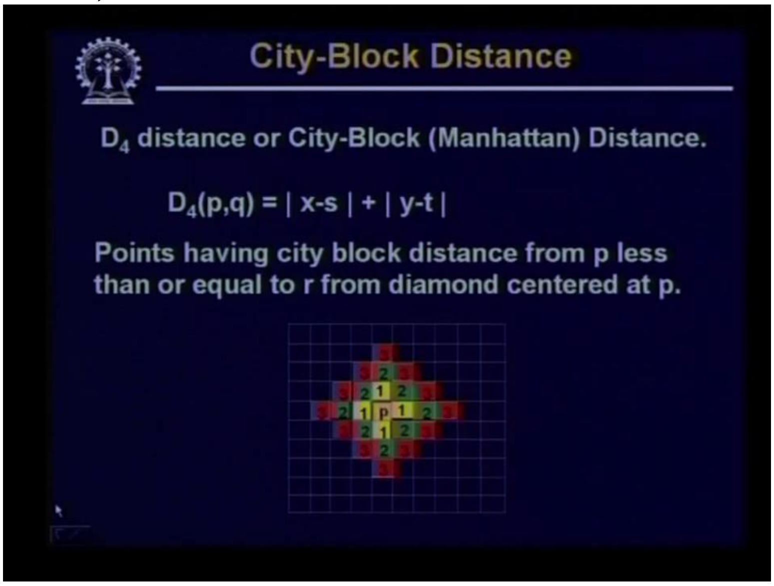

That here you find that if p is the point at the center then all the points having city block distance, they are just the 4 neighbors of the point p. Similarly all the points having the city block distance is equal to 2, they are simply the points which are at a distance 2, that is the distance taken in the horizontal direction plus the distance taken in the vertical direction, that becomes equal to 2 and set of all these points with city block distance is equal to 2, that simply forms a diamond of radius 2 and similarly other points at distances 3, 4 and so on.

Now we come to the third distance measure

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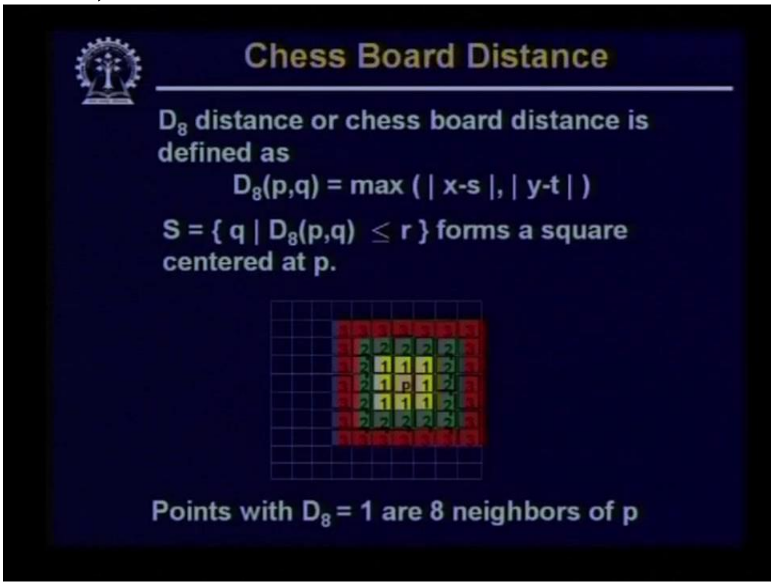

which is the chess board distance

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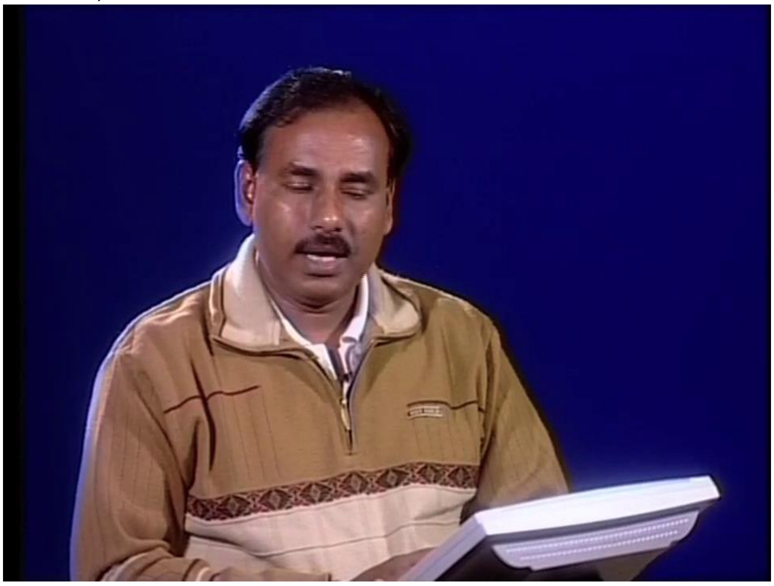

As you have seen in case of city block distance, the distance between two points was defined as the sum of the distances that you cover along x direction plus the distance along the y direction. In case of chess board distance it is the maximum of the distances that you cover along x direction and y direction. So this is D 8 p q which is equal to

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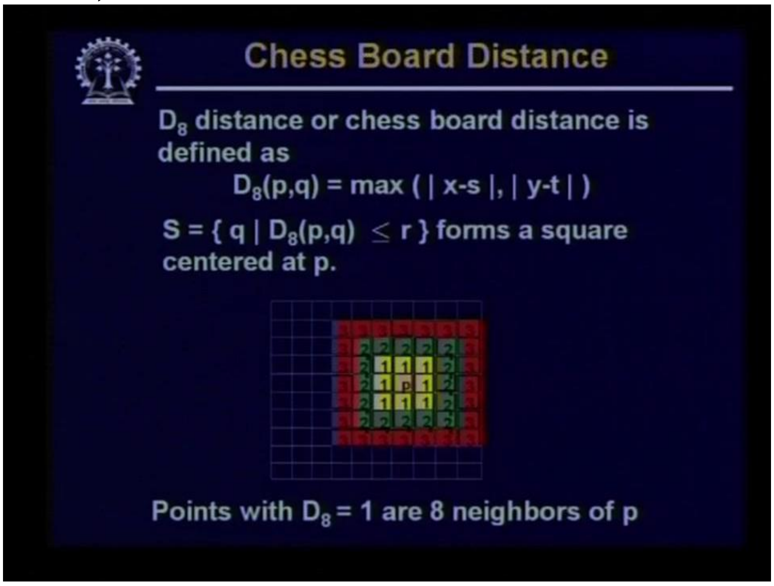

max of x minus s and y minus t where we take the absolute value of both x minus s and y minus t. And following the same argument, here you find that the set of points with a city block with a chess board distance of less than or equal to r now forms a square centered at point p. So here all the points with a chess board distance of equal to 1 from point p, they are nothing but the 8 neighbors of point p.

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Similarly the set of points with a chess board distance will be equal to 2 will be just the points outside the points having a chess board distance equal to 1. So if you continue like this you will find that all the points having a chess board distance of less than or equal to r from a point p will form a square with point p at the center of the square. So these are the distance different distance measures that can be used in the digital domain.

Now let us see that what is the application of this distance measure; one of the obvious application is if I want to find out the distance between 2 points I can make use of either Euclidean distance or city block distance or the chess board distance. Now let us see one particular application other than

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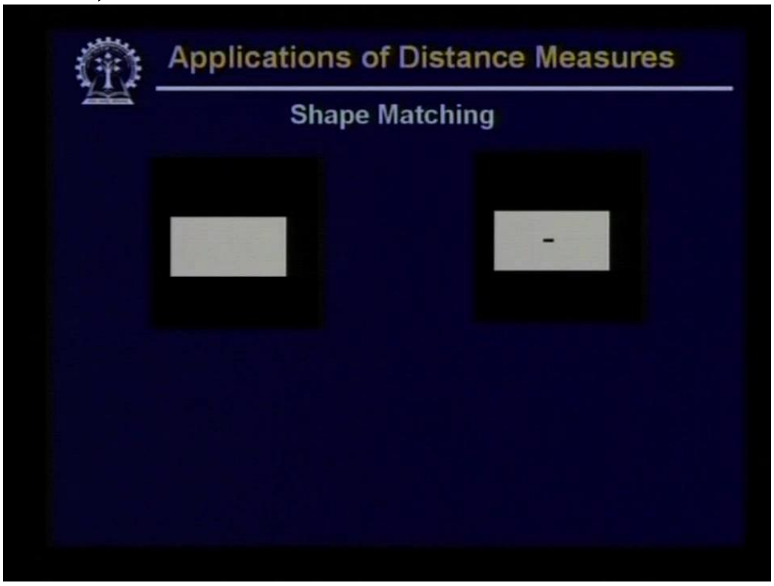

just finding out the distance between 2 points Say for example here I want to match two different shapes which are shown in this particular diagram. Now you will find that these two shapes are almost similar except that you have a hole in the second shape. But if I simply go for matching these two shapes they will be almost similar. So just by using these original figures

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I cannot possibly distinguish between these two shapes. So if I want to say that these two shapes are not same, that they are dissimilar in that case, I cannot work on these original shapes but I can make use of some other feature of this particular shape. So let us see what is that other feature

If I

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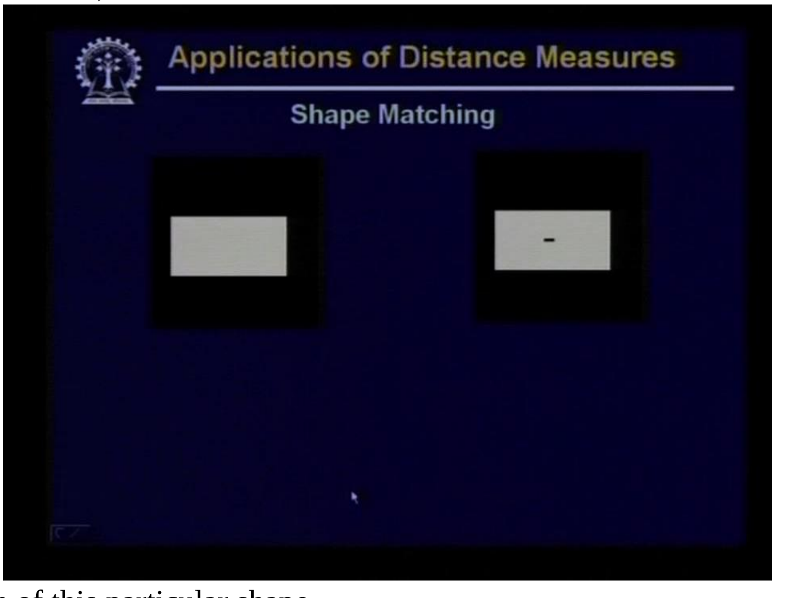

take the skeleton of this particular shape

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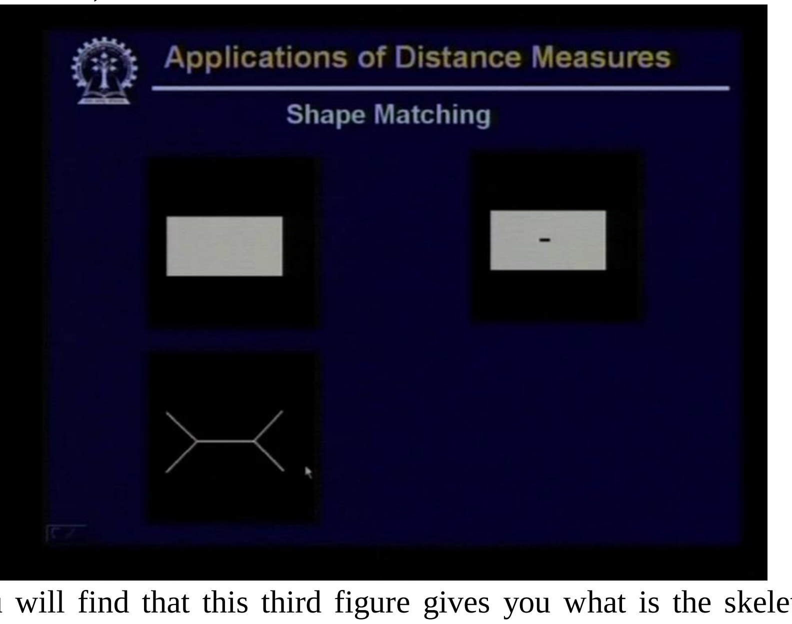

in that case, you will find that this third figure gives you what is the skeleton of the first shape. Similarly the fourth figure gives you

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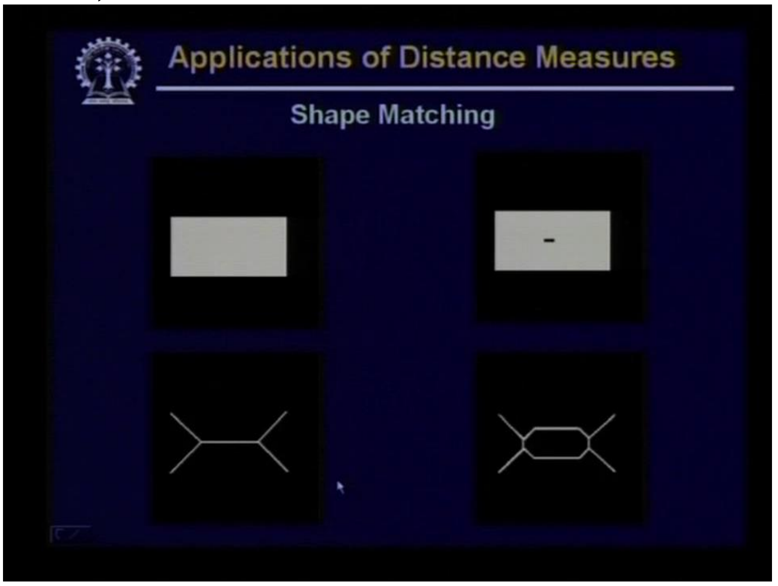

what is the skeleton of the second shape Now if I compare these two skeletons rather than comparing the original shapes you find that there is lot of difference between these two skeletons

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So I can now describe the shapes with the help of the skeletons in the sense that I can find out that how many line segments are there in the skeleton. Similarly I can find out that how many points are there where more than 2 line segments meet. So by this if I compare the two skeletons you will find that for the

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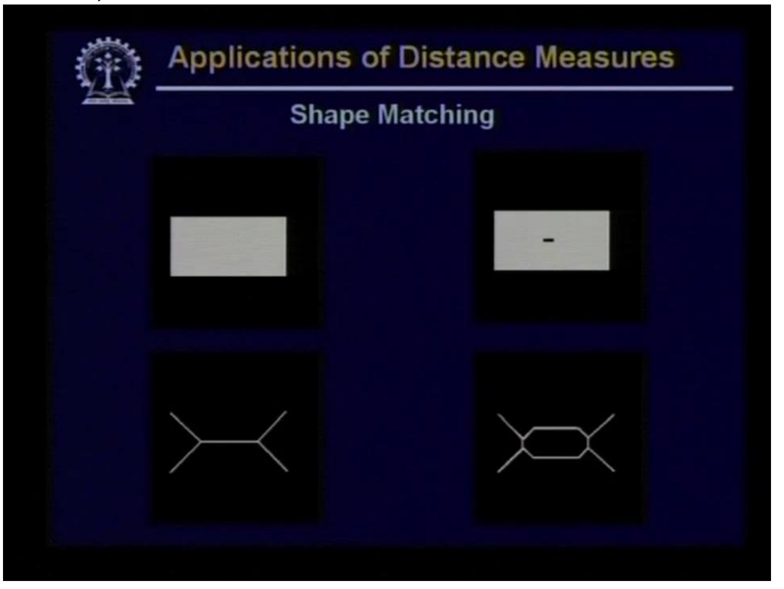

skeleton of the first shape there are only 5 line segments where as for the skeleton of the second shape there are 10 line segments. Similarly the number of points where more than 2 line segments meet; in the first skeleton there are only two such points where as in the second skeleton there are 4 such points. So if I compare using the skeleton

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rather than comparing using the original shape you will find that there is lot of difference that can be found out both in terms of the number of line segments the skeleton has and also in terms of the number of points where more than one line segments meet. So using these descriptions which I have obtained from the skeleton, I can distinguish between the two shapes as shown in this particular figure.

Now the question is how do we get this skeleton

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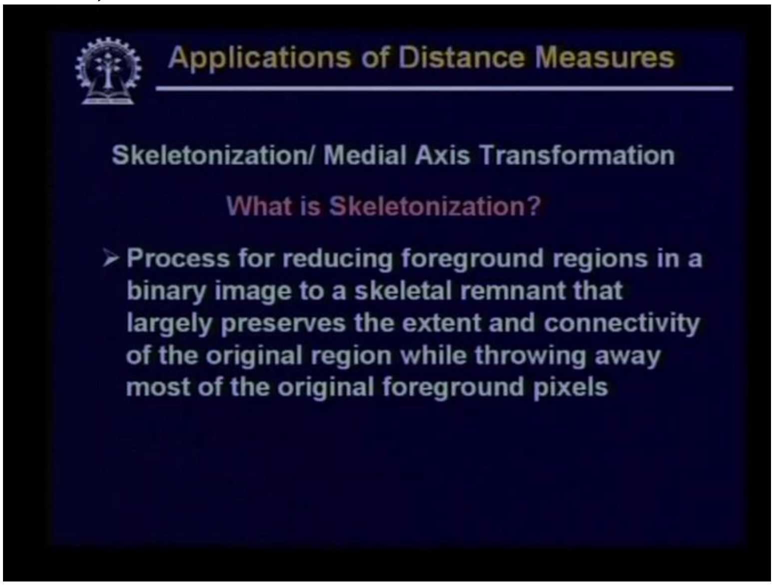

and what is this skeleton? So you will find that if you analyze the skeletons you will find that the skeletons are obtained

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by removing some of the foreground points but the points are removed in such a way that the shape information as well as the dimensionality that is what is the length or breadth of that particular shape is more or less retained in the skeleton. So this is how this is what is the skeleton of the particular shape and now the question is how do you obtain the skeleton

Now before coming to how do you

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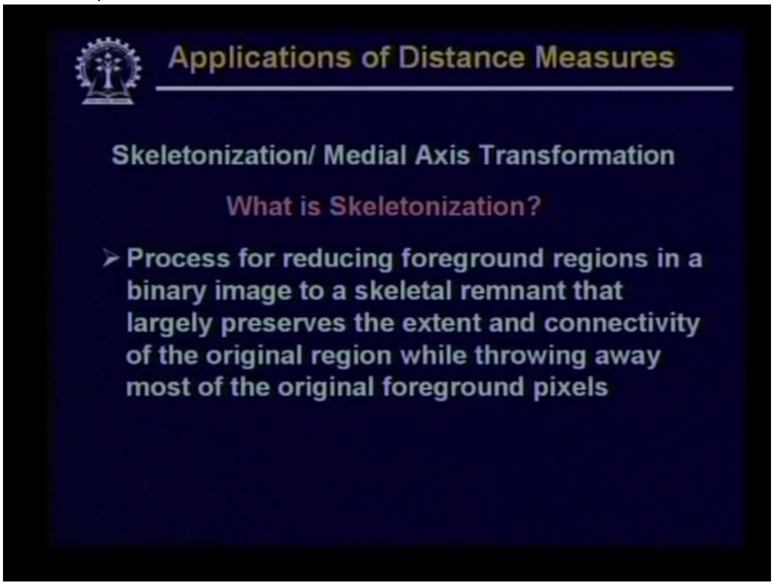

obtain the skeleton

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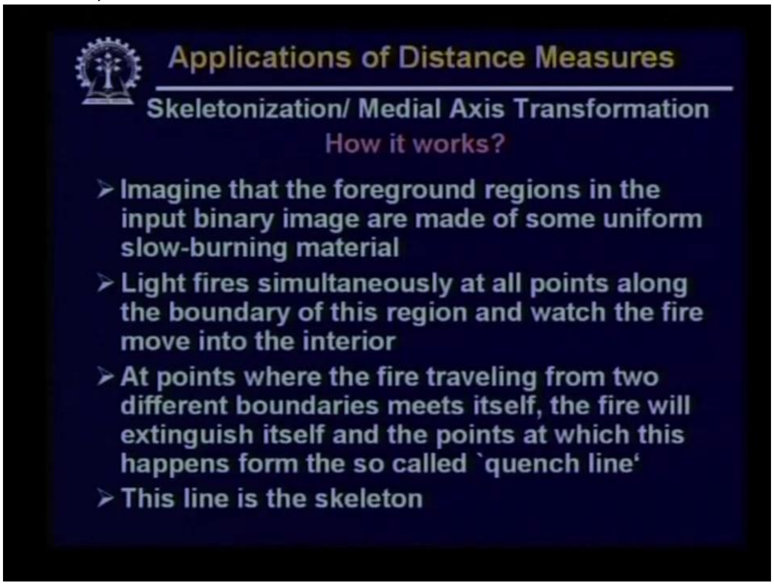

let us see that how the skeleton can be found out. So the skeleton can be found out in this manner. If I assume that the foreground region in the input binary image is made of some uniform slow burning material and then what I do is

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I light fire at all the points across the boundary of this region that is the foreground region. Now if I light fire across the boundary points simultaneously then the fire lines will go in slowly because the foreground region consists of slow burning material. Then you will find that as the fire lines they go in, there will be some points in the foreground region where the fire coming from two different boundaries will meet and at that point the fire will extinguish itself. So the set of all those points is

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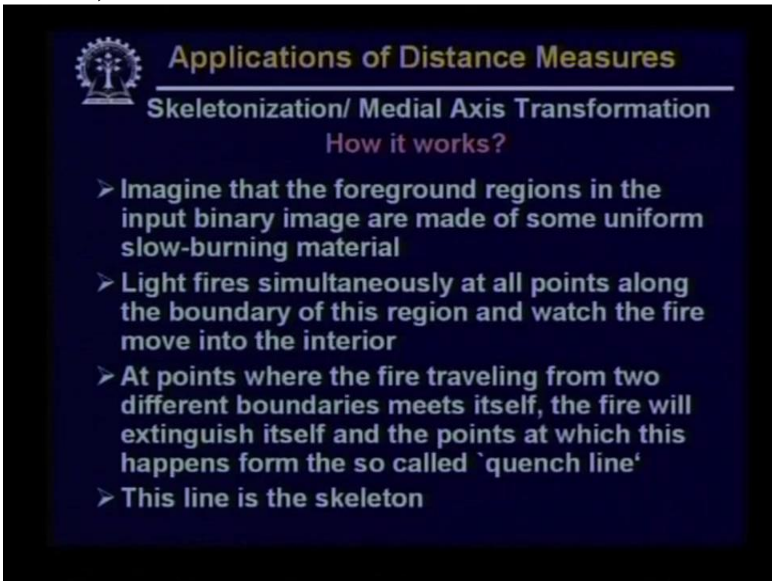

what is called the quench line

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and the skeleton of the region is nothing but

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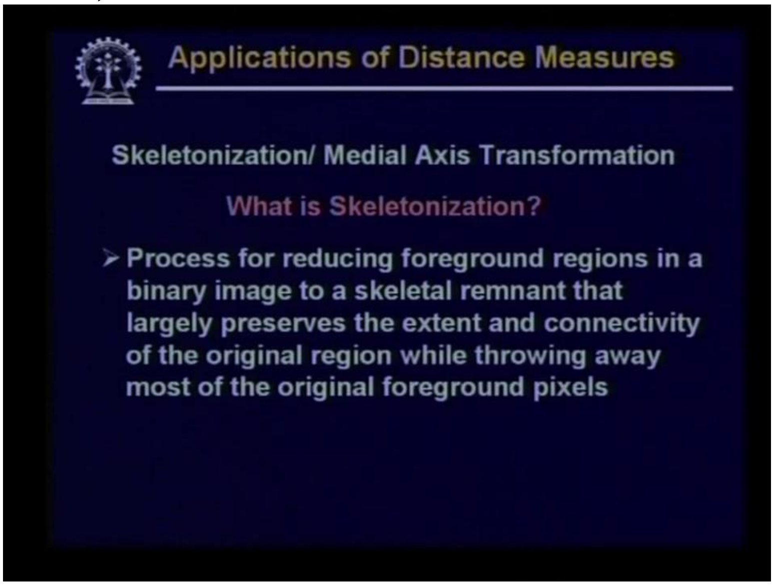

the quench line that we obtain by using this fire propagation concept

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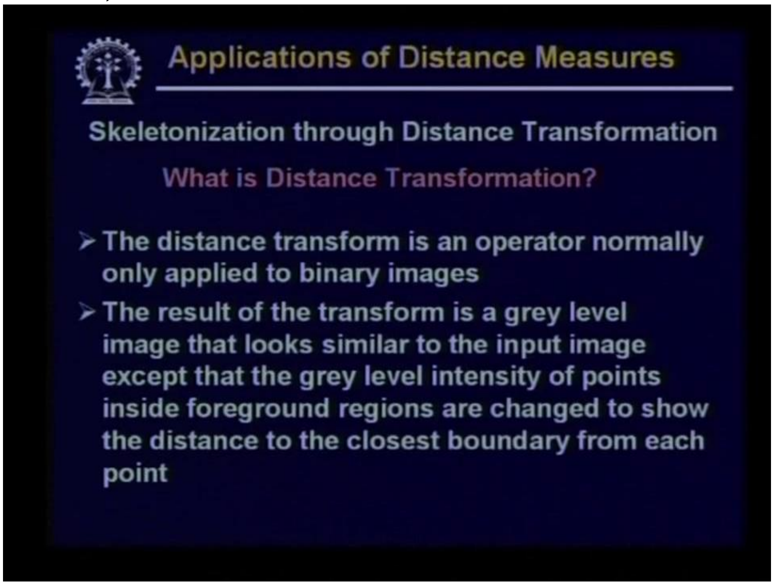

Now to obtain this kind of skeleton you will find that this simple description of movement of the fire line

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does not give you an idea of how to compute the skeleton of a particular shape So for that what we can use is something called distance measure. Now the distance measure is in the same manner we can define that when we are lighting the fire across all the boundary points simultaneously and the fire is moving inside the foreground region slowly we can note at every point that how much time the fire takes to reach that particular point. That is the minimum time the fire takes to reach that particular point and at every such foreground point, if we note this time taken to reach, the time the fire takes to reach that particular point then effectively what we get is a distance transformation of the image. So in this case, you will find that

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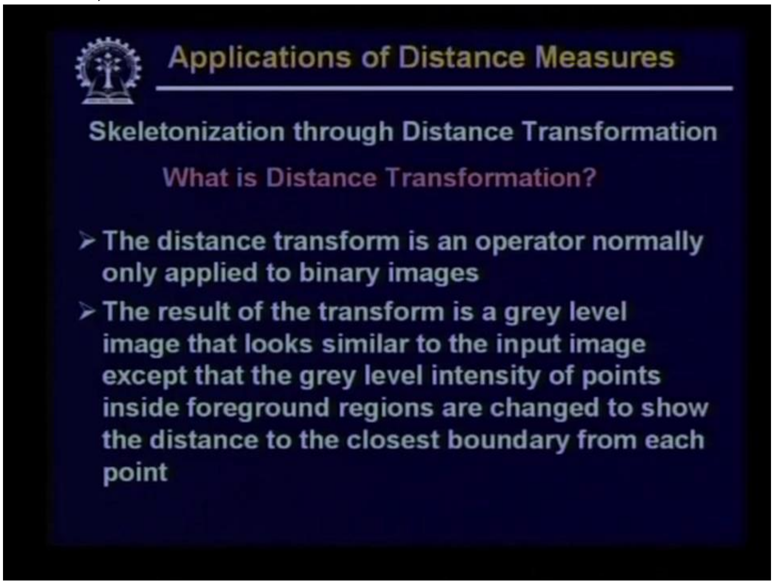

this distance transform is normally used for binary images and because at every point we are noting the time the fire takes to reach that particular point so by applying distance transformation what we get is an image where the shape of the image is similar to the input binary image but in this case the image itself will not be a binary but it will be a gray level image where the gray level intensity of points in the, inside the foreground region are changed to show the distance of that point from the closest boundary point. So let us see

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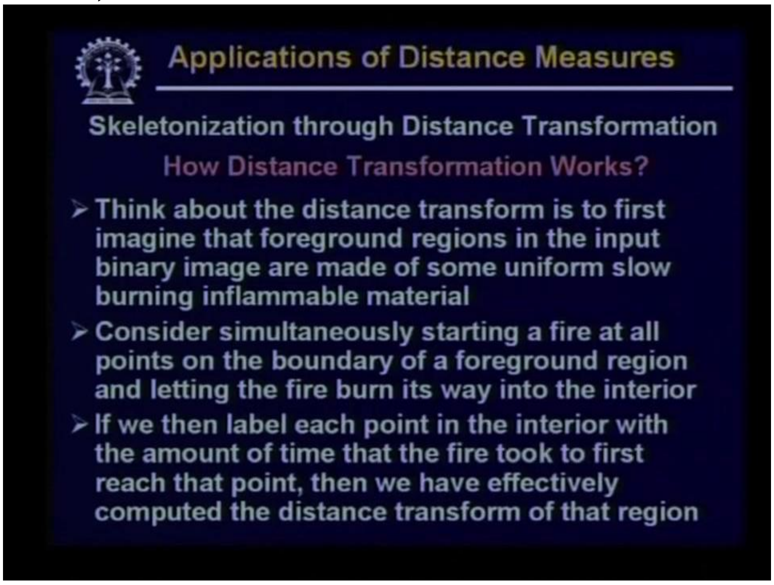

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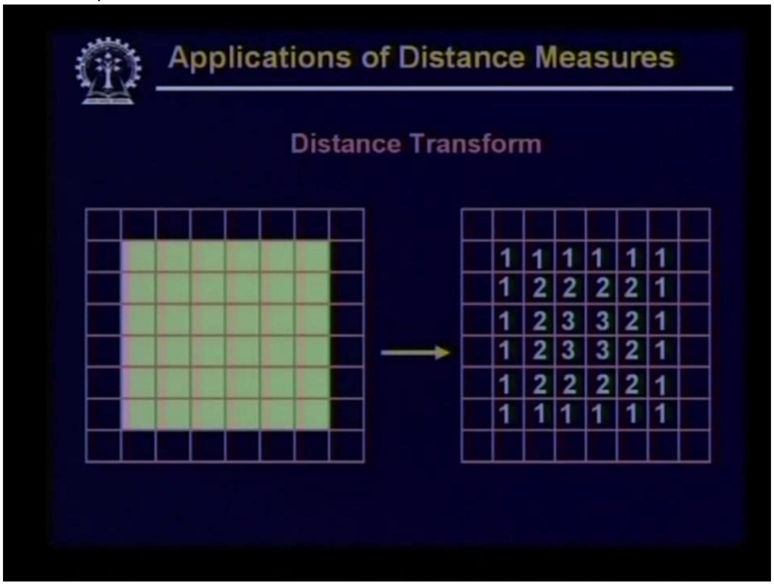

that what this distance transform means.

Here you find that here we have shown a particular binary image where the foreground region is a rectangular region and if I take the distance transform of this, the distance transformed image is shown in the right hand side. Here you find that all the boundary points, they are getting a distance value equal to one. Then the points

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inside the boundary points, they get the distance value equal to 2

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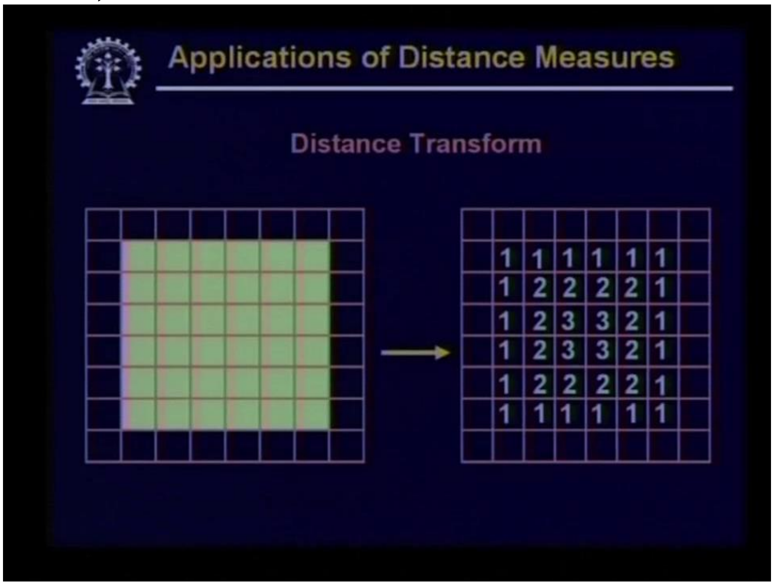

and the points further inside, they get the distance value equal to 3. So you will find that the intensity value that we are assigning to different points within the foreground region, the intensity value increases slowly

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from the boundary to the interior points

So this is nothing but a gray level image which you get after performing the distance transformation. So now you find that if I apply the distance transformation to the

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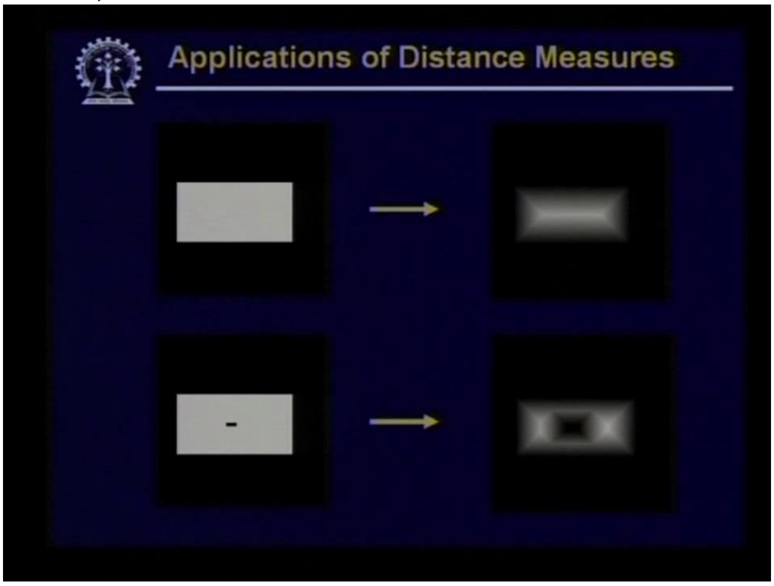

shapes that we have just discussed, the two rectangular shapes, on the left hand side you have the original rectangular shape or the binary image. On the right hand side what is shown is the distance transformed image. And here you find again in this distance transformed image, as you move inside, inside the foreground region that distance value increases gradually. And now if you analyze this distance transformed image you find that there are few points at which there is some discontinuity of the curvature. So from this distance transformed image, if I can identify the points of discontinuity or curvature discontinuity those are actually the points which lies on the skeleton of this particular shape.

So as shown

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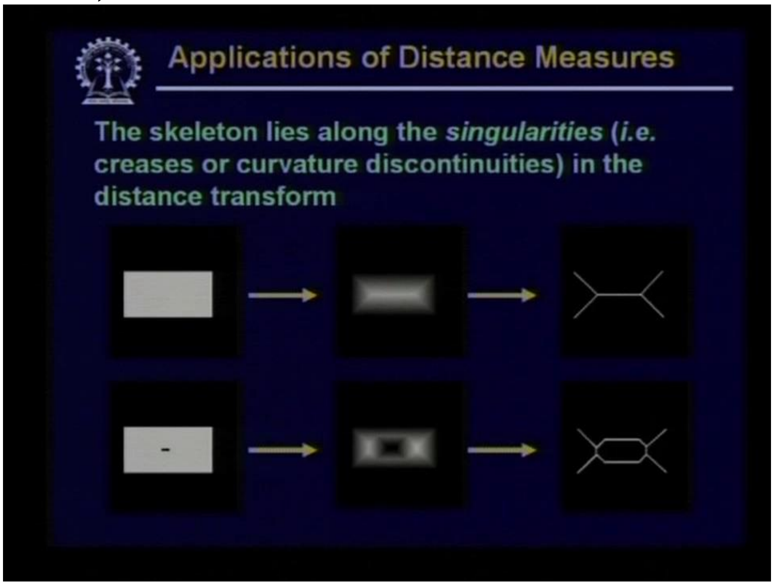

in the next slide you find that on the left hand side we have the original image. The middle column tells you the distance transformed image and the rightmost column tells you the

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skeleton of this particular image And if you now correlate this rightmost column with the middle column, you will find that the skeleton in the rightmost column can now be easily obtained from the middle column which tells you that what is the distance transform of the shape that we have considered.

So this shows some more skeleton

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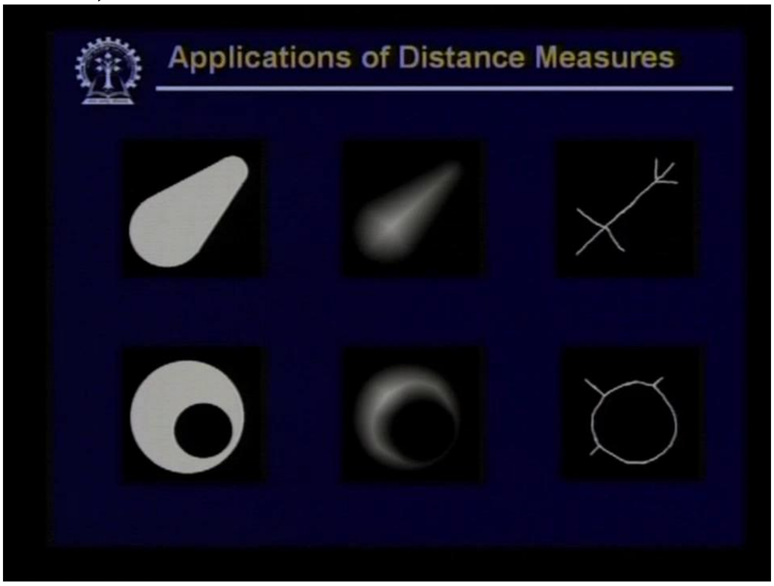

of some more shapes. Again on the left hand side we have the original image. In the middle column we have the distance transformed image and on the rightmost column we have the skeleton of this particular shape. So here again you can find that the relation between the skeleton and the distance transformed image, they are quite prominent. So for all such shapes or wherever

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we go for some shape matching problem or shape discrimination problem; in that case instead of processing on the original shapes, if we compare the shapes using the skeleton then the discrimination will be better than if we compare the original shapes. Now here when I am going for distance transformation of a particular shape, as you have seen that we can have different kinds of distance measures or distance metrics like Euclidean distance metric, we can have city block distance metric or even we can have chess board distance metric, similarly when I take this distance transformation for each of the distance metrics there will be different transformations, different distance transformations. And obviously the different distance transformations will produce different results. But all the results that we will get, similarly from the distance transformed image when we get the skeleton, all the skeletons that we will get using different metrics, they will be slightly different but they will be almost similar. So this can be just another application of the distance metric and you find that here

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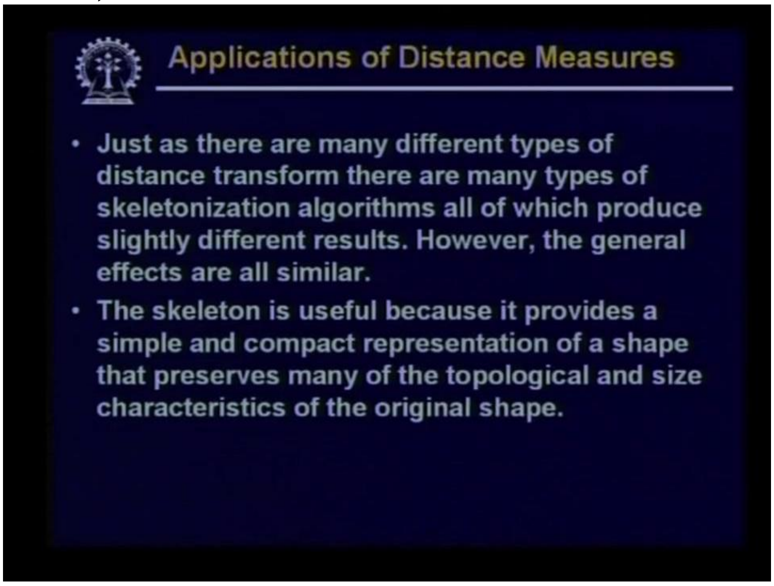

the skeleton is very, very useful because it provides a simple and compact representation of shape that preserves many of the topological and size characteristics of the original shape. Also from this skeleton

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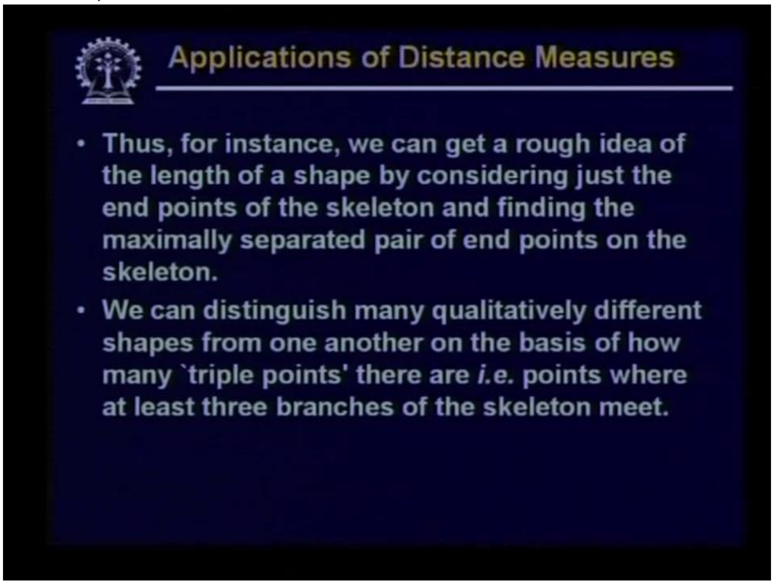

we can get a rough idea of the length of a shape. Because when I get the skeleton

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I get different end points of the skeleton and if I find out the distances between every pair of end points in the skeleton then the maximum of all those pair wise distances will give me an idea of what is the length of that particular shape and as we have already said, that using this distance, using this skeleton we can qualitatively differentiate between different shapes

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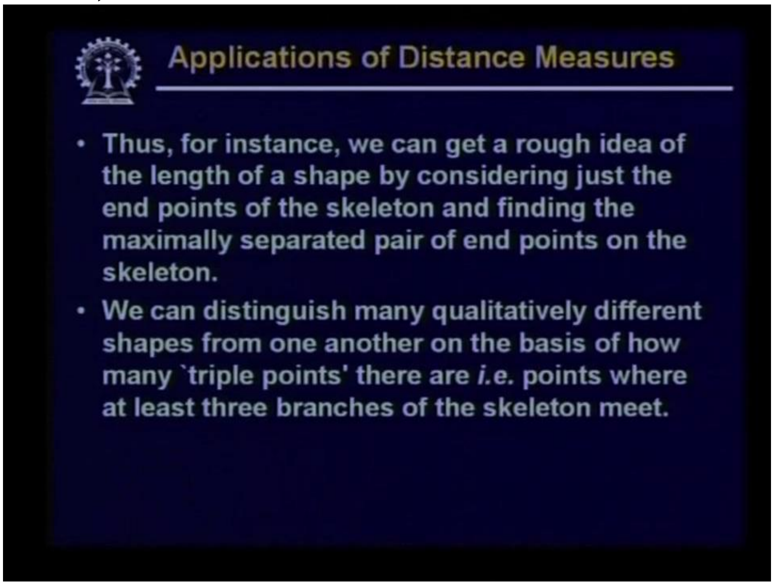

because here we can find out, we can get a description of the shape from the skeleton in terms of the number of line segments that the skeleton has and also in terms of the number of points in the skeleton where more than two line segments meet.

So as we have said that the distance metric or the distance function is not only

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useful for finding out the distance between two points in an image but the distance metric is also used for, useful for other applications though here also we have found out the distance measure between different pair of points and for skeletonization what we have used is first we have taken the distance transformation and in case of distance transformation we have taken the distance of every foreground pixel from its nearest boundary pixel and that is what is gives you a distance transformed image and from the distance transformed image we can find out the skeleton of that particular shape considering the points of curvature discontinuity in the distance transformed image. And later on also we will see that this distance metric is useful in many other cases.

Now after our discussion on these distance metrics

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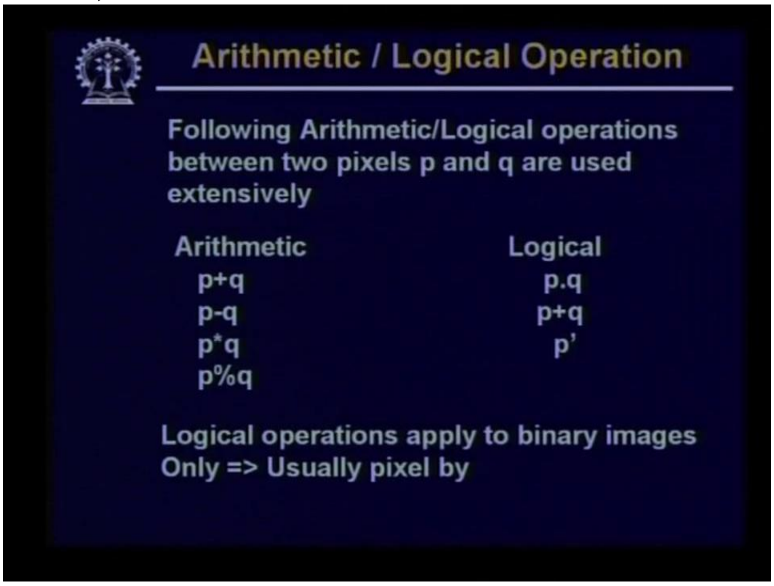

let us see what are simple operations that we can perform on the images. So as you have seen that in case of numerical system whether it is decimal number system or binary number system, we can have arithmetic as well as logical operations. Similarly for images also

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we can have arithmetic and logical operations. Now coming to images, I can add two images, pixel by pixel, that is pixel from an image can be added to the corresponding pixel of a second image. I can subtract two images pixel by pixel, that is pixel of an image can be subtracted from the corresponding pixel of the, of another image. I can go for pixel by pixel multiplication. I can also go for pixel by pixel division. So these are different arithmetic operations that I can perform on two images and these operations are applicable both in case of gray level image as well as in case of binary image. Similarly

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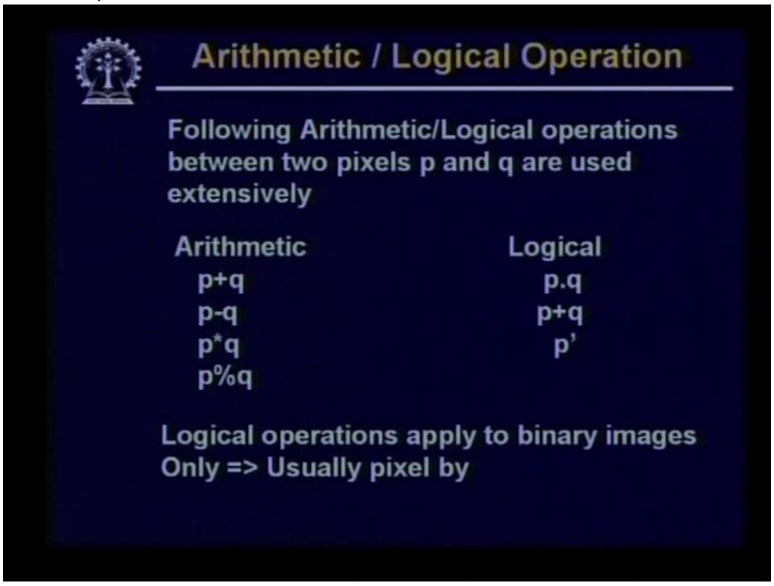

in case of binary images we can have logical operations, the logical operation in terms of anding pixel by pixel, oring pixel by pixel, and similarly inverting pixel by pixel. So these are the different arithmetic, logical operations that we can do on gray level image and similarly logical operations on binary image

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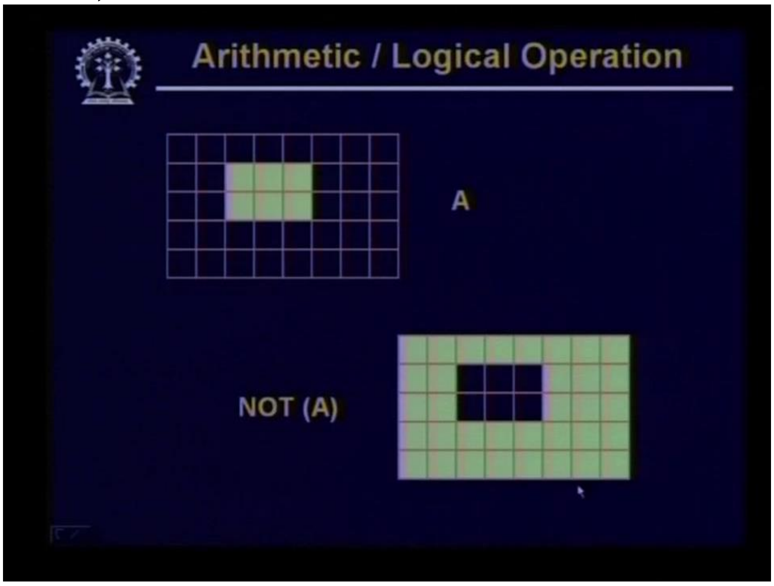

So here is an example that if I have a binary image A where again all the pixels with value equal to 1 are shown as shown in green color and the pixels with value equal to 0 are shown in black color, then I can just invert this particular binary image, that is I can make a not operation or invert operation. So not of A is another binary image

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where all the pixels in the original image which was black now becomes white or 1 and pixels which were white or 1 in the original pixel, those pixels become equal to 0. Similarly I can perform other operations like

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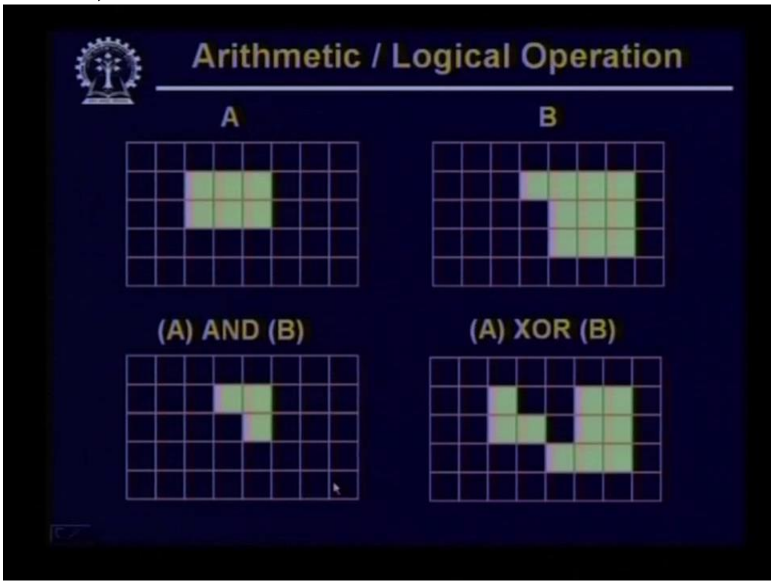

given two images A and B, I can find out A and B, the logical anding operation which is shown in the left image. Similarly I can find out the xor operation and after xor the image that I get is shown in the right image.

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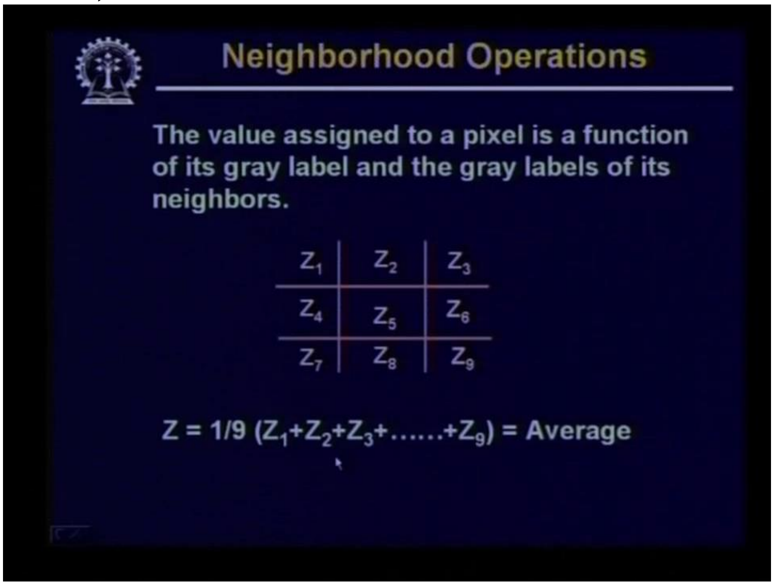

So these are the different pixel operations or pixel by pixel operations that I can perform.

In some other applications

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we can also perform some neighborhood operations, that is, the intensity value at a particular pixel may be replaced by a function of the intensity values of the pixels which are neighbors of that particular pixel p. Say for example, in this particular case, if this 3 by 3 matrix, this represents

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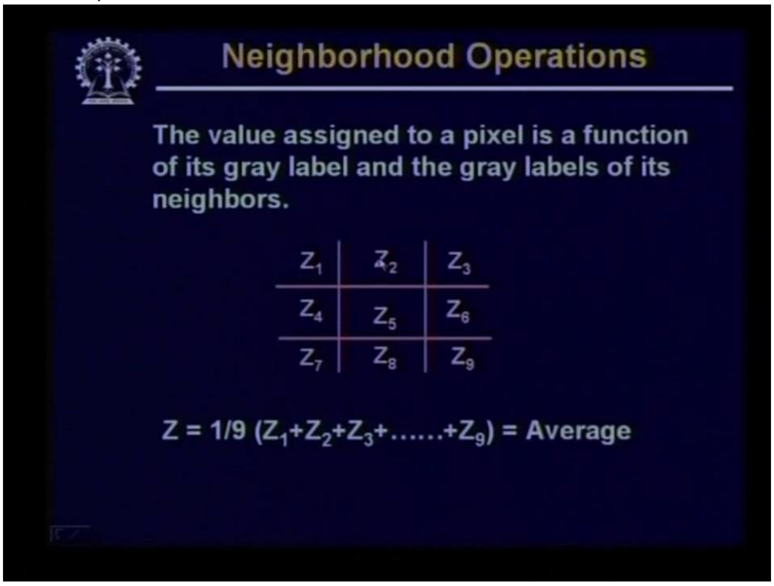

the part of an image, so which has got 9 pixel elements z 1 to z 9 and I want to replace every pixel value by the average of its neighborhood considering the pixel itself. So you find that at location z 5 if I want to take the average, the average is simply given by z 1 plus z 2 plus z 3 plus z 4 up to plus z 9 that divided by 9. So this is a simple average operation at individual pixels that I can perform which is nothing but a neighborhood operation because at every pixel level we are replacing the

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intensity by a function of the intensities of its neighborhood pixels And this averaging operation, we will see later, that this is the simplest form of low pass filtering to remove noise from a noisy image.

Now this kind of

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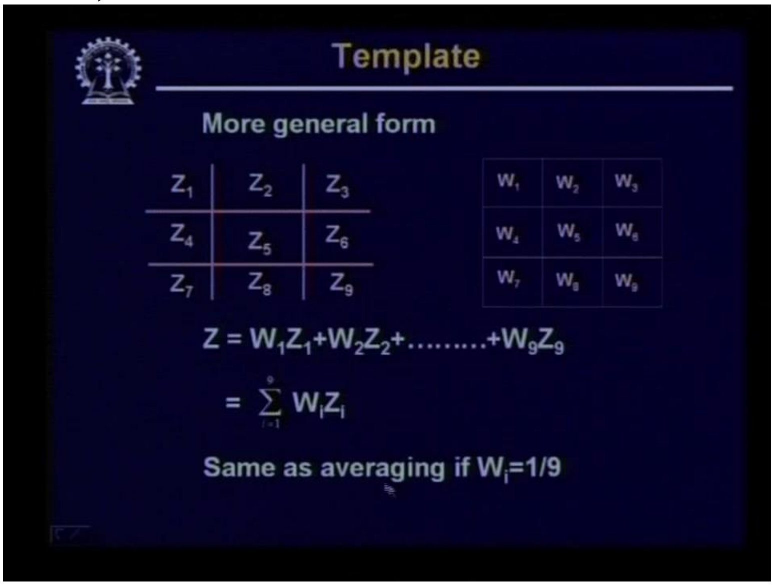

neighborhood operation can be generalized with the help of templates. So here what we do is, we define a 3 by 3 template which is shown on this right hand figure where the template has contains 9 elements w 1 to w 9. And if I want to perform the neighborhood operation, what we do is you put this template, this particular template on the original image in such a way that the pixel at which I want to replace the value, the center of the template just coincides with that pixel. And then at the particular location, we replace the value with the weighted sum of the values taken from the image and the corresponding point from the template. So in this case the value will be which will be replaced is given by z equal to w i z i summation of this i varying from 1 to 9 and here you find if I simply put w i equal to 1 by 9, that is all the points in the template have the same

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value which is equal to 1 by 9, then the resultant image that I will get is nothing but the averaged image which we have cons we have done just in the previous slide. So this neighborhood operation using the template is a very, very general operation. It is useful not only for averaging purpose. It is useful for many other neighborhood operations and we will see later that

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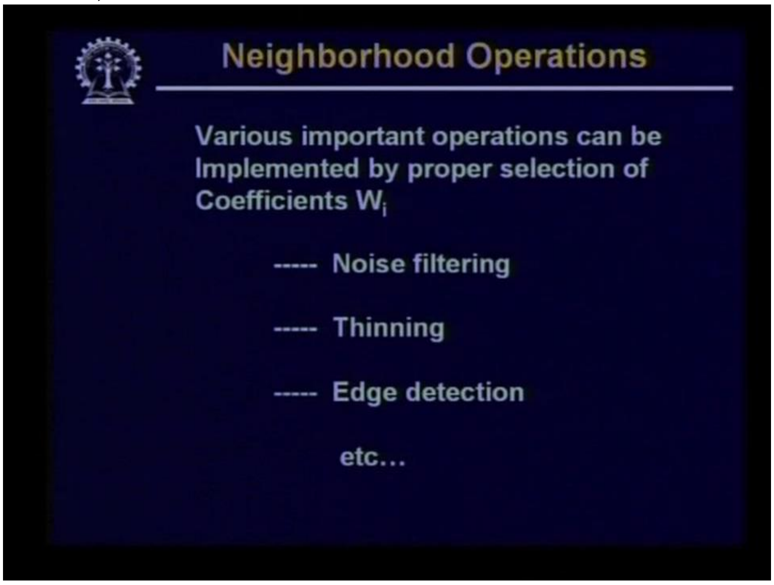

this can be used for noise filtering, it can be used for thinning of binary images, this same template operation can also be used for edge detection operation in different images.

So with this we complete our lecture today.

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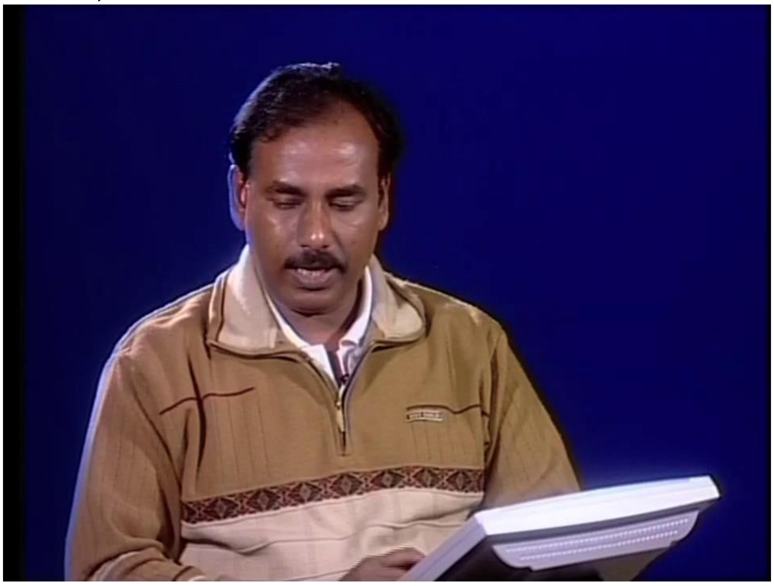

Thank you.