

Digital Image Processing
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Module 02 Lecture Number 08
Relationship of Adjacency and Connected Components Labeling
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Hello, welcome to the video lecture series on Digital Image Processing.

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Adjacency

Two pixels p and q are adjacent if they are connected

- 4-adjacency
- 8-adjacency
- m -adjacency

-- depending on type of connectivity used.

Two image subsets S_i and S_j are adjacent if $\exists p \in S_i$ and $\exists q \in S_j$ such that p and q are adjacent

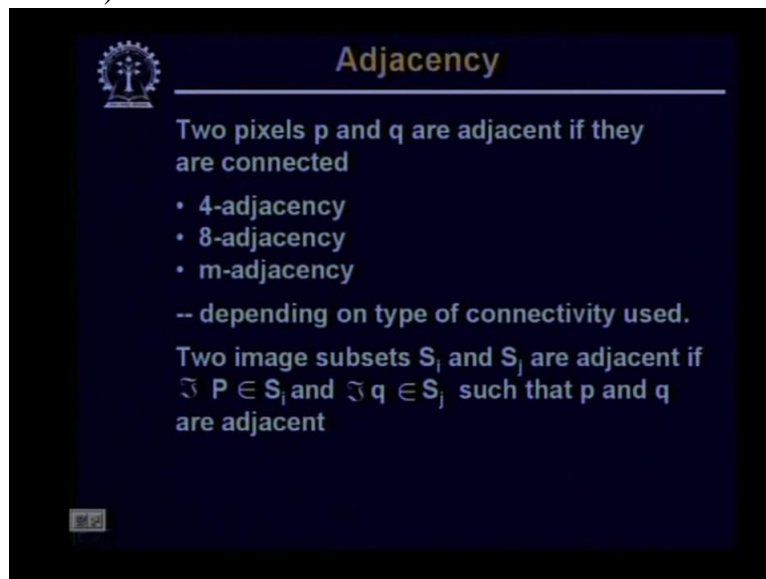
Now from the connectivity, we come to the relationship of adjacency.

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So we say that 2 pixels p and q are adjacent if

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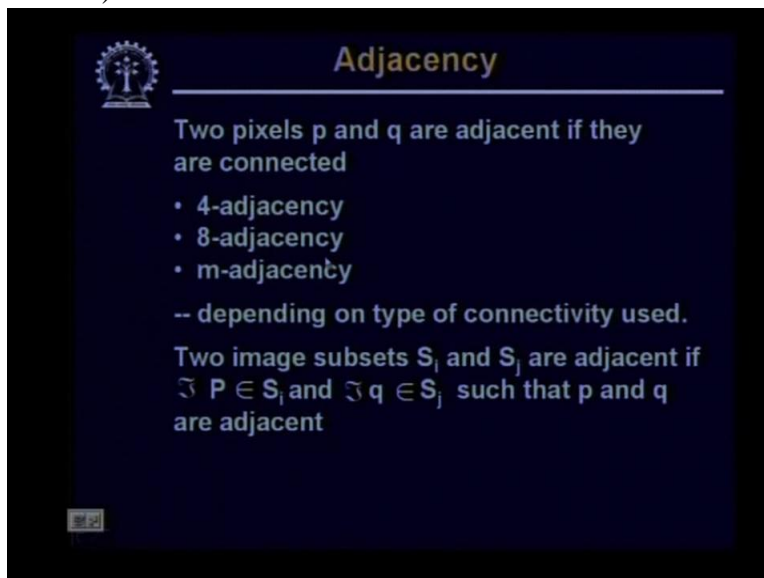
they are connected So since for connectivity we have introduced 3 different types of connectivity that is 4-connectivity,

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8-connectivity and m-connectivity So for all these 3 different types of connectivity, we will have 3 different types of adjacency because our adjacency definition is two points are adjacent if they are connected. So just by extension of this definition you find that we have got

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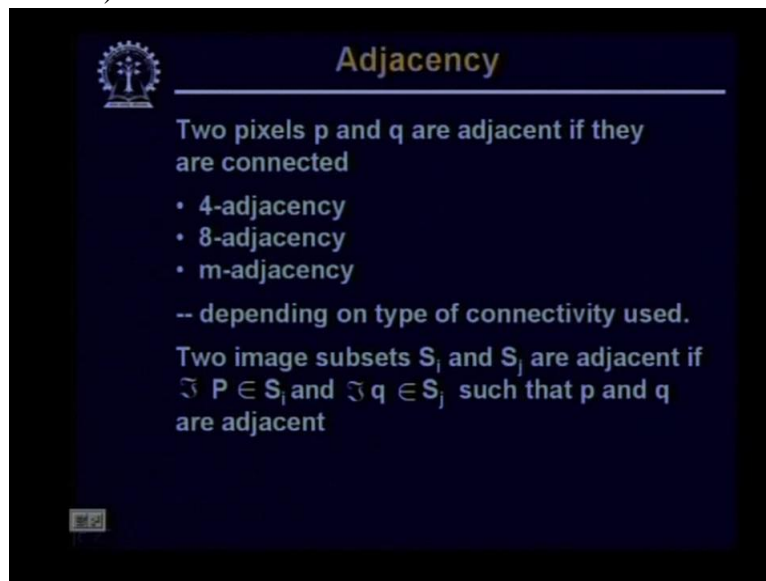
3 different types of adjacency, the first one is 4-adjacency, the second one is 8-adjacency, and the third one is m-adjacency. And you define this type of adjacency depending upon the type of connectivity that is used. Now this is about the adjacency of two different

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points or two or more different points We can extend this concept of adjacency to image regions, that is we can also say, that 2 image regions may be adjacent or they may not be adjacent. So what is the condition for adjacency of 2 image regions? In this case you find that we define that adjacency for 2 image regions

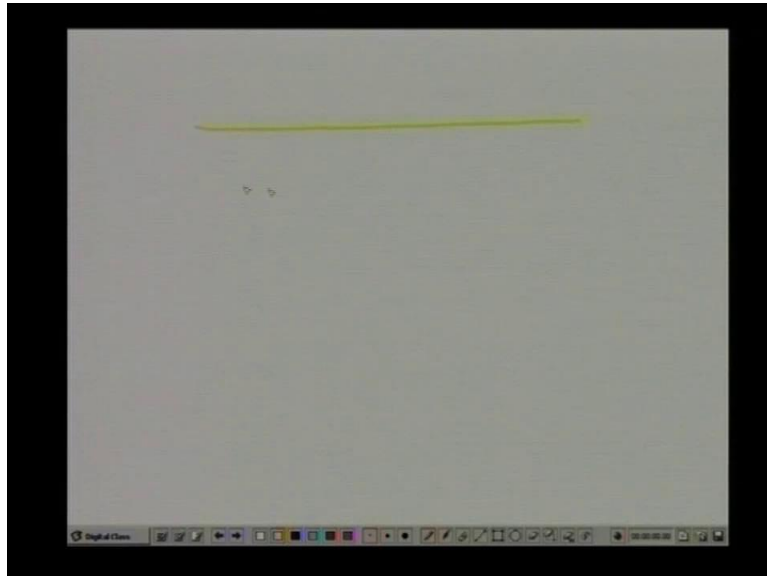
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like this, that if there are 2 image subsets S_i and S_j , we say that S_i and S_j will be adjacent if there exists a point p in image region S_i and point q in image region S_j such that p and q are adjacent ok.

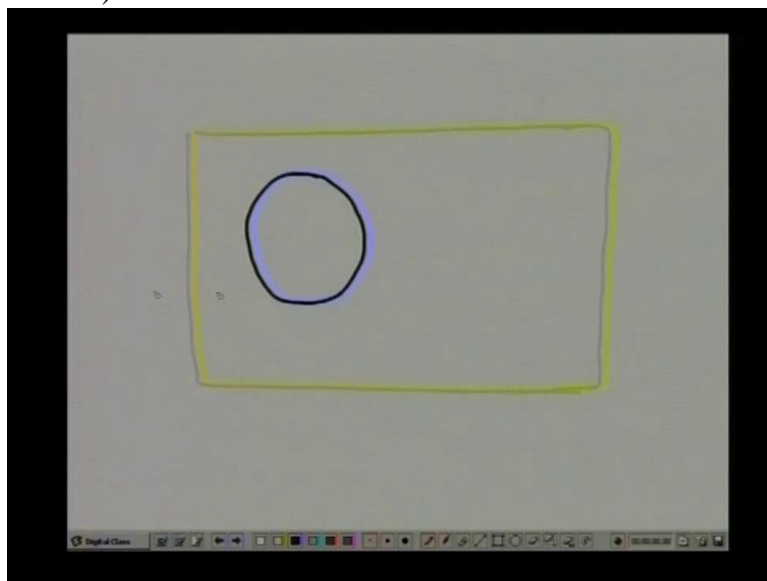
So just let us try to elaborate this. So I have this overall image region. This is the

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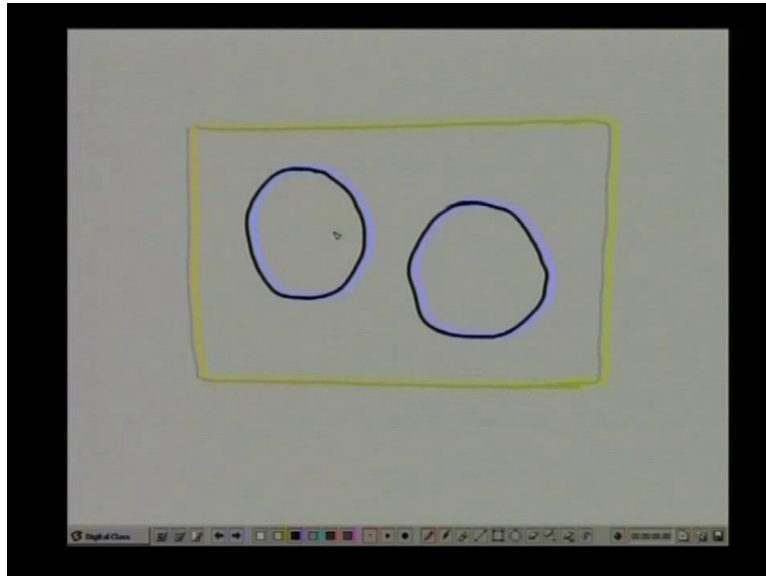
This is the whole image and within this image I have two image regions, one is here

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the other one is here.

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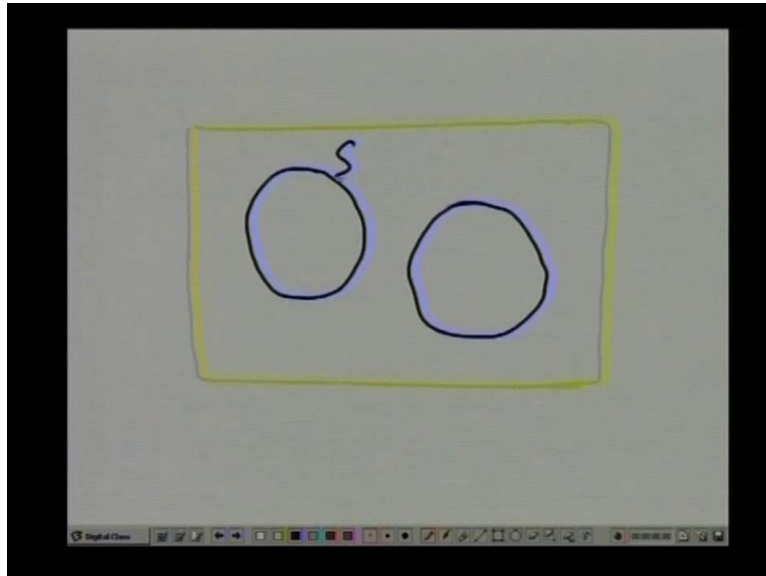
So the adjacency relation between two image regions is defined like this

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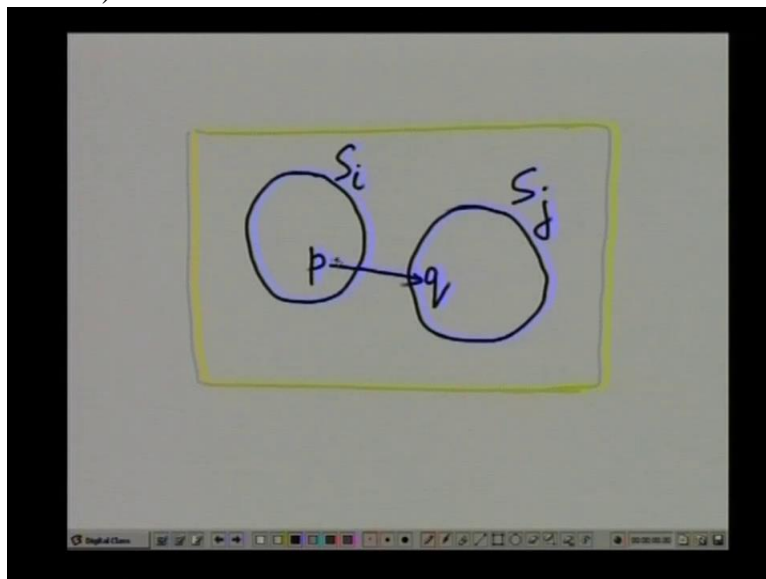
that I have to have some point in one region which is adjacent to a point in the other region
So if I call say this is image region S_i

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and this is image region S_j then I must have some point p in S_i and some other point q in image S_j so that this p and q they are adjacent

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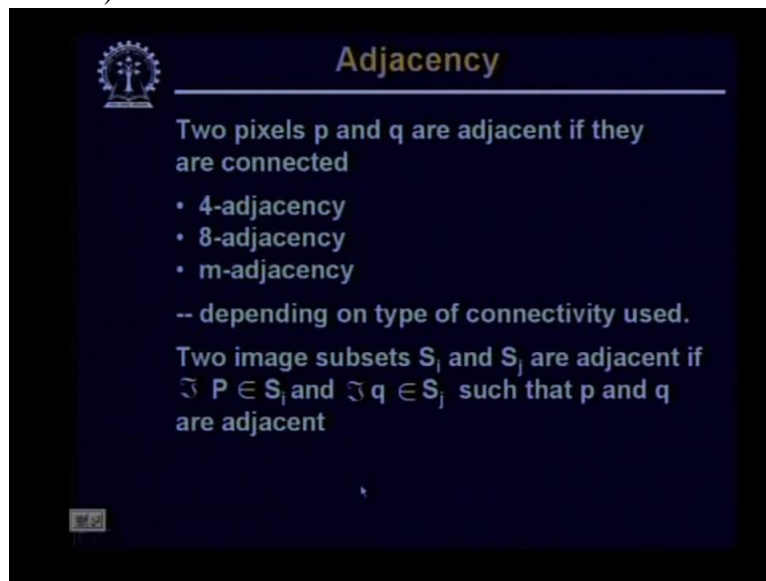
so if p and q are adjacent then I say that this image region S_i is adjacent to image region S_j that means S_i and S_j they must

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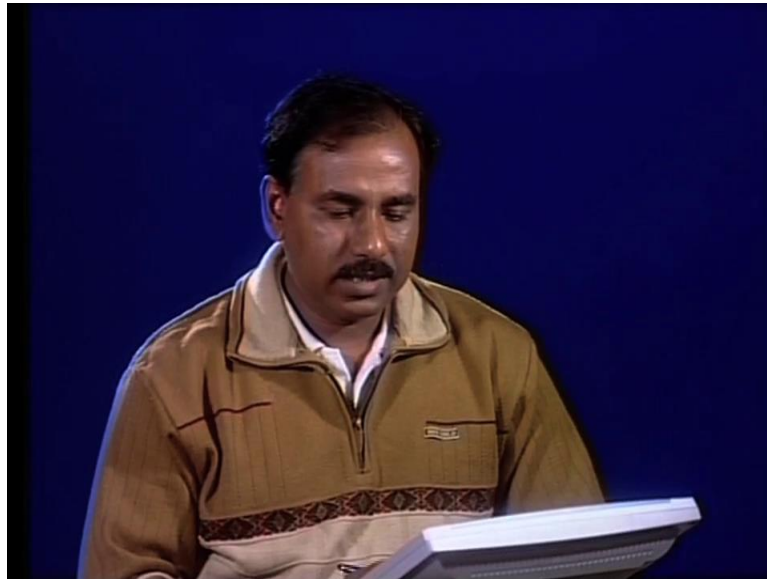
appear one after the other, one adjacent to the other

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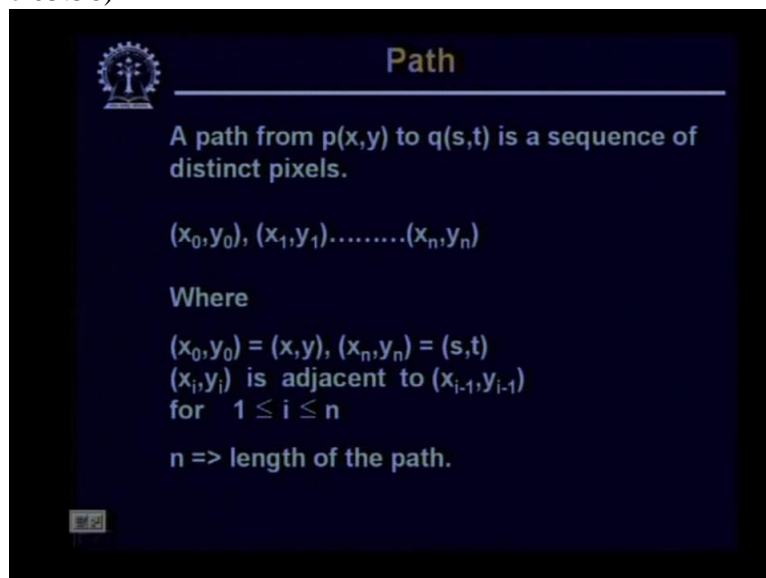
So this is the adjacency relation. After the adjacency relation

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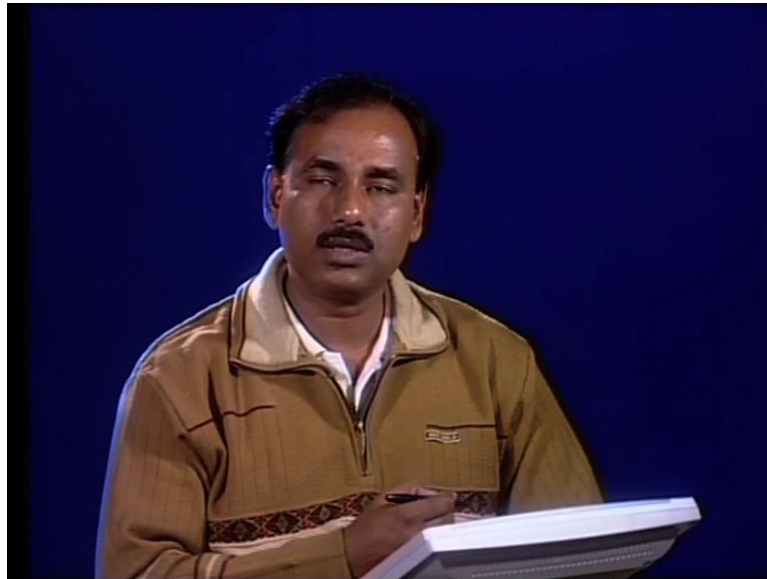
we can also define a path between 2 points p and q . So the definition of a path is like this. That we say

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a path exists from a point p having a coordinate x, y to a point q having coordinate s, t if there exist a sequence of distinct pixels say $x_0, y_0, x_1, y_1, x_2, y_2$ and so on up to x_n, y_n where x_0, y_0 is equal to x, y that is the same as point p and x_n, y_n is equal to s, t which is the same as point q and all the other intermediate points like x_1, y_1, x_2, y_2 they must be adjacent the subsequent points must be adjacent in the sense that x_i, y_i has to be adjacent to x_{i-1}, y_{i-1} for all values of i lying between 1 and n . So if I have such a sequence of points between

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p and q such that all the points which are traversed in between p and q, all those subsequent points, they are adjacent then we say that a path exists between from point p to point q and we also define the length of the path to be the n

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Path

A path from $p(x,y)$ to $q(s,t)$ is a sequence of distinct pixels.

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where

$$(x_0, y_0) = (x, y), (x_n, y_n) = (s, t)$$
$$(x_i, y_i) \text{ is adjacent to } (x_{i-1}, y_{i-1})$$

for $1 \leq i \leq n$

$n \Rightarrow$ length of the path.

that is considering both point p and q if I have n plus 1

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number of points including the end points p and q and all the points in between the length of the path is said to be n .

So this is what we define as path. Now very important concept that can arise

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Path

A path from $p(x,y)$ to $q(s,t)$ is a sequence of distinct pixels.

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where

$$(x_0, y_0) = (x, y), (x_n, y_n) = (s, t)$$
$$(x_i, y_i) \text{ is adjacent to } (x_{i-1}, y_{i-1})$$
$$\text{for } 1 \leq i \leq n$$

$n \Rightarrow$ length of the path.

from here that is how to define a

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connected region We have said two pixels are connected we said two pixels are connected if they are adjacent in some sense that is they are neighbors and their intensity values are also similar. We have also defined two regions to be adjacent if there is a point in one region which is adjacent to some other point in another region. And we have also defined a path between a point p and q if there are a set of points in between which are adjacent to each other. Now this concept can be extended to define what is called a connected component.

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Connected component

Let
 $S \subseteq I$ and $p, q \in S$

Then p is connected to q in S if there is a path
From p to q consisting entirely of pixels in S

For any $p \in S$, the set of pixels in S that are
Connected to p is call a connected component
of S .

\Rightarrow Any two pixels of a connected component
are connected to each other
Distinct connected components are disjoint

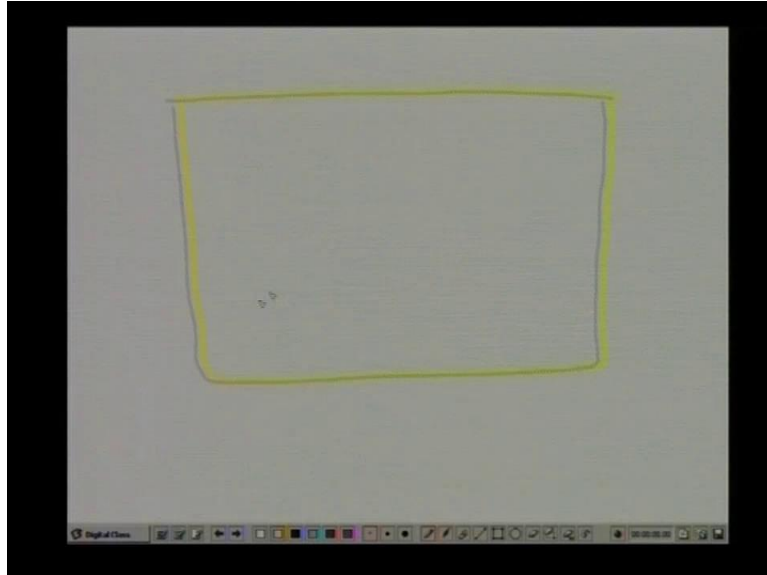
So let us see what is a connected component?

We take a subset s of an image i and we take two points p and q which belong to this subset s of image i . Then we say that p is connected to q in S so you just mind this term that p is

connected to q in the subset s if there exists a path from p to q consisting entirely of pixels in S . For any such p belonging to s , the set of pixels in s that are connected to p is called a connected component of S . So the concept is like this.

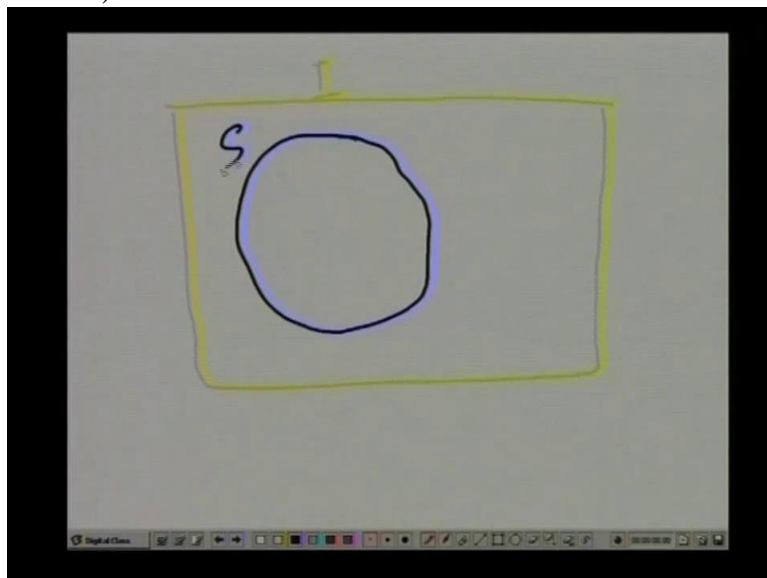
Say this is my entire image i

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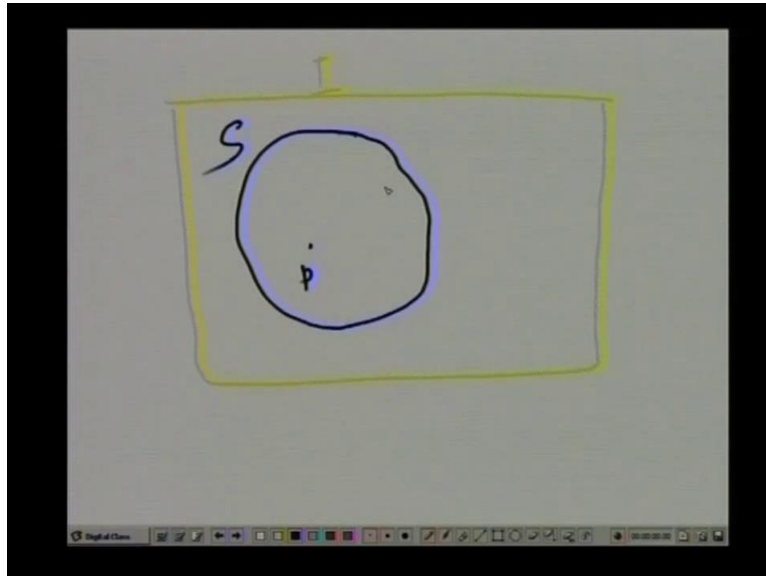
and within this image I take a sub image say s

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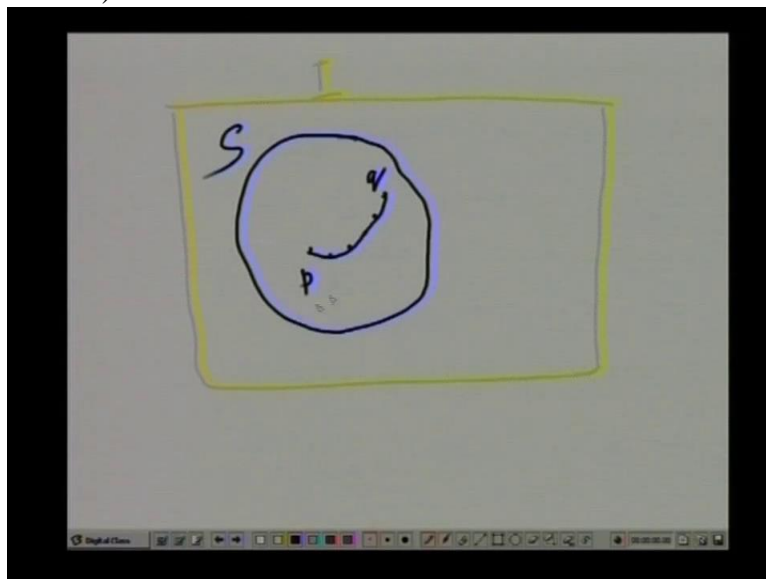
Ok and here say we have a point p and we say that take any other point q .

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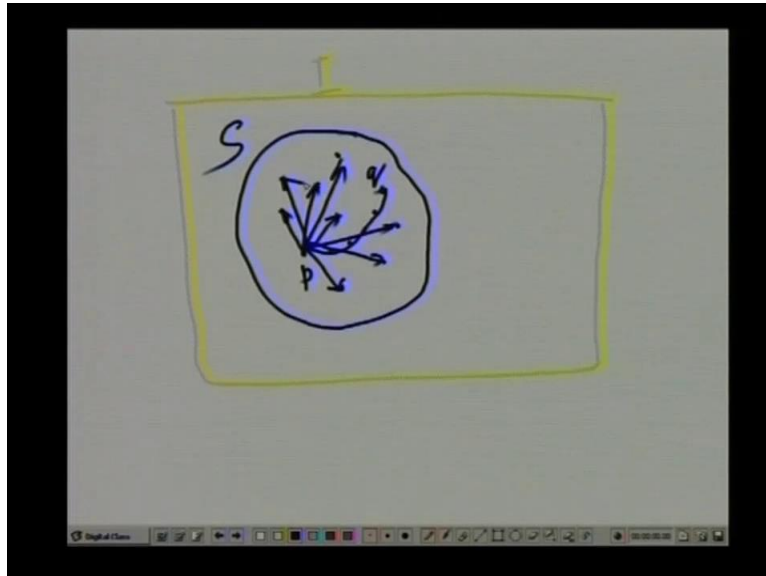
If there exists a path from p to q consisting of all other intermediate points, these intermediate points must also belong to the same subset s so there exists a path between p and q

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consisting of intermediate points belonging to the same subset s then we say that the point p and q , they are connected and if there are a number of such points to which a path exists from p then set of all these points are said to be connected to p and they form a connected component of s .

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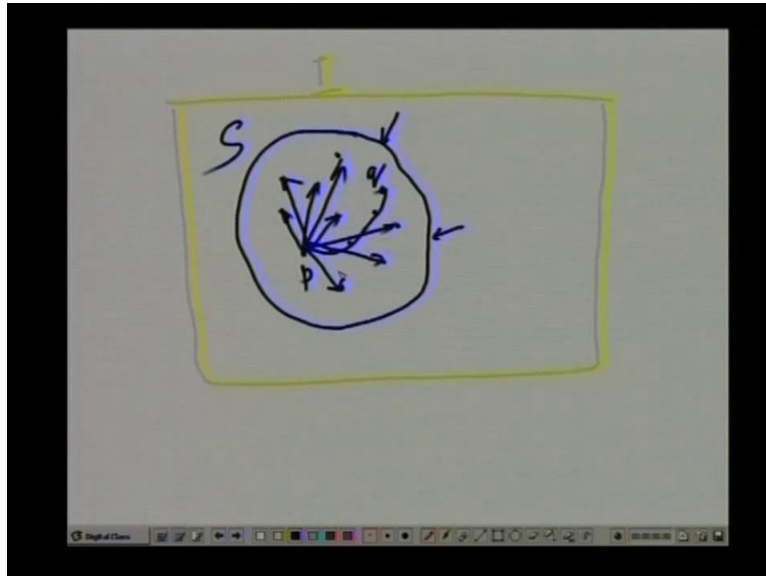
So find just by using this concept of connected component I can identify

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a region in an image So going back to our earlier example where we have said that simply by identifying that a pixel belongs to an object does not give me the entire solution because I have to group the pixels which belong to the same object and give them some identification, that these are the group of pixels which belong to the same object and then I can go for extracting the region property which will tell me what is the property of that particular object. And now that belongingness to a particular object

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can be found out by using this concept of connected component

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 **Connected component**

Let
 $S \subseteq I$ and $p, q \in S$

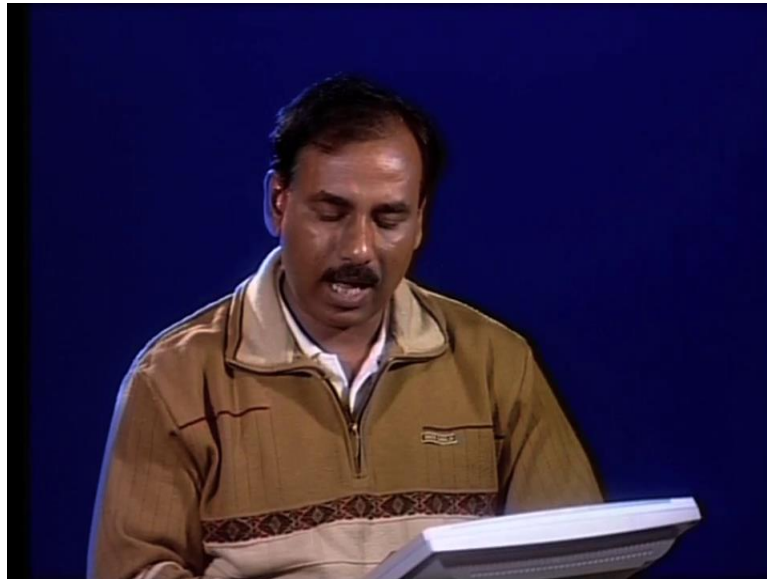
Then p is connected to q in S if there is a path
From p to q consisting entirely of pixels in S

For any $p \in S$, the set of pixels in S that are
Connected to p is call a connected component
of S .

\Rightarrow Any two pixels of a connected component
are connected to each other
Distinct connected components are disjoint

So any two pixels of a connected component, we say they are connected to each other and distinct connected components are disjoint. Obviously the points belonging to one particular region and points belonging to another

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particular region, they are not connected but the points belonging to a particular region, they are connected with each other.

So for this identification or group identification what we have to do is when we identify a set of pixels which are connected, then for all those set, all those points belonging to a particular group we have to assign a particular identification number.

Say for example in this particular figure you find that there are

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Connected component labeling

Ability to assign different labels to various disjoint connected components of an Image.

The slide shows a 10x10 grid of pixels. On the left, a set of white pixels forms a connected shape. An arrow points to the right, where the same shape is shown with different colors (yellow, orange, red) assigned to different connected components. Below the grid, the text reads: "Connected component labeling is a fundamental step in automated image analysis".

- Shape
- Area
- Boundary
- Shape/Area/Boundary based features

two groups of pixels So in the first figure we have a set of pixels here; we have another set of pixels here. So find that these set of pixels are connected, these set of pixels, they are also connected; so this forms one connected component. This set of pixels form another

connected component. So this connected component labeling problem is that I have to assign a group identification number to each of these pixels. That means the first set of pixels which are connected to each other; I had to give them one group identification number. In this particular case, all these pixels are identified to be yellow and I have to give another group identification to this second set of pixels. So in this particular case, all these pixels are given the color red.

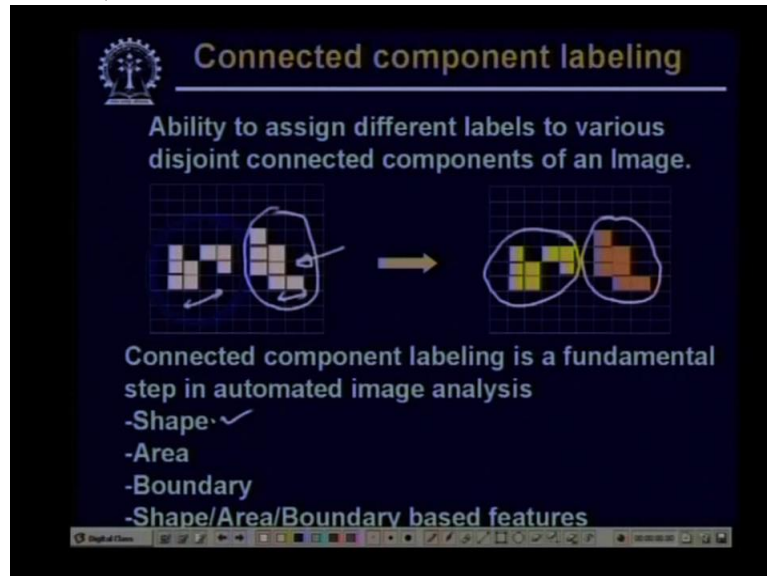
Now once we identify these group of

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pixels to belong to the particular belong to a particular region then we can go for finding of some region properties and those region properties may be shape of that particular

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Connected component labeling

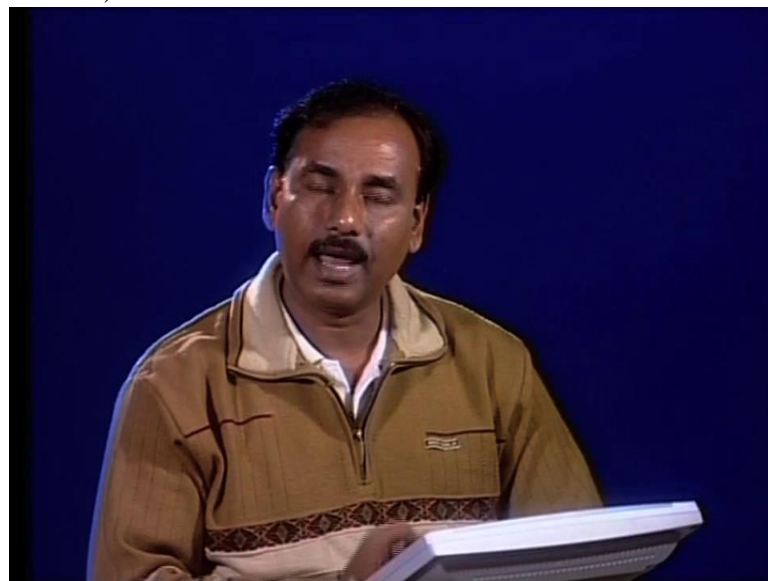
Ability to assign different labels to various disjoint connected components of an Image.

Connected component labeling is a fundamental step in automated image analysis

- Shape ✓
- Area
- Boundary
- Shape/Area/Boundary based features

region, it may be area of that particular region, it may be the boundary, the length of the boundary of this particular region and many other shape, area or boundary based features can be extracted

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once we identify

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Connected component labeling

Ability to assign different labels to various disjoint connected components of an Image.

Connected component labeling is a fundamental step in automated image analysis

- Shape ✓
- Area ✓
- Boundary ✓
- Shape/Area/Boundary based features

these different region properties

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Connected component labeling

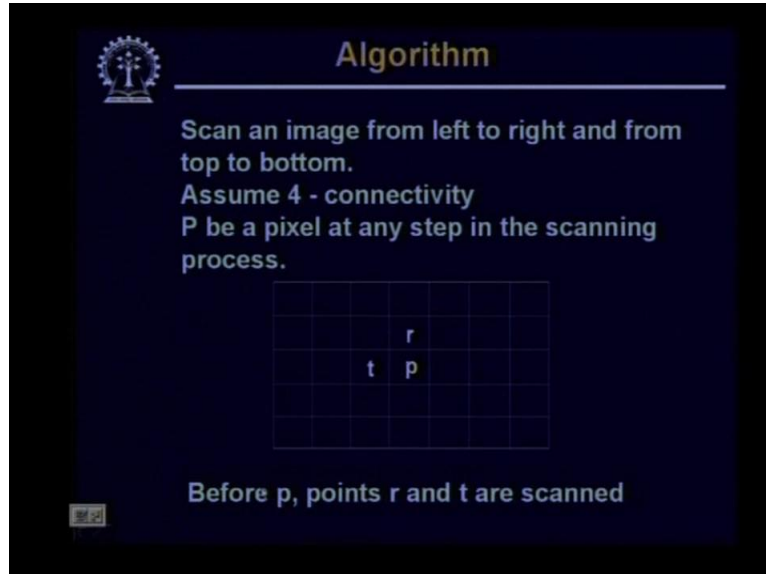
Ability to assign different labels to various disjoint connected components of an Image.

Connected component labeling is a fundamental step in automated image analysis

- Shape
- Area
- Boundary
- Shape/Area/Boundary based features

So

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The slide is titled "Algorithm" and features a logo in the top left corner. The text on the slide reads: "Scan an image from left to right and from top to bottom. Assume 4 - connectivity P be a pixel at any step in the scanning process." Below the text is a 5x5 grid. In the center of the grid, the letter 'r' is positioned above the letter 'p', and the letter 't' is positioned to the left of 'p'. Below the grid, the text states: "Before p, points r and t are scanned".

now let us see that what will be the algorithm that has to be followed

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for to find out what is the group identification for a particular region or the pixels belonging to a particular region So idea would be, the algorithm will be like this. That you scan the image from left to right and from top to bottom. So as shown in this particular figure, if I scan the image like this from left to right and from top to bottom, so this will be our scanning order and for the time being let us assume that we are

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Algorithm

Scan an image from left to right and from top to bottom.
Assume 4 - connectivity
P be a pixel at any step in the scanning process.

Before p, points r and t are scanned

using 4-connectivity So by using this 4-connectivity whenever we reach a particular point, say p, you find that before reaching this particular point p and following this order of scanning the other 4 neighbors of point p which will be scanned is the point which is above this that is point r in this particular figure and the point which is just left to p that is point t before this figure so before this particular point p is scanned the point belonging to 4 neighbors of p which will be scanned are point r and point t. So by using this particular fact that which are the points which will be scanned before you scan point p

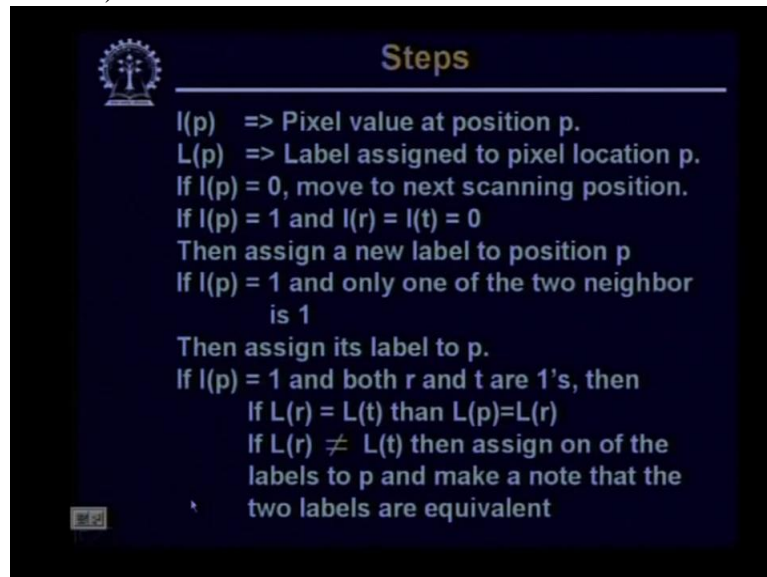
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we can develop a component labeling algorithm. The purpose of component labeling algorithm is to assign an identification number to each of the pixels, connected pixels which will identify it to belong to a particular region.

So

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the steps involved will be like this. So when I consider a point p I assume that $i p$ is the pixel value at that particular location. And I also say that $l p$ will be the label assigned to the pixel at location p . Then the algorithm steps will be like this. That if $i p$ equal to zero, because as we have seen in the previous case, that after segmentation we say

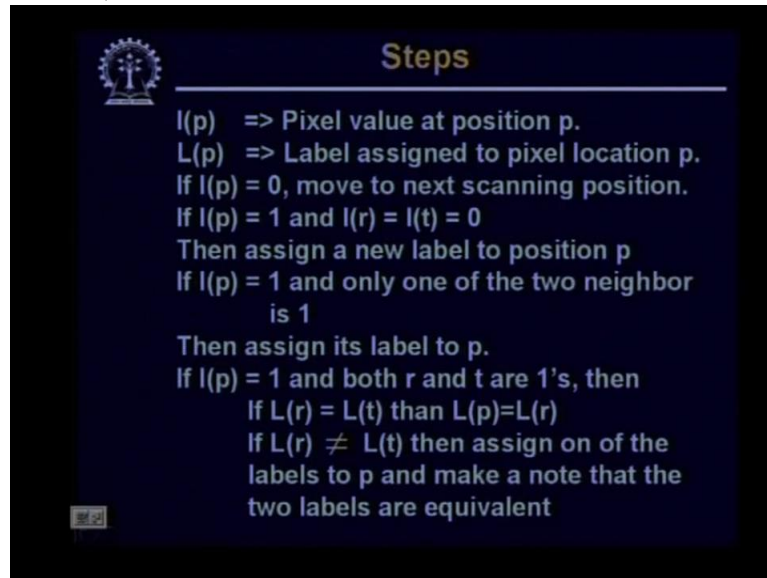
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that whenever the intensity value at a particular location is above certain threshold we assign value 1 to that particular location, where as if the intensity value is less than the threshold we assign the value 0 to that particular location. So by using this convention, when I want to find out the region property based on the shape the pixels which are of importance or the points which are of importance are the points having a value equal to 1 and we assume the points

having a value equal to 0, they belong to the background so they are not of importance. So just by this, if a point

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has a value equal to 0, that is $i p$ equal to 0, we don't assign any label to this, so we just move to the next scanning position either to the left, either to the right or to bottom.

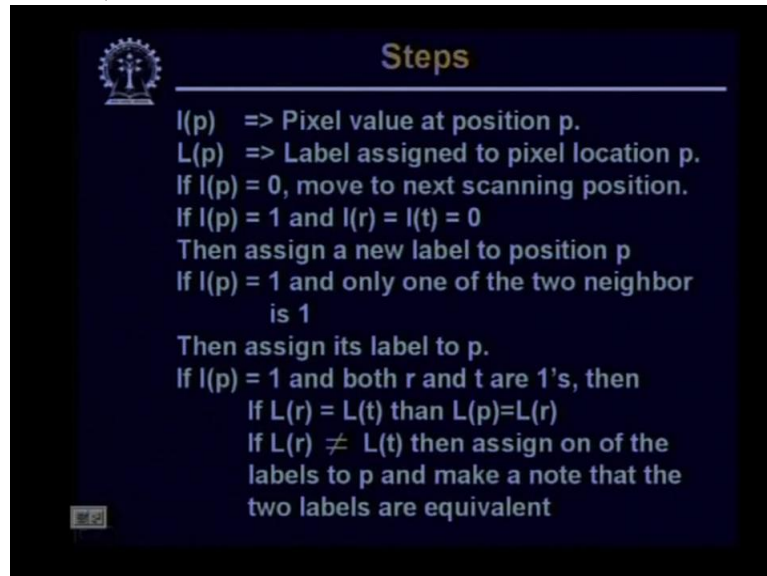
But if $i p$ equal to 1, that is the value at a particular point equal to 1 and we find while scanning this we come across 2 points r and t . So when I find a point p for which value equal to 1 and the values at both the points r and t equal to 0,

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then we assign a new label

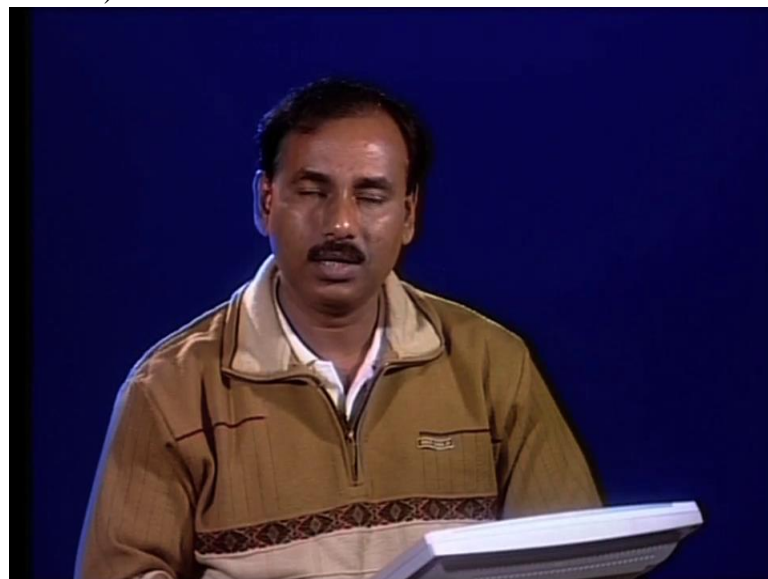
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to position p

If i p equal to 1, and only one of the two neighbors that is r and t is 1 and because r and t has already been scanned so both r and t , if those values were equal to 1

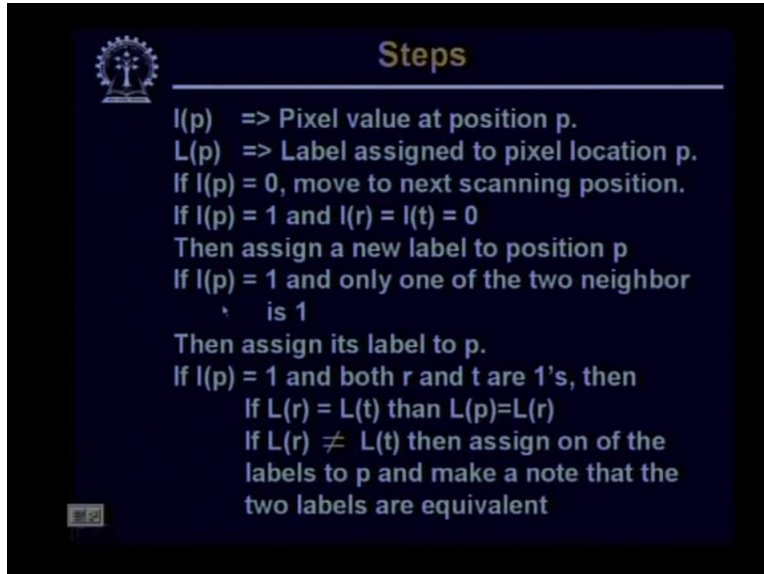
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should have got a particular label l

So in this particular case, if i p equal to 1 and one of r and t is equal to 1, in that case, to p we assign the same label which was assigned to r or t whichever was 1 before, Ok .

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Steps

- I(p) => Pixel value at position p.
- L(p) => Label assigned to pixel location p.
- If I(p) = 0, move to next scanning position.
- If I(p) = 1 and I(r) = I(t) = 0
- Then assign a new label to position p
- If I(p) = 1 and only one of the two neighbor is 1
- Then assign its label to p.
- If I(p) = 1 and both r and t are 1's, then
 - If L(r) = L(t) then L(p)=L(r)
 - If L(r) ≠ L(t) then assign one of the labels to p and make a note that the two labels are equivalent

So if $I(p)$ equal to 1 and only one of the two neighbor is 1, then assign the label of that neighbor to point p.

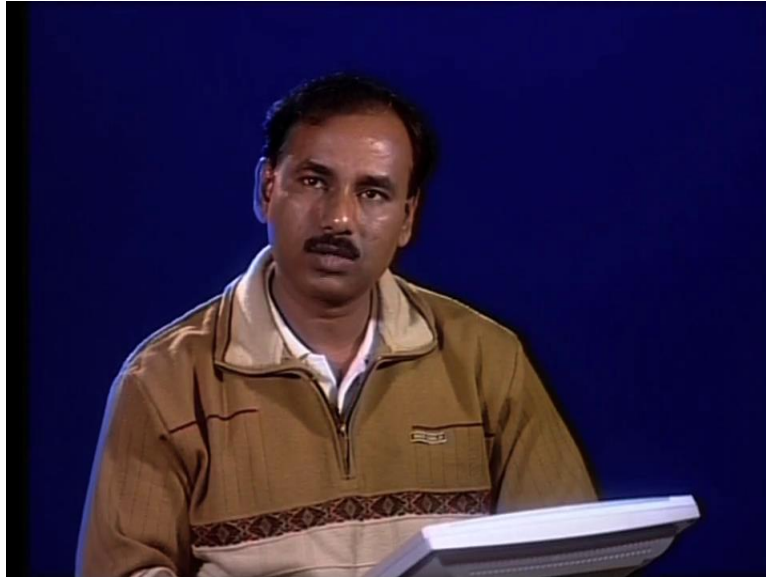
But the problem comes, if $I(p)$ equal to 1 and both r and t are equal to 1. So the problem becomes simpler, our assignment is simple if the label which was assigned to t and the label which was assigned to r, that were same. So if $I(r)$ equal to $I(t)$, then you assign the same label to point p.

So you see in this particular case that if $I(r)$ equal to $I(t)$ then $I(p)$ gets $I(r)$ which is obviously same as $I(t)$.

But the problem comes if the label assigned to r and the label assigned to t, they were not same. So in this particular case we have, what we have to do is we have to assign one of the two labels to point p and we have to note that these two labels are equivalent because p and t or p and r, they are adjacent and for r and t, the labels were different.

So after doing this initial labeling, we have to do some post-processing

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so that all these pixels p, r and t, they get the same label. So here what we have to do is, we have to assign to point p one of the labels, the label of r or the label of t and we have to keep a note that the label of the other pixel and the label of this, the label which is assigned to p, they are equivalent so that in the post processing stage this anomaly that has been generated, that can be avoided.

So

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A slide with a dark blue background and white text. In the top left corner, there is a small logo of a tree inside a circle. The title "Connected component labeling" is in a yellow font. Below the title, there are four lines of white text, each starting with a bullet point.

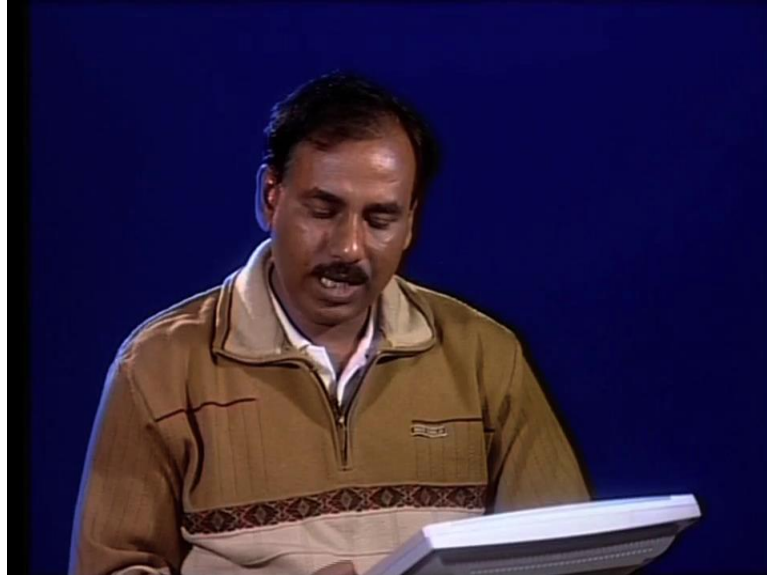
Connected component labeling

- At end of the scan all pixels with value 1 are labeled.
- Some labels are equivalent.
- During second pass process equivalent pairs to form equivalence classes.
- Assign a different label to each class. In the second pass through the image replace each label by the label assigned to its equivalence class.

at the end of the scan, all pixels with value 1 will have some label and some of the labels will be equivalent. So during post-processing or the second pass what we will do is we will

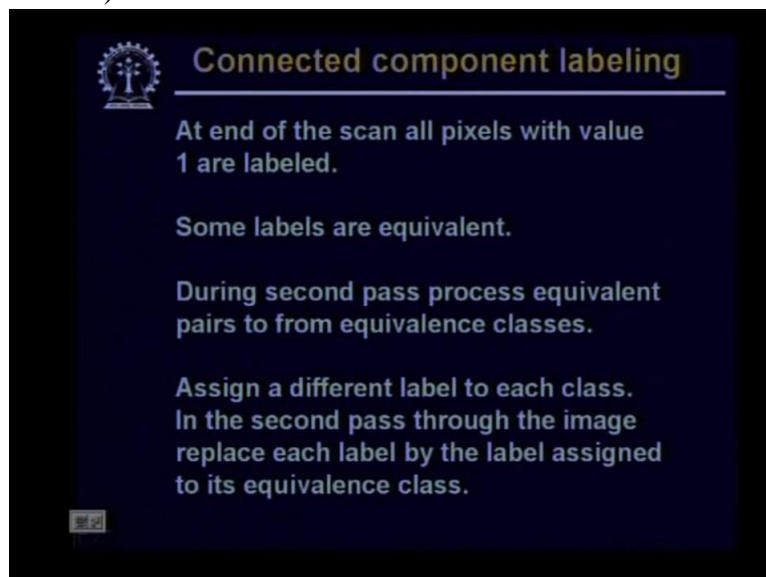
identify all the equivalent pairs to form an equivalent class and we can assign a different label to each of these

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equivalent classes And in the second pass you go through the image once more and

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for all the labels which belongs to a particular equivalent class you replace its original label by the label that has been assigned to the equivalent class.

So these two passes

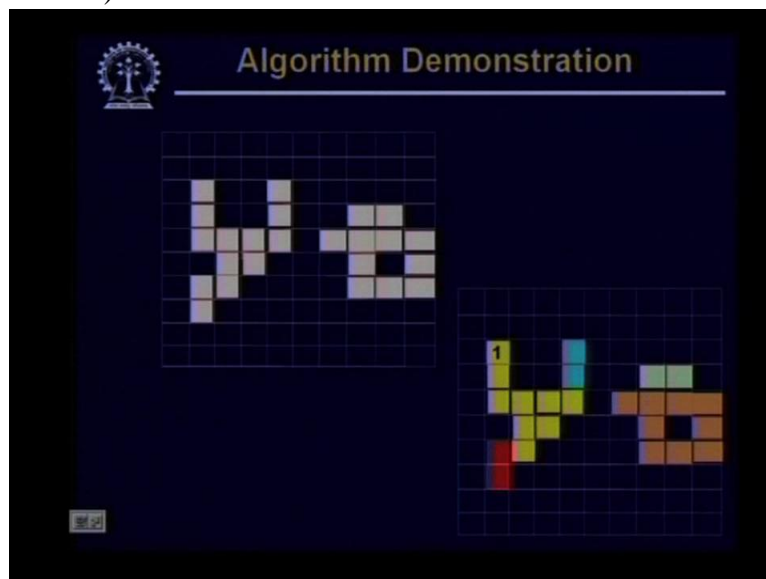
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gives, at the end of the second pass, you get a labeled image where you maintain, where you identify the region belongingness of a particular pixel by looking at what is the label assigned to that particular pixel.

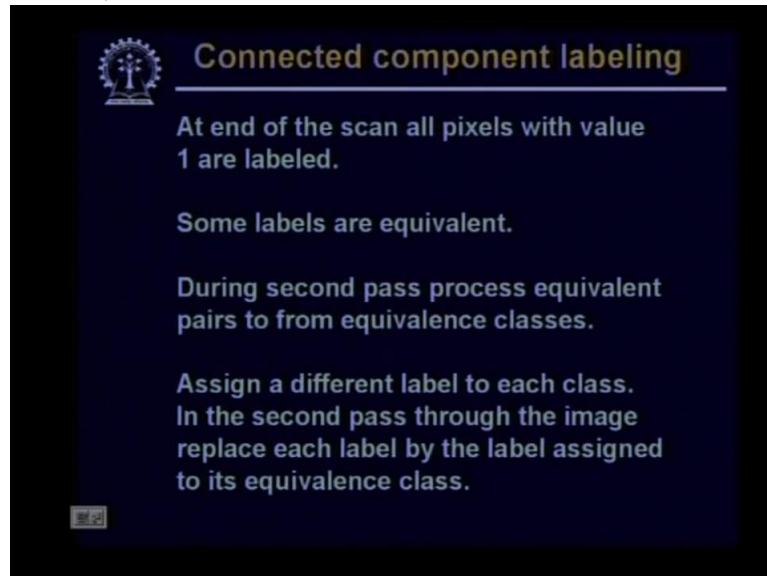
So here

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let us

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Connected component labeling

At end of the scan all pixels with value 1 are labeled.

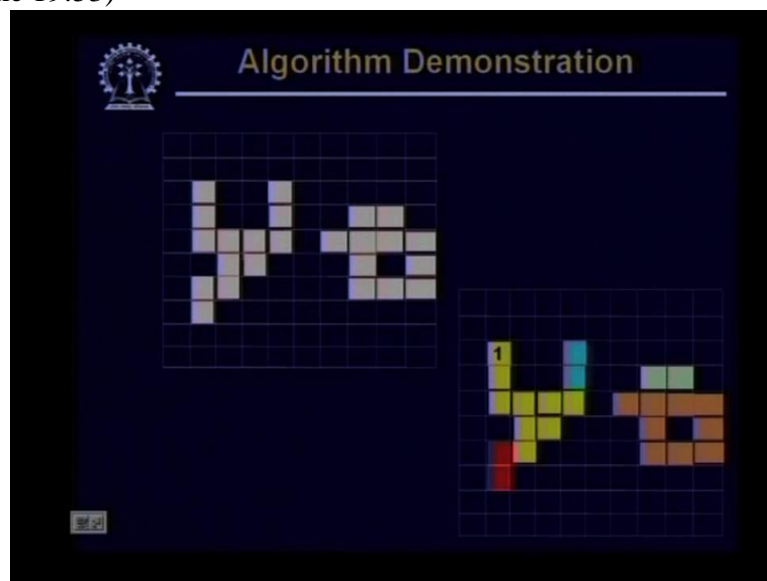
Some labels are equivalent.

During second pass process equivalent pairs to form equivalence classes.

Assign a different label to each class. In the second pass through the image replace each label by the label assigned to its equivalence class.

come to say an example

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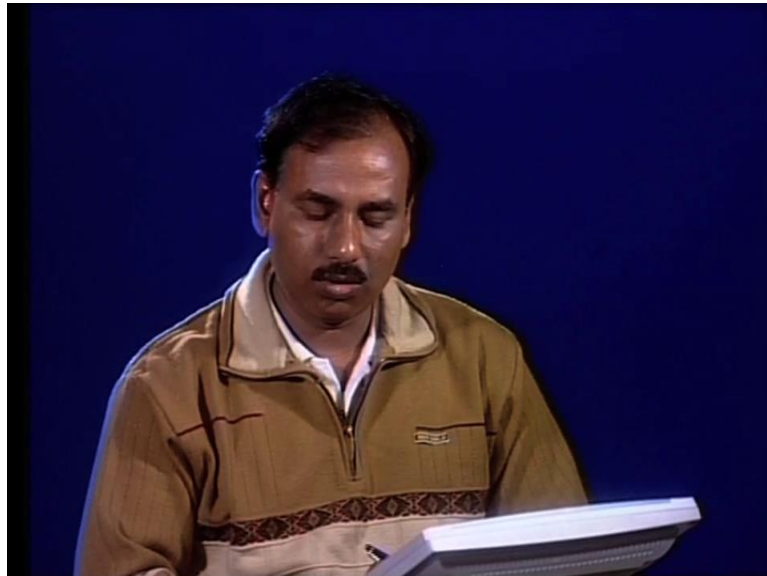


Algorithm Demonstration

So in this particular example you find that we have shown two different regions or two different connected regions of pixels having value equal to 1. So here what we do is, during the initial pass, as we scan this image from left to right and from top to bottom what we do is, the first image that I get I assign a label 1 to it. Then you continue you, your scanning. When I, when you come to the second pixel i , second white pixel you find that by connectivity it belongs to the same region but when I come to this particular pixel i , if I go to the top pixel and the left pixel, I find that there is no other pixel which is having a value equal to 1. So I have to assign a new value to this particular pixel and this pixel gets a value equal to 2.

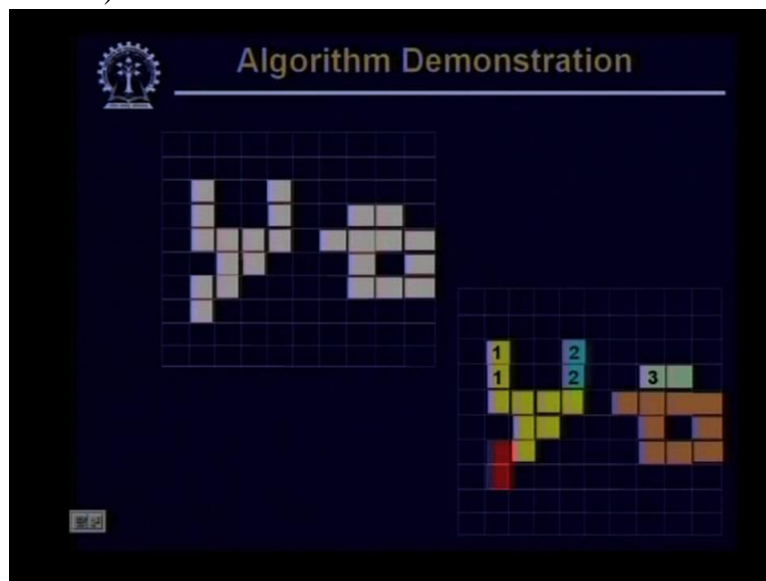
Come to the next one. This pixel again gets the value equal to 1 because its top pixel equal to 1. Coming to the next one, this one gets the value

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equal to 2 because its top pixel has a value equal to 2. Come to the next one. This gets the value 3 because it has to be a new label, because neither its top neighbor nor the left neighbor has any other label.

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So the next one again gets the value 3 because its left neighbor has the label 3. Again this gets the value 1 because top neighbor is equal to 1. This gets the value 1 because the left neighbor is equal to 1. This gets the value 1. This gets the value 1. Now you find that in this case, there is an anomaly. Because for this pixel the top pixel has label equal to 2 and the left pixel has value equal to 1. So I have to assign one of these two labels to this particular pixel. So here we have assigned label 1 to this pixel and after this what we have to do is, we have to declare that 2 and 1, they are equivalent.

Then you continue your processing. Here again I have to assign a new label because none of the top or the left pixel of this, neighbors of this have got any labels so it gets a label 4. Coming to the next one, here you find that its top one has got label 3 and left one has got label 4. So I have to assign one of the labels and in this case the label assigned is 4 but at the same time, I have to keep a note that label 3 and label 4, they are equivalent. So you mark 3 and 4, equivalent.

So if you continue like this, the next one is again 4, the next one again gets a 4, this one gets a 1, this one again gets 1, this one gets 4, this one gets 4, this one gets a 5, that's a new label because for this particular pixel

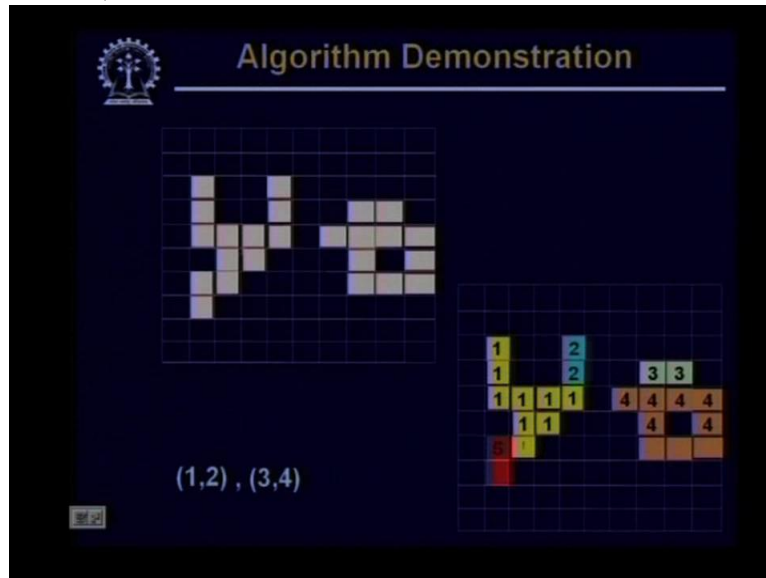
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its top or left, does not have any label.

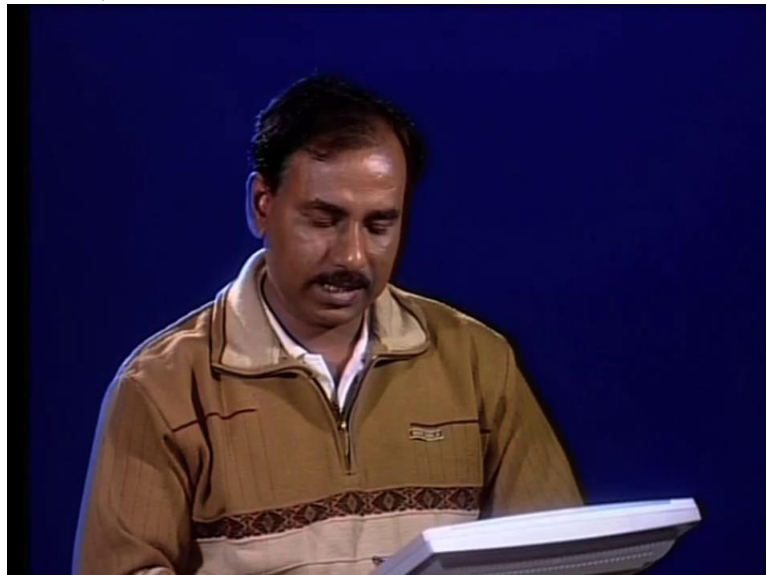
Coming to the next one

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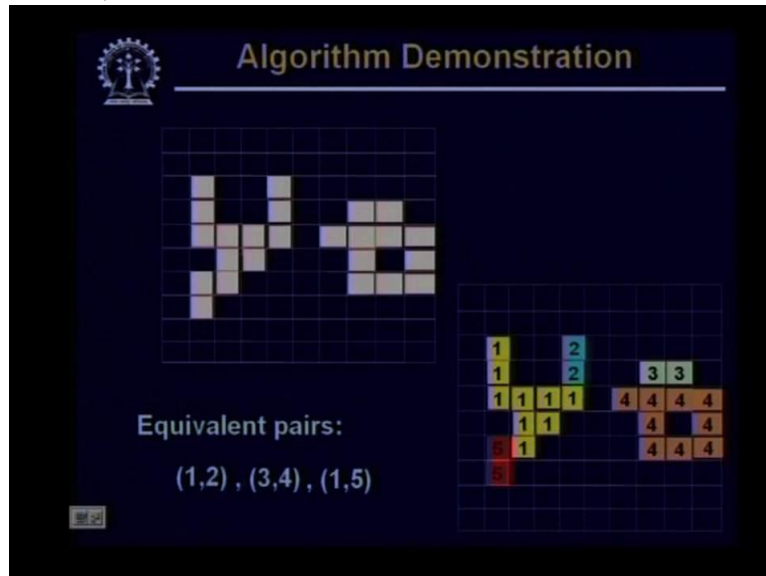
it gets a label 1 because here the top pixel has already have a label 1 but at this particular point we have to keep a note that 5 and 1 are equivalent. So you note 5 and 1 to be equivalent. You continue like this. The other pixel gets a label 4. This pixel gets a label 4. This pixel gets a label 4. This pixel again gets a label 5 because its top pixel is already having a label equal to 5. So at the end of this scanning

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I get

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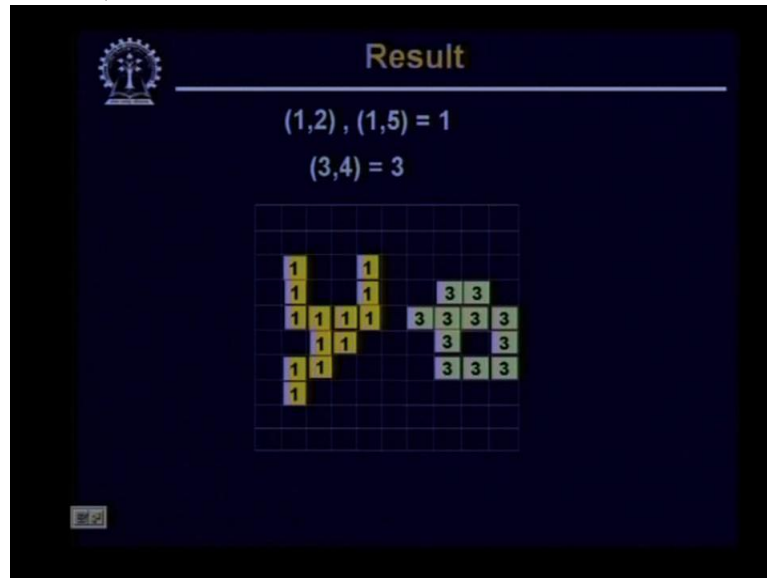
3 equivalent pairs One equivalent pair is 1, 2. The other equivalent pair is 3, 4 and the third equivalent is 1, 5.

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So in the second pass what I have to do is I have to process these equivalent pairs To identify the equivalence classes, that means the labels, all the labels which are equivalent. So by processing this you find

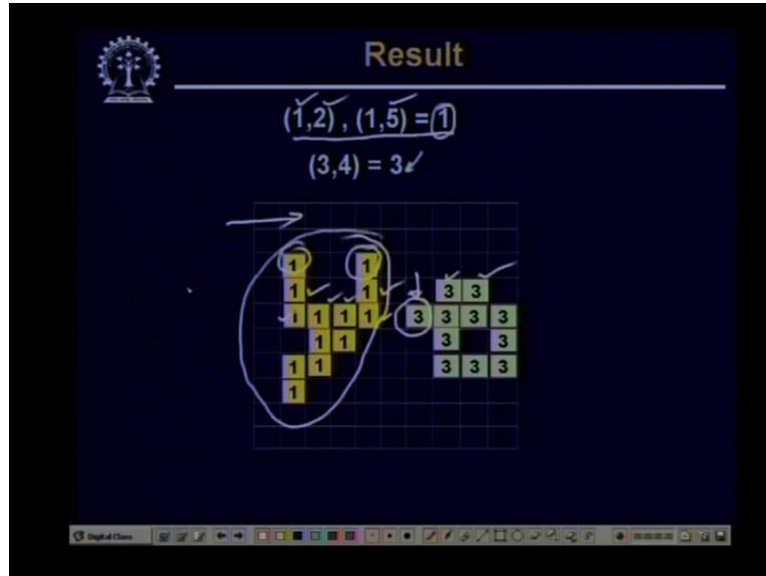
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that 1 and 2, and 1 and 5, they are equivalent, Ok So 1, 2 and 5, these two labels form a particular equivalence class. And similarly 3 and 4, they are equivalent forming an equivalence class. So if I assign a label 1 to the equivalence class containing the labels 1, 2 and 5, and at the same time I assign a label 3 to the equivalence class containing labels 3 and 4 then during the second pass, what I will do is I will scan over this labeled image which is already labeled and I will reassign the labels. So wherever the label was equal to 1, I will maintain that equal to 1. And wherever I get a label which is 2 or 5, I will reassign that label to be equal to 1. So in this particular case if you remember, this pixel had got the label equal to 2. I reassign because 2 belongs to an equivalence class consisting of the labels 1, 2 and 5 to which we have assigned the label equal to 1. So wherever I get the label equal to 2, I reassign that label equal to 1. So continue this way. This was already 1. This was 2 which has been reassigned to be 1. This was 3 which remains because 3 and 4 form equivalence class and the label assigned to this equivalence class is 3. This is also 3. That remains. This was 1 that remains. This was 1 that remains. This was 1 that remains. This was say possibly 2 or 1, so I make it equal to 1. This had got a label equal to 4, so that has been reassigned the label value equal to 3.

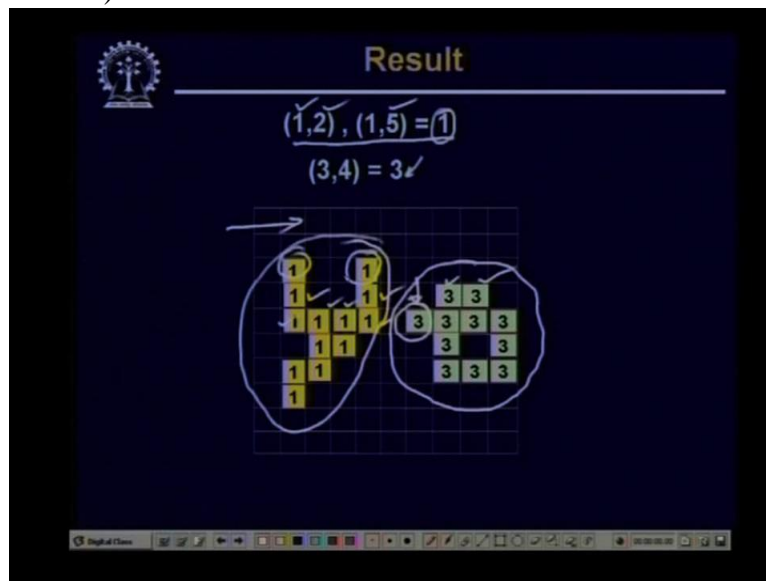
So will find that at the end of the second pass, I identify all the pixels belonging to a particular group to have a single label

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and similarly all the pixels belonging to this particular group to have

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another label So I will stop here today. I will continue with this

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lecture in the next class. Thank you