## Digital Image Processing Prof. P. K. Biswas Department of Electronics and Electrical Communications Engineering Indian Institute of Technology, Kharagpur Module 01 Lecture Number 07 Relationship between Pixels

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Hello, welcome to the video lecture series on Digital Image Processing.



So in today's lecture, we will see, we will try to see that what are the relationships that exist among the pixels of an image. And among these relationships the first relationship that we will talk about is the neighborhood and we will also see that what are different types of neighborhood of a pixel in an image. Then we will also try to explain that what is meant by connectivity in an image. We will also learn the connected component leveling algorithm, the

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importance of this connected component leveling algorithm and the properties, that is connectivity, we will discuss about later. We will also explain what is adjacency and we will see what are the different types of adjacency relationships. Then we will also learn different distance measures and towards the end of today's lecture we will try to find out

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what are different image operations We will try to see that what are pixel by pixel operations and what are the neighborhood operations in an image.

So, as we say that the first relationship is the pixel relationship

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or the neighborhood relationship Now let us first try to understand that what is meant by neighborhood.

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We say that the people around us are our neighbors. Or we say that a person who is living in the house next to mine is my neighbor. So it is the closeness of the different persons which forms the neighborhood of the persons. So it is the persons who are very close to me, they are my neighbors. Similarly in case of an image also, we say that pixels are neighbors if the pixels are very close to each other.

So let us try to see

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formally what is meant by neighborhood in case of an image pixel Here let us consider a pixel at location, a pixel p at location x y as shown is this middle pixel. Now find that because the image in our case is represented by a

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two-dimensional matrix, so the matrix will have a number of rows and a number of columns. So when I consider this pixel p whose location is x y that means the location of the pixel in the matrix is in row number x in row x and in column y. Obviously there will be a row which is just before x that is row x minus 1. There will be a row just after the row x which is row x plus 1. Similarly there will be a column just before the column y that is column y minus 1 and there will be a column just after column y which is column y plus 1. (Refer Slide Time 03:43)



So come back to this figure. Coming to this particular pixel p which is at location x y I can have 2 different pixels, one is in the row just above row x, other one in the row just below row x but in the same column location y. So I will have 2 different pixels, one is in the vertically upward direction, the other one is in the vertically downward direction. So these are the 2 pixels which are called the vertical neighbors of point p. Similarly if I consider the columns, there will be a column pixel at location x i minus y minus 1 that is in row number x column number y minus 1. There is a pixel x and y plus 1 that is row number x and column number y plus 1.

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So in this case these are the two pixels which are the horizontal neighbors of the point p. So in this case these are not 4,

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rather these should be 2; here this will also be 2. So this pixel p has two neighbors in the horizontal direction and two neighbors in the vertical direction. So these total 4 pixels are called 4 neighbors of the point p and is represented by p equal to and is represented by

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n 4 p, that is these pixels are 4 neighbors of the pixel p or point p. Each of these neighbors, if you find out the distance between these neighboring pixels you will find that each of the neighbors is at a unit distance from point p. Obviously if p is a boundary pixel then it will have less number of neighbors. Let us see why.

Say I have a

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two-dimensional image where this image is represented in the form of a matrix So I have pixels in different rows and pixels in different columns Now if this pixel p, the point p is one of the

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boundary pixels, say I take this corner pixel, then as we said that for a pixel p usually there are

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4 different pixels taken from a row above it, a row below it, the column before it and the column after it But when we consider

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this particular pixel p, this pixel p does not have any pixel in the row above this pixel it does not have any pixel in the column before this particular column. So for this particular pixel p I will have only 2 neighboring pixels, one is in this location, the other one is in this location which are part of 4 neighbor or n 4 p. So find that for all the pixels which belong to the boundary of an image, the number of neighboring pixels is less than 4 where as (Refer Slide Time 07:24)



for all the pixels which belong to, which is inside an image, the number of neighborhood pixels is equal to 4. So this is what is the 4 neighborhood of

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a particular pixel

Now as we have done, as we have taken the points from vertically upward direction and vertically downward direction or horizontally from the left as well as from right, similarly we can find that there are 4 other points which are in the diagonal direction. So those points are here.

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Again I consider this point p at location x y

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but now if I consider the diagonal points you find that there are 4 diagonal images, there are 4 diagonal points, one at location

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x minus 1 y minus 1, the other one at location x plus 1 y plus 1,

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one at location x minus 1 y plus 1 and

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the other one is at location x plus 1 and y minus 1.

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Now we say these 4 pixels, because they are in the diagonal directions, these 4 pixels are known as diagonal neighbors of point p.

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And it is represented by n d p. So I have got 4 pixels which belong to n 4 p that is those are the 4 neighbors of point p and I have got 4 more points are the diagonal neighbors or represented by n d p. Now I can combine these 2 neighborhoods and I can say an 8 neighborhood.

So again coming back to this, if I take the points both from n 4 p

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and n d p, they together are called 8 neighbors of point p and represented by n 8 p. So obviously, this n 8 p is the union of n 4 p and n d p.

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And naturally as we have seen in the previous case,

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that if the point p belongs to the boundary of the image, then n d p, the number of diagonal neighbors of point p will be less than 4, similarly the points belonging to n 8 p or the number of 8 neighbors of the point p will be less than 8 where as if p is inside an image, it is not a boundary point, in that case there will be 8 neighbors, 4 in the horizontal and vertical directions and 4 in the diagonal directions so there will be 8 neighbors of point p if point p is inside an image. So these are the different neighborhoods of the point p.

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Now

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Now after this, we have another property which is called as connectivity. Now the connectivity of the points

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in an image is a very, very important concept which is used to find the region property of the image or the property of the particular region within the image. You will recollect that in our introductory lectures, we have given an example of segmentation that is we had an image of certain object and we wanted to find out the points which belong to the object and the points which belong to the background. And for doing that, we had used a very, very primitive operation called the thresholding operation. So here in this particular case, we have shown that

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if the intensity value or if F x y at a particular point x y is greater than certain threshold say t h then in that case we decided that the point x y belongs to the object.

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Where as if the point or the intensity level at the point x y is less than the threshold, then we have said, we have decided that the point x y belongs to the background.

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So by simply by performing this operation and if you represent every object point as a white pixel or assign a value 1 to it and

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every background pixel as a black pixel or assign a value 0 to it, in that case the type of image that we will get after the thresholding operation is like this.

So here

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this is the original image.

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And you find that for all the points which belong to the object, the intensity value is greater than the threshold. So the decision that we have taken is, these points belong to the object, so in case of the segmented image or the thresholded image (Refer Slide Time 12:40)



we have assigned value 1 to each of those image points where as in other regions, region like this we decided that these points belonged to the background, so we have assigned a value 0 to these particular points. Now just by performing this operation what we have done is,

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we have identified certain number of pixels, the pixels which belong to background, and the pixels which belong to the object. But just by identification of the pixels belonging to the background, I cannot, or the pixels belonging to the object, I cannot find out what is the property of the object until and unless I do some more processing to say that those pixels belong to the same object. That means I have to do some sort of grouping operations.

Not only this, I can have a situation like this.

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Say for example, I have in this entered image, 2 different objects

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One object may be in this location and

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another object may be, say somewhere here.

So just by using this thresholding operation what I have done is, I have decided that all the pixels in this region will also get a value 1

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all the pixels in this region will also get a value 1. So both these two sets of pixels, they belong to the object. But

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here our solution does not end there. We have to identify that a set of pixels of this particular set of pixels belong to one object

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this particular set of pixels belong to another object.

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They don't belong to the same object.

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So for this what is needed is I have to identify

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that which pixels are connected and I also have to identify which pixels are not connected

So I will say that the pixels having value equal to 1 which are connected, they belong to one region and another set of pixels or points having value equal to 1 but not connected to the other set, they belong to some other object. So this connectivity property between the pixels is a very, very important property and using this connectivity property

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we can establish the object boundaries. We can find out what is the area of the object and likewise we can find out many other properties of the object or the descriptors of the object which will be useful for further

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high level processing techniques where we will try to recognize or identify a particular object So now let us try to see that what is this connectivity property. What do we mean by connectivity?

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We say that 2 pixels are connected if they are adjacent in some sense. So this term some sense is very, very important. So this adjacent means that they have to be neighbors, that means one pixels,

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if I say two points p and q are connected, then by adjacency we mean that p must be a neighbor of q or q must be a neighbor of p. That means q has to belong to n 4 p, or n d p or n 8 p. And in addition to this neighborhood, one more constant that has to be put is that the intensity values

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or the gray levels of the two points p and q must be similar.

So let us take this example. Here we have shown 2, 3 different situations where we have taken points p and q. So here we find that point q, it belongs to

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the 4 neighborhood of point p Here point q belongs to

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the diagonal neighborhood of point p, here again

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point q belongs to the 4 neighborhood of point p. And in this case we will say that point p and q are connected, obviously the neighborhood restriction holds true because q and p, they are neighbors and along with this we have said that another

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restriction or another constraint must be satisfied that their intensity values must be similar. So in this particular case, because we are considering

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a binary image, so we will say that if q belongs to the neighborhood of p, or p belongs to the neighborhood of q and the intensity value at point p is same as the intensity value at point q. So because it is binary image

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this value will be either 0 or 1.

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So in this case, if I assume that if the pixels have value equal to 1, then we will assume that that those 2 pixels to be connected. So in this case, if for both p and q

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the intensity value is equal to 1 and since they are the neighbors so we will see, say that points p and q are connected. So from this connectivity

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can be defined in a more general way

So the earlier example that we had taken is the connectivity of in case of a binary image, where the intensity values

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are either 0 or 1 This connectivity property can also be defined in case of gray level image so how do you define connectivity in case of a gray level image? In case of a gray level image, we define a set of gray levels. Say for example in this case, we have defined v

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to be a set of gray levels which is used to define the connectivity of two points p and q So the, if intensity values at points p and q belongs to the set v so this is not point p by p and q but the intensity values of f p and f q. So the intensity values at the points p and q belong to set v and points p and q are neighbors then we can say that points p and q are connected. And here again, we can define 3 different types of connectivity. One is 4 connectivity that is in this case, the intensity values at p and q must be from the set b and p must be a 4 neighbor or q

must be a 4 neighbor of p. In that case we define 4-connectivity. Similar we define 8connectivity. If the intensity values at point p and q

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belong to the set v and p is an 8-neighbor of q or q is an 8-neighbor of p. There is another type of connectivity which is defined which is called m-connectivity or mixed connectivity. So in case of

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m-connectivity it is defined like this, that p and q, the intensity values at points p and q, obviously they have to be from the same set v and if q belongs to neighborhood of p or q belongs to n d p that is diagonal neighborhood of p and n 4 p intersection with n 4 q is equal to phi. So this concept extends or puts some restriction on the 8-connectity in the sense that here we say that either q has to be a 4-neighbor of p or p has to be a 4 neighbor of q or q has

to be a diagonal neighbor of p but at the same time, n 4 p intersection with n 4 q must be equal to phi.

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And you find that this n 4 p intersection with n 4 q this indicate the set of points which are 4 neighbors

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of both the points p and q So this says that if in, if the point q belongs to the diagonal neighbor of p and there is a common set of points which are 4 neighbors to both the points p and q, then m-connectivity is not valid. So the reason why this m-connectivity is introduced is to avoid some problems that may arise with simple 8-connectivity concept. Let us see what are these problems.

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$(\hat{n})$	Connectivity
14	Let V be the set of gray levels used to define Connectivity for two points $p,q \in v$ , three types of Connectivity are defined
	• 4-connectivity=>p,q $\in$ v & p $\in$ N <sub>4</sub> (q) • 8-connectivity=>p,q $\in$ v & p $\in$ N <sub>8</sub> (q) • M-connectivity (mixed connectivity) p,q $\in$ v are m-connected if (i) q $\in$ N <sub>4</sub> (p) Or (ii) q $\in$ N <sub>D</sub> (p) and N <sub>4</sub> (p) $\cap$ N <sub>4</sub> (q)= $\phi$
<b>회</b> 관	$N_4(p) \cap N_4(q) \Longrightarrow$ set of pixels that are 4-neighbors Of both p and q and whose values are from v.

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EX:	$V = \{1\}$	0 1 1	0 1 1
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0	1	0 0 1	0 1
4 - c	onnected	8 - connected	m - connected

So the problem is like this. Here again we have taken the example from a binary image and in case of a binary image we say that two points may be connected if the values of both the points equal to 1, so set v contains a single intensity value which is equal to 1.

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Now

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here we have depicted one particular situation where we have shown the different pixels in a binary image. So you find that if I consider this point at the middle of this image which is having the value 1

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there is one more pixel on the row above this which is also having the value 1 and a diagonal pixel which is having a value 1 and a diagonally downward pixel which is also having a value equal to 1. Now if I define 4-connectivity, then you find that this point is 4-connected to this point. This point is 4 connected to this point because this particular point is member of the 4-neighbor of this particular point. This point is not connected because this is not a 4-neighbor of this point. But by 4-connectedness, this point is not connected because this is not a 4-neighbor of any of these points. Now from 4-connectivity, if I move to 8-connectivity, then what I get?

Time 24:14)ConnectivityConnectivityMixed connectivity is a modification of<br/>8-connectivity- Eliminates multiple path connections<br/>that often arise with 8-connectivity.Ex:  $V = \{1\}$ 0101010110110110110101011010110101101011010110101101011011011011101101101110111</td

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Again I have the same set of points. Now you find that we have defined 8-connectivity

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to be an union of or 8-neighborhood to be an union of 4-neighborhood and diagonal neighborhood So because this is union of 4-neighborhood and diagonal neighborhood, so I will have set of points which are connected through 4 neighbors I will also have set of connections or set of points which are connected through diagonal neighbors.

So as shown in the second figure here you find

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that when I consider this central pixel, again these 2 connectivities which are 4-connectivity, they exist. In addition to this, this point which was not connected considering the 4-neighborhood now gets connected because this belongs to the diagonal neighborhood of this central point. So these two points are also connected.

Now the problem arises here. This point was connected through 4-neighborhood and at the same time, this point, because this is a diagonal neighbor of the central point, so this point is also connected through this diagonal neighborhood. So if I consider this situation and I simply have 8-connectivity, I consider 8-connectivity, then you find that multiple number of paths for connection exist in this particular case.



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So the m-connectivity or mixed connectivity has been introduced to avoid this multiple connection path. So you just recollect the restriction we have put in case of mixed connectivity. In case of mixed connectivity we have said that 2 points are m-connected if one is the 4-neighbor of other or one is 4-neighbor of other and at the same time they don't have any common 4-neighbor. So just by extending this concept in this case, you find that

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for m-connectivity, these are diagonal neighbors so they are connected but these 2 points, though they are diagonal neighbors but they are not m-connected because these 2 points have a point here. This point is a 4-neighbor of this, at the same time this point is 4-neighbor of this. So when I introduce this 4-connectivity concept, you find

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that the problem that arises, that is the multipath connection which have come in case of 8connectivity no more exist in case of m-connectivity. So in case of m-connectivity, even if we consider the diagonal neighbors but the problem of multiple path does not arise. So this is the advantage that you get in case of m-connectivity. (Refer Slide Time 27:22)

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Ex	: V = {1}		
0	1-1	0 1-1	0 1-1
0	1 0	0 1 0	0 1 0
0	1	0 0 1	0 1
4 -	connected	8 - connected	m - connected

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Now from the connectivity we come to the relationship of adjacency.

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So we say 2 pixels p and q are adjacent if

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<ul> <li>4-adjacency</li> <li>8-adjacency</li> <li>m-adjacency</li> <li>m-adjacency</li> <li> depending on type of connectivity used.</li> <li>Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>i</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> and S<sub>j</sub> are adjacent in Two image subsets S<sub>j</sub> are adjacent in Two image subjacent in Two image subsets S<sub>j</sub> are</li></ul>	T a	wo pixels p and q are adjacent if they are connected
depending on type of connectivity used. Two image subsets $S_i$ and $S_j$ are adjacent i		4-adjacency 8-adjacency m-adjacency
Two image subsets $S_i$ and $S_j$ are adjacent i		- depending on type of connectivity used.
$\Im P \in S_i$ and $\Im q \in S_j$ such that p and q are adjacent	Ta	wo image subsets S <sub>i</sub> and S <sub>j</sub> are adjacent if $\Im P \in S_i$ and $\Im q \in S_j$ such that p and q are adjacent

they are connected.

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Thank you.