Digital Image Processing
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Module 01 Lecture Number 05
Signal Reconstruction from Image

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Hello, welcome to the course on Digital Image Processing.

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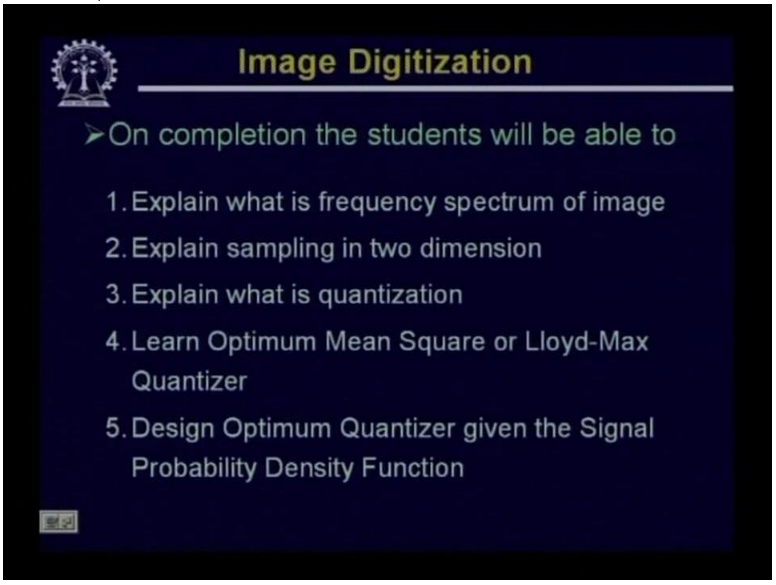

We will also talk about the Optimum Mean Square Error or Lloyd-Max Quantizer. Then we will also talk about that how to design an optimum quantizer which, with the given signal probability density function.

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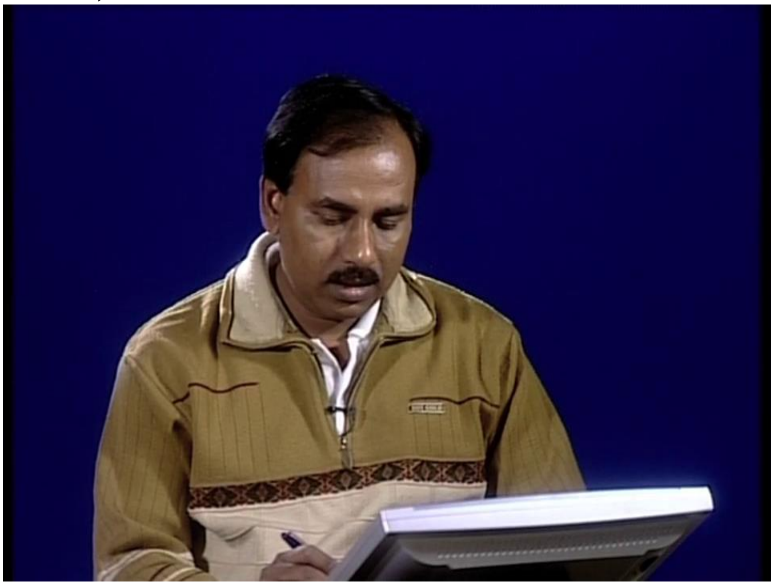

Now let us briefly recapitulate that what we have done in the last class.

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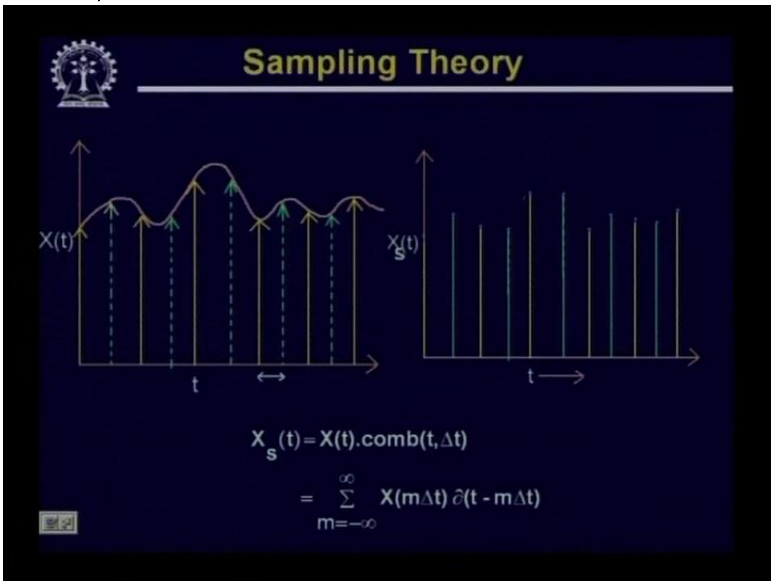

This is a signal x t

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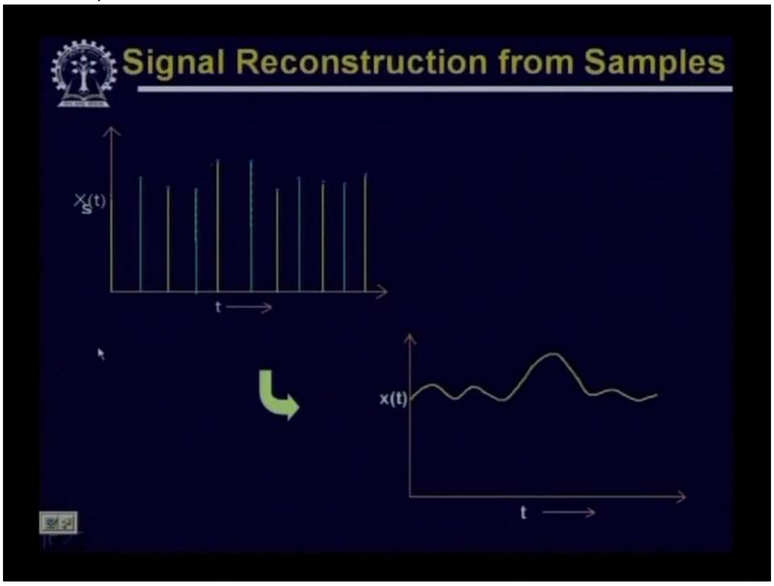

which is a function of

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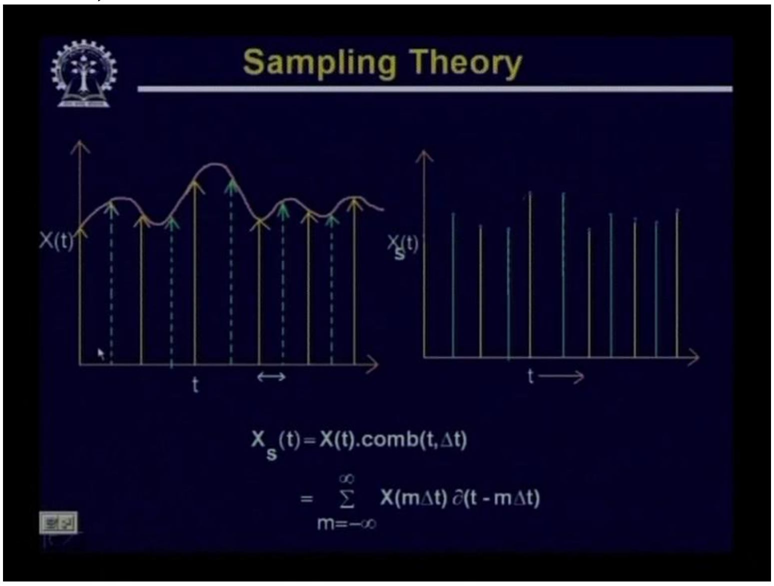

a single variable, say t And then what we have done is, we have sampled this one dimensional signal with a sampling function which is represented in the form of combo function, say comb of t delta t and we get sample values as represented by "x s" t. And we have also said that this "x s" t can be represented in the form of multiplication of x t by comb function of t delta t. Now the same function can also be represented in the form of summation of x m delta t into delta t minus m delta t where this t minus m delta t when you take the summation from m equal to minus infinity to infinity, this gives you what is the comb function. So this "x s" of t, that is the sampled value, that is the sample version of the signal x t can be represented in the form of x m delta t into delta t minus m delta t where m varies from minus infinity to infinity.

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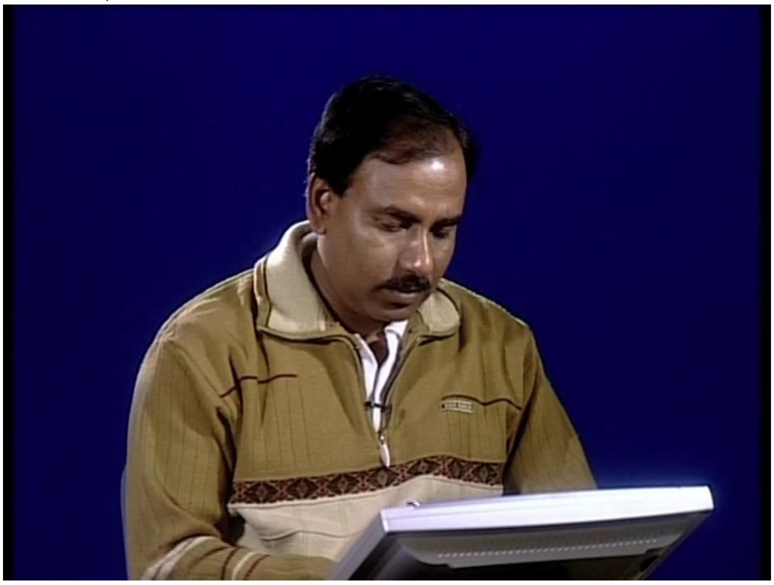

Now our problem is, that given these sample values, how to reconstruct the original signal x t from the sample values of

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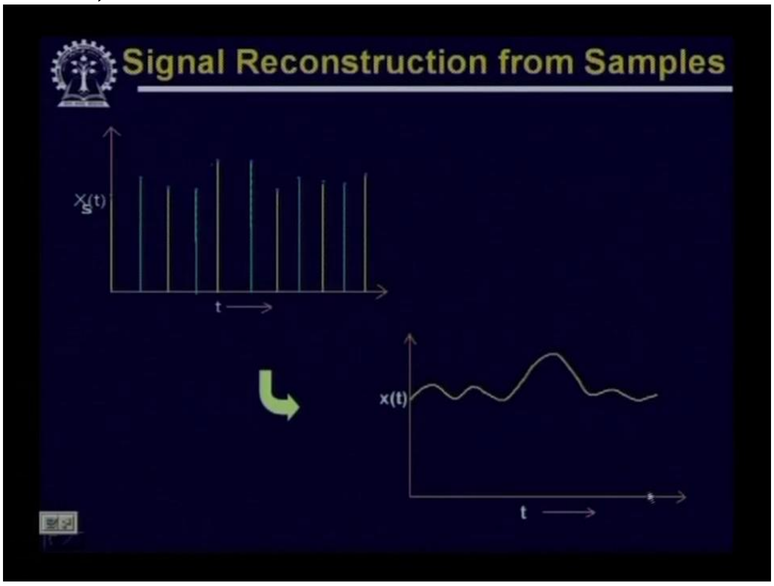

x t that is “x s” t

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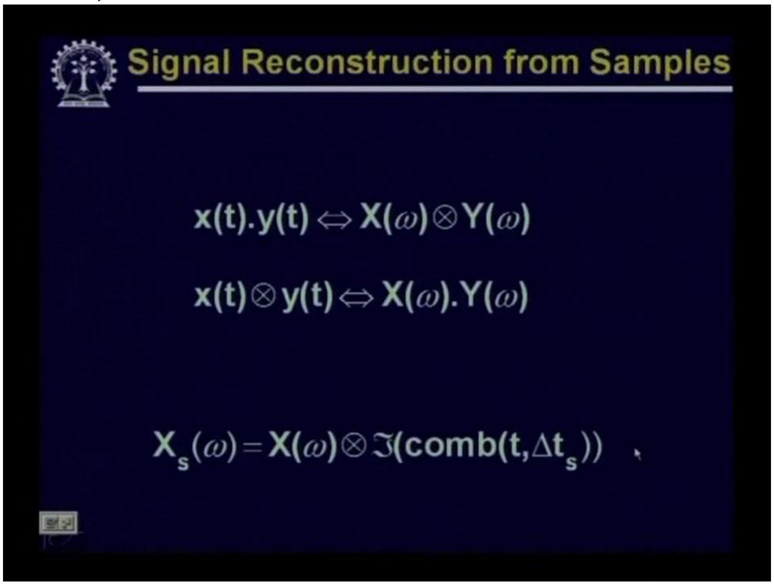

and for this purpose we have introduced what is known as Convolution Theorem. The Convolution Theorem says if you have two signals x t

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and y t in time domain then the multiplication of x t and y t in time domain is equivalent to, if you take the convolution of the frequency spectrum of x t and frequency spectrum of y t in the frequency domain. So that, that is to say that x t into y t is equivalent to

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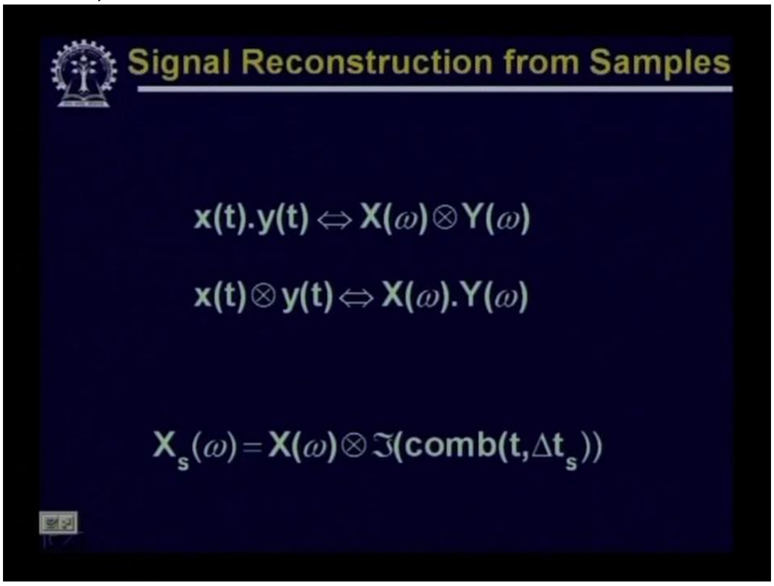

X omega convoluted with Y omega; similarly if you take the convolution of x t and y t in time domain that is equivalent to multiplication of X omega and Y omega in the frequency domain. So by using this concept of the convolution theory, we will see that how to reconstruct the original signal x t from the sampled values of "x s" t. Now as per this Convolution Theorem, we have seen that "x s" of t is nothing but multiplication of x t into the comb function comb of t delta t. So in the frequency domain that will be equivalent to "X s" of omega is equal to X omega convoluted with the frequency spectrum of comb of t delta "t s" where delta "t s" is the sampling interval.

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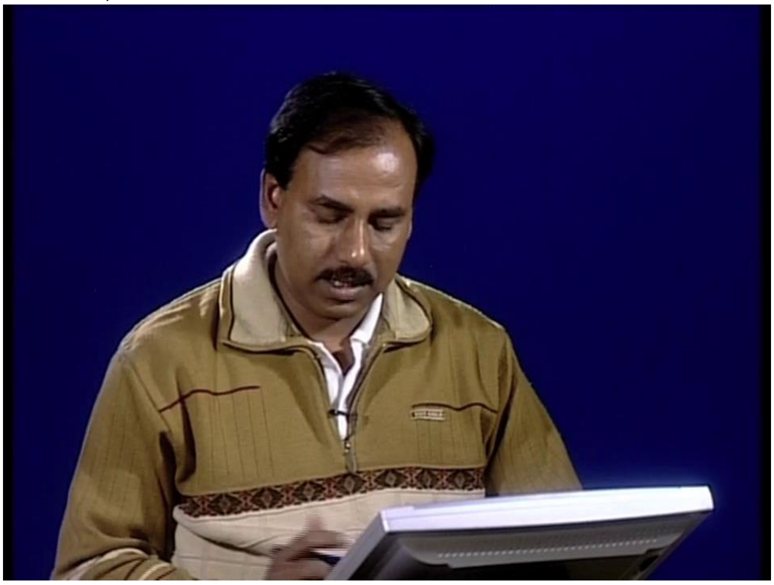

We have also seen that if X omega is the frequency spectrum

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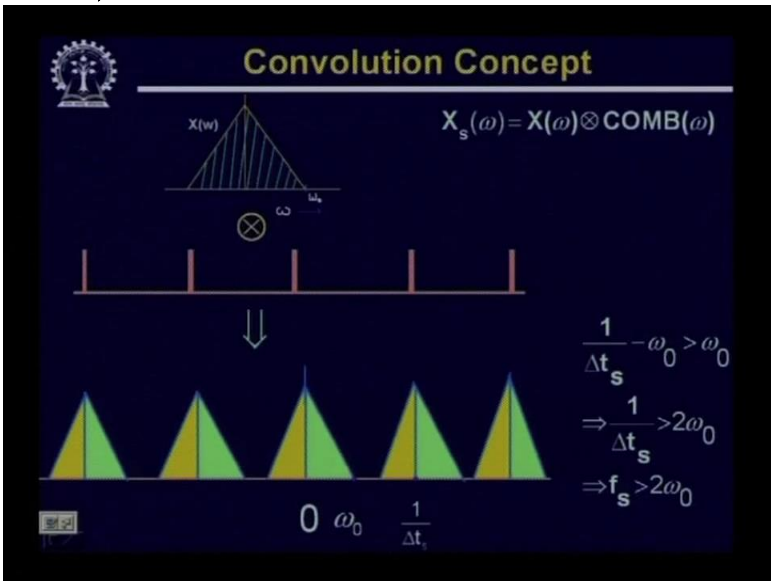

or the bandwidth of the signal, frequency spectrum of the signal which is presented here and this is the frequency spectrum of the sampling function, then when these two, I, we convolute, the convolution result will be like this where the original frequency spectrum of the signal gets replicated along the frequency axis at an interval, at a interval of 1 upon delta "t s" where 1 upon delta "t s" is nothing but the sampling frequency f of s. And here you find that for proper reconstruction what you have to do is, this original spectrum, the spectrum of the original signal has to be taken out and if we want to take out this then we have to make use of a filter which will only take out this particular band and the remaining frequency components will simply be discarded. And for this filtering operation to be successful, we must need that 1 upon delta "t s" minus "omega naught" where "omega naught" is the bandwidth of the signal or the maximum frequency component present in the signal x t so 1 upon delta "t s" minus "omega naught" must be greater than or equal to "omega naught". And that leads to the condition that the sampling frequency "f s" must be greater than twice of "omega naught" where "omega naught" is the bandwidth of the signal and this is what is the Nyquist rate.

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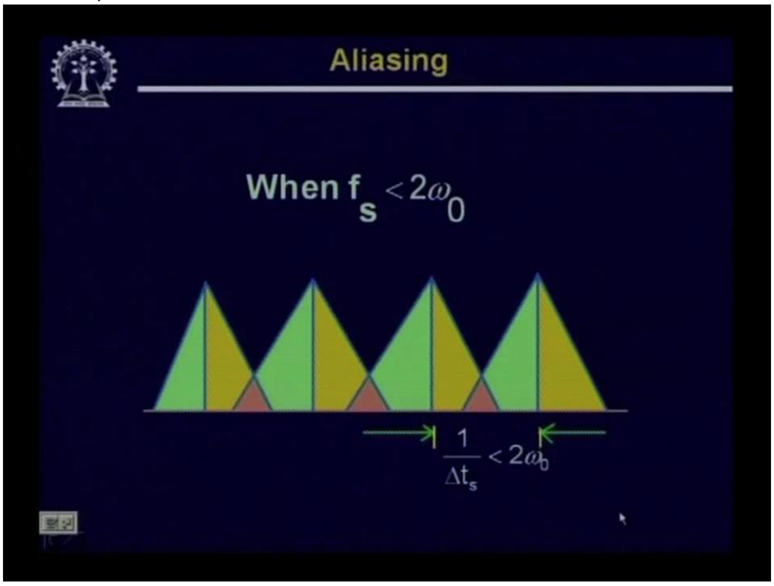

Now what happens if the

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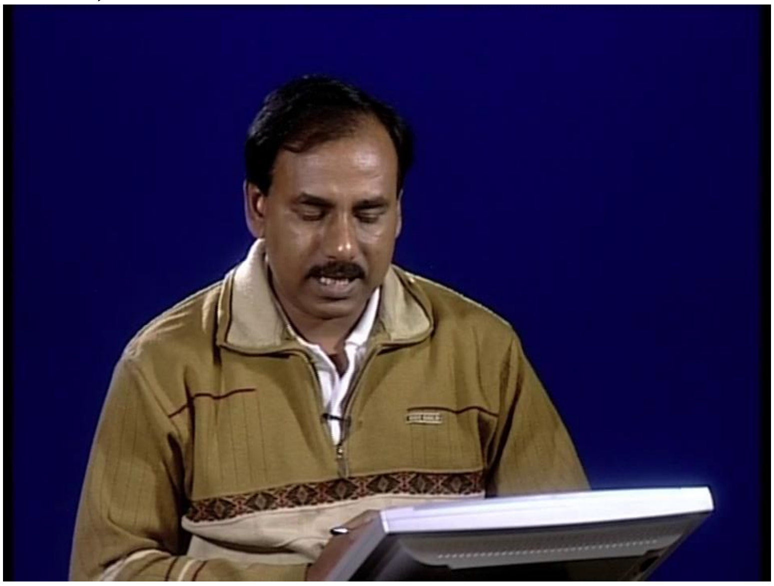

sampling frequency is less than twice of "omega naught"? In that case as it is shown in this figure, you will find that

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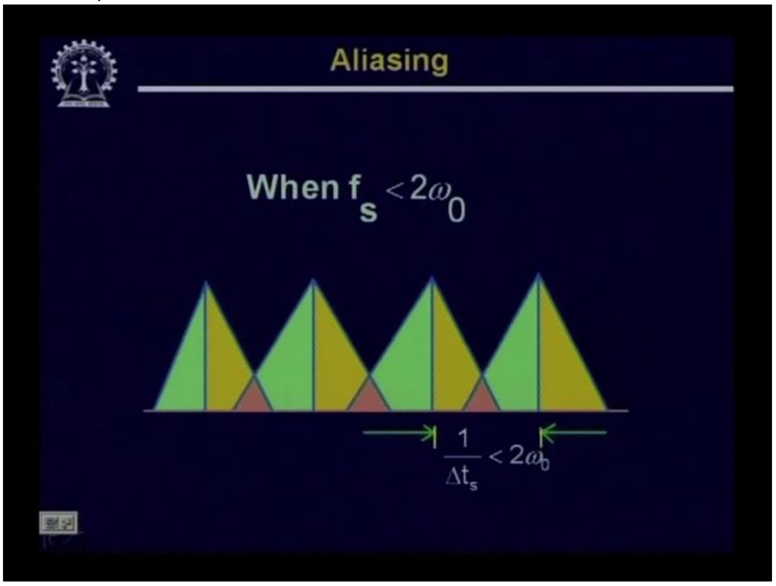

subsequent frequency bands after sampling, they overlap. And because of this overlapping, a single frequency band cannot be extracted using any of the low-pass filters. So effectively, as a result what we get is,

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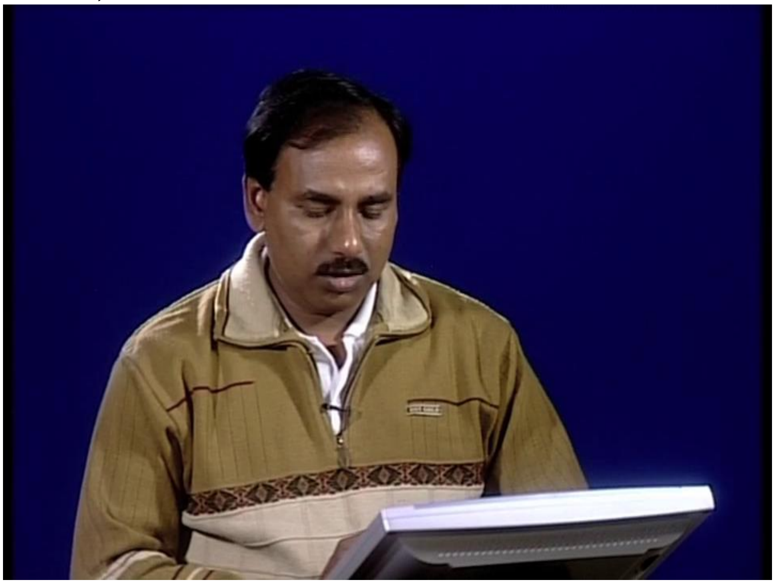

after low pass filtering the signal which is reconstructed is a distorted signal, it is not the original signal. And this effect is what is known as aliasing.

So now let us see what happens in case of two-dimensional image which is a function of two variables x and y. Now find that here, in this slide

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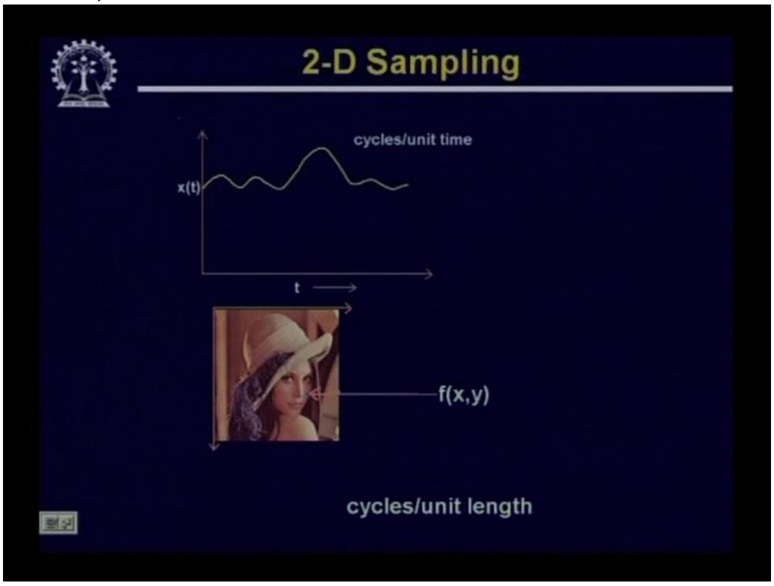

we have shown two figures. On the top we have shown the same signal x t which we have used earlier which is a function of t and the bottom figure is an image which a function of two variables x and y. Now if t is time, in that case x t is a signal which varies with time. And for such a signal the frequency is measured as you know in terms of Hertz which is nothing but cycles per unit time. Now how do you measure the frequency in case of an image? You will find that in case of an image, the dimension is represented either in the form of say 5 centimeter by 5 centimeter or say 10 centimeter by 10 centimeter and so on. So for an image, when we measure the frequency, it has to be cycles per unit length, not the cycles per unit time as is done in case of a time varying signal.

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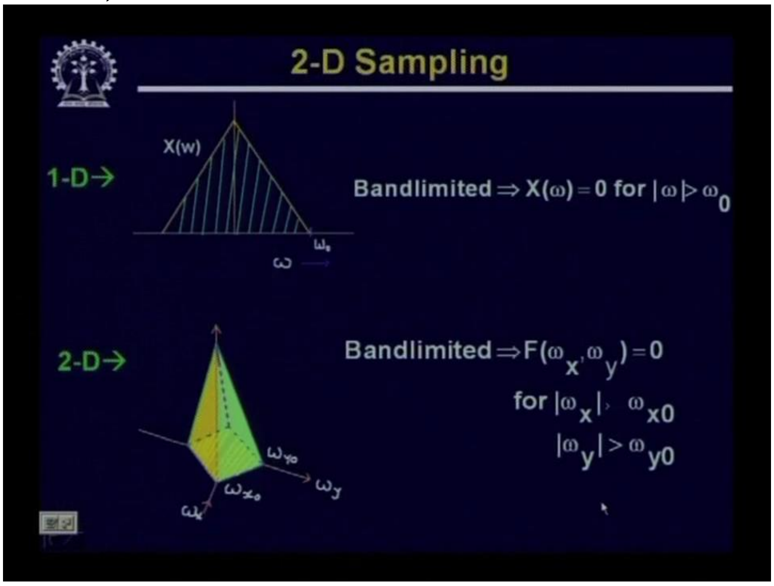

Now in this figure we have shown that as we in case of the signal x t we had its frequency spectrum represented by X of omega and we say that the signal x t is band limited if X of omega is equal to 0 for omega is greater than “omega naught” where “omega naught” is the bandwidth of the signal x t. Similarly in case of an image, because image is a two-dimensional signal which is a variable, which is a function of two variables x and y, so it is quite natural that in case of image we will have frequency components which will have two components, one in the x direction and other in the y direction. So we call them “omega x” and “omega y”. So we say that the image is band limited if

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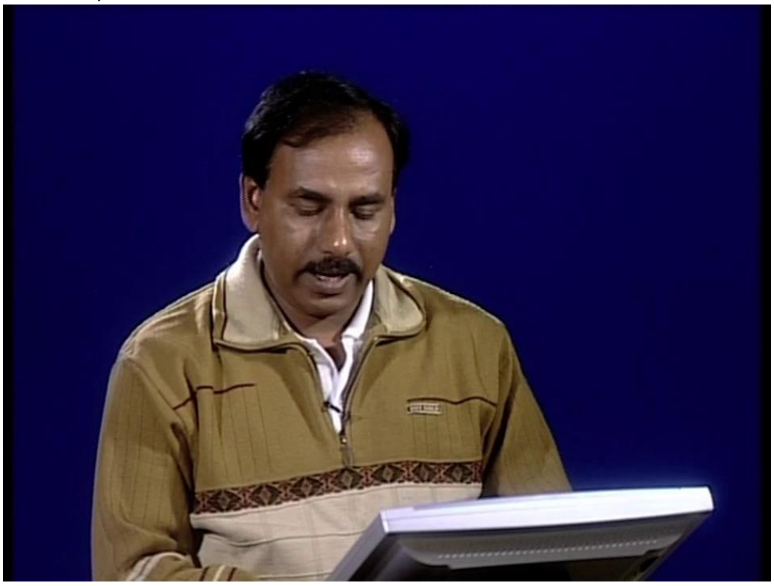

f of “omega x” “omega y” is equal to 0 for “omega x” greater than “omega x naught”

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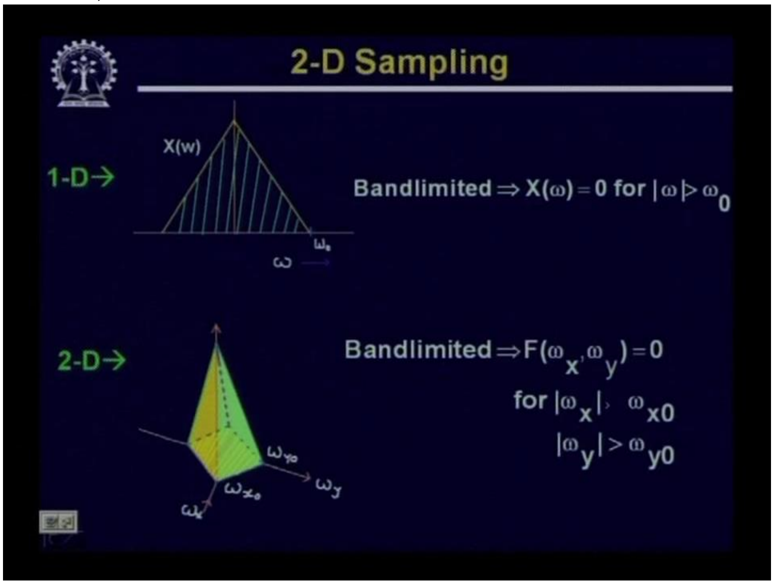

and “omega y” greater than “omega y naught”; so in this case, the maximum frequency component in the x direction is “omega x naught” and the maximum frequency component in the y direction is “omega y naught”. And this figure on the bottom left shows how the frequency spectrum of an image looks like.

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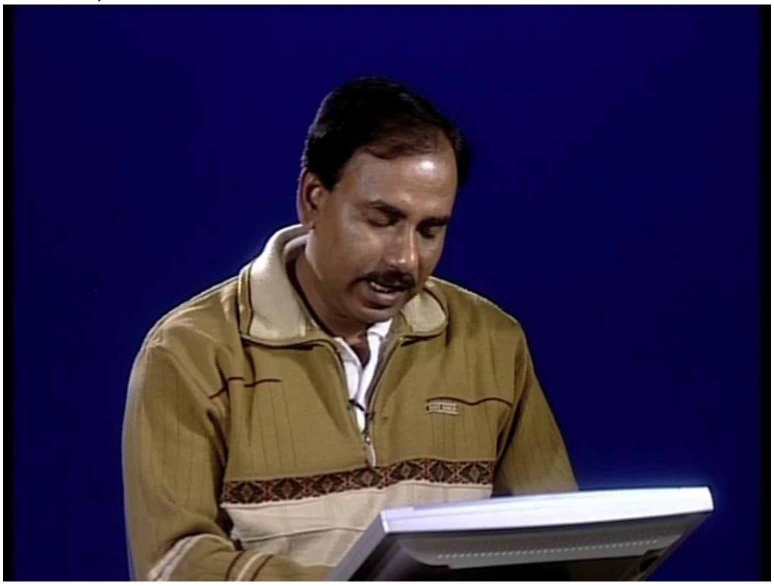

And here you find that

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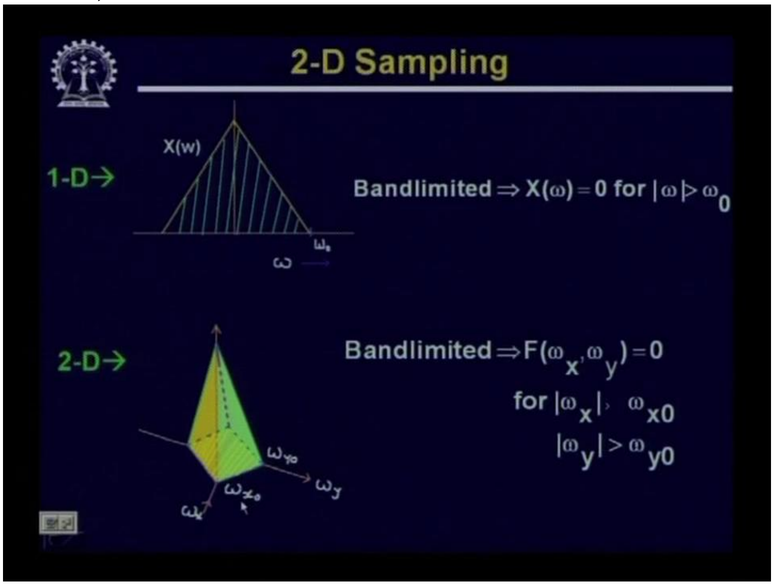

the base of this frequency spectrum on the “omega x” “omega y” plane is what is known as the region of support of the frequency spectrum of the image.

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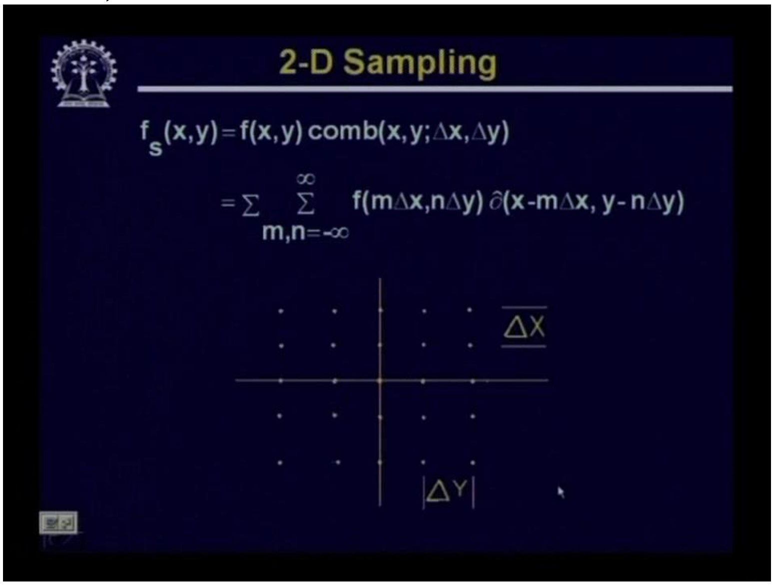

Now let us see what happens in case of two dimensional sampling or when we try to sample an image . The original image is represented by the function f of “x, y”, Ok and as we have seen in case of a two-dimensional, one-dimensional signal that f x of t is multiplied by comb of t delta t for the sampling operation, in case of image also, f of “x, y” has to be multiplied by comb of “x, y” delta “x, y” to give you the sample signal f s “x, y”. Now this comb function, because it is again a function of 2 variables x and y is nothing but a two dimensional array of the delta functions where along x direction the spacing is delta x, along y direction the spacing is del y. So again as before , this f s “x, y” can be represented in the form f m delta x n delta y multiplied by delta function x minus m delta x, y minus n delta y where both m and n varies from minus infinity to infinity.

So as we have done in case of one-dimensional signal,

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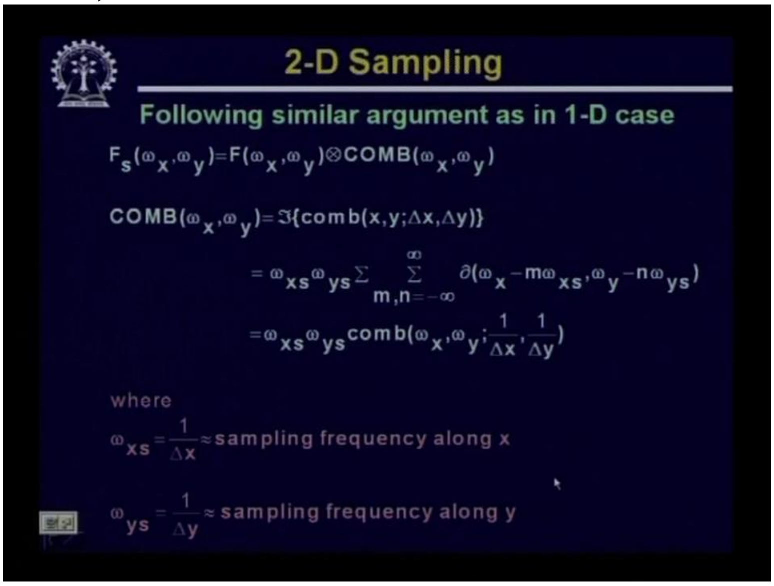

if we want to find out frequency spectrum of this sampled image, then this frequency spectrum of the sampled image F s “omega x” “omega y” will be same as F “omega x” “omega y” which is the frequency spectrum of the original image f “x, y” which has to be convoluted with COMB “omega x” “omega y” where COMB “omega x” “omega y” is nothing but the Fourier Transform of comb “x, y” delta x delta y, Ok and if you compute this Fourier Transform you find that COMB “omega x” “omega y” will come in the form of “omega x s” “omega y s” comb of “omega x” “omega y” 1 upon delta x, 1 upon delta y. Here this “omega x s” and “omega y s”; omega “x s” is nothing but 1 upon delta x which is the sampling frequency along the x direction and “omega y” s is equal to omega 1 upon delta y which is nothing but the sampling frequency along the y direction.

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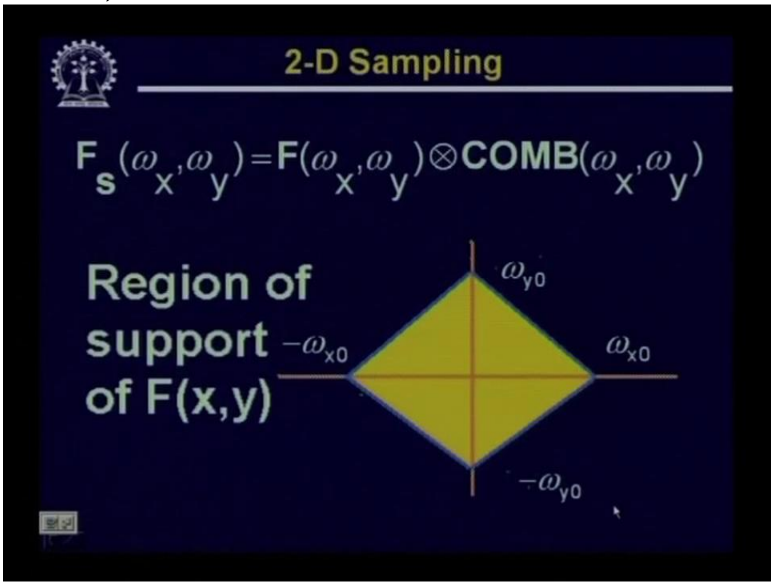

So coming back to similar concept as we have done in case of one-dimensional signal x t that F s “omega x” “omega y” which is now the convolution of F “omega x” “omega y” which is the frequency spectrum of the original image convoluted with COMB “omega x” “omega y” where COMB “omega x” “omega y” is the Fourier Transform of the sampling function in two-dimension. And as we have seen earlier that such a type of convolution operation replicates the original, the frequency spectrum of the original signal in along the omega axis in case of one dimensional signal, so here again

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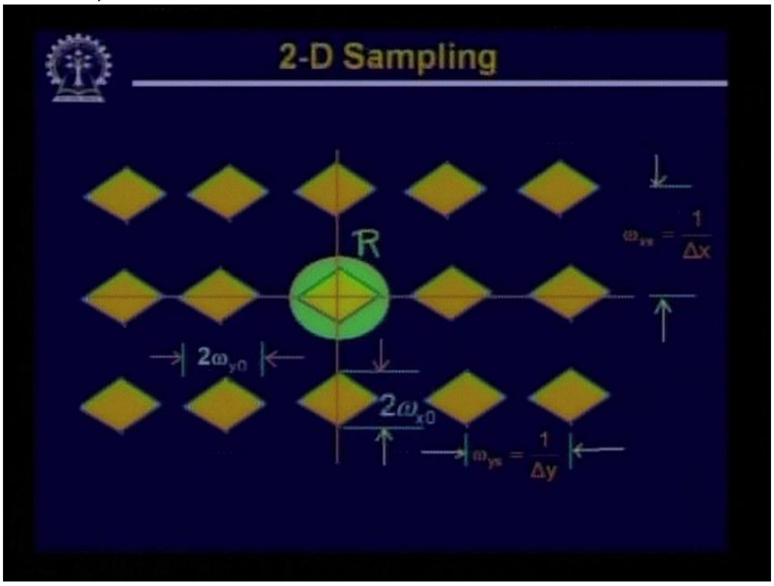

in case of two-dimensional signal this will be replicated, the original signal will be replicated

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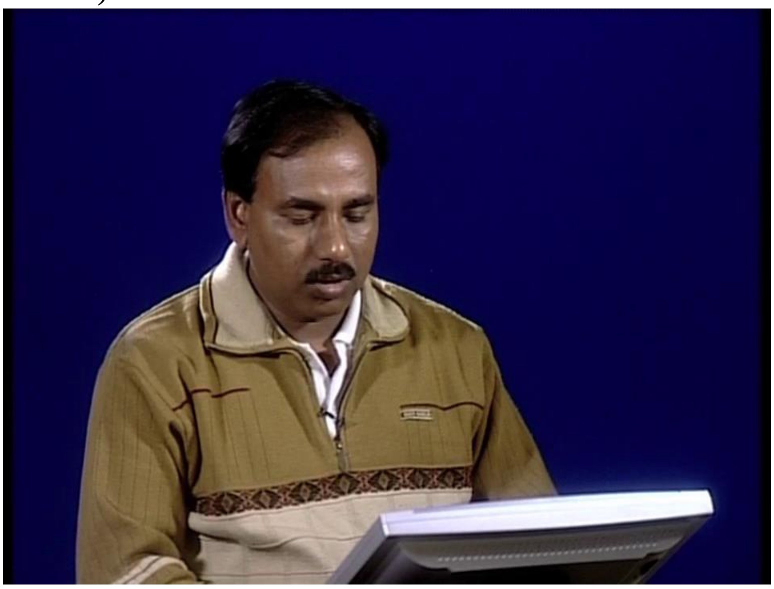

along both x direction and y direction So as a result what we get is

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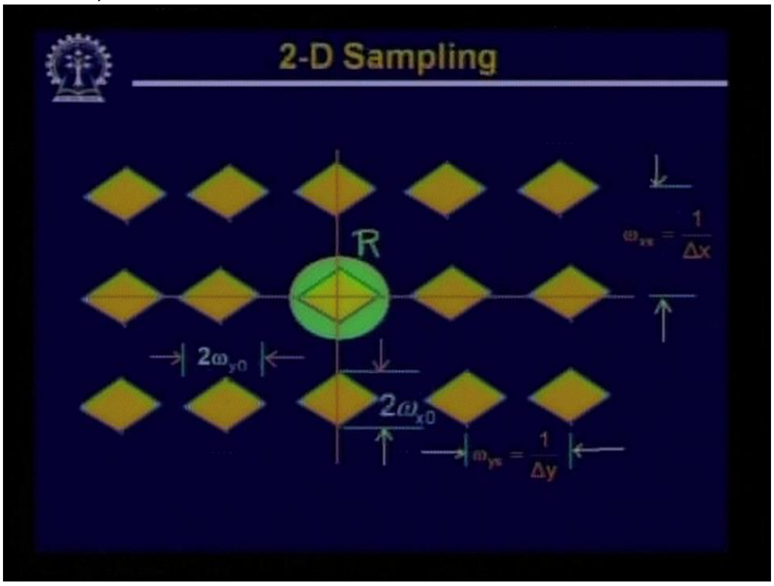

a two-dimensional array of the spectrum of the image as shown in this particular figure

So here again you find that the, we have simply shown the region of support getting replicated, that you find that along y direction and along x direction the spectrum gets replicated and the spacing between two subsequent frequency bands along the x direction is equal to “omega x s” which is nothing but 1 upon delta x and along y direction the spacing is 1 upon delta y which is nothing but “omega y s” but which is the sampling frequency along the y direction.

Now if we want to reconstruct the original image from this particular spectrum, then what you have to do is

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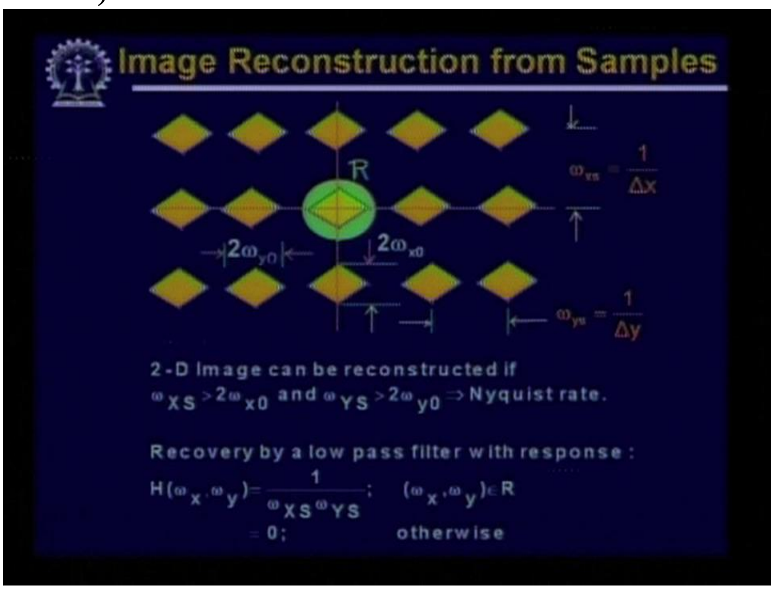

we have to take out a particular frequency band, say a frequency band which is around the origin in the frequency domain. And if we want to take out this particular frequency band, then as we have seen before that this signal has to be low-pass filtered and if we pass to this to a low-pass filter whose response is given by h “omega x” “omega y” is equal to 1 upon “omega x s” into “omega y s” for “omega x” “omega y” in the region R where region R just covers this central band. And it is equal to 0 outside this region R. In that case it is possible that we will be able to take out just these particular frequency components within this region R by using this low-pass filter. And again for taking out this particular frequency region the same condition of the Nyquist rate applies, that is sampling frequency in the x direction must be greater than twice of “omega x naught” which is the maximum frequency component along x . And sampling frequency along the y direction again has to be greater than twice of “omega y naught” which is the maximum frequency component along direction y.

So let us see some result.

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This is, here we have shown

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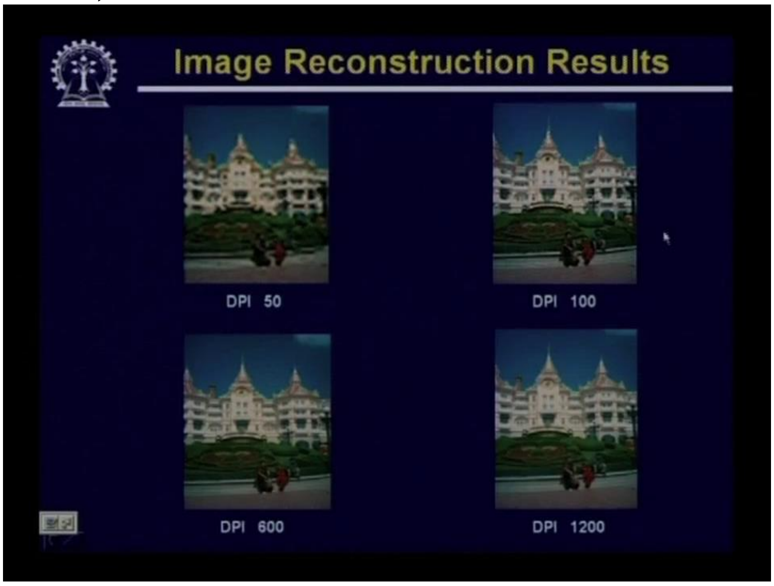

4 different images So here you will find that the first image which is shown here was sampled with 50 dots per inch or 50 samples per inch. Second one was sampled with 100 dots per inch, third one with 600 dots per inch and fourth one with 1200 dots per inch. So out of these 4 images you find the quality of first image is very, very bad. It is very blurred and the details in the image are not at all recognizable. As we increase the sampling frequency, when we go for the second image where we have 100 dots per inch, you find that the quality of the reconstructed image is better than the quality of the first image. But here again, still you find that if you study this particular region or wherever you have edges, the edges are not really continuous. They are slightly broken. So if I increase the sampling frequency further, you will find that these breaks have been smoothed out. So at, with a sampling frequency of 600 dots per inch, the quality of the image is quite acceptable. Now if we increase the sampling frequency further when we go from 600 dots per inch to 1200 dots per inch sampling rate, you find that the improvement in the image quality is not that much as the improvement we have got when we moved from say, 50 dots per inch to 100 dots per inch or 100 to 600 dots per inch. So it shows that when your sampling frequency is above the Nyquist rate you are not going to get any improvement of the image quality, where as when it is less than the Nyquist rate, the sampling frequency is less than the Nyquist rate, the reconstructed image is very bad. So till now

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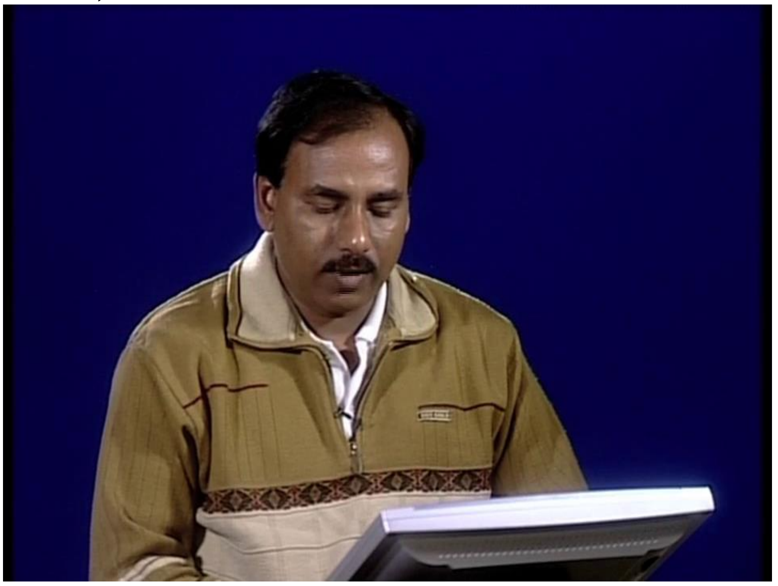

we have covered the first phase of the image digitization process, that is quantization and we have also through the examples of the reconstructed image that if we vary the sampling frequency below and above the Nyquist rate, how the quality of the reconstructed image is going to vary. Thank you.