Digital Image Processing
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Module 10 Lecture Number 49
Image Registration - 2

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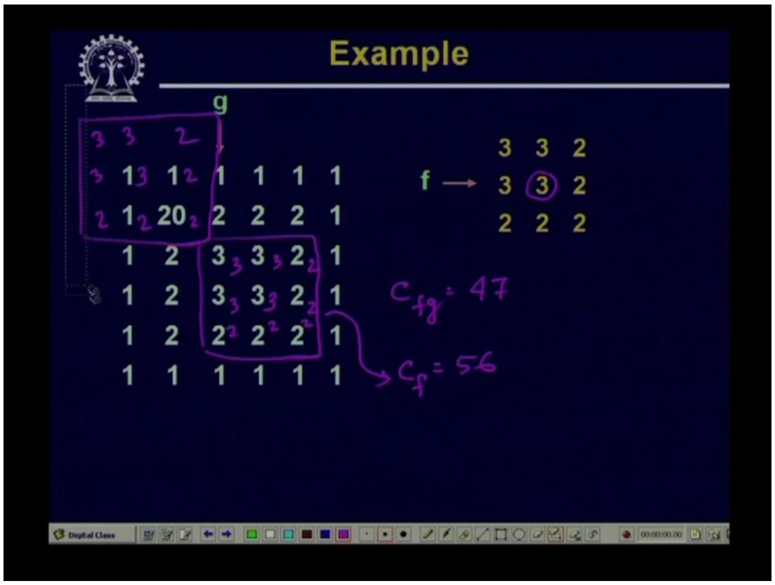

Say here this is the image g which is given in the form of a 2 dimensional matrix. 2 dimensional array of size 6 by 6 and our template is of 3 by 3 matrix which is given on the right hand side, ok. Now, if I calculate the Cfg for this, so what I have to do is, to find out the match location, I have to take the template, shift it all possible location in the given image g and find out the cross correlation or the similarity measure at that for that particular shift.

So initially let us put this template at the left most corner, and what we do is because the template 3,3,2 3,3,2 2,2,2. Let us match let us place the center of the template at the location at which we want to find out the similarity measure. So because of this 3 will be placed over here. this particular element will be placed over here this 2 will go here, this 2 will come here, this 2 will come here. And on the left hand side the other part of the template will be like this 2,3,3,3,2.

So this will be the position of the template and at this location we have to find out what is the similarity value. So for that let us find out what is Cfg, the cross correlation between f and g for this particular shift. Now, here if you compute you find that all this elements of the template are going beyond the image. So if I assume that the image components are zero beyond this boundary of the image then this element will not take part in the computation of cross correlation.

The cross correlation will be computed only considering this elements, nd here if you compute you will find that this Cfg will for this particular position for this particular sheet will attain a value of 47, because here it is 40 +2+2+3 so this becomes assume the value of 47. Similarly, if I want to find out the cross correlation at this particular location over here where the template is placed, the center of the template is placed over here. The other components of the template will come like this, you will find that cross correlation at this location is given by Cfg is equal to 56.

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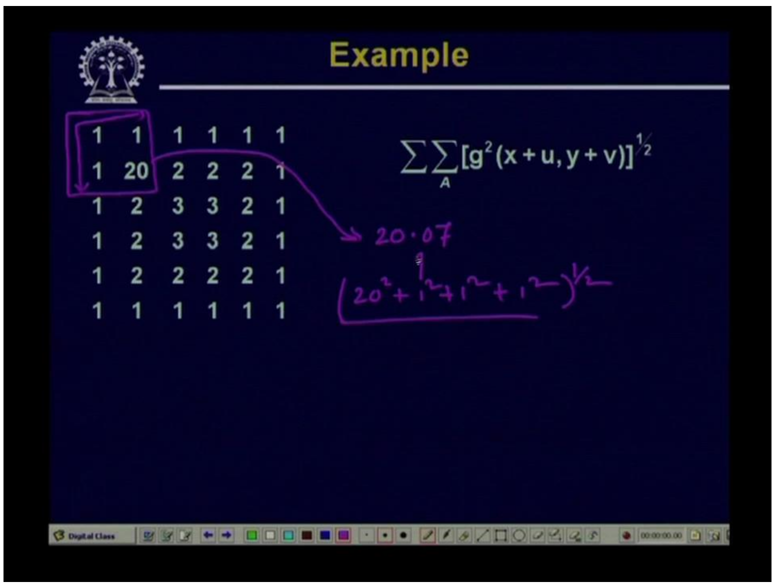

So like this for all possible shifts within this particular image I have to find out what is the cross correlation value. And if I complete this particular cross correlation computation in that case you will find that finally I get a cross correlation matrix which is like this. So this gives the complete cross correlation matrix when this template is shifted to all possible locations in this given image and the cross correlation value is computed for all such possible shifts.

Now from this you find that here the maximum cross correlation value is coming at this particular location which is given by 107 and if I take this 107 to be the similar or the cross correlation values to be the similarity measure, that means this is where I get the maximum similarity. And because of this it gives a falls match that it we it appears that the template is matching the best in this particular, as shown by this red rectangle.

But that is not the case because if I just by checking visually we can see that the template is best match in this location. So that is why we said that the cross correlation measure directly cannot not be used as a similarity measure.

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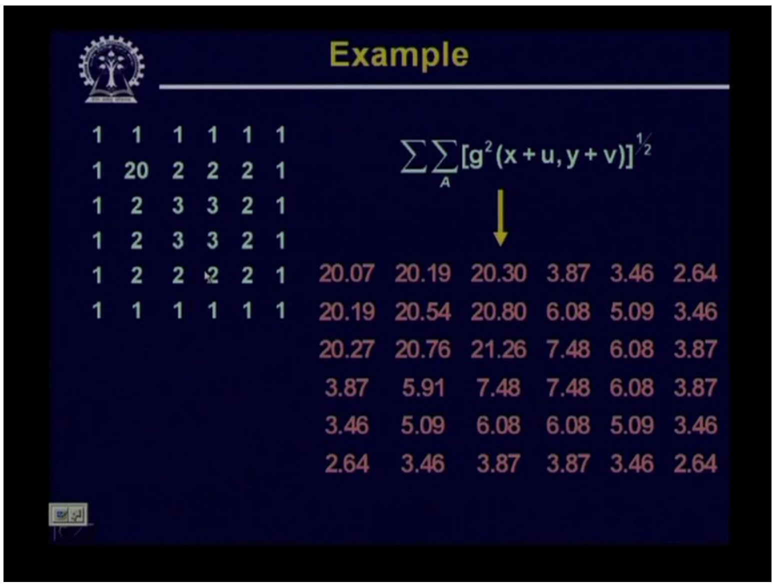

So over here as we have said that we cannot use the cross correlation measure directly as a similarity measure. So what we have to do is we have to compute the normalized cross correlation value. So for commutation of the normalized cross correlation we have to compute this particular components that is g square (x+u,y+v) double summation over the region A and square root of that. So this is the one that we have to compute for all possible shifts in the given image g and we have to normalized the cross correlation that we compute with the help of this quantity.

So if I compute this again let us take the same location, if I compute this g square (x+u,y+v) summation over the region A, square root of that summation over the A then we will find that this particular value will come out to be something like 20.07 because this is nothing but 20 square + 1 square + 1 square + 1 square, other elements for this shifts within this 3 by 3 wing window is equal to zero. So this is what we get I have to take the square root of this.

So if I compute these components I get the value of 20.07. So this is how if I compute this quantity. That is g square (x+u,y+v) over the region A square root of this and summation over the region A for all possible u and v that is for all possible shifts, then finally this normalization components that I get is given by this given by this particular matrix.

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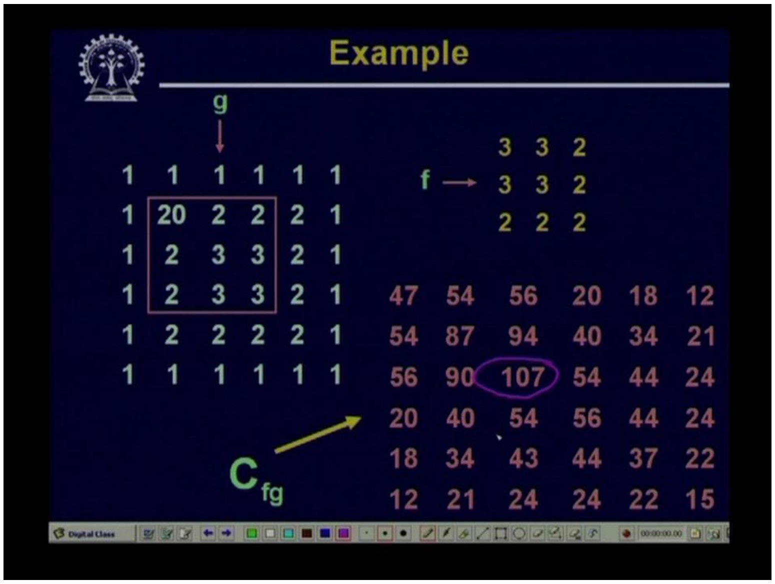

So this cross correlation normalization coefficient have been computed for all possible values of u and v, and then what we have to do is we have to normalized the cross correlation with this normalization factors.

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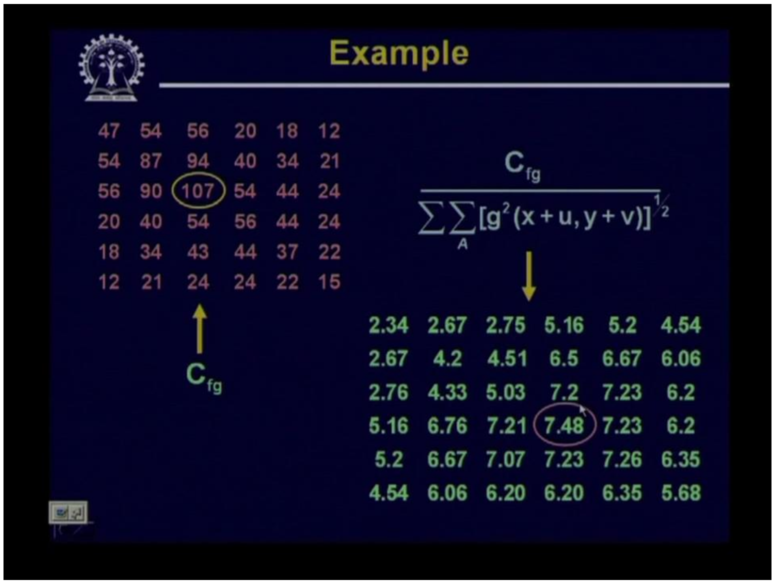

So if I do that normalization this is my original cross correlation coefficient matrix that I have computed earlier, and by using the normalization what I get is the normalization normalized cross correlation coefficient matrix which comes like this.

And now you find the difference in the original cross correlation matrix the maximum was given in this location which is 107. And in the normalized cross correlation matrix the maximum is coming at this location which is 7.48. So now let us see that using this as the similarity measure that is normalized cross correlation as the similarity measure, what happen to our matching.

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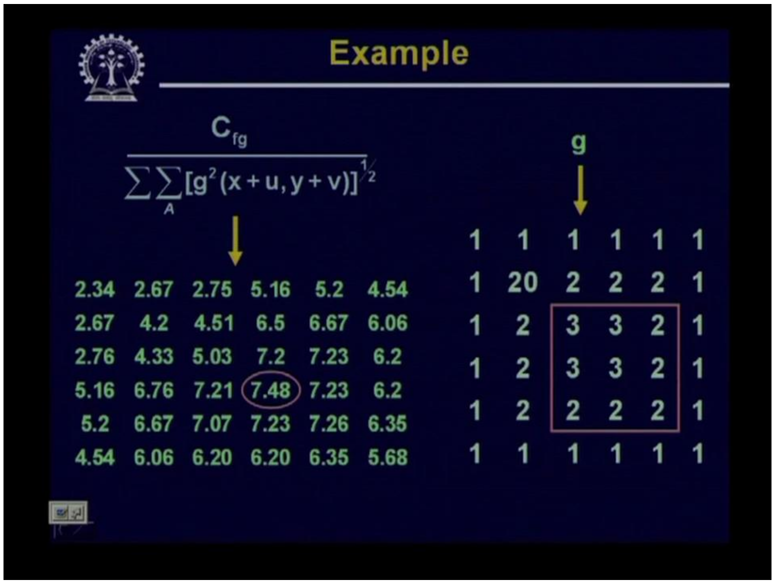

So this is the same matrix the normalized cross correlation matrix, the maximum 7.48 is coming at this location and for this maximum the corresponding match location of the template within the image given over here as shown by this red rectangle. So now you find that this is a perfect matching where the template is where the similarity measure given the perfect measure where the template matches exactly.

So that is possible with the normalized cross correlation but it is not possible using the simple cross correlation. So the simple cross correlation cannot be used as a similarity measure but we can use this normalized cross correlation as a similarity measure.

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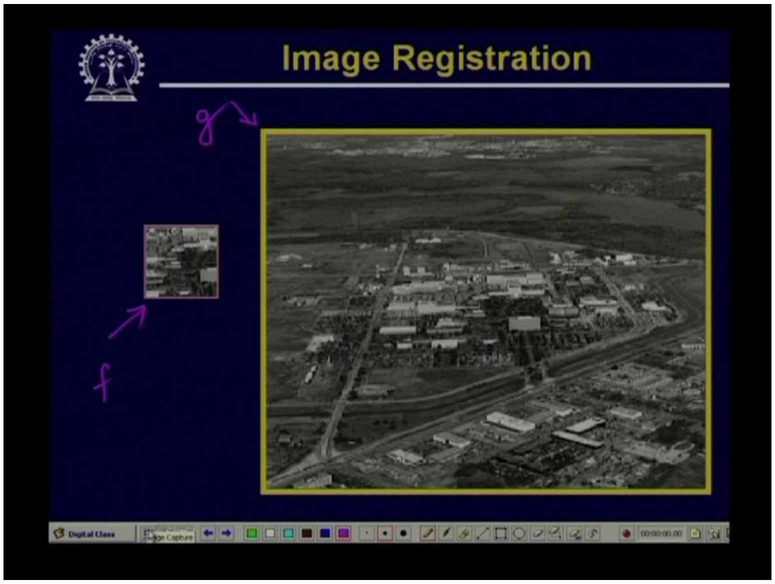

Now coming to application of this so this here we have shown that application over real image. So this is an aerial image taken from the satellite, and the smaller image on the left hand side which is the a part of the image which is cut out from this original image, and we are using this as a template. The smaller image we are using as a template. So this is our template f which we want to match against this image g. So we want to find out where in this given image g this template f matches the best.

So for doing that as we have said earlier that what we have to do is we have to paste this template A that is we have to shift this template A at all possible location in the given image g at for all possible such location we have it find out what is the normalized cross correlation? And wherever we get the normalized cross correlation to be maximum that is location where this template matches the best. So let us placed this template at different location in the image and find out what is the corresponding similarity measure we are getting.

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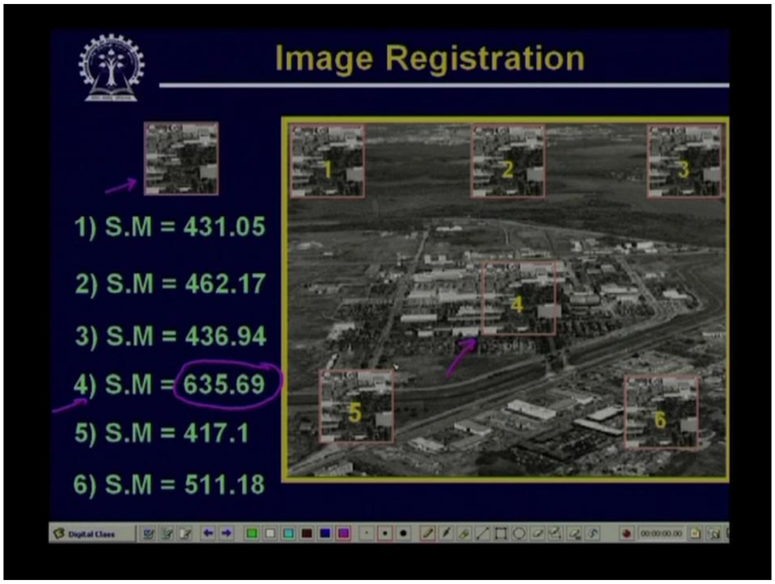

So this is the template and this the given image. If I place this template over here we are calling this as location1. So this location is given by this red rectangle. Then the similarity measure that we are getting is a value 431.05. If I place it location2, which is over here the similarity measure, or the normalized cross correlation is get is given by 462.17. If I place it at location 3 then at location 3 the similarity measure is 436.94. If I place the template at location 4 the corresponding similarity measure is coming out to be 635.69. If I place it location 5 the corresponding similarity measure is coming-out to be 417.1. If I place it at this location 6 the corresponding similarity measure comes out to be 511.18.

So you see that from its similarity measure value and if I compute for all possible location, of course for all possible location cannot show it on the screen. So if I compute this for all possible location then you will find that the similarity measure value is coming out to be maximum that is 635.69 for this location which is location 4 in the given image. And exactly this is the location from where this particular template f was cut out.

And if you look at this picture after placing this template you find that there is a almost a perfect match. So for a given image and a given template if I find out the normalized cross correlation for all possible shifts u and v then for the shift u and v where you get the normalized cross correlation to be maximum, that is the location where the template matches the best. So obviously this is a registration problem where we want to find out, where the given template matches the best in a given image.

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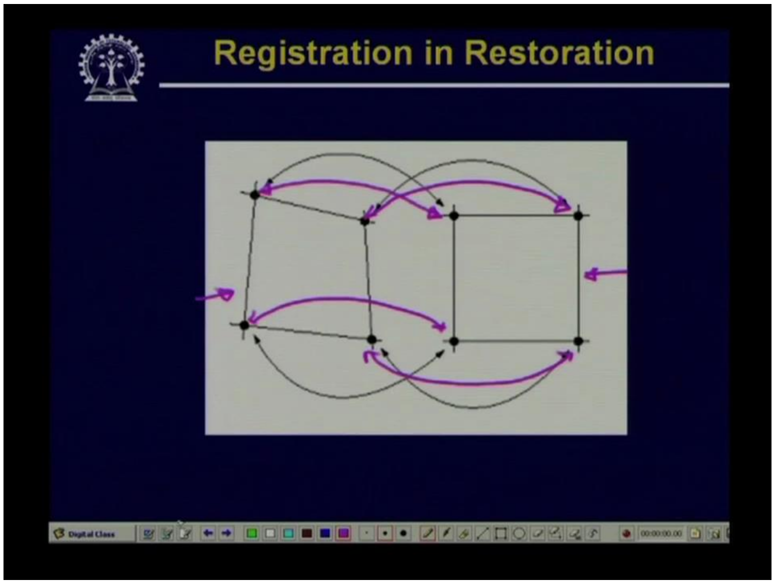

Now to come to the other application of this registration you find that this registration is also applicable for image restoration problem. Earlier we have talked about the image restoration problem, where you where you have it estimate that what is the degradation model which degrades the image. And by making use of this degradation model we can restore the image by different type of operation like inverse filtering, wiener flirting constrained least square estimation and all those defined kinds of techniques.

Now here we are taking about a kind of degradation where the degradation is given in the form a geometric distortion. And that is a distortion, which is introduced by optical system of the camera. So if you take an image of a very large area you might have noticed that as you move away from the center of the image the points try to become closer to each other. So that is something which leads to a distortion in the image as the point goes away from the center of the image.

So here we have shown one such distortion. So suppose this is the figure that we want to or for which we want to take the image. But actually the image comes in this particular form then this distortion, which is introduced in the image that can be corrected by applying this image registration technique. So for doing this what we have to do is we have to register different points in the expected image and the different points in the restored image in the degradation image of the distortion image.

So if I have once I can do that kind of registration, so is this case this is a point which corresponds to this particular point. So if somehow we can register this two that is we can find out we can establish the correspondence between this point and this point, we can establish the correspondence between this point and this point, we can establish the correspondence between this point and this point, and we can establish the correspondence between this point and this point then it is possible to estimate the degradation model.

So here you find that find out for estimating this degradation we have to go for registration. So this registration is also very very important for restoring a degraded image or the degradation is the introduced by the camera optical system. So the kind of restoration that will be, that can be applied here is something like this.

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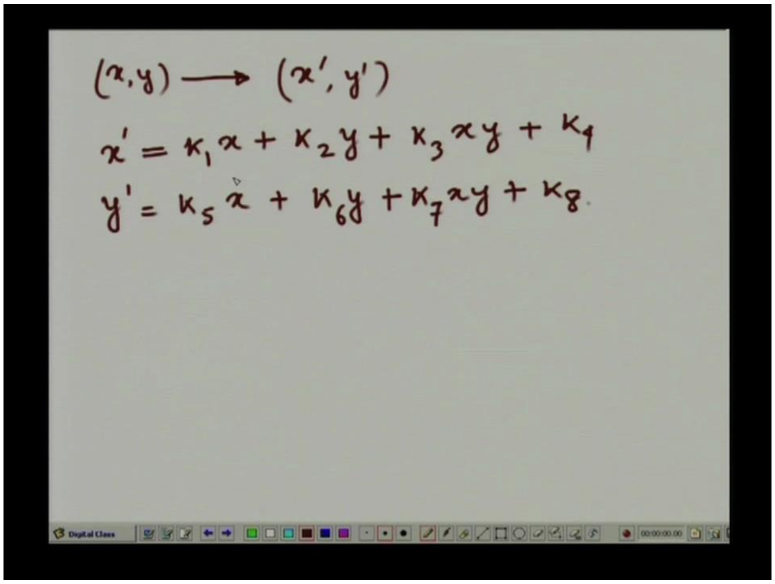

Say if I have an original point say (x,y) in the original image and this point after distortion is mapped to location say x time y time then what we can do is, we can go for estimation of a polynomial degradation model. In the sense that I estimate that x time is a polynomial function of x and y. So I write in this form that x time is equal to say some constant k1 times x + a constant k 2 times y + a constant k3 times xy + a constant k4. Similarly y time can be written as some constant k5 times x + k6 times y+ k7 times xy+ say k8.

So from this you find that if we can estimate this constant coefficient k1 to k8 then for any given point in the original image we can estimate what will be the corresponding point in the degraded image. So for estimation, for computing this k1 to k8 this a constant coefficient because there are 8 such unknown, I have to have 8 such equation. And those equations can be obtained by 4 pairs of corresponding point from the 2 images.

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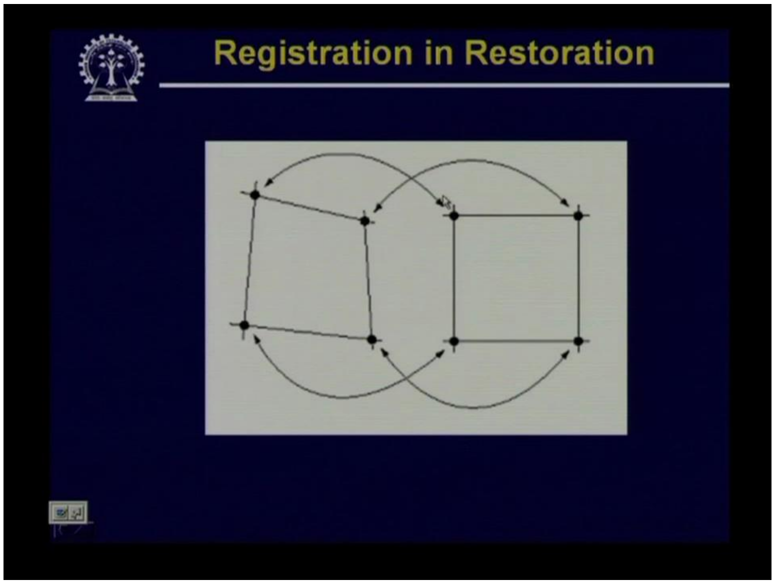

So that is what if I look at this figure. I have to get 4 such correspondence pair so once I have 4 such correspondence pairs I can generate 8 equations, and 8 using those 8 equations I can solve for all those constant coefficient k1 to k8. And once I get that then what I can do is I can say that I want an original image and undistorted image, and to a particular point to the undistorted image I apply that distortion find out a point in distorted image and whatever is intensity value at that location in the distorted image I simply copy that in my estimated location in the original image.

So I can find out a restored image from the distorted image. Obviously while doing this we will find that there will be some location where I don't get any information that is for a particular location, say pq in the estimates undistorted image, when I apply the distortion then in the distorted image I don't get any point at that particular location. So in such cases we have to go for interpolation techniques to estimate what will be the intensity value at that point location in the distorted image. So for that the different interpolation operation that we discussed earlier can be used.

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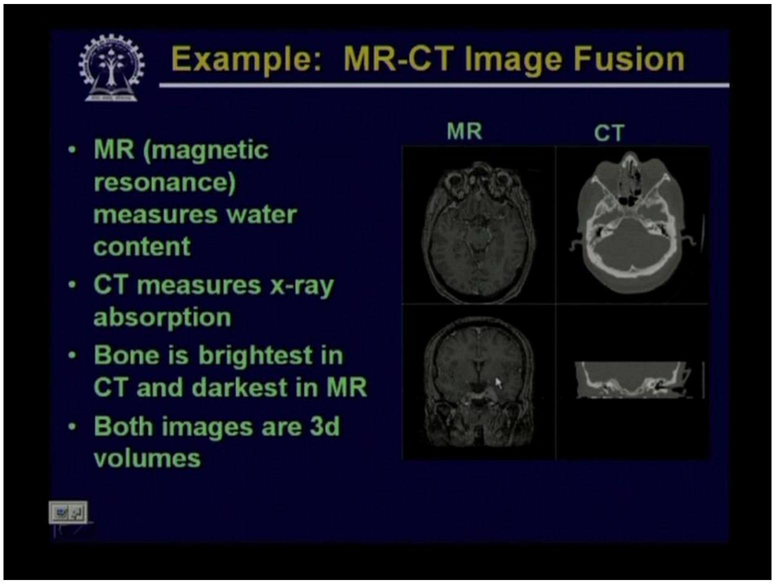

So this is how this image restoration technique is also playing a measure role in restoration of distorted images. This image registration technique is also very very useful as I said in image fusion or combining different images. And for that also we have to go for image registration. Say for example here we have given shown 2 types of images, one at magnetic resonance images and CT scan images.

Now magnetic images that give you a measure of water contained whereas in case of CT images, CT XT images is that gives you a brightest region for the bone region ok. So if I combine, if I can combine the magnetic resonance image along with the CT XT image. The fused image that you get where you can get both the information that is the water contained as well as the nature of the bone in the same image. So naturally the information extraction is much more easier in the fused image.

Now again for doing this the first operation has to be image registration because the alignment of images of the MR images and the CT images even if there same region they may not be proper aligned or they may not have properly scaled. Then may not be uh there may be a some distortion images. So the first operation we have to do is we have to go for registration, using registration we have to get that what is the transformation that can be applied to align properly align the 2 images. And after that transformation applying the transformation when we align the 2 images then only they can fused properly.

So here there are 2 MR images and there are 2 CT images and obviously you find that this MR images and this CT images though they are of the same region but they are not properly align. Similarly on the bottom row this MR images and this CT images they are not properly aligned. So first operation we have do is alignment.

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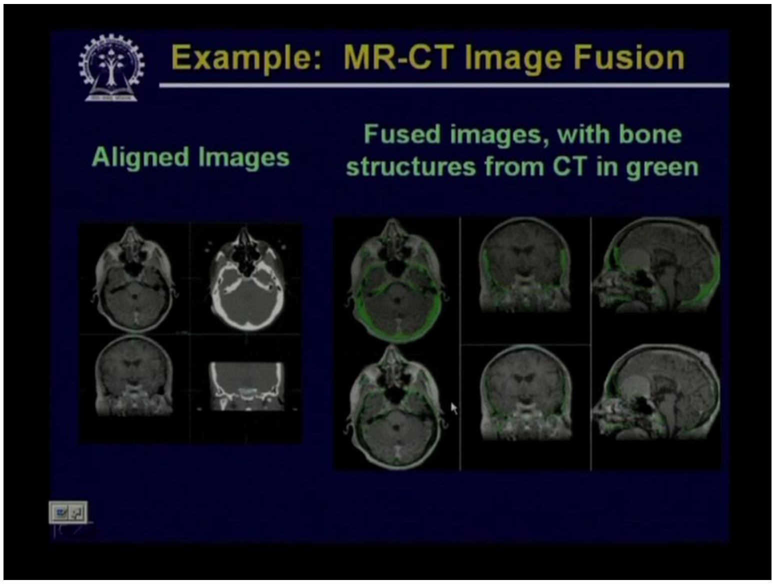

So in this slide form the right hand side what we have got is the result after alignment, and of the right hand side it is the result after fusion. So in this fused image you find that the green regions this shows you what is the bone structure and this bone structure has been obtained from the CT image. And the other regions can get the information from MR image. So this is much more convenient to interpret then taking the MR image and the CT image separately.

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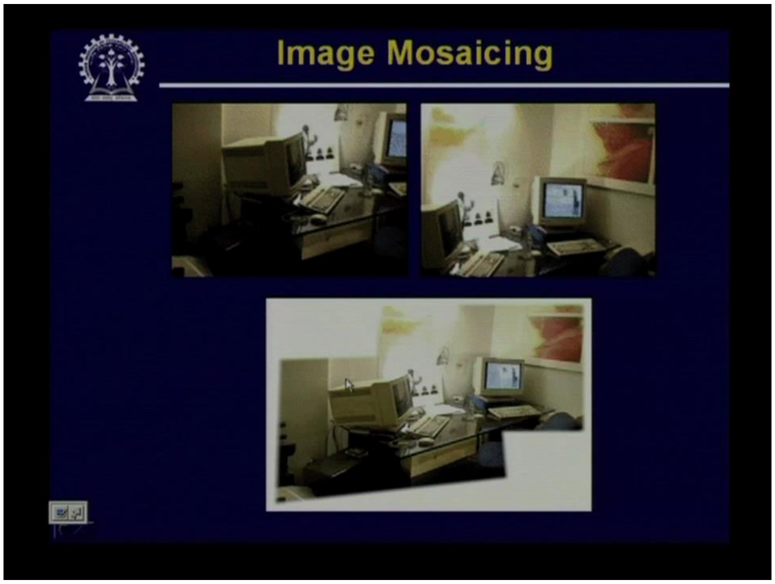

The other application of this is for image mosaicking that is the normally the cameras have very narrow field of view so using a camera which is having a narrow field of view you cannot image a very large area.

So what can be done is you take the images, smaller images of different region of in the scene and then you try to stretch those smaller images to give you a large field of view image. So that is what has shown here on the top there are two smaller images. These 2 images are combined to give a big a bigger image as shown on the bottom. So this is a problem which is called image mosaicking and here again because these are 2 images they may be skilled differently, their orientation may be different so firstly we have to go for normalization and alignment, and for this normalization and alignment again the first operation has to image registration.

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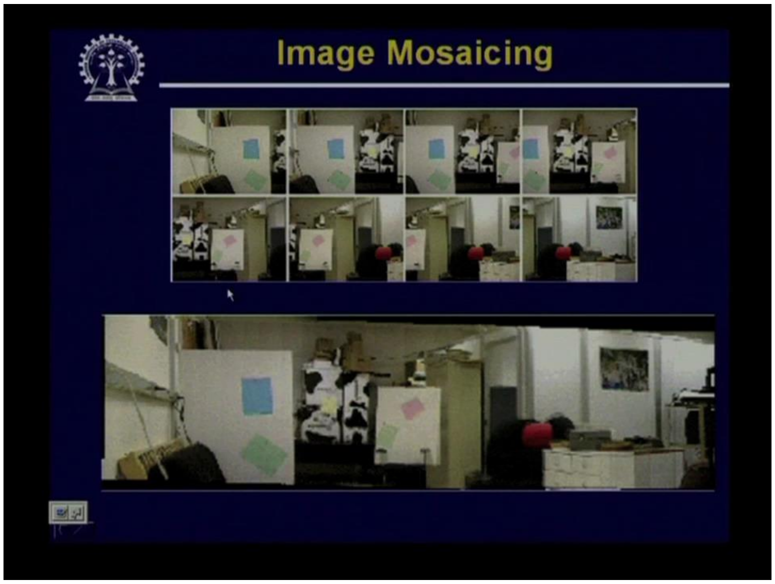

this shows another mosaicking example where this bottom image has been obtained from top 8 images. So all this top 8 image have been combined properly to give you the bottom images. So this is the mosaicking that image get. So with this we complete our discussion on image registration. Thank you.