Digital Image Processing
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Module 10 Lecture Number 48
Image Registration - 1

Hello, welcome to the video lecture series on Digital Image Processing. For our last few classes we have talked about different types of image restoration techniques. In today's lecture we are going to talk about another topic which is called Image Registration.

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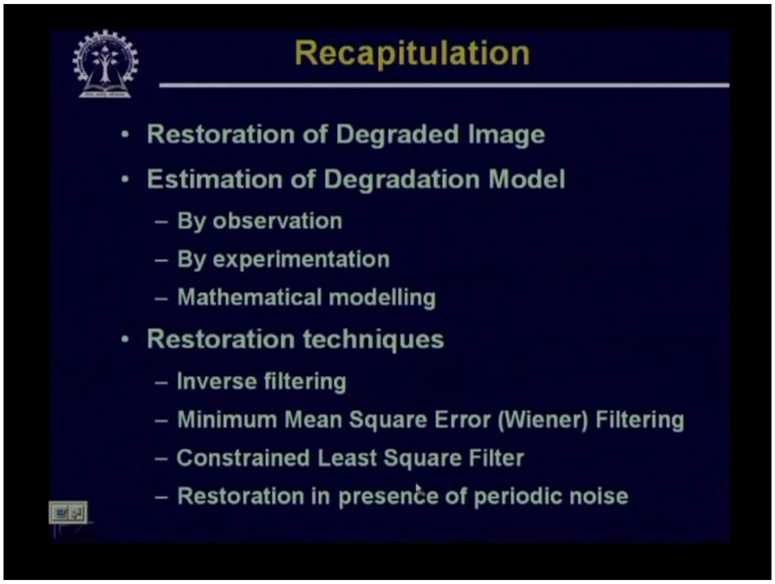

So in our last few lectures we have talked about restoration of degraded image. We have seen different techniques for estimation of the degradation model. And the different model estimation techniques we have discussed or the estimation of the degradation model by observation. Estimation of the degradation model by experimentationa and the mathematical modeling of degradation.

And once you have the model the degradation model then we have talked about the restoration techniques for restoring a degraded image. So among the different restoration techniques we have talked about the inverse filtering techniques. We have talked about minimum mean square error or wiener filtering technique. We have talked about the constrained least square filtering approach. And we have also talked about the restoration of an image if the image is contaminated by periodic noise.

So in such case we have seen that if I take the Fourier transform of the degraded image, if the is image actually degraded by a periodic noise or a combination of periodic noise then those noise components, the noise frequency appears as very bright spots, very bright dots in the Fourier transformation or in frequency plane. So there we can apply a band reject filter or some time notch filter to remove that particular frequency component of the Fourier transform, and after performing the band reject operation whatever is remaining if we take the inverse Fourier transformation of that then we get the restored image which is free from the periodic noise.

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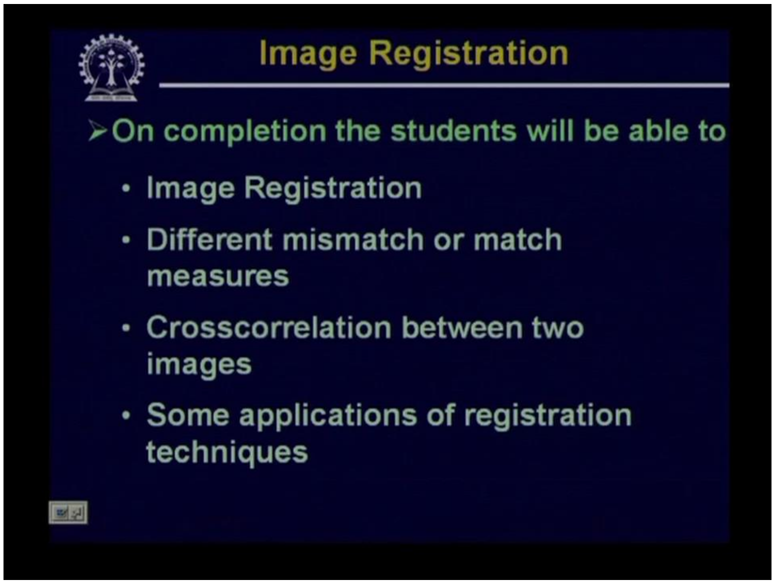

So in today's lecture we will talk about the image registration techniques. So you see what is Image Registration, then we go for image registration and then we have to think of the mismatch or match measures. So will talk about the different mismatch or match measures or similarity measures. We will see that whether the cross correlation or will see what is the cross correlation between two images.

And we will also see whether this cross correlation can be used as a similarity measure when we go for image registration. And then will talk about some application of this image registration techniques with examples.

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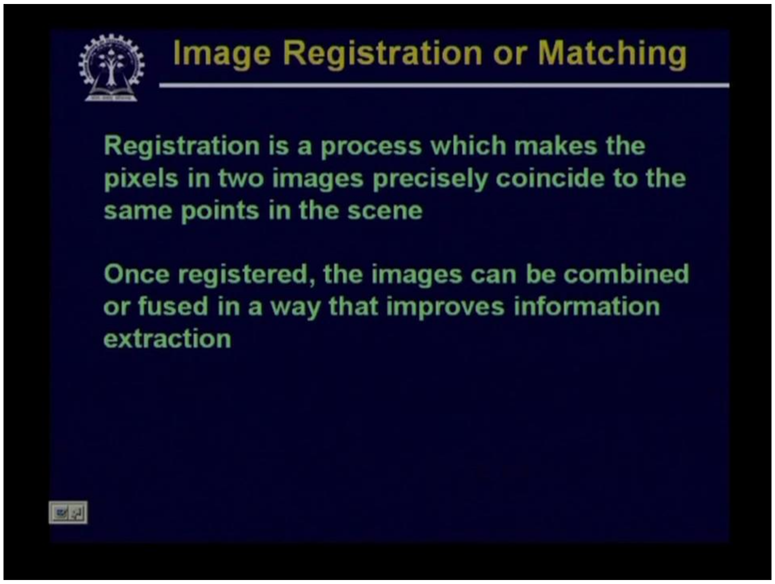

So by image registration what we mean is that the registration is a process which makes the pixels in two images precisely coincide to the same points in the scene. So by registration what you mean is, if we are having two images of the same scene, may be the images, two or images are acquired with different sensor located at different positions or maybe the images are acquired using the same sensor but at different time instance at different instant of time.

So in such cases if we can find out for every pixel in one image, the corresponding pixel in the other image or images that is the process of registration or the process of matching. And this has various applications that we will talk about a bit later. So once registered the images can be combined or fused, this is called fusion or combination.

So once we have the images from different sensors may be located at different location or maybe if it is remote sensing Image taken trace of light where we have images taken in different bands of frequencies, then if we register all those different images then those images can be combined or they can fused. So that the information extraction or the image the fused image becomes more rich in the information contained.

So once registered the images can be combined or fused in a way that improves the information extraction process. Because this fused image now they have combination of the information from different images or different bands of frequencies they will have more information or there will be more rich in the information contained.

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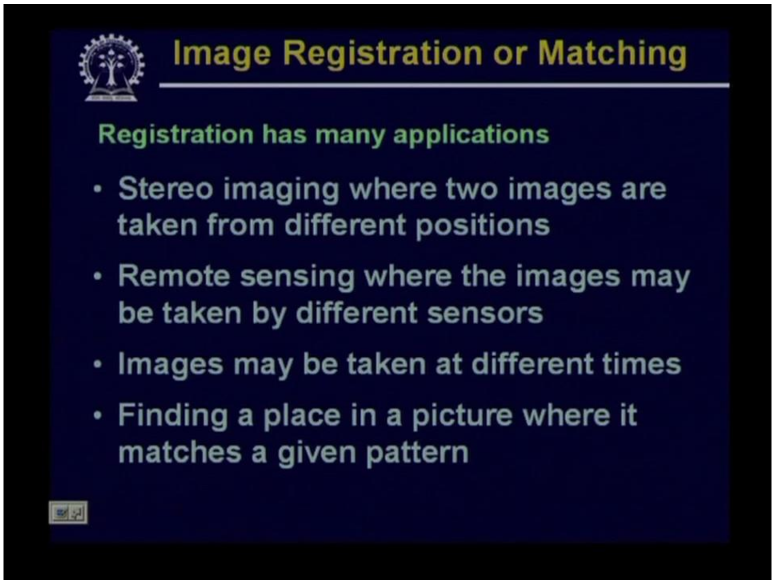

So this image registration technique has many applications. The first application we have already said that the stereo imaging techniques. In stereo imaging what we do is, we take the image of the same scene or the images of the same object by two cameras which are slightly displaced and they are we are assumed that the cameras are otherwise identical that is only apart from the displacement along a particular axis y axis or x axis, the feature of the camera are identical that is they have same focal length same view angle, and all this things.

So once i acquired these images that one of them we call as left image and the other one is called as right image, then if I go for this point by point correspondence that is for a particular point in the left image if I can find out, what is the corresponding point in the right image then from this two I can find out what is the disparity for that particular point location.

And if this disparity is obtained for all the point in the images then from the disparity we can find out what is the depth or the distance of the different object points from the camera. So that is in case of stereo imaging we have to find out this point correspondence, or the point registration or this is also called point matching.

The other application that we have just said that in remote sensing what the images can be taken by different sensors, working different bands, even the sensors may be located in different locations. So the images of the same scene are taken by different sensors working in different bands, they are also in different geometrical location. They are also, if we go for image registration that is point by point correspondence among different images, then we can fuse those different images or you can combine those different images so that fused image become more rich in terms of information contain. So the information extraction from such fused images becomes much more easy.

The other application again the images may be taken at different times. So if the images taken from different times and in all those images if we can register those images that is we can find out which is the point that a at a particular for a particular point in a given image what is the correspondence point in the other image which is taken at some other time instant.

Then by this registration we can find out what is the variation at different point location in the scene. And from this variation we can a extract many information like vegetation growth or may be the land erosion or deforestation or the occurrence of a fire so all this different information can be obtained when we go for registration of the images which area taken at different instance of time.

There are other application like finding a place in picture where it matches a given pattern. So here we have a small image which is known as a pattern or a template and we want to find out that in another image which is usually of bigger size where does this template match in the best. Now this as various application like automated navigation where want to find out that what is the location of a particular object with respective map.

So in such automated navigation applications this pattern matching or template matching is very very important. So this image registration techniques of the image registration method as various other application and which can be exploited once the registration techniques are known.

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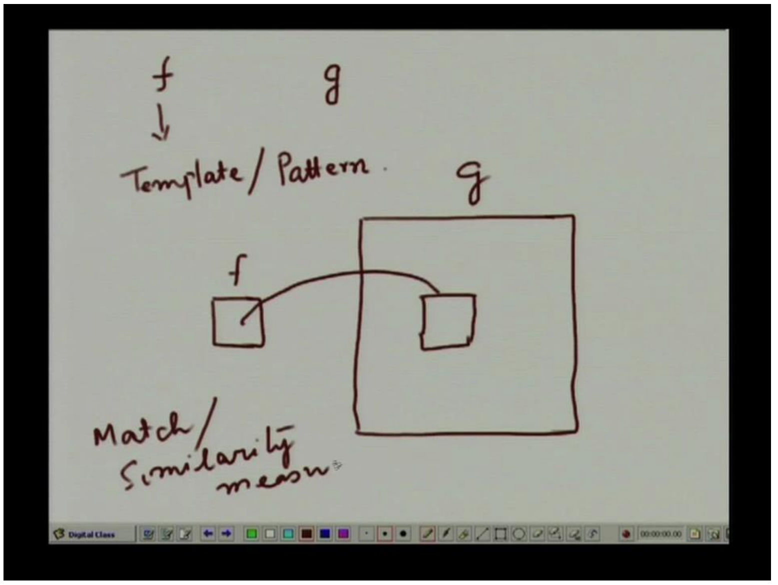

Now to explain the registration techniques let us take the first example that example of that the template matching. So for this template matching we take a template of a smaller size. We called the template to be a template F. Say F is a two dimensional image of a smaller size and we have an image g which is a bigger size. So the problem of template matching is to find out what this F, the template F matches best in the given image g.

So this F is called a template or it is also called a pattern. So our aim is that given a very big image say g, a two directional image g and a template F which is usually of size smaller than that of g. We want to find out that where this template f is matches best in the image g. So to find out what the template f matches best in the given image g we have to have some measure of imagine, which is called which may be called as mismatch measure or the opposite of this that is the match measure or the simulating measure.

So we have to take different match or similarity measure, so you call them match measure or similarity measure to find out where this template F matches the best in the given image g. So there are various such match or mismatch measures. And let us see that what are the different measures that can be used.

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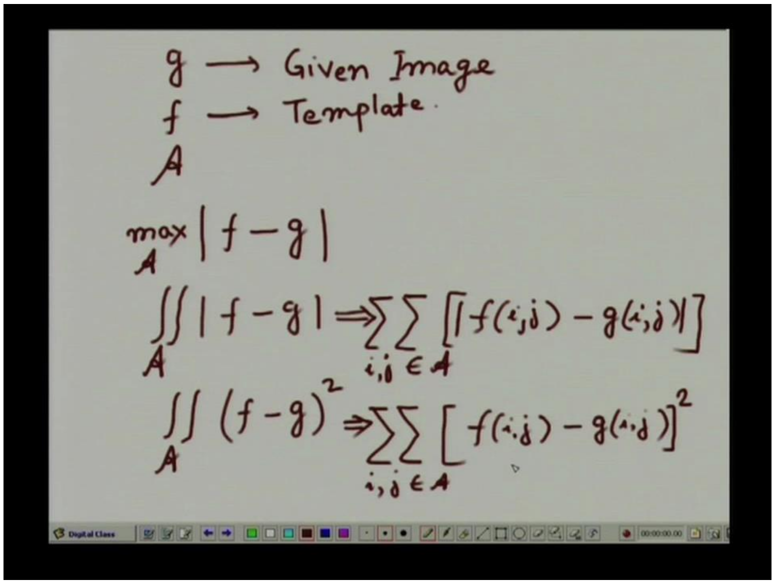

So we have the given image g. G is the given image. And we have the temp template f. So f is the template. So we have to find out the measure of similarity between a region of the image g and the template f. So there are various ways in which this similarity can be measured so that we can find out the match between the f and g over a certain region say given by A. So one of the similarity image or the simplest similarity image is we take the difference of F and g. So you take the absolute difference between f and g.

And find out the maximum of the absolute difference. And this maximum has to be computed over the region say A. The other simulate image can be that again we take the absolute difference of f and g. And then integrate this absolute difference over the same region A. The other similarity measure can be you take f-g the difference between f and g and the square of this and take the integration of this over the same region A.

So you find that in case when it is the difference between f and g so when I'm talking about the difference it is pixel by pixel difference. So it takes the difference between f and g take the absolute value and the maximum of that computed over the given region A. In the second case it is f-g again the absolute value and this is integrated over the region A. So this is in the analog case if I convert this in the digital form this will take the form as f (ij) – g (IJ) where ij is the pixel location.

Absolute value of this and you take the double summation for all I and j in the given region A. So you find that this is nothing but what is called the sum of the absolute difference. So sum of the absolute difference between the image g and the template f over the region A, and if I convert this expression again in the digital form this becomes f (ij)–g (ij) square of this. And take the double summation for all ij belonging to that region A.

So this is nothing but some of the difference square so if I say that f (ij) – g (ij) is the difference between two images or the error so this last expression that is a f - g square integration over A double integration over A. In the digital domain it becomes f (ij) – g (ij) square, take the double summation for all ij in the region A, this is something equivalent to some of square error.

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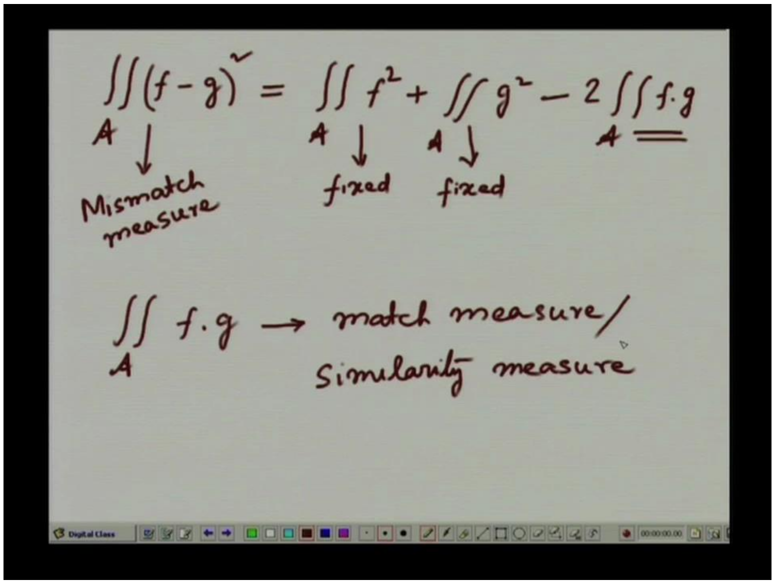

Now out of these three different measures. It is the last one gf- g square integration, this is very very interesting. So if I expand this terms f- g square double integration over the region A, if I expand this then this becomes double integration f square plus double integration g square minus 2 into double integration f into g. So all these double integrations are to be taken over the given region A.

Now find that for a given image this double integration f square this is fixed, for a given template at this is fixed and also for a given image over the region A. This double integration g square this is also fixed. Now, what is this f - g square sum of this, this is nothing but a sum of the square differences that means this gives the degree of mismatch, or this is nothing but the mismatch measure, ok. So if this f-g square integration over the region A is minimum because this is the mismatch measure so wherever this is the minimum at that particular location f matches the base over that region that particular region of g.

Now when I expand this, this becomes integration double integration f square plus double integration g square minus twice into double integration f into g, and as I said for a given template f square is fixed at for a given image and a given region g square is also fixed that means this f-g square integration this will be minimum, when this f into g double integration this term will be maximum.

So whenever the mismatch measure is minimum that will lead to f into g double integration over the region F this will be maximum. So when f - g square double integration is taken as the measure of mismatch we can take the integration f into g over the region given region A, this to be the match measure or the similarity measure. So this means that whenever the given template matches the best in a particular region in a particular portion of the given image g in that case f into g double integration over the region A will have the maximum value and we take this as the similarity measure or the match measure.

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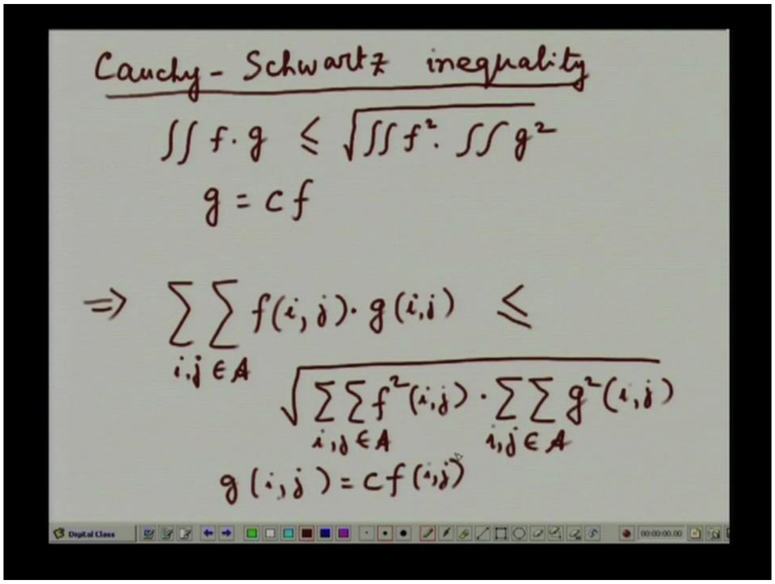

Now the conclusion can also be drawn from what is called Cauchy Schwartz Inequality. So this Cauchy Schwartz inequality says that double integration of f into g, this is less than or equal to square root of double integration of f square into double integration of g square. And these two terms will be equal only when g is equal to some constant times f. So this Cauchy Schwarz inequality says that f into g double integration will be less than or equal to square root of double integration x square into double integration g square, and the left hand side and the right hand side will equal, whenever g is equal to c times f otherwise left hand side will always be less than the right hand side.

So this also says that whenever f or the template is similar to a region of the given image g within with a multiplicative factor of constant C then this f into g integration will take on the maximum value, otherwise it will be less. If I convert this in to the digital case then the same expression is taken, can be written in the form that double summation f (I, j) into g (I, j) where I and j belongs to the given region A. This should be less than or equal to square root of double summation f square (I, j) into g square (I, j), again.

Sorry this has to be double summation g square I, j. For this I and j belonging to the region A, here also for I and j belonging to the region A. And this left hand side and the right hand side will be equal only when g (I, j) is equal to sum constant into f (I, j) and this has to be true for all values of I, j within the given region. So for this template matching problem we have assume that f is the given template and g is the given image and we also assumed that f is less, than the size of the given image g.

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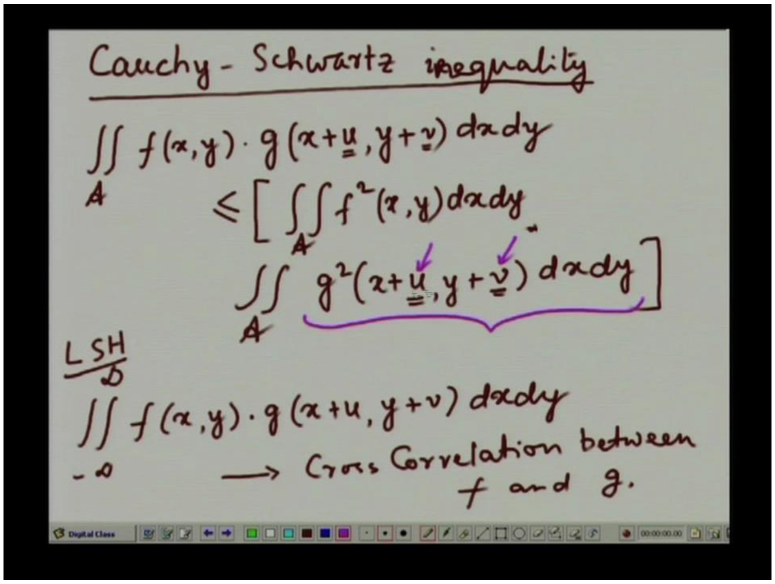

Now, again from the Cauchy Schwartz inequality, so I go back to this Cauchy Schwartz inequality, what we get that double integration f (x, y) into g (x + u, y + v) into dxdy should less than or equal to double integration f square (x , y) dxdy into double integration g square (x, y) into dxdy. Sorry this should be this into double integration g square (x+u, y+v) dxdy. Now the reason we are introducing term this two variables u and v is that whenever we try to match the given pattern f against the given image g we have to find out what is the match measure or the measure similarity measure at different location g.

So for this f has to be shifted at all possible location in the given image g and that shift the amount of shift that has to be given to the pattern f to find out the similarity measure at that particular location we introduced this two shift components u and v. So this says that shift along x direction u and shift along y direction is v. So here, in this expression the similarity between the given template F and the image g with shift u and v is computed. And this is computed over the given region A.

Now because this f (x,y) is small at the value of f (x,y) is zero outside the region A, so we can replace this left hand side by an integration of this form F (x,y) into g (x+u, y+v) into dxdy. And as I said that because f(x,y) is zero outside the region A. So this definite integral the integral over

A, can now be replaced by an integral from minus infinity to infinity. So this is what we get from this left hand side of the expression.

Now if you look this particular expression that is f (x,y) into g (x+u,y+v) dxdy double integral from minus infinity to infinity, this is nothing but the cross correlation between f and g. And then if you look at the right hand side this f square (x,y) dxdy integral over A. This is a constant for a given template, whereas this particular components that is g square into (x+u,y+v) dxdy this is not a constant, because the value of this depends upon the shift u and v.

So though from the left hand side we have got that this is equivalent to cross correlation between the function f and g but this cross correlation directly cannot be used as similarity measure because the right hand side is not fixed, though f square (x,y) dxdy is fixed but g square x, y (x+u, y+v) dxdy integral is not fixed, it depends upon the shift u and v. So because of this the cross correlation measure cannot be directly used as a simulate measure or a match measure. So what we have to go for is, what is called a normalized cross correlation

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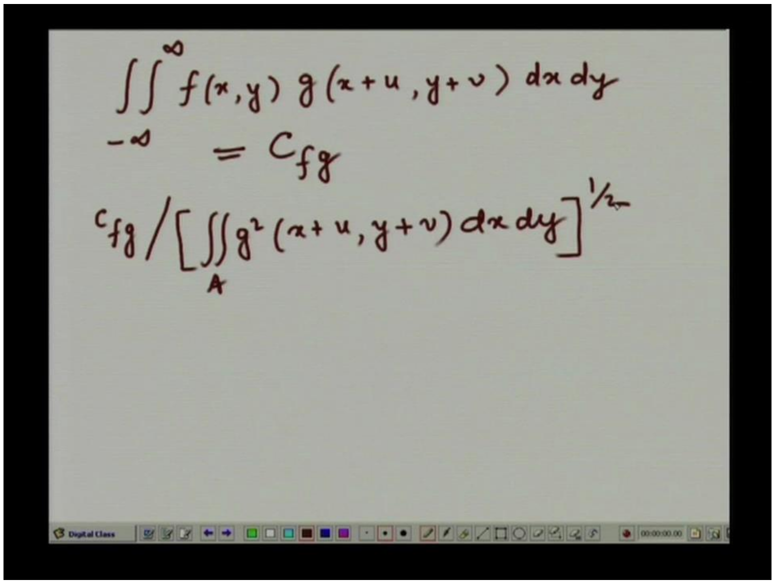

So if we call this cross correlation measure f (x,y) g (x+u,y+v) dxdy integration from minus infinity to infinity, if I represent this as the cross correlation Cfg then the normalized cross correlation will be given by Cfg divided by g square (x+u,y+v) dxdy double integral over the region A and square root of this.

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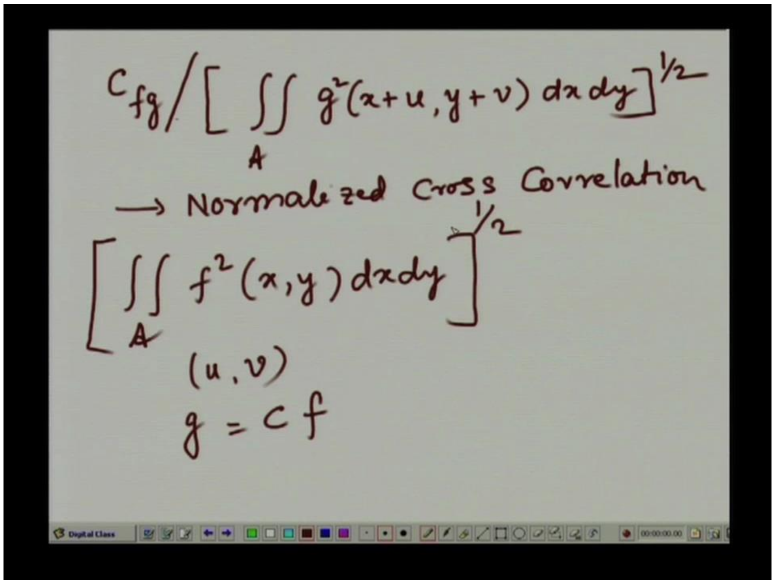

So in our previous one, so what is said is that Cfg has to be, the normalized cross correlation will be Cfg divided by double integral g (x+u, y+v) dxdy where the integration is taken over the region A and square root of this. And you see that once I take this normalized cross correlation this is what we are calling as normalized cross correlation. So once we consider this normalized cross correlation you find that Cfg will take the maximum possible value which is given by double integral f square (x,y) dxdy, take the integral over the region A and because this is fixed so this region of integration is not very important.

Square root of this, this is the maximum value which will be attained by this normalized cross correlation for a particular value of u and v, for the value of u and v for which this function g becomes some constant C times f. So for that particular value, that particular shift u, v where g equal to some constant C, C times f, this normalized cross correlation will take the maximum value and the maximum value of this normalized cross correlation is given by double integral f square (x,y) dxdy and square root of this ok. Thank you.