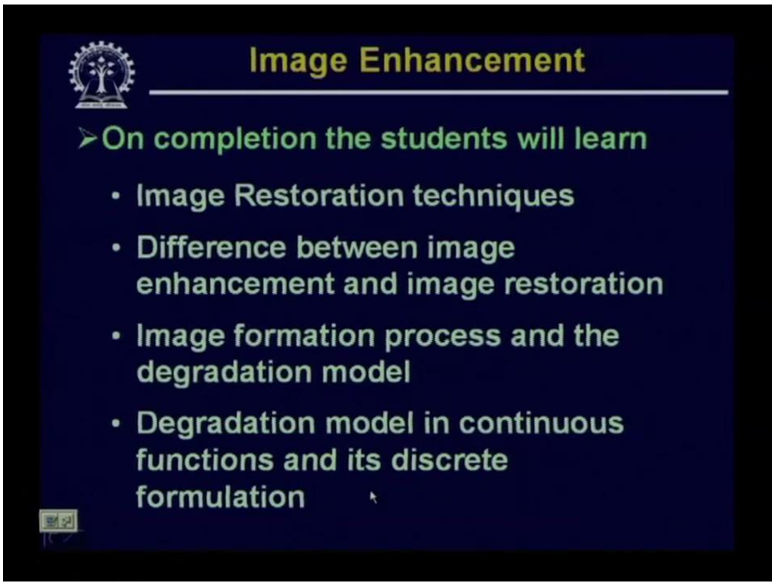
Digital Image Processing.
Professor P. K. Biswas.
Department of Electronics and Electrical Communication Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-42.
Image Restoration Techniques-I.

(Refer Slide Time: 0:31)



Hello, welcome to the video lecture series on digital image processing. Now in today's lecture, or in a number of lectures starting from today, we will talk about image restoration techniques. So we will talk about image restoration techniques, and we will see what is the difference between image enhancement and image restoration, we will talk about image formation process and the degradation model involved in it, and we will see the degradation model and the degradation operation in continuous functions and how it can be formulated in the discrete domain.

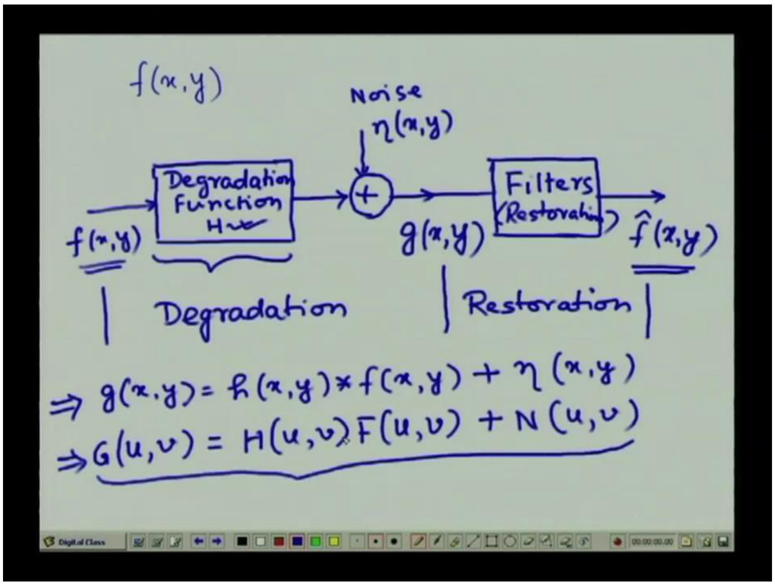
Now when we have talked about the image enhancement, particularly using a lowpass filter or using smoothings masks in the spatial domain, we have seen that one of the effect of using a lowpass filter or the effect of using a smoothing mask in the spatial domain is that the noise content of the image gets reduced. The simple reason is the noise content leads to high frequency components in the displayed image. So if I can remove or reduce the high frequency components that also leads to reduction of the noise.

Now this type of reduction of the noise is also a sort of restoration. But these are not usually termed as restoration, rather a process which tries to recover or which tries to restore an image which has been degraded by some knowledge of a degradation method which has degraded the image. This is an operation which is known as image restoration. So in case of image restoration, the image degradation model is very very important. So we have to find out what is the phenomenon or what is the model which has degraded the image and once that model the degradation model is known, then we have to apply the inverse process to recover or restore the desired image.

So this is the difference between an image enhancement or simple noise filtering in terms of image enhancement and image restoration, that is in case of image enhancement or simple noise filtering, we do not make use of any of the degradation model or we do not bother about what is the process which is degrading the image. Whereas in case of image restoration, we will talk about the degradation model, we will try to estimate the model that has degraded the image and using that model we apply the inverse process and try to restore the image.

So the degradation modeling is very very important in case of image restoration. And when we try to restore an image, in most of the cases we define some goodness criteria. So using this goodness criteria, we can find out an optimally restored image which more or less which is almost same as the original image. And we will see later that image restoration operations can be applied as in case of image enhancement both in the frequency domain as well as in the spatial domain.

(Refer Slide Time: 4:15)



So first of all, let us see that what is the image degradation model that we will consider in our subsequent lectures. So let us see the image degradation model first, so here we assume that our input image is image f(x,y) it is a two dimensional function as before and we assume that this f(x,y) the input image f(x,y) is degraded by a degradation function H. so we will put it like this, that we have a degradation function H which operates on the input image f(x,y).

Then the output of this degradation function is added to an additive noise, so here we add a noise term which we represent by, say eta(x,y) which is added to the degradation output, and this finally gives us the output image g(x,y). So this g(x,y) is the degraded image which we want to recover, so from this g(x,y) we want to recover the input image the original input image f(x,y) using the image restoration techniques. So for recovering, this f(x,y) what we have to do is, we have to perform some filtering operation and we will see later that these filters, they are actually derived using the knowledge of the degradation function that is H. And output of the filters is our restored image and let us put it as f hat(x,y) and we put it as f hat(x,y) because in most of the cases, we are unable to restore the image exactly. That means it is very difficult to get the exact image f(x,y).

Rather, by using the goodness criteria that we have just mentioned, what we can do is, we can get an approximation of the original image f(x,y). So that is this reconstructed image f hat(x,y) which is an approximation of the original image f(x,y). So the blocks from here to here, that is up to obtaining g(x,y) this is actually the process of degradation. So you find that in the degradation, we first have a degradation function H which operates on the input image f(x,y), then the output of this degradation function block that is added with an additive noise which in this particular case we have represented as eta(x,y) and this degradation function output added to this additive noise, that is what is the degraded image that we actually observed.

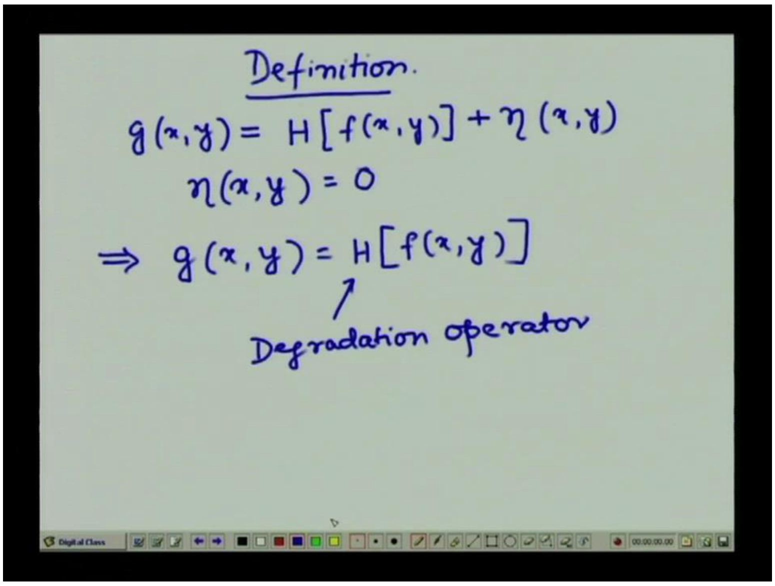
And this degraded image is filtered by using the restoration filter, so this filters that we use they are actually restoration filters. So this g(x,y) is passed through the restoration filters where we get the filter output as the reconstructed image f hat (x,y). And we as we have just said that this f(x,y) is an approximation of the original image f(x,y). So this particular block which represents an operation this is a restoration operation and as we have said that the process we call as restoration, in that the knowledge of the degradation model is very very essential.

So one of the fundamental task, one of the very important task in the restoration process is to estimate the degradation model of, the degradation model which has degraded the input image. And later on we will see various techniques of how to estimate the degradation model, that is how to estimate the degradation function H. And we will see in a short while from now that this particular operation, that is the conversion from f(x,y) to g(x,y) this can be represented in spatial domain as g(x,y) = h f(x,y) convolution with f(x,y) + the noise eta(x,y).

So this is the operation which is done in the spatial domain and the corresponding operation in frequency domain will be represented by G(u,v) = H(u,v) into F(u,v) + N(u,v), where H(u,v) is the Fourier transformation of h(x,y), F(u,v) is the Fourier transformation of the input image f(x,y), N(u,v) is the Fourier transform of the additive noise eta(x,y) and G(u,v) is the Fourier transform of the degraded image g(x,y).

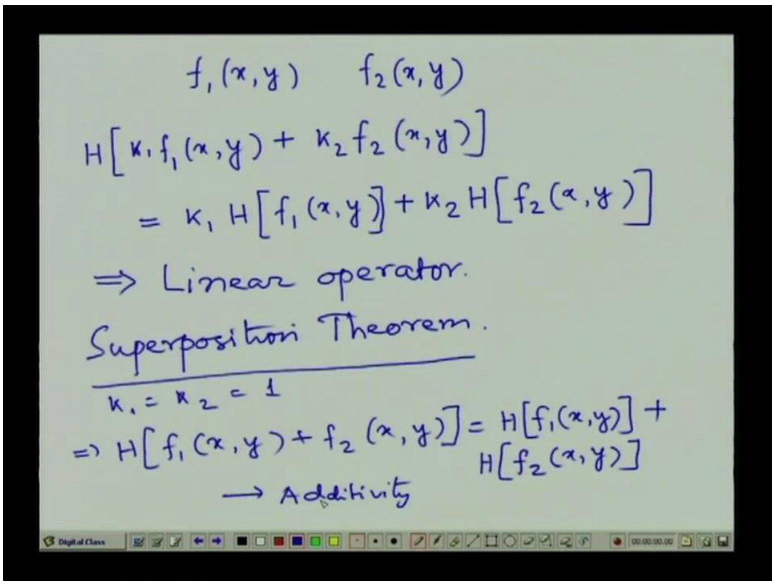
And this operation is the frequency domain operation and the equivalent operation in the spatial domain it is the upper one and here you see that in the spatial domain we have represented this operation as a convolution operation and we have said earlier that a convolution in the spatial domain is equivalent to multiplication in the frequency domain. So that is what this second term that is G(u,v) = H(u,v) into F(u,v) + N(u,v). So here the convolution in the spatial domain is replaced by the multiplication in the frequency domain. So these two are very very important expressions and we will use, make use of this expressions subsequently more or less throughout our discussion or image restoration process.

(Refer Slide Time: 11:23)



Now before we proceed further, let us try to recapitulate some of definitions. So first we will look at some of the definitions that will be used throughout our discussion on image restoration. So here what we have is, we have a degraded image g(x,y) which now let us represent it like this, H of [f(x,y)] + eta(x,y), where in this particular case, we assume that this H is the degradation operator which operates on the input image f(x,y) and that when added with the additive noise eta(x,y) gives us the degraded image g(x,y). Now here if we assume or for the time being, if we neglect the term eta(x,y), or we set eta(x,y) = 0 for the time being for simplicity of our analysis, then what we get is g(x,y) = H [f(x,y)] and as we said that here this H we assume that this is the degradation operator.

(Refer Slide Time: 13:28)

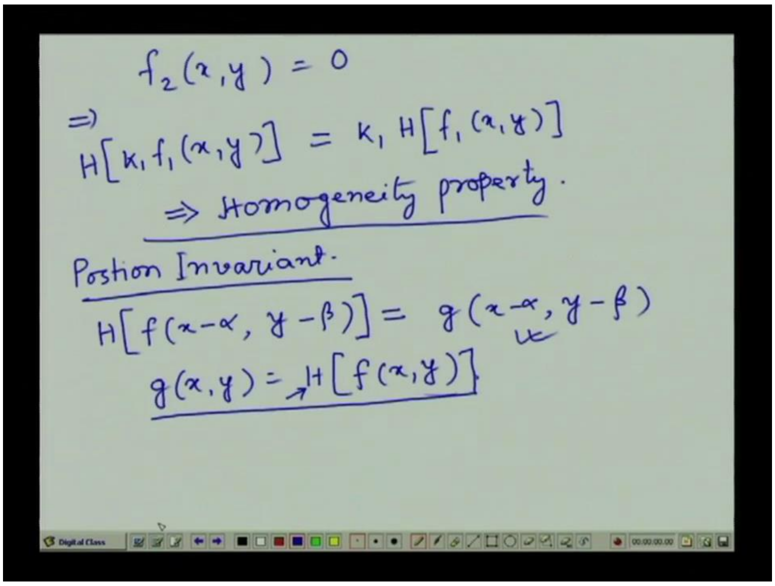


Now the first term that we will define in our case is what is known as linearity. So what do we mean by the linearity? Or we say that this degradation operator H is a linear operator. So for defining linearity, we know that if we have two functions, say f1(x,y) and f2(x,y), then we say that if H[k1 f1(x,y) + some constant k2 f2(x,y)] this is equal to k1 H[f1(x,y)] + k2 H[f2(x,y)]. So if for this two functions f1(x,y) and f2(x,y) and for these two constants k1 and k2, this particular relation is true, that is H k1 f1(x,y) + k2 f2(x,y) = k1 H f1(x,y) + k2 H f2(x,y) if this relation is true, then the operator H is set to be a linear operator..

And we know very well from our linear system theory that this is nothing but the famous superposition theorem, so this is what is known as the superposition theorem and as per our definition of a linear system, we know already that the superposition theorem must hold true, if the system is a linear system. Now using this same equation, if I set say k1 = k2 = 1 then, the same equation leads to H[f1(x,y) + f2(x,y)] this is nothing but = H[f1(x,y)] + H[f2(x,y)]. Simply we have replaced k1 and k2 by 1, and this is what is known as additivity property.

So the additivity property simply says that the response of the system to the sum of two inputs is same as the sum of their individual responses. So here, we have two inputs f1(x,y) and f2(x,y), so if I take the summation of f1(x,y) and f2(x,y), and then allow H to operate on eight, then whatever result we will get, that will be same as, when H operates on f1 and f2 individually, and we take the sum of those individual responses.

(Refer Slide Time: 17:20)

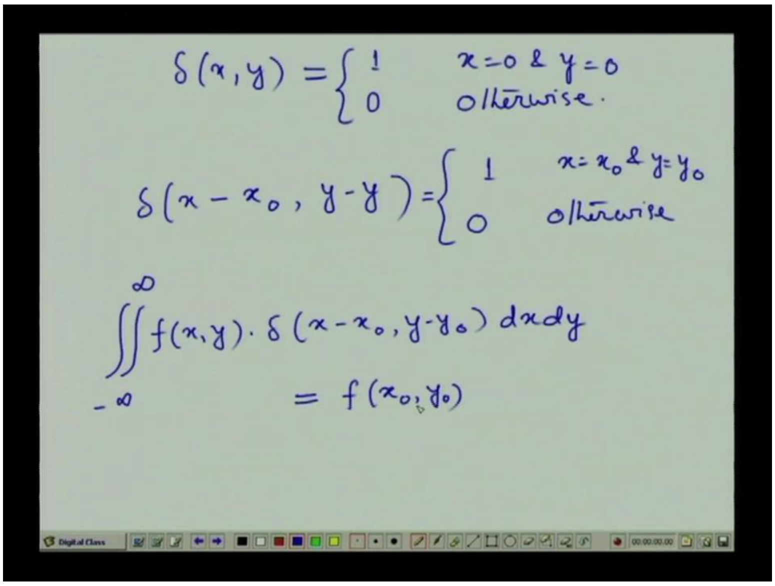


And these two must be equal to 2 for a linear system and this is what is known as the additivity property. So this is what is the additivity property in this particular case. Now here, again if I assume that f2(x,y) = 0, so this gives H[k1 f1(x,y)] should be = k1 H[f1(x,y)], and this is a property which is known as homogeneity property. So these are the different properties of a linear system, and the system is also called position invariant, if certain properties hold. The system will be position invariant or location in variant, if H[f(x -alpha, y -beta)] is same as = g(x -alpha, y -beta).

So in this case obviously, what we have assumed is g(x,y) = H[f(x,y)], so when this is true, that g(x,y) = H[f(x,y)], then this particular operator H will be called to be position invariant, if H[x -alpha, y -beta] is equal to g(x -alpha, y -beta) and that should be true for any function f(x,y) and any value of alpha beta. So this position invariant property, this simply says that the response at any point in the image, the response of H at any point in the image should solely depend upon the value of the pixel at that particular point, and the response will not depend upon the position of the point in the image.

And that is what is given by this particular expression, that is H[f(x -alpha, y -beta)] = g(x -alpha, y -beta). Now given this definition, let us see that what will be the degradation model for a what will be the degradation model in case of continuous functions.

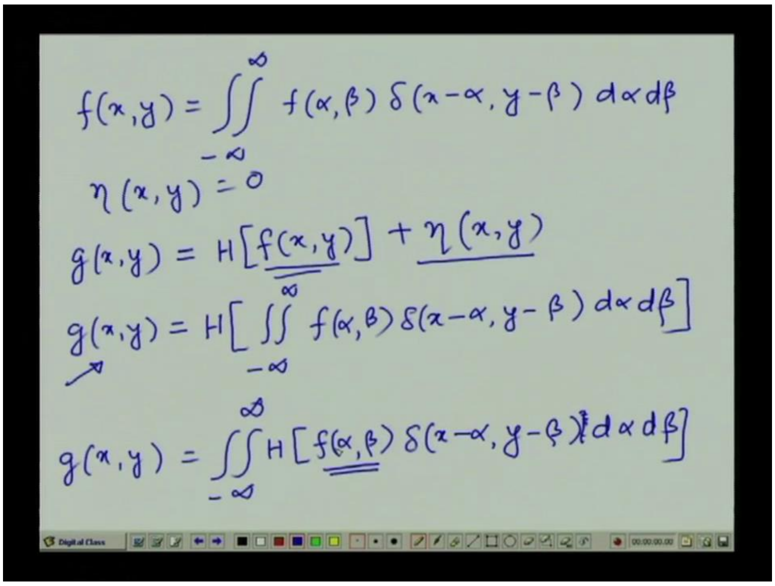
(Refer Slide Time: 20:37)



So to look at the degradation model in case of continuous functions, we make use of an old mathematical expression where we have seen that if I take a delta function say, delta(x,y) and the definition of delta(x,y) we have seen earlier that this is equal to 1, if x=0 and y=0, and this is equal to zero, otherwise. So this is the definition of a delta function that we have already used, and we can use a shifted version of this delta function, that is delta(x -x0 and, y -y0) will be equal to 1, if x = x0 and y = y0, and it will be zero otherwise.

So this is the definition of a delta function. Now earlier we have seen that if we have an image say, f(x,y) or a two dimensional function f(x,y), then multiply this with delta(x -x0, y -y0) and integrate this product over the interval minus infinity to infinity, then the result of the integral will be simply equal to f(x0, y0). So this says that if I multiply a two dimensional function f(x,y) with the delta function delta(x -x0, y -y0) and integrate the product over the interval minus infinity to infinity, then the result will be simply the value of the two dimensional function f(x,y) at location x0, y0.

(Refer Slide Time: 22:46)



So by slightly modifying this particular expression, we can have an equivalent expression which is given by, I can formulate the two dimensional function f(x,y) as a similar integral operation and in this case I will take f(alpha, beta) delta(x -alpha, y -beta) dalpha dbeta and take the integral from minus infinity to infinity.

So we find that we have an equivalent mathematical expression which is equivalent to just the earlier expression that we have said, and in this case we can formulate f(x,y), the two dimensional function f(x,y) in terms of the value of the function at a particular point alpha beta, and in terms of the delta function delta(x -alpha, y -beta).

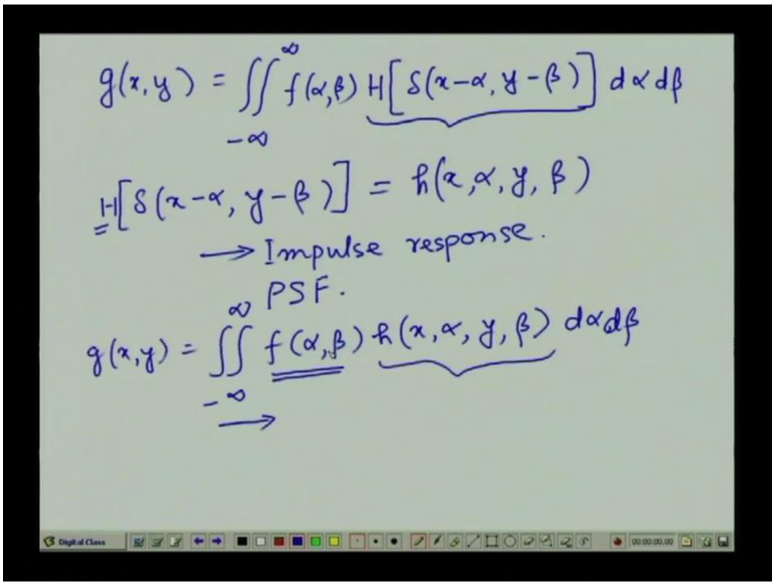
Now, for the time being if we consider, say the noise term ita(x,y) = 0 for simplicity, then we can write the degraded image g(x,y), we have seen earlier that g(x,y) we have written as

H[f(x,y)] + eta(x,y). So for the time being, we are assuming that this additive noise term etat(x,y) is zero or it is negligible, then the degraded image g(x,y) can now be written in the form H of I replace this f(x,y) by this integral term. So this will be simply H of double integral f(alpha, beta) delta(x -alpha, y -beta) dalpha dbeta, but the integral has to be taken from minus infinity to infinity.

So I can write, I can get an expression of the degraded image g(x,y) in terms of this integral definition of the function f(x,y) which is operated by the degradation operator H. Now once I get this kind of expression, now if I apply the linearity and additivity property of the linear system, then this particular expression gets converted to g(x,y) is equal to, I can take this double summation outside it becomes H of [f(alpha, beta) delta(x -alpha, y -beta)] dalpha dbeta, take the integral from minus infinity to infinity.

And this is what we have obtained by applying the linearity and additivity property to this earlier expression of this degraded image. Now here you find that this term f(alpha, beta) this is independent of the variables x and y. So because the term f(alpha, beta) is independent of the variables x and y, this same expression can now be rewritten in a slightly different form.

(Refer Slide Time: 26:31)



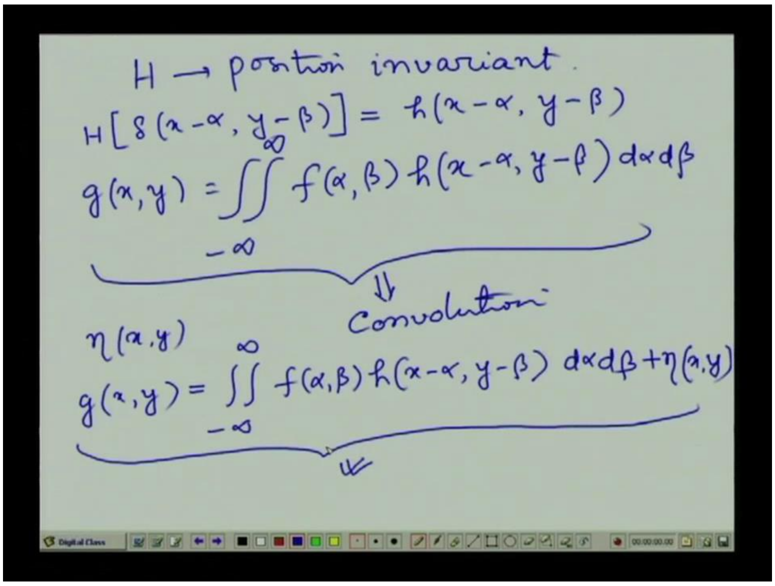
So that form gives us that g(x,y) can now be written as, same double integral we take f(alpha, beta) outside the scope of the operator H, so this simply becomes f(alpha, beta) then H[delta(x -alpha, y -beta)] dalpha dbeta, take the integral over minus infinity to infinity.

Now this particular term H of [delta(x -alpha, y -beta)] we can write this as = h(x, alpha, y, beta) and this is nothing but what is known as the impulse response of H. So this is what is known as the impulse response. That is the response of the operator H when the input is an impulse, given in the form delta(x -alpha, y -beta). And in case of optics, this impulse response is popularly known as point spread function or PSF. So using this impulse response, now the same g(x,y), we can write as double integral again f(alpha, beta) h(x, alpha, y, beta) dalpha dbeta integral from minus infinity to infinity.

And this is what is popularly known as superposition integral of first kind, now this particular expression is very very important, it simply says that if the impulse response of the operator H is known, then it possible to find out the response of this operator H to any arbitrary input f(alpha, beta). So that is what has been done here, that using the knowledge of this impulse response h(x, alpha, y, beta), we have been able to find out the response of this system to an input f(alpha, beta).

And this impulse response is the one which uniquely or completely characterizes a particular system, okay. So given any system, if we know what is impulse response of the system, then we can find out what will be the response of that system to any other arbitrary function. Now in addition to this, if the function H, this operator H is position invariant, so we use H to be position invariant.

(Refer Slide Time: 30:06)



So if H is position invariant, then obviously H [delta(x -alpha, y -beta)] as per our definition of position invariance will be same as h(x -alpha, y -beta) this is as per the definition of position invariance of a system. Now using this position invariance property, now we can write g(x,y) that is the degraded image as simply, double integral f(alpha, beta) into h(x -alpha, y -beta) dalpha dbeta take the integral from minus infinity to infinity. And if you look at this particular expression, you find that this expression is not, is nothing but the convolution operation, this is nothing but the convolution operation of the two functions f(x,y) and h(x,y).

And that is what we said that when we have drawn our degradation model, we have said that input image f(x,y) is actually convolved by the degradation process that is g(x,y). So this is nothing but that convolution operation. And now if I take you find that earlier we have considered this noise term eta(x,y) to be equal to zero. So now if I consider this noise term eta(x,y), then our degradation function, or the degradation model becomes simply g(x,y) = f(alpha, beta) h(x -alpha, y -beta) dalpha dbeta take the integral from minus infinity to infinity plus the noise term eta(x,y).

So this is the general image degradation model, and you find that here we have assumed that the degradation function H is linear and position invariant. And it is very important to note that many of the degradation operations which are, which we encounter in reality can be approximated by such linear space invariant or linear position invariant models. The advantage is, once a degradation model is can be approximated by a linear position invariant model.

Then the inter mathematical rule of linear system theory can be used to find out the solution for such image restoration process, that means we can use all those tools of linear system theory to estimate what will be the restored image f(x, y) from given degraded image g(x,y) provided we know we have some knowledge of the degradation function, that is h(x,y) and we have some knowledge of what is the noise function eta(x,y). Thank you.