Digital Image Processing.
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Lecture-41.
Frequency Domain Processing Techniques.

Hello, welcome to the video lecture series on digital image processing. Now we start discussion on the frequency domain processing techniques. Now so far, you must have noticed that these mask operations or the spatial domain operations using the masks whatever we have done that is nothing but a convolution operation in two dimension.

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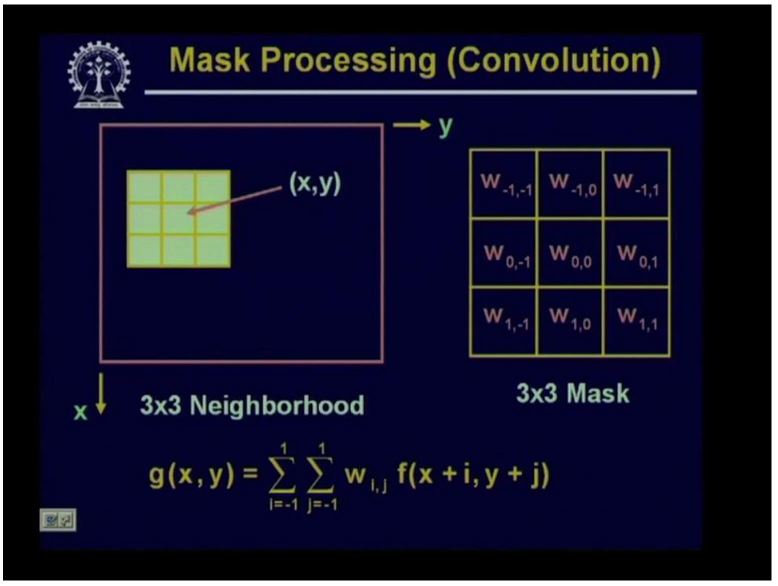

So what we have done is we have the original image f(x,y), we define a mask corresponding to the type of operation that we want to perform on the original image f(x,y) and using this mask, the kind of operation that is done, the mathematical expression of this is given on the bottom, and if you analyze this, you will find that this is nothing but a convolution operation.

So using this convolution operation, we are going for spatial domain processing of the images. Now we have seen, we have already seen during our earlier discussions that a convolution operation in the spatial domain is equivalent to multiplication in the frequency domain. Convolution in the spatial domain is equivalent to multiplication in the frequency domain. Similarly a convolution in the frequency domain is equivalent to multiplication in the spatial domain.

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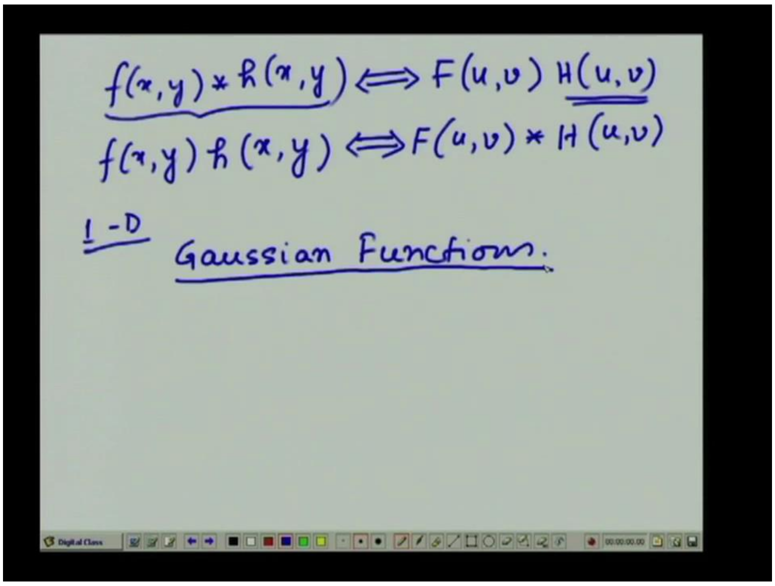

So, what we have seen is that if we have a convolution of, say two functions f(x,y) and h(x,y) in the spatial domain. The corresponding operation in the frequency domain is multiplication of F(u,v) and H(u,v), where F(u,v) is the Fourier transform of this spatial domain function f(x,y) and H(u,v) is the Fourier transform of the spatial domain function h(x,y). Similarly, if we multiply two functions f(x,y) and h(x,y) in the spatial domain, the corresponding operation in the frequency domain is the convolution operation of the Fourier transforms of f(x,y) which is F(u,v) that has to be convolved with H(u,v).

So these are the convolution theorems that we have done in during our previous discussions. So to perform this convolution operation, the equivalent operation can also be done in the frequency domain, if I take the Fourier transform of the image f(x,y) and I take the Fourier transform of the spatial mask, that is h(x,y). So the Fourier transform of the spatial mask h(x,y) as we have said that this is nothing but H(u,v) in this particular case.

So the equivalent filtering operations, we can do in the frequency domain by choosing the proper filter H(u,v), then after taking the product of F(u,v) and H(u,v) if I take the inverse Fourier transform, then I will get the processed image in the spatial domain. Now to analyze this further, what we will do is, we will take the cases in one dimension and we will consider the filters based on Gaussian functions for analysis purpose. The reasons we are choosing these filters based on Gaussian function is that, the shapes of such functions can be easily specified and easily analyzed.

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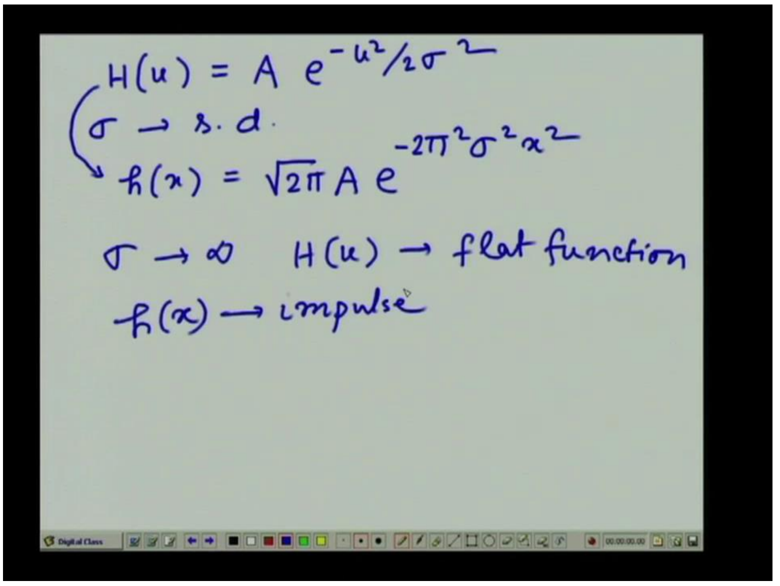

Not only that, the forward transformation, the forward Fourier transformation and the inverse Fourier transformation of Gaussian functions are also Gaussian. So if I take a Gaussian filter in the frequency domain, I will write a Gaussian filter in the frequency domain as H(u) = some constant A e to the power minus u square by 2 sigma square, where sigma is the standard deviation of the Gaussian function. And if I take the inverse Fourier transform of this, then the corresponding filter in the spatial domain will be given by h(x)= root over 2 pi A e to the power minus 2 pi square, sigma square x square.

Now if you analyze these two functions, that is H(u) in the frequency domain and h(x) in the spatial domain, you find that both this functions are Gaussian as well as real. And not only that, both these functions they behave reciprocally with each other, that means, when H(u) has a broad profile, this particular function H(u) in the frequency domain, it has a broad profile, that is it has a large value of standard deviation sigma.

The corresponding h(x) in the spatial domain will have a narrow profile. Similarly, if H(u) has a narrow profile, h(x) will have a broad profile. Particularly, when this sigma tends to infinity, then this function H(u), this tends to be a flat function, and in such case the corresponding spatial domain filter h(x) this tends to be an impulse function. So, this shows that both H(u) and h(x) they are reciprocal to each other. Now let us say what will be the nature of these functions nature of such lowpass filter functions.

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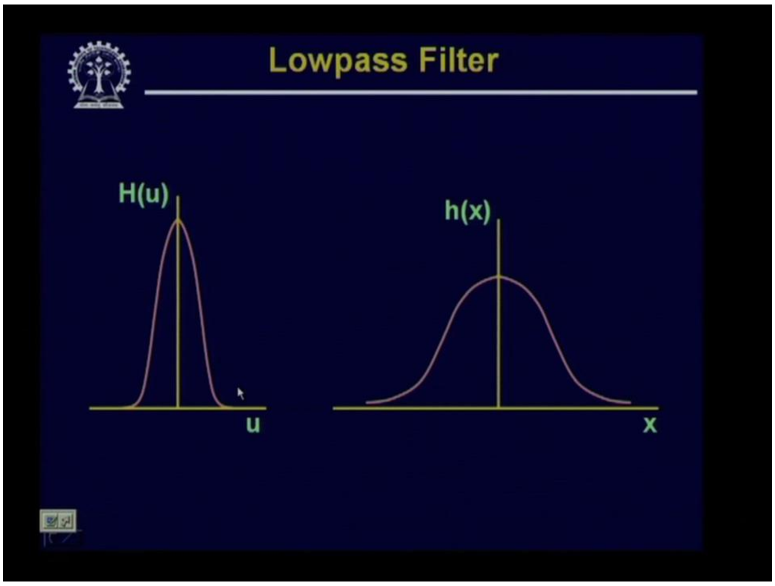

So here, on the left hand side we have shown the frequency domain filter H(u) as a function of you and on the right hand side we have shown the corresponding spatial domain filter h(x) which is a function of x. Now from these filters, it is quite obvious that all the values once I specify filter H(u) as a function of you in the frequency domain, the corresponding filter h(x) in the spatial domain, they will have all positive values, that is none h(x) never becomes positive negative for any value of x.

And the narrower the frequency domain filter, more it will attenuate the lowpass frequency components resulting in more blurring effect. And if I say make the frequency domain filter narrower, that means the corresponding spatial domain filter or spatial domain mask will be flatter, that means the mask size in the spatial domain will be larger.

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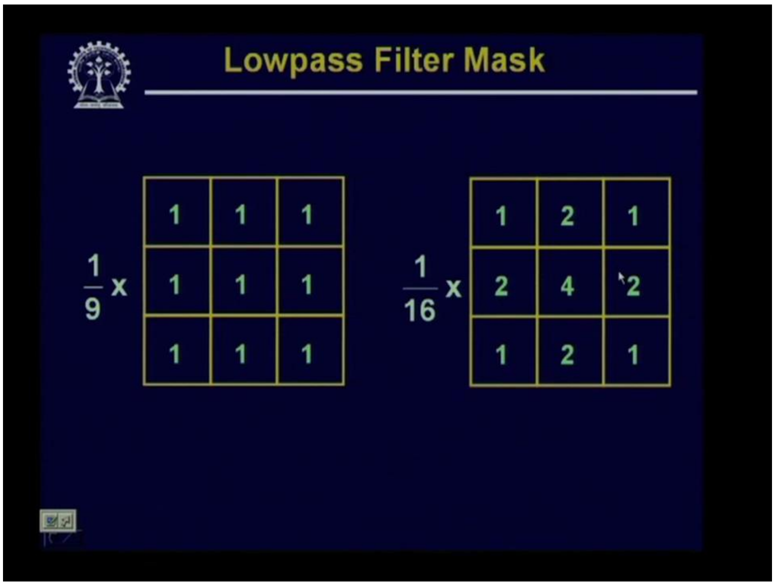

So this slide shows two such masks that we have already discussed during our previous discussion. So this is a mask where all the coefficients are positive and same, and in this mask the coefficients are all positive, but the variation shows that it is having some sort of Gaussian distribution in nature.

And we have already said that if the frequency domain filter becomes very narrow, it will attenuate even the low frequency components leading to a blurring effect of the processed image. Correspondingly in the highpass correspondingly in the spatial domain, the mask size will be larger and we have seen through are results that if I use a larger mask size for smoothing operation, then the image gets more and more blurred. Now, in the same manner as we have said the lowpass filter, we can also make the highpass filters again in the Gaussian domain.

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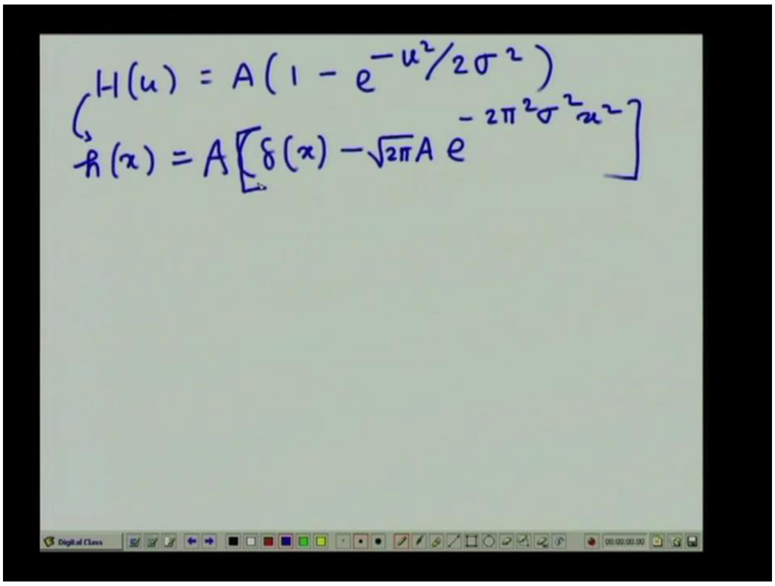

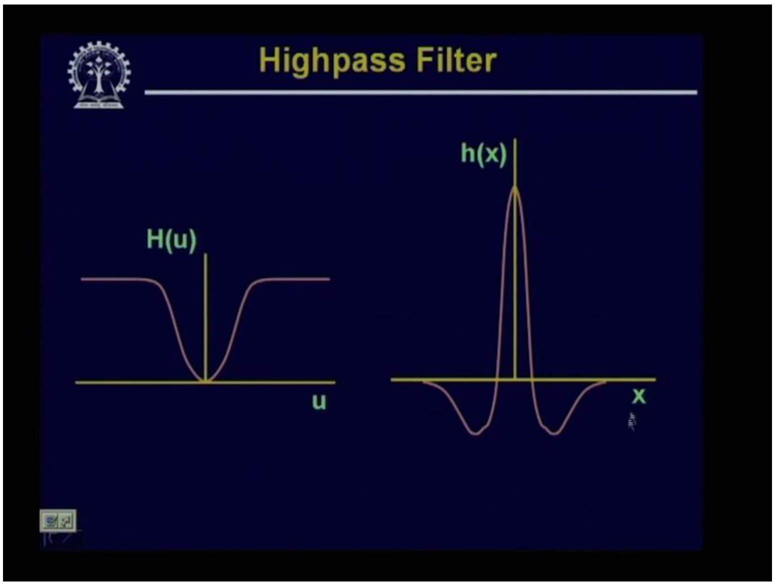

So in this case, in case of Gaussian domain, using the Gaussian function a highpass filter H(u) can be defined as A(1 - e to the power – u square by 2 sigma square). So this is the highpass filter, which is defined using the Gaussian function. If I take the inverse Fourier transform of this, the corresponding spatial domain filter will be given by h(x) = A(delta (x) – the same square root of 2 pi into A into e to the power minus 2pi square sigma square x square). So if I plot this in the frequency domain, this shows the highpass paltar filter in the frequency domain, so as it is quite obvious from this plot that it will attenuate the low frequency components, whereas it will pass the high frequency components.

And the corresponding filter in the spatial domain is having this form which is given by h(x) as a function of x. Now as you note, from this particular figure, from this particular function h(x) that h(x) can assume both positive as well as negative values. And an important point to note over here is, once h(x) becomes negative, it will remain negative it does not become positive any more. And in the spatial domain, the laplacian operator that we have used earlier, the laplacian operator was of similar nature.

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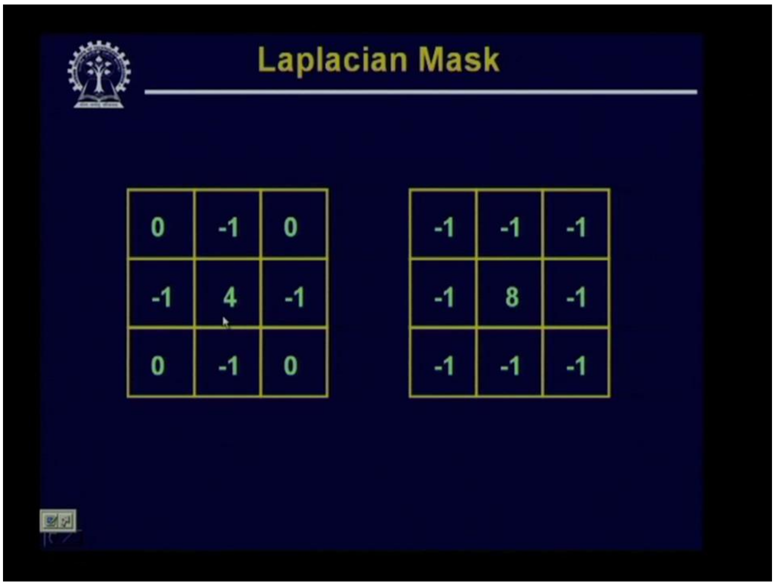

So the laplacian mask that we have used, we have seen that the center pixel is having a positive value, whereas all the neighboring pixels have the negative values, and this is true for both the laplacian masks if I consider only the vertical and horizontal components or whether along with vertical and horizontal components, I also consider the diagonal components. So these are the two laplacian masks where the center coefficient is positive and the neighboring coefficients once they become negative, they will remain negative.

So this shows that using the laplacian mask in the spatial domain, the kind of operation that we have done is basically a highpass filtering operation. So now first of all we will consider the smoothing frequency domain filters or lowpass filters in the frequency domain. Now as we have already discussed, that edges as we as sharp transitions like noises, they lead to high frequency components in the image. And if we want to reduce these high frequency components, then the kind of filter that we have used is a lowpass filter.

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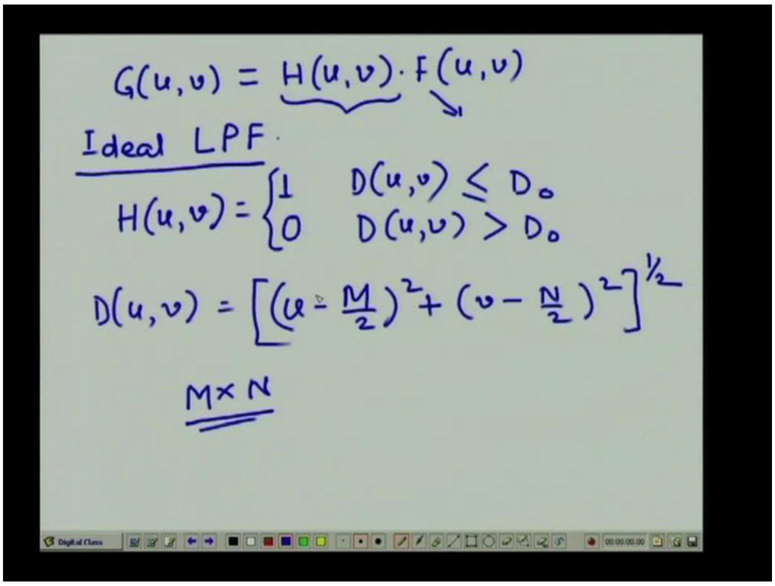

For the lowpass filter, we will allow the low frequency components of the input image to be passed to the output and it will cut off the high frequency components of the input image which will not be passed to the output. So our basic model for this filtering operation will be like this that we will have the output in the frequency domain, which is given by G(u,v) which is equal to H(u,v) multiplied by F(u.v), where this F(u,v) is the Fourier transform of the input image and we have to select a proper filter function H(u,v) which will attenuate the high frequency components and it will let the low frequency components to be passed to the output.

Now here we will consider an ideal lowpass filter, where we will assume the ideal lowpass filter to be like this, that H(u,v) = 1, if D(u,v), where D(u,v) is the distance of the point u,v in the frequency domain from the origin of the frequency rectangle. So if D(u,v) is less than or equal to some value say D0, than H(u,v) H(u,v) will be equal to 1 and this will be equal to zero if the distance from the origin of the point u,v is greater than D0.

So this clearly means, that if I multiply F(u,v) with such an H(u,v), than all the frequency components lying within a circle of radius D0 will be passed to the output and all the frequency components lying outside this circle of radius D0 will not be allowed to be passed to the output.

Now if the Fourier transform F(u,v) is centered, is the centered Fourier transform, that means the origin of the Fourier transform rectangle is set at the middle of the rectangle, then this D(u,v) the distance value instantly computed as [(u -M by 2 square)+(v -N by 2) square]

square root of this, where we are assuming that we have an image of size M x N. So for an M x N image size, D(u,v) will be computer like this, if the Fourier transform F(u,v) is the centered Fourier transformation.

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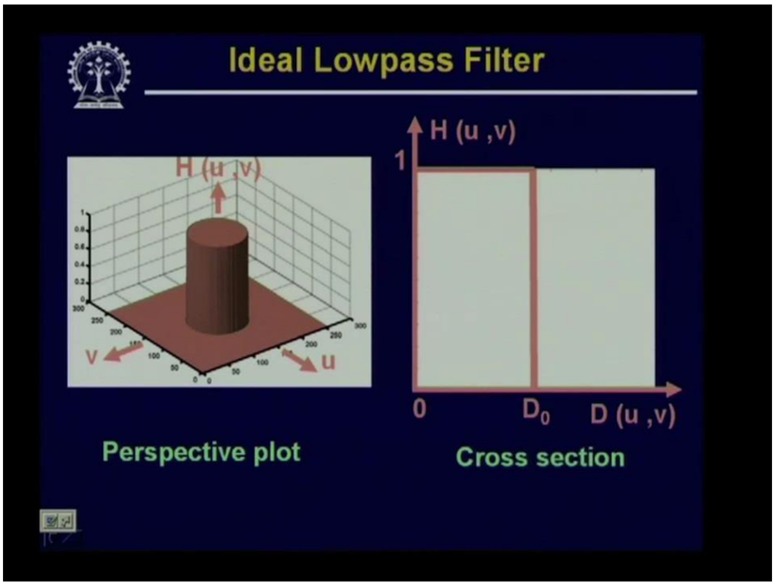

A plot of this kind of function is like this, so (he) here you find that in the left hand side shows the prospective plot of such an ideal filter, whereas on the right hand side we just show the cross section of such an ideal filter. And in such cases, we define a cut off frequency of the filter to be the point of transition between H(u,v)=1 and H(u,v)=0. So in this particular case, this point of transition is the value D0 so we consider D0 to be the cut off frequency of this particular field.

Now it may be noted that such a sharp cut off filter is not realizable using the electronic components. However using software, using computer program it is different, because we are just letting some values to be passed to the output and we are making the other values to be zero. So this kind of ideal lowpass filter can be implemented using software, whereas using electronic components, we may not be, we are not able to implement such ideal lowpass filters.

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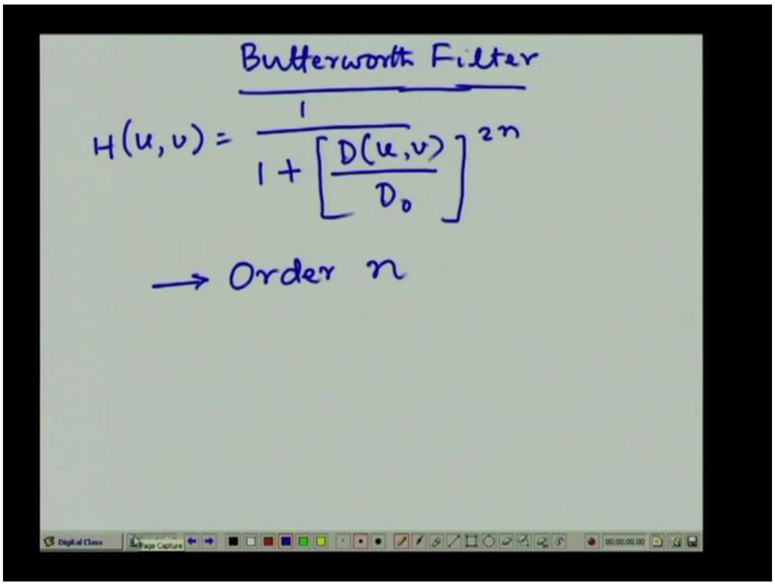

So a better approximation of this is a filter which is called a Butterworth filter. So a Butterworth filter, a Butterworth low pass filter is the response, the frequency response of this is given by H(u,v)=1 upon 1+[D(u,v) by D0] to the power 2 n. So this is a Butterworth filter of order n.

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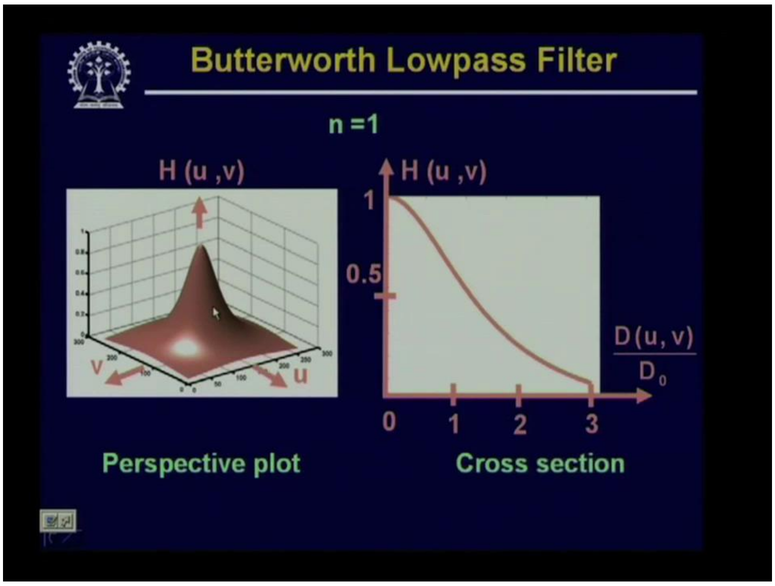

The response of or the plot of such a Butterworth filter is shown here, so here we have shown the Butterworth Butterworth filter the prospective plot of the Butterworth filter, and on the right hand side we have shown the cross section of this Butterworth filter. Now if I apply the ideal lowpass filter and the Butterworth filter on an image, let us see what will be the kind of the output image that we will get. So in all these cases, we assume that first we take the Fourier transform of the image, then multiply that Fourier transformation with the frequency response of the filters, then whatever the product that we get, we take the inverse Fourier transformation of this to obtain our processed image in the spatial domain.

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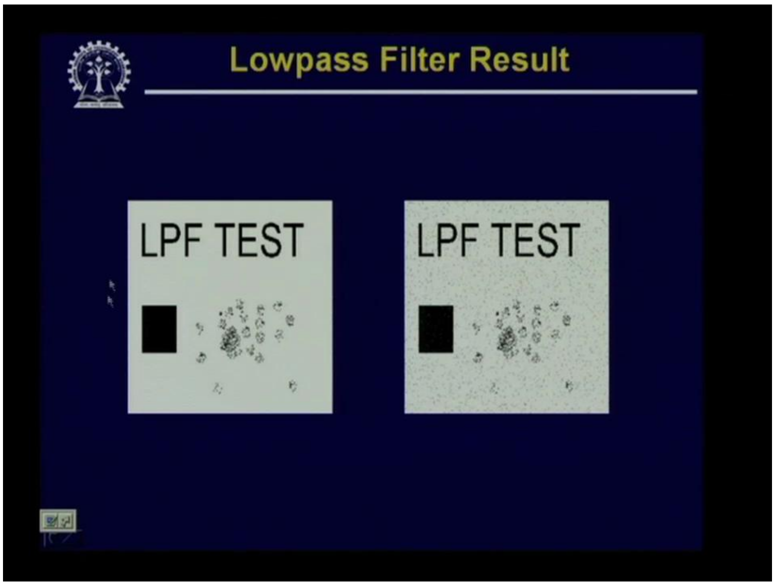

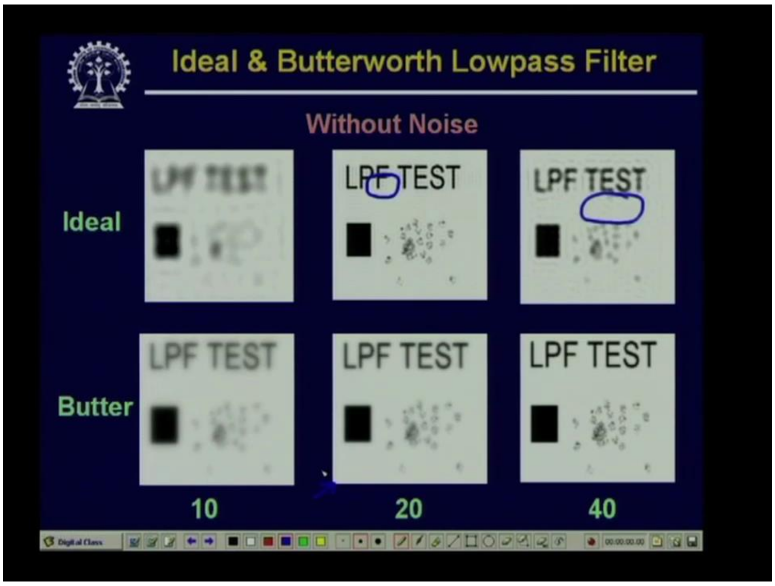

So here we use two images for test purpose, on the left hand side we have shown an image without any noise, and on the right hand side we have shown an image where we have added some amount of noise. Then if I process that image, using the ideal lowpass filter, and using the Butterworth filter, the top row shows the results with ideal lowpass filter when the image is without noise and the bottom row shows the result by applying the Butterworth filter again when there is no noise contamination in the image.

Here you find, as the top row shows that if I use the ideal lowpass filter, for the same cut off frequency, say 10, the blurring of the image is very high compared to the blurring which is introduced by the Butterworth filter. If I increase the cut off frequency, when I go for cut off frequency of twenty, in that case you find that in the original image, in the ideal lowpass filtered image, the image is very sharp but the disadvantage is that, if you simply look at these locations, say along these locations you find that there is some ringing effect. That means there are a number of lines undesired lines which are not present in the original image.

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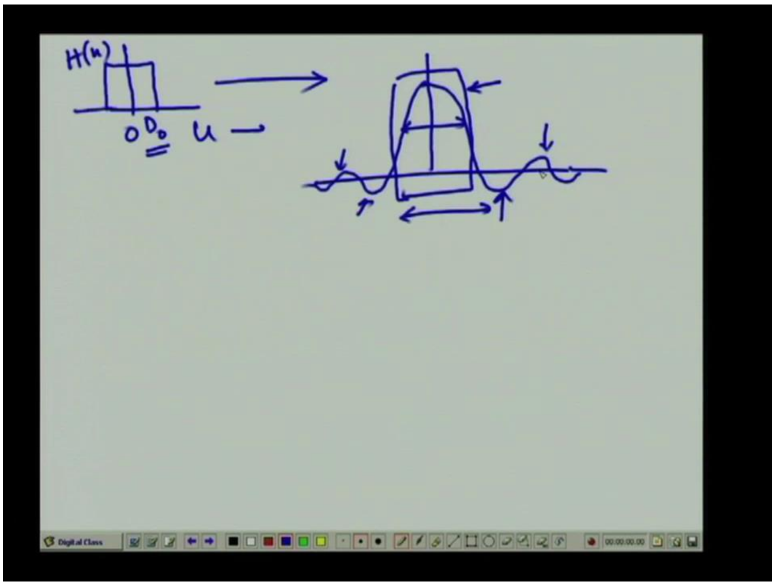

Same is the case over here, so the Butterworth filter, Butterworth lowpass filter, it introduces the ringing effect, the ringing effect which are not visible in case of Butterworth filter. Now the reason why the ideal lowpass filters introduces the ringing effect is that, we have seen that for an ideal lowpass filter, in the frequency domain the ideal lowpass filter response was something like this, so if I plot u versus H(u) this was the response of the ideal lowpass filter. Now if I take the inverse Fourier transform of this, corresponding h(x) will have a function of this form. Like this.

So here, you find that there is a main component which is the central component and there are other secondary components. Now the spread of this main component is inversely proportional to D0 which is the cut off frequency of the (butter) of the ideal filter, ideal lowpass filter. So as I reduce D0, this spread is going to increase and that is what is responsible for more and more blurring effect of the smoothed image. Whereas all the secondary components, the number of these components again over an unit length is again an inverse function, inversely proportional to this cut off frequency D0.

And these are the ones which are responsible for ringing effect. When I use Butterworth filter, the outputs that we have shown here using the Butterworth filters, these outputs are obtained using Butterworth filter of order one, that is value of n = 1. So Butterworth filter of order 1 does not lead to any kind of ringing effect. Whereas, if go for butter Butterworth filter of higher order that may leads to the ringing effect. In the same manner we can also go for Gaussian lowpass filter.

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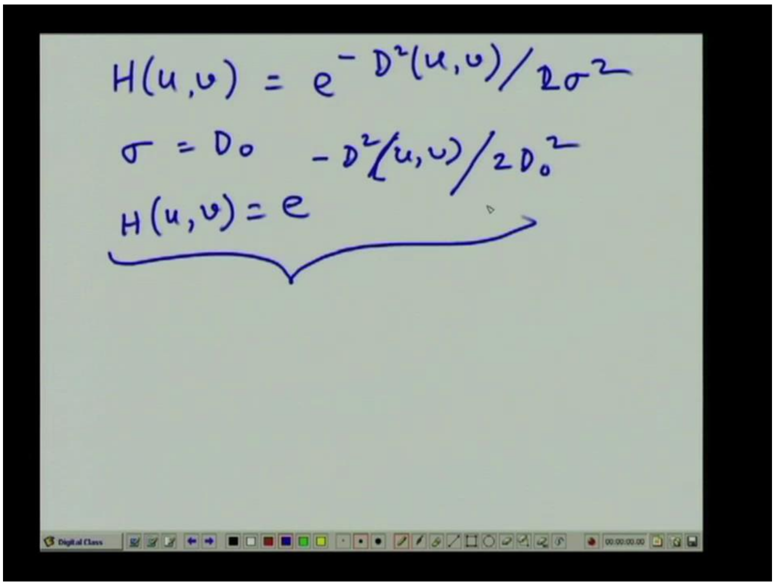

And we have already said, that for a Gaussian lowpass filter, the filter response H(u,v) is given by, e to the power minus D square(u,v) upon 2 sigma square, and if I allow sigma to be equal to the cut frequency say D0, then this H(u,v), the filter response will be, e to the power minus D square (u,v) upon 2 D0 square. Now if I use such a Gaussian lowpass filter for filtering operation and as we have already said the inverse Fourier transform of this is also Gaussian in nature, so using the Gaussian filters, we will never have any ringing effect in the processed image.

So this is the kind of lowpass filtering filtering operation or smoothing operations in the spatial domain that we can have. We can also have the high frequency operation or sharpening filters in the frequency domain. So as lowpass filters give the smoothing effect, the sharpening effect is given by the highpass filter. Again we can have the ideal highpass filter, we can have the Butterworth highpass filter, we can also have the Gaussian highpass filter.

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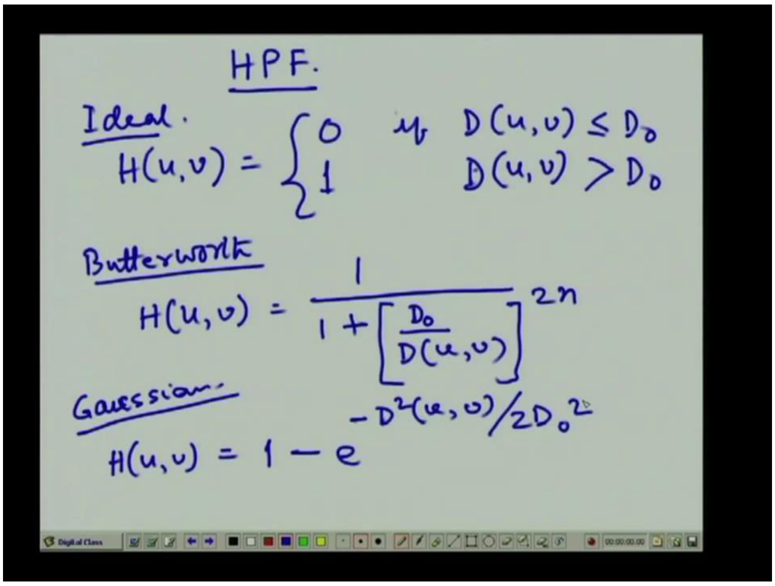

So just in the reverse way, we can define an ideal highpass filter as, for an highpass filter, the ideal highpass filter will be simply H(u,v) = 0, if D(u,v) is less than or equal to D0 and this will be equal to one, if D(u,v) is greater than D0. So this is the ideal highpass filter. Similarly we can have Butterworth highpass filter, where H(u,v) will be given by the expression 1 upon 1 + D0 by D(u,v) to the power 2n, and we can also have the Gaussian highpass filter, which is given by H(u,v) = 1 – e to the power – D square(u,v) upon 2D0 square.

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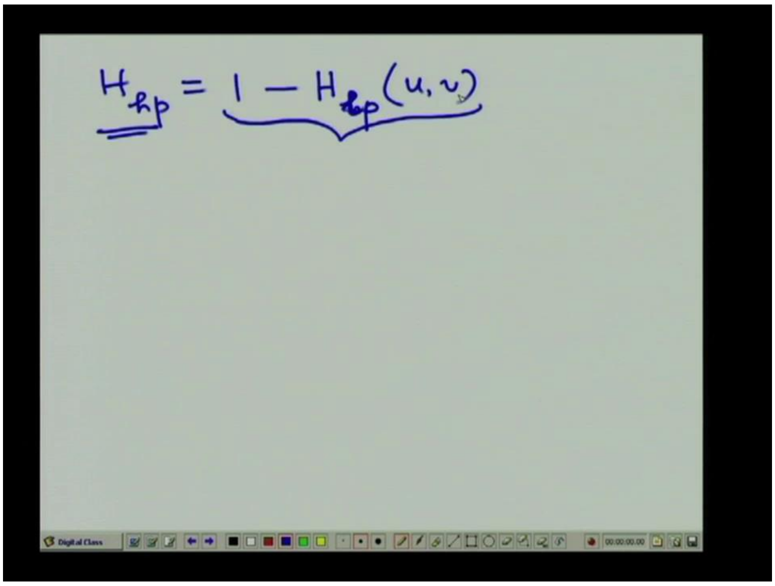

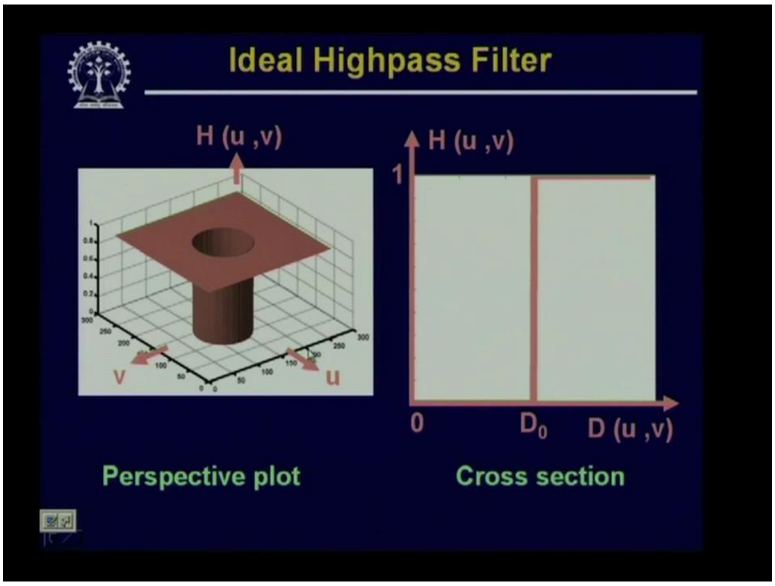

And you find that in all this cases, the response, the frequency response of an highpass filter, if I write it thas, write it as a Hhp is nothing but 1 – the response of a lowpass filter, so the highpass filter response can be obtained by the lowpass filter response where the cut off frequencies are same. Now using such highpass filters, the kind of results that we can obtain is given here, so this is the ideal highpass filter response, where the left hand side gives you the prospective plot and the right hand side gives you the cross section.

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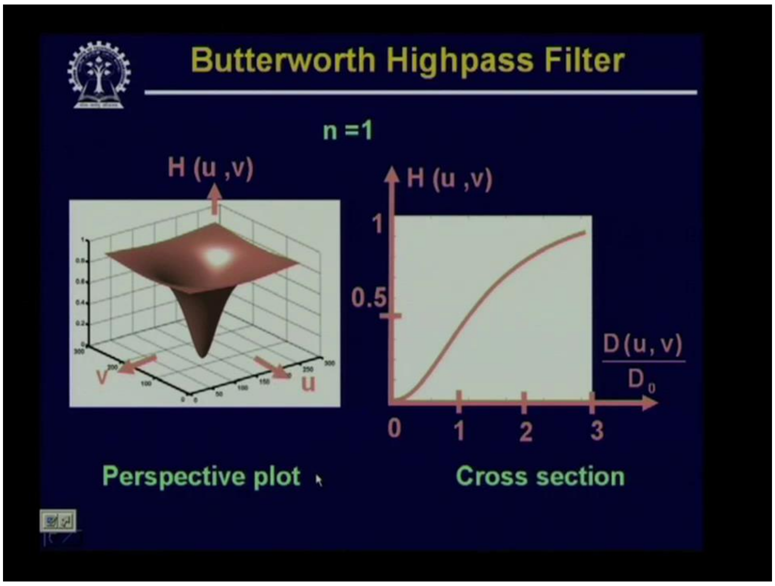

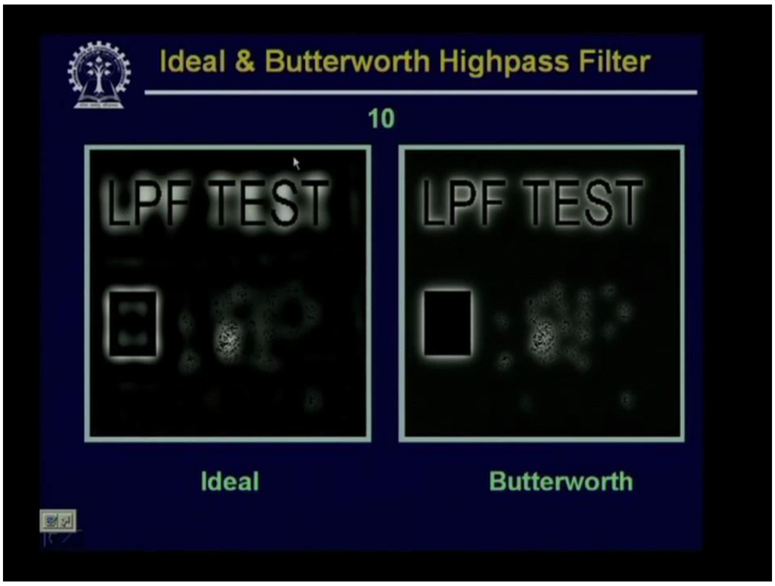

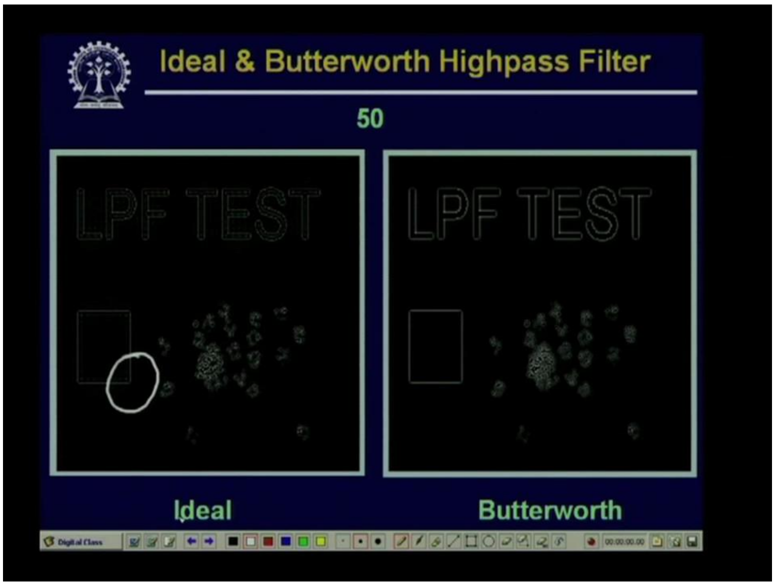

This shows the Butterworth filter prospective plot as well as cross section of a Butterworth filter of order one. And if I apply such highpass filters to the image, to the same image then the results that we obtain is something like this. So here on the left hand side, this is the response of an ideal highpass filter, on the right hand side we have shown the response of a Butterworth highpass filter and in both this cases, the cut off frequency was taken to be equal to 10. This one, where the cut off frequency was taken to equal to fifty and if you close the look at the ideal filter output, here again you find that you can obtain, you can find that there are ringing effects around these boundaries.

Whereas in case of Butterworth filter, there is no ringing effect, and again we said that this is a Butterworth filter of order one if I go for higher order Butterworth filters that also may lead to ringing effects. Whereas, if I go for a highpass filter which is a Gaussian highpass filter, the Gaussian highpass filter does not leads to any ringing effect. So using these highpass filters I can go for smoothing operation using the lowpass filters I can go for the smoothing operation and using the highpass filters I can go for image sharpening operation.

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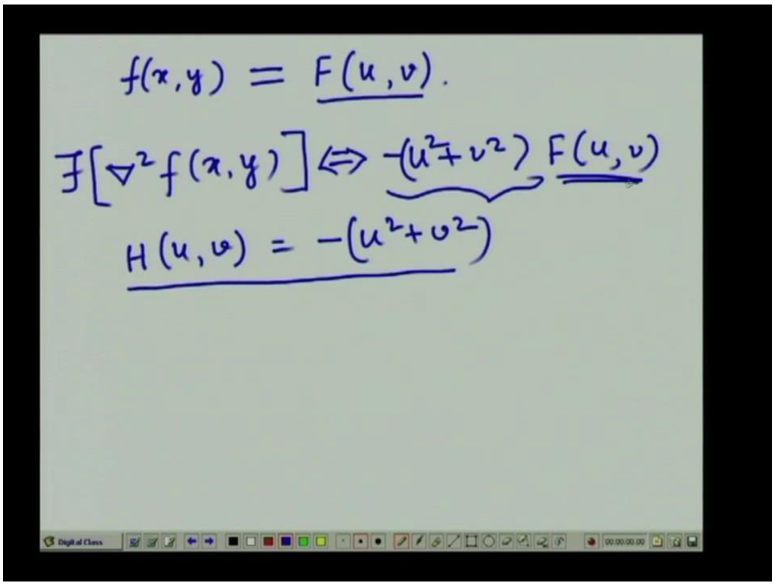

The same operation can also be done using the laplacian in the frequency domain. It is simply because if I take the laplacian of a function, if for a function f(x,y), I get the corresponding frequency domain say F(u,v) the corresponding fourier transform, then the laplacian operator, if I perform dell square f(x,y) and take the fourier transform of this, this will be nothing but, it can be shown, it will be equal to (–u square + v square) into F(u,v). So, using this operation if I consider say H(u,v) = - u square + v square and using this as a filter, I filter this F(u,v) and after that I compute the inverse fourier transformation then the output that we get is nothing but a laplacian operated output which will obviously be an enhanced output.

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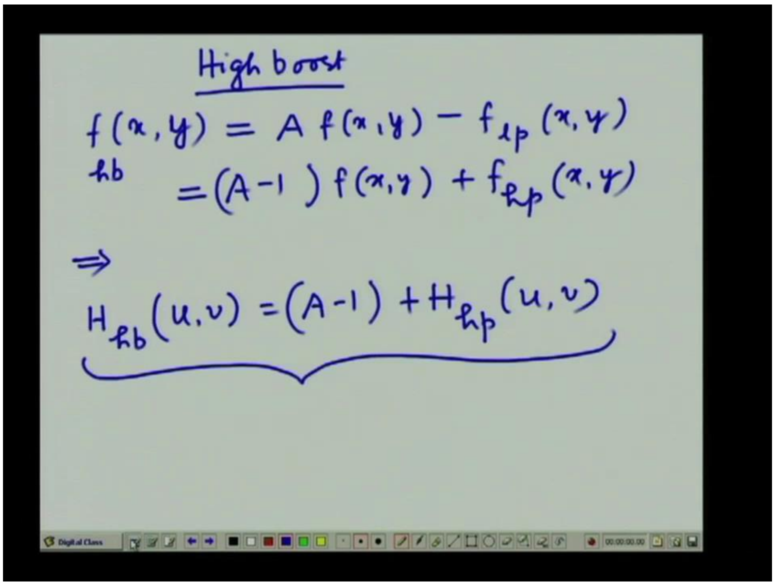

Another kind of filtering that we have already done during a in connection with our spatial domain operation that is high boost filtering. So there we have said that in spatial domain, the high boost filtering operation, the high boost filtering output f(x,y) if I represented, represent this as fhb(x,y) is nothing but A into f(x,y) – flp(x,y) and which is can be represented as, (A – 1) into f(x,y) + fhp(x,y). In the frequency domain, the corresponding operation the corresponding filter can be represented by Hhb(u,v) = (A – 1) + high pass filter Hhp(u,v). So this is what is the high boost filter response in the frequency domain.

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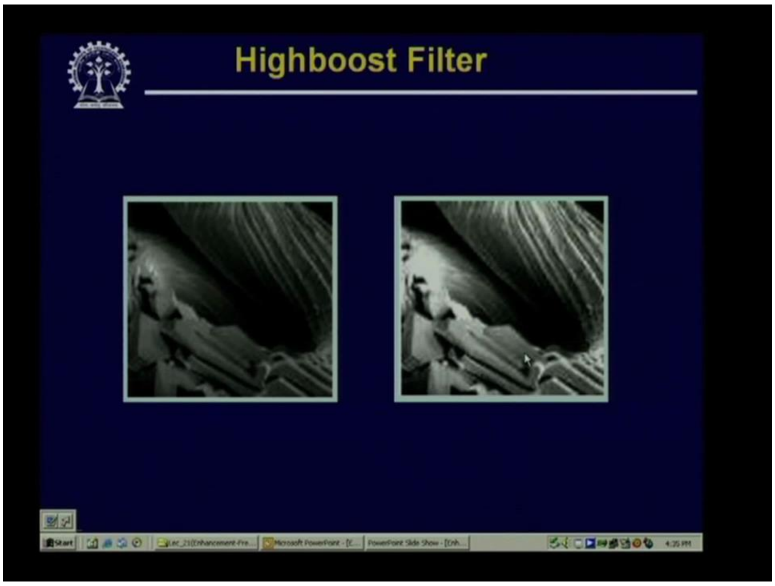

So if I apply this highboost filter to an image, the kind of result that we get is something like this, where again on the left hand side is the original image, and on the right hand side it is the highboost filtered image.

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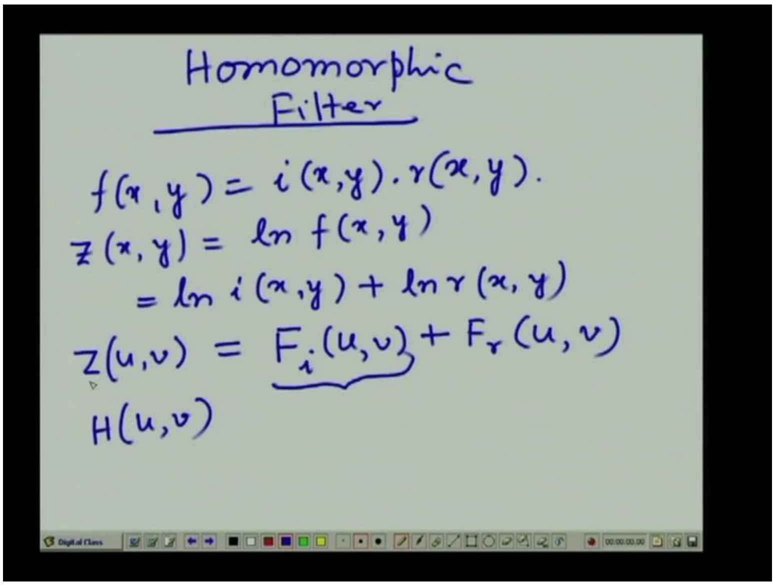

Now let us consider another very very interesting filter, which we call as homomorphic filter, Homomorphic filter. The idea aims from our one of the earlier discussions where we have said that the intensity at a particular point in the image, is a product of two terms, one is the illumination term, other one is the reflectance term. That is f(x,y) we have earlier said that it can be represented by an illumination term i(x,y) multiplied by r(x,y), where r(x,y) is the reflectance term.

Now coming to the corresponding frequency domain, because this is a product of two terms, one is the illumination, other one is the reflectance, taking the Fourier transform directly on this product is not possible. So what we do is, we define a function, say z(x,y) which is logarithm of f(x,y) and this is nothing but logarithm of i(x,y)+logarithm of r(x,y). And if I compute the fourier transform, then the fourier transform of z(x,y) will be represented by z(u,v) which will have two components, Fi(u,v) + Fr(u,v).

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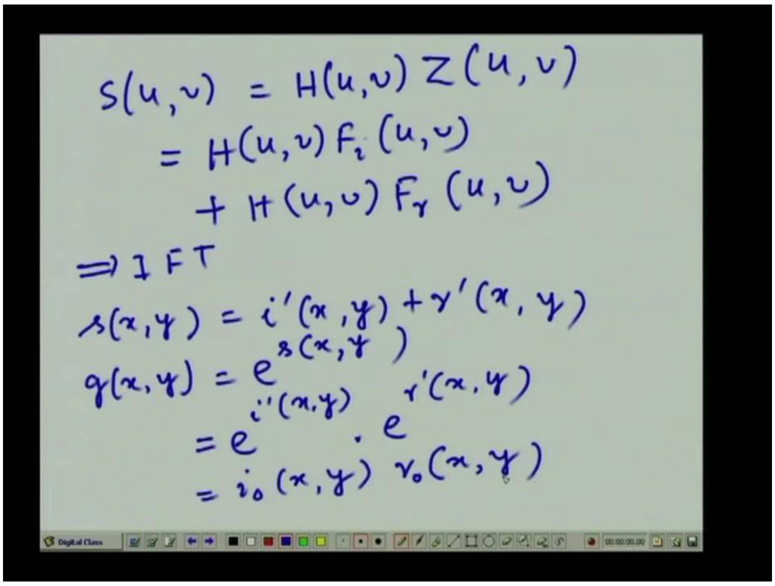

Where this Fi(u,v) is the fourier transform of ln i(x,y) and Fr(u,v) is the Fourier transform of ln r(x,y). Now if I define a filter, say H(u,v) and apply this filter on this z(u,v), then the output that I get is, say S(u,v) which is equal to H(u,v) times Z(u,v) which will be nothing but, H(u,v) times Fi(u,v)+H(u,v) times Fr(u,v). Now taking the invers fourier transform, I get s(x,y) = i dash(x,y) + r dash(x,y) and finally, I get g(x,y), which is nothing but e to the power s(x,y) which is nothing but e to the power i dash(x,y) into e to the power r dash(x,y) which is nothing but i0 (x,y) into r0(x,y).

So the first term, is the illumination component, and second term is the reflectance component. Now because of this separation, it is possible to design a filter which can enhance the high frequency components and it can attenuate the low frequency components. Now it is generally the case that in an image the illumination component leads to low frequency components, because illumination is slowly varying, whereas the reflectance components leads to high frequency components particularly at the boundaries of two reflecting objects.

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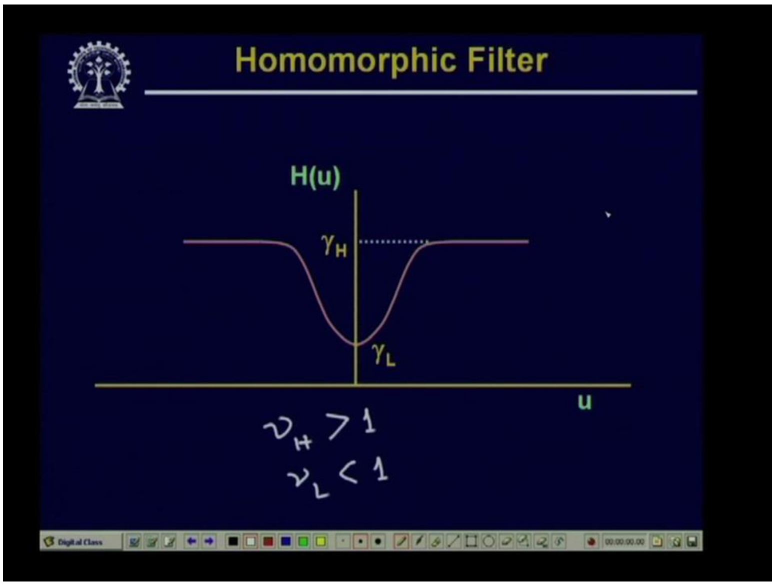

As a result, the reflectance term leads to high frequency components and illumination term leads to low frequency components. So now if we define a filter like this, a filter response like this, and here if I say that I will have, say gamma H greater than 1, and gamma L less than 1, this will amplify all the high frequency components that is the contribution of the reflectance and it will attenuate the low frequency components that is contribution due to the illumination.

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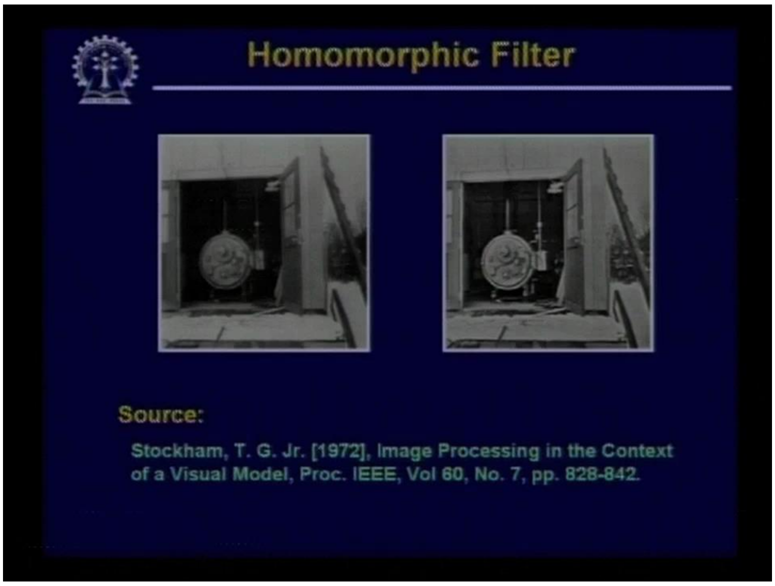

Now using this time, type of filtering, the kind of result that we get is something like this, here on the left hand side is the original image, and on the right hand side is the enhanced image, and if you look in the boxes you find that many of the details in the boxes which are not available in the original image, is now available in the enhanced image. So using such homomorphic filtering, we can even go for this kind of enhancement or the illumination, the contribution due to illumination will be reduced. So even in the dark areas we can take out the details.

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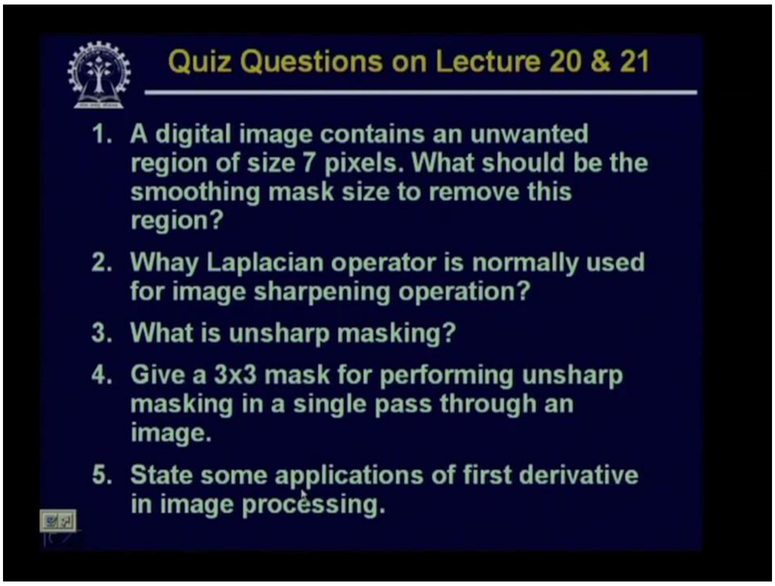

So with this, we come to an end to our discussion on image enhancements. Now let us go to some questions of our today's lecture. The first question is a digital image contains an unwanted region of size 7 pixels. What should be the smoothing mask size to remove this region? Why laplacian operator is normally used for image sharpening operation? Third question, what is unsharp masking? Fourth question, give a 3x3 mask for performing unsharp masking in a single pass through an image. Fifth, state some applications of first derivative in image processing.

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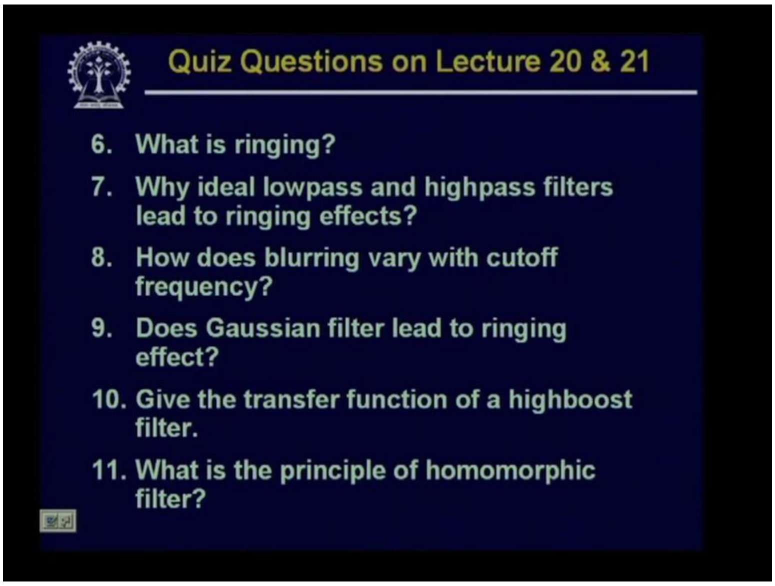

Then, what is ringing? Why ideal lowpass and highpass filters lead to ringing effects? How does blurring vary with cutoff frequency? Does Gaussian filter lead to ringing effect? Give the transfer function of a highboost filter. And what is the principle of homomorphic filter? Thank you.