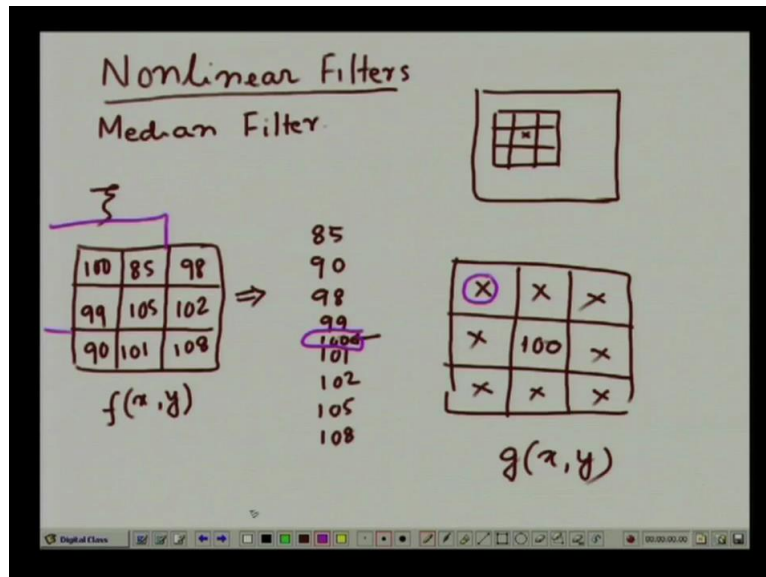


Digital Image Processing.
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Lecture-39.
Image Enhancement: Mask Processing Techniques-II.

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Hello, welcome to the video lecture series on digital image processing. A widely used filter under this order statistics is what is known as a median filter. So in case of a median filter, what we have to do is, I have an image, and what I do is around point x, y , I take a 3x3 neighborhood and consider all the nine pixels intensity values of all the nine pixels in this 3x3 neighborhood. Then I arrange this pixel values, the pixel intensity values in a certain order and take the median of this pixel intensity values.

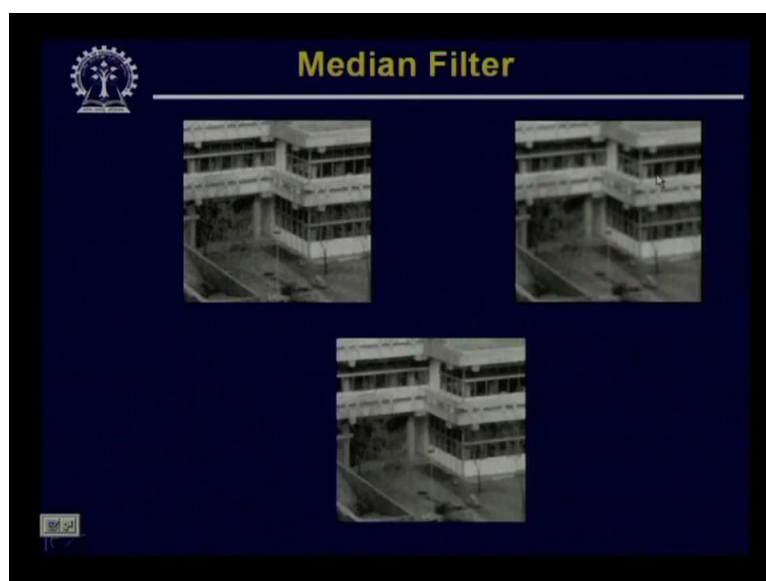
Now how do you define the median? We define the median say, ζ of a set of values, such that half of the values in the set will be less than or equal to ζ , and the remaining half of the values of the set will be greater than or equal to zero. So let us take a particular example, suppose I take a 3x3 neighborhood around a pixel location x, y and the intensity values in this 3x3 neighborhood, let us assume that this is 100, this is say 85, this is say 98, this may have a value 99, this may have a value say 105, this may have a value say 102, this may have a value say 90, this may have a value say 101, this may have a value say 108. And suppose this represents a part of my image say $f(x,y)$.

Now what I do is, I take all this pixel values, all this intensity values and put them in ascending order of magnitude. So if I put them in ascending order of magnitude, you find that the minimum of these values is 85, the next value is say 90, the next one is 98, the next one is 99, the next one is 101, the next one is 102, the next one is 105, and the next one is 108. So all these nine intensity values, I have put in ascending order of magnitude, there will be one more, so there is one more value 100.

So these are the nine intensity values which are put in the ascending order of magnitude. So once I put them into ascending order of magnitude, from this, I take the fifth maximum value, which is equal to hundred. So if I take the fifth maximum value, you find that there will be equal number of values which is greater than this fifth, greater than or equal to this fifth number, and there will be same number of values which will be less than or equal to this fifth number.

So I consider, this particular pixel value of 100, and when I generate the image $g(x,y)$, in $g(x,y)$ at location x,y , I put this value 100, which is the median of the pixel values within this neighborhood. So this gives my processed image $g(x,y)$, of course the intensity is in other locations, in other pixel regions will be decided by the median value of the neighborhood of the corresponding pixels, that is if I want to find out what is the, what will be the pixel value at this location, then the neighborhood that I have to consider will be this particular neighborhood.

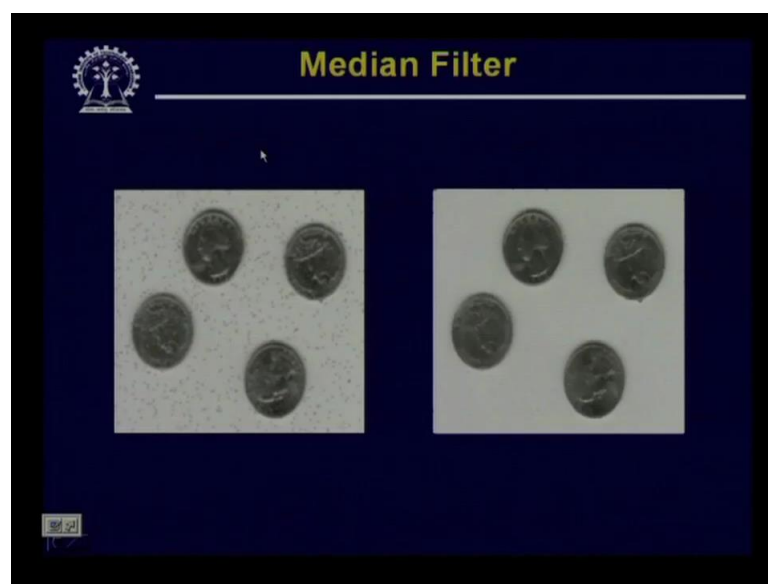
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So this is how I can get the median filtered output and as you can see that this kind of filtering operation is based on statistics. Now let us see that what kind of result that we can have using this median filter. So here you find that it is again on the same building image, the left top is our original noised image, on the right hand side, it is the smooth image using box filter, and on the bottom we have the image using this median filter.

So here again, as you see that the image obtained using the processed image, obtained using the median filtered operation, maintains the sharpness of the image to a greater extent than that obtained using the smoothing images.

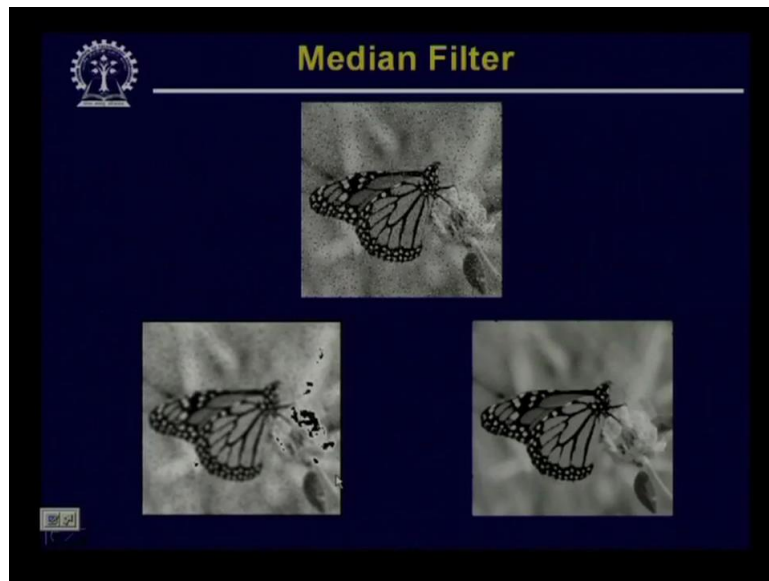
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Coming to the second one, again this is one of the image that we have shown earlier, a noisy image having four coins. Here again you, you find that after doing the smoothing operation, the edges becomes blurred, and at the same time the noise, noises are not reduced to a great extent till the this particular image is noisy.

So if I want to remove all this noise, what I have to do is, I have to smooth these images using higher neighborhood size and the moment I go for the larger neighborhood size, the blurring effect will be more and more. On the right hand side, the image that we have, so this particular image is also the processed image, but here the filtering operation which is done is median filtering. So here you find that because of the median filtering operation, the output image that we get, the noise in this output image is almost vanished, but at the same time the contrast of the image or the sharpness of the image remains more or less intact.

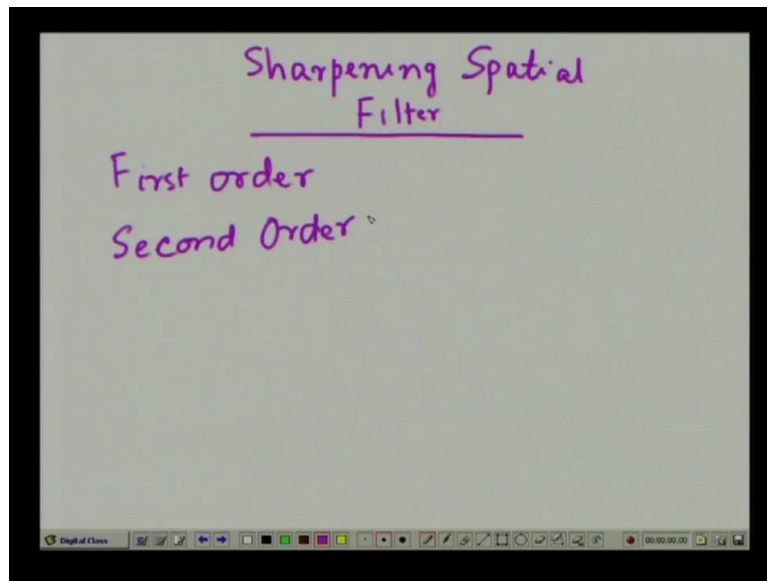
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So this is an advantage that you get if we go for median filtering rather than smooth smoothing filtering or averaging filtering. To show the advantage of this median filtering, we take another example, so this is the image of a butterfly, a noisy image of a butterfly on bottom left the image that is shown, this is an averaged image or the averaging is done over a neighborhood of size 5x5. On the bottom right is the image which is filtered by using median filtering.

So this particular image clearly shows, this result clearly shows the superiority of the median filtering over the smoothing operation or averaging operation. And such median filtering is very very useful for a particular kind of noise, for the noise is a random noise which are known as salt and pepper noise because of the appearance of the noise in the image. So these are the different filtering operations which reduces the noise in the particular image or the filtering operations which introduced blurring or smoothing over the image.

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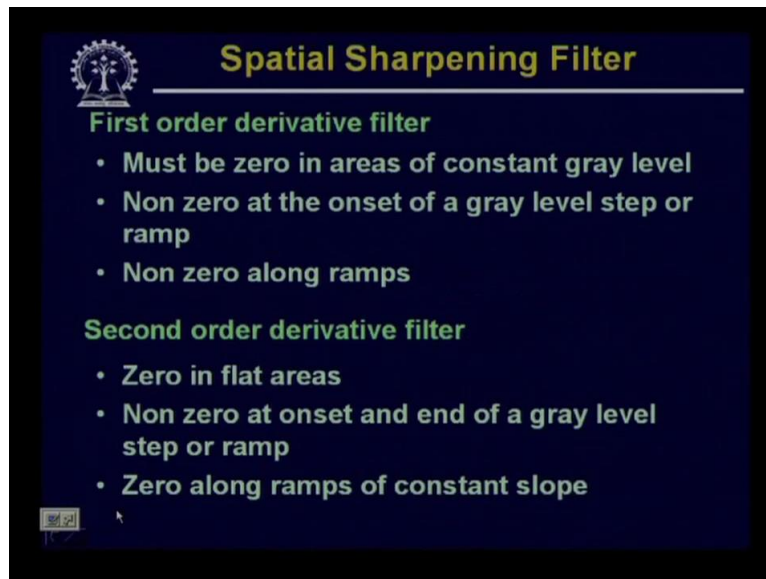


We will now consider another kind of spatial filters which increases the sharpness of the image. So the spatial filter that we will consider now is called sharpening spatial filter, so we will consider sharpening spatial filter. So the objective of this sharpening spatial filter is to highlight the details, the intensity details or variation details in an image. Now through our earlier discussion we have seen that if I do averaging over an image, or smoothing over an image, then the image becomes blurred, or the details in the image are removed.

Now this averaging operation is equivalent to integration operation. So if I integrate the image, then what I do is, what I am going to get is a blurring effect or a smoothing effect of the image. So if integration gives a smoothing effect, so it is quite logical to think that if I do the opposite operation that is, instead of integration, if I do differentiation operation, then the sharpness of the image is likely to be increased. So it is the derivative operations or the differentiations which are used to increase the sharpness of an image.

Now when I go for the derivative operations, I can use two types of derivatives, I can use the first order derivative, or I can also use the second order derivative. So I can either use the first order derivative operation, or I can use the second order derivative operation to obtain the, or to enhance the sharpness of the image. Now let see what are the desirable effects that this derivative operations are going to give.

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Spatial Sharpening Filter

First order derivative filter

- Must be zero in areas of constant gray level
- Non zero at the onset of a gray level step or ramp
- Non zero along ramps

Second order derivative filter

- Zero in flat areas
- Non zero at onset and end of a gray level step or ramp
- Zero along ramps of constant slope

If I use a first order derivative operation or a first order derivative filter, then the desirable effect of this first order derivative filter is, it must be zero, the response must be zero in areas of constant gray level in the image.

And the response must be non-zero, at the onset of a gray level step or at the onset of a gray level ramp. And it should be non-zero along ramps. Whereas if I use a second order derivative filter, then the second order derivative filter response should be zero in the flat areas, it should be non-zero at the onset and end of gray level step or gray level ramp. And it should be zero along ramps of constant slope. So these are the desirable features or the desirable responses of a first order derivative filter and the desirable response of a second order derivative filter. Now whichever derivative filter I use, whether it is a first order derivative filter, or a second order derivative filter, I have to look for discrete domain formulation of those derivative operations.

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The image shows handwritten mathematical derivations on a whiteboard. At the top, the function $f(x)$ is written in purple. Below it, the first-order derivative is defined as $\frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$. Then, for a discrete domain where $\Delta x = 1$, the first-order derivative is approximated as $\frac{\partial f}{\partial x} = f(x+1) - f(x) \Rightarrow 1^{st}$. This is followed by the text "2nd Order." and the second-order derivative approximation: $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$. The whiteboard also shows a toolbar at the bottom with various drawing tools.

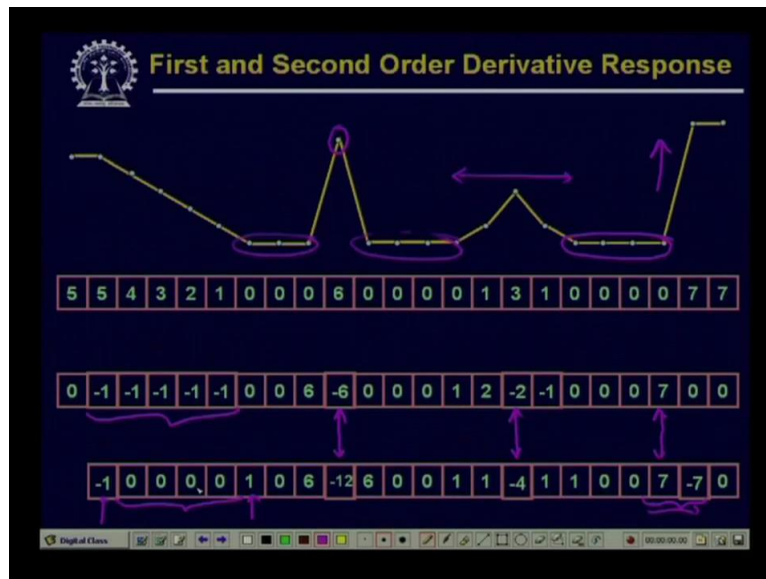
So let us see, how we can formulate the derivative operations, the first order derivative or the second order derivative in discrete domain. Now we know, that in continuous domain that derivative is given by let us consider a one dimension case that if I have a function $f(x)$ which is a function of variable x , then I can have the derivative of this which is given by $\frac{df(x)}{dx}$ which is given by limit Δx tends to zero $\frac{f(x+\Delta x) - f(x)}{\Delta x}$.

So this is the definition of derivative in continuous domain. Now when I come to discrete domain, in case of our digital images, the digital images are represented by a discrete set of points or pixels, which are represented at different grid locations, and the minimum distance between two pixels is equal to one. So in our case, we will consider the value of Δx equal to one and this derivative operation, in case of one dimension, now reduces to $\frac{df}{dx} = f(x+1) - f(x)$.

Now here I use the partial derivative operation because our image is a two dimensional image. So when I take the derivative in two dimension we will have partial derivatives along x , and we will have partial derivatives along y . So the first derivative, the first order derivative in for one dimensional discrete signal is given by this particular expression. Similarly, the second order derivative, of a discrete signal in one dimension can be approximated by $\frac{\partial^2 f}{\partial x^2}$ upon Δx^2 which is given by $f(x+1) + f(x-1) - 2f(x)$.

So this is the first order derivatives, and this is the second order derivative. And you find that these two derivations, these two definitions of the derivative operations this, they satisfy the desirable properties that we have discussed earlier.

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Now to illustrate the response of these derivative operations, let us take an example, say this is a one dimensional signal, where the values of the one dimensional signals for various values of x are given in the form of array like this.

And the plot of these functional values, these discrete values are given on the top. Now if you take the first order derivative of this as we have just defined, the first order derivative is given in the second array, and the second order derivative is given in the third array. So if you look at this functional values, the plot of this functional value, this represents various regions, say for example, here this part is a flat region, this particular portion is a flat region, this is also a flat region, this is also a flat region.

This is a ramp region, this represents an isolated point, this area represents a very thin line and here we have a step kind of discontinuity. So now if you compare the response of the first order derivative and the second order derivative of this particular discrete function, you find that the first order derivative is non-zero during ramp, whereas the first order derivative is zero along a ramp, the second; second order derivative is zero along a ramp, the second order derivative is non-zero at the onset and end of the ramp.

Similarly coming to this isolated point, if I compare the response of the first order derivative and the response of the second order derivative, you find that the response of the second order derivative for an isolated point is much stronger than the response of the first order derivative. Similar is the case for a thin line, the response of the second order derivative is greater than the response of the first order derivative. Coming to this step edge, the response

of the first order derivative and the response of the second order derivative is almost the same, but the difference is in case of second order derivative, I have a transition from a positive polarity to a negative polarity.

Now because of this transition from positive polarity to negative polarity, the second order derivatives normally lead to double lines the moment in case of a step discontinuity in a image. Whereas, the first order derivative that leads to a single line, of course this double line getting this double line usefulness of this we will discuss later. Now but as we have seen, the first order, the second order derivative gives a stronger response to isolated points or to thin lines and because the details in an image normally has the property that it will be either isolated points or thin lines to which the second order gives us, second order derivative gives a stronger response.

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Observations

- First order derivative generally produce thicker edges in an image
- Second order derivatives give stronger response to fine details such as thin lines and isolated points
- First order derivative have stronger response to gray level step
- Second order derivative produce a double response at step edges

→ Second order derivatives are better suited for image enhancement

So it is quite natural to think that the second order derivative based operator will be most suitable for image enhancement operations. So our observation is, as we have discussed previously that first order derivative generally produce a thicker edge because we have seen that during a ramp or along a ramp the first order derivative is non-zero, whereas the second order derivative along a ramp is zero but it gives non zero values at the starting of the ramp and the end of the ramp.

So that is why the first order derivatives generally produce a thicker edge in an image. The second order derivative gives stronger response to fine details such as thin lines and isolated points. The first order derivative have stronger response to gray level step. And the second

order derivative produce a double response at step edges. As we have already said that as the details in the image are either in the form of isolated points or thin lines, so the second order derivatives are better suited for image enhancement operations.

So we will mainly discuss about the second order derivatives for image enhancement but to use this for image enhancement operation obviously because our images are digital and as we have said many times that we have to have a discrete formulation of this second order derivative operations. And, the filter that we design that should be isotropic, that means the response of the second order derivative filter should be independent of the orientation of the discontinuity in the image.

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$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

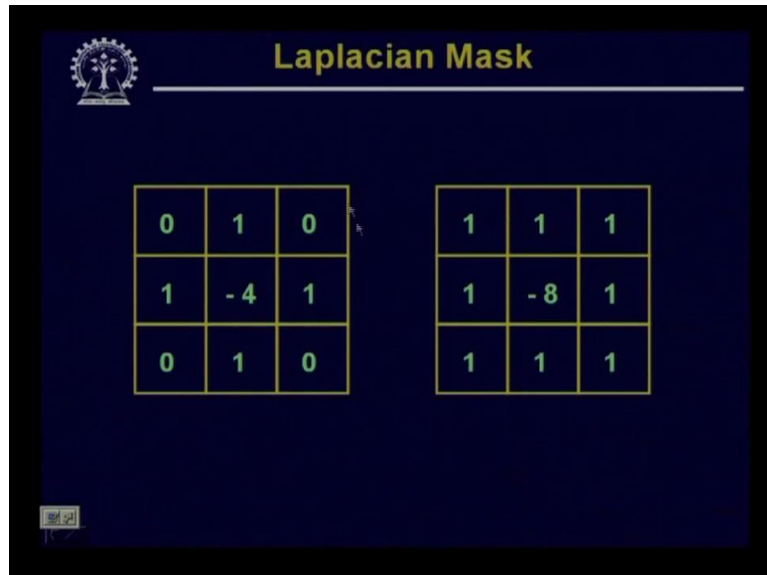
$$= [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

And the most widely used or the popularly known second order derivative operator of isotropic nature is what is known as a laplacian operator, so we will discuss about the laplacian operator. And as we know that the laplacian of a function is given by $\text{del}^2 f = \text{del}^2 f \text{ del} x^2 + \text{del}^2 f \text{ del} y^2$. So this is the laplacian operator in continuous domain. But what we have to have is the laplacian operator in the discrete domain, and as we have already seen that $\text{del}^2 f \text{ del} x^2$ or in case of discrete domain is approximated as $f(x-1) + f(x+1) - 2f(x)$. So this is in case of a one dimensional signal.

In our case, our function is a two dimensional function, that is a function of variables x and y . So for this, we can write for two dimensional signal $\text{del}^2 f \text{ del} x^2$, which will be simply $f(x+1, y) + f(x-1, y) - 2f(x, y)$. Similarly, $\text{del}^2 f \text{ del} y^2$ will be given by $f(x, y+1) + f(x, y-1) - 2f(x, y)$, and if I add this two, I get the laplacian operator in discrete domain which is given by $\text{del}^2 f =$

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ and you will find that this will be given as $[f(x+1,y)+f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$ and this particular operation can be represented again in the form of a two dimensional mask, that is for this laplacian operator, we can have a two dimensional mask.

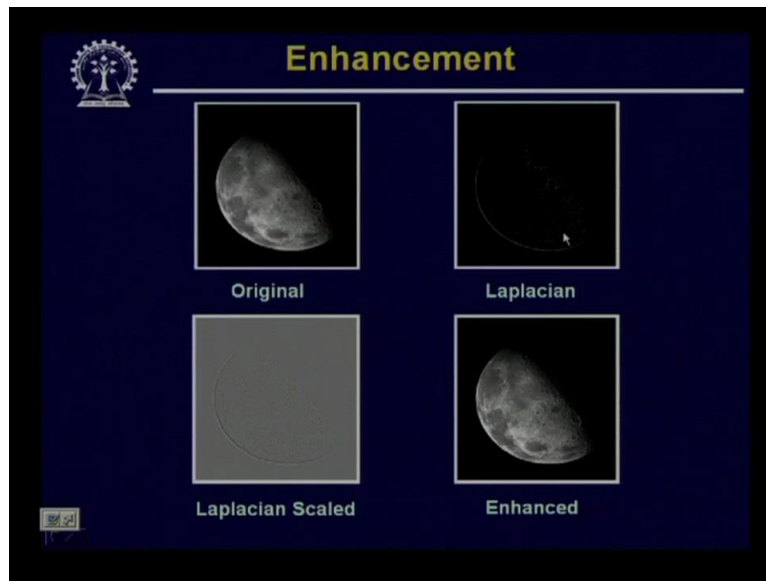
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And the two dimensional mask in this particular case will be given like this, so on the left hand side the mask that is shown, this, this mask considers the laplacian operation only in the vertical direction and in the horizontal direction, and we if we also include the diagonal directions, then the laplacian mask is given on the right hand side. So you find that using this particular mask which is shown on the left hand side, I can always derive the expression that we have just shown.

Now here I can have two different types of mask depending upon polarity of the coefficient at the center pixel, I can have the center pixel to have a polarity either negative or positive. So, if the polarity is positive say of the center coefficient, then I can have a mask of this form, but the center pixel will have a positive polarity but otherwise the nature of the mask remains the same.

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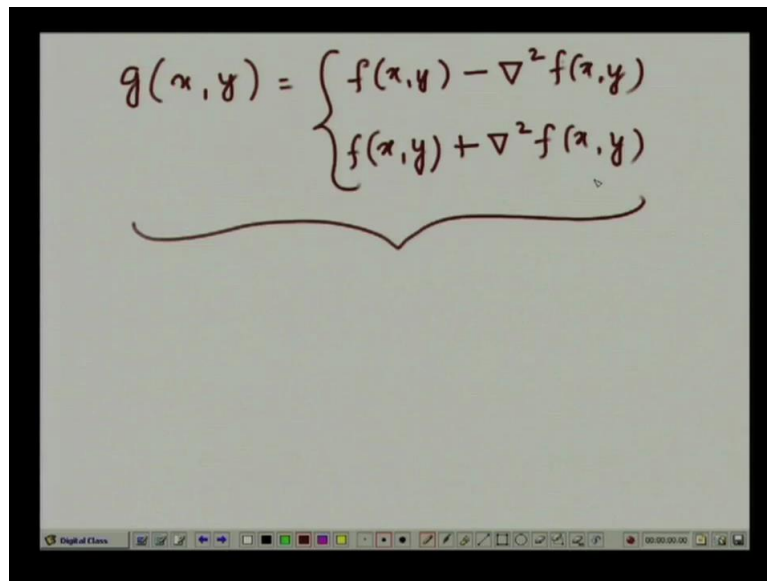


Now if I have this kind of operation, then you find that the image that you get, that will have a that will just highlight the discontinuous regions in the image, whereas all the smooth regions in the image will be suppressed. So this shows an original image, on the right hand side we have the output of the laplacian, and if you closely look at this particular image you will find that all the discontinuous regions will have some value. However this is this particular image cannot be displayed properly, so we have to have some scaling operation because output of the laplacian will be will have both positive values as well as the negative values.

So for scaling what we have to do is, the minimum negative value, we have to add to that particular image than we have to scale it down so that the image can be fit or can be displayed properly on the given terminal. So this shows the scaled laplacian image. And the right hand side shows an enhanced image, now what is this enhancement? By enhancement what we mean in this particular case? So as we have just said that this laplacian operator simply enhances the discontinuities in the image, whereas all the smooth regions in the image will be suppressed.

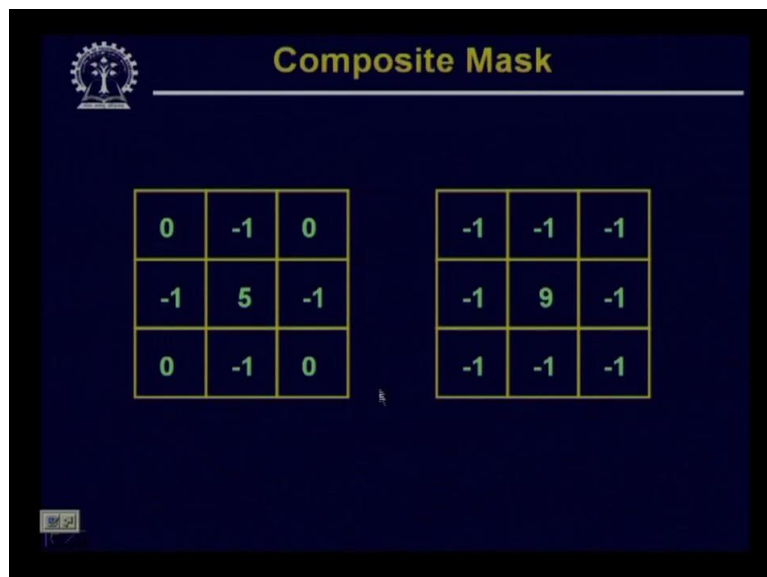
But if we want to superimpose this discontinuities, this enhanced discontinuities of the image, enhanced discontinuities on the original image, in that case what we will have is an image which is an enhanced version of the original image. Now how we can obtain such enhancement? This can be obtained simply by adding the scaled laplacian output that you get either subtracting that from the original image or adding that to the original image.

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$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) \\ f(x, y) + \nabla^2 f(x, y) \end{cases}$$

So what we can have is, we can have a function of this form $g(x,y) = f(x,y) - \text{del square } f(x,y)$ so this will be done when the center coefficient is negative or we have to perform this operation $f(x,y) + \text{del square } f(x,y)$ when the center coefficient of the laplacian mask is negative. So if I do this, then all the details in the image will be added to the original image so the background smoothness will be maintained, however the image that we will get is an enhanced image. So that is what we have got in this particular case that on the right bottom, the image that we have got is an enhanced image.

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Composite Mask

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

And the composite mask after performing this operation, we get as given over here you find that the center coefficient in both the cases has been incremented by one, so that indicates that whatever the laplacian operator that we have got the laplacian output will be added to pixel location $f(x,y)$. So we will stop our discussion today and we will continue with our discussion on the same topic as well as our frequency domain techniques in the next class. Thank you.