Digital Image Processing.
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Lecture-35.
Histogram Equalization and Specifications-II.

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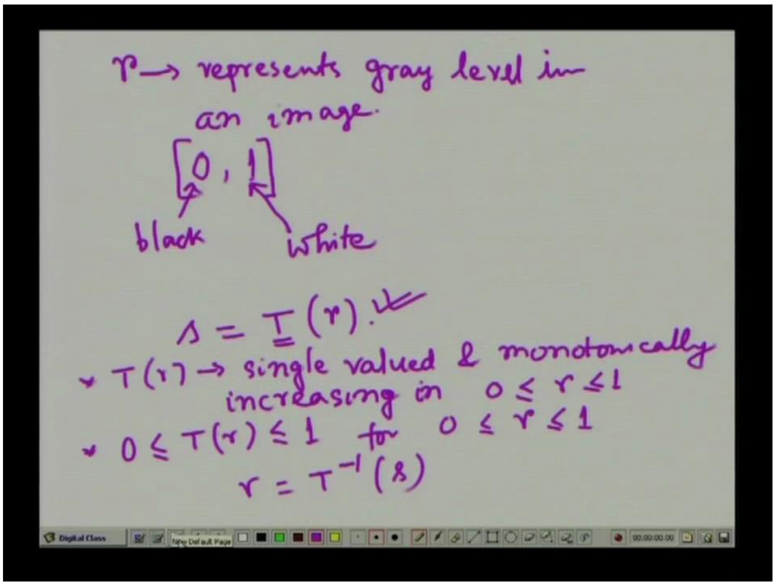

Hello, welcome to the video lecture series on Digital Image Processing.

We will also satisfy this particular condition. Now let us see how the histograms help us to get a Transformation function of this form. So now we assume, so as we said that we assume that the images assume the intensity values normalized intensity values in the range 0 to 1 and as we said that r is an intensity value in the original image s is an intensity value in the processed image.

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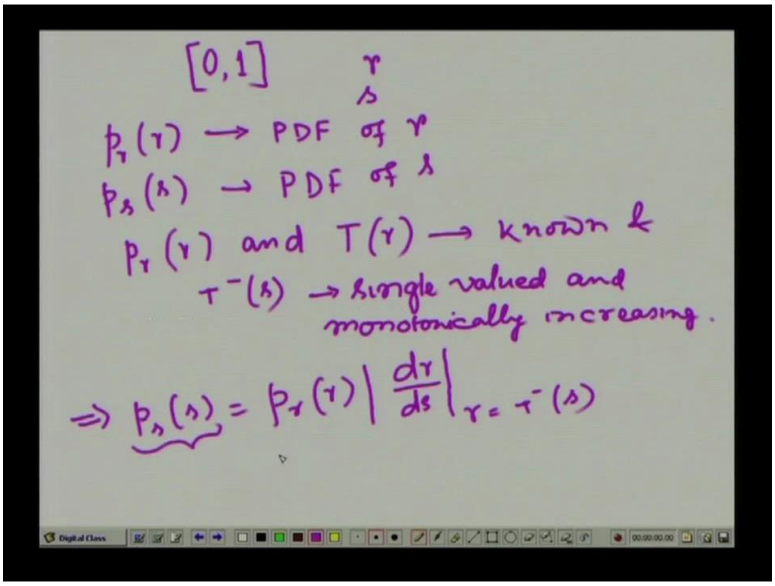

We assume pr(r) to be the Probability Density Function of r where r is a variable representing intensity values in the original image and we also assume ps(s) to be the PDF or Probability Density Function of s where s is a variable representing intensity values in the processed image. So these are the two probability functions PDFs that we assume.

Now given this, from elementary probability theory, we can we know that if pr(r) and the transformation function T(r), they are known and T inverse(s) is single valued and monotonically increasing. T inverse(s) is single valued and monotonically increasing. Then we can obtain the PDF of s, that is ps(s) is given by pr(r) into dr ds where at r equal to T inverse(s).

So this is what is obtained from elementary probability theory that if we know p r(r) and we also know T(r) and T inverse(s) is single valued and monotonically increasing then p s(s) can be obtained from p r(r) as p s(s) is equal to p r(r) into dr ds. Now all the histogram processing techniques they try to modify the probability density function PDF p s(s) so that the image gets a particular appearance and this appearance is obtained via the transformation function T(r).

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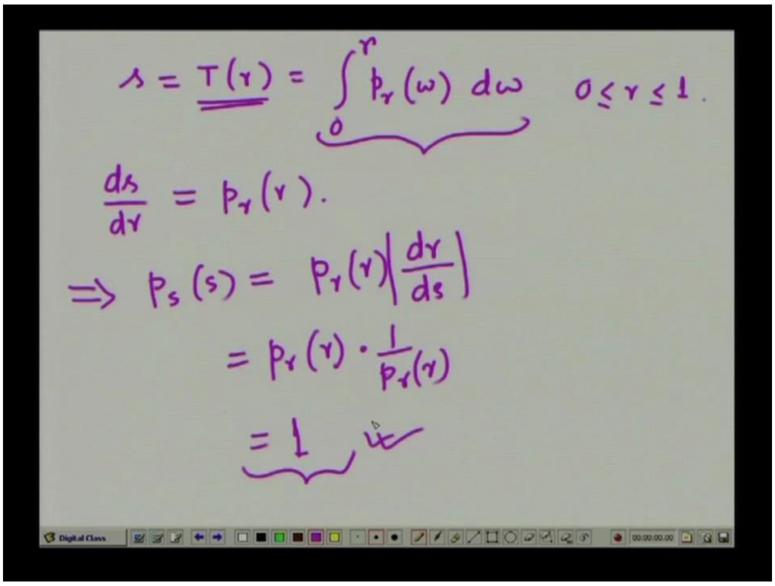

So now what is that type of T(r), the transformation function T(r) that we can have. So let us consider a particular transformations function. Say we take a transformation function of this form say s is equal to T(r) is equal to integral p r(w) dw where the range of integration varies from 0 to r and r varies in the range 0 to 1. So find that this integral gives the cumulative distribution function of the variable r. Ok?

Now if I take T(r) of this particular form, then this particular T(r) will satisfy all the conditions, both the conditions that we have stated earlier and from this we can compute ds upon dr which is nothing but p r(r). So by substitution in our earlier expression you will find that p s(s) as we have said is nothing but p r(r) into dr ds; this we have said earlier, this was obtained from elementary probability theory.

And in this particular case this will be p r(r) into 1 upon p r(r) which will be equal to 1. So you find that if we take this particular transformation function which is nothing but cumulative distribution function of the variable r. Then using this transformation function, the transformation that we get, this generates an image which has an uniform probability density function of the intensity values s.

And we have seen earlier, that an image high contrast image have a probability distribution function or has a histogram which has intensity values, pixels having intensity values over the entire range 0 to 255 of the pixel values. So if I go for this kind of transformation, as we are getting a uniform Probability Distribution Function, Probability Distribution function of the processed image.

Uhh then this is expected, then this is what is going to enhance the contrast of the image. And this particular result is very very important. That p s(s) is equal to 1 and you find that we have obtained this result irrespective of T inverse(s). And that is very very important because it is, it may not always be possible to obtain T inverse analytically. So whatever be the nature of T inverse(s) if we take the cumulative distribution function of r and use that as the transformation function T(r), then the image is going to be announced.

So this simply says that using CTF the Cumulative Distribution Function as the transformation function. We can enhance the contrast of an image and by this contrast enhancement what you mean is the dynamic range of the intensity values is going to be increased. Now what we have discussed till now, this is valid for the continuous domain but the images that we are going to consider. All the images are discrete images so we must have a discrete formulation of whatever derivation that we have done till now.

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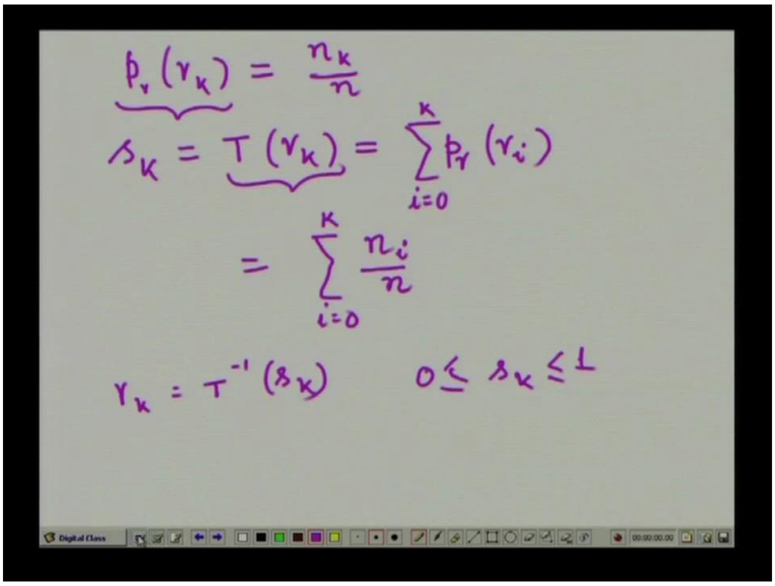

So now let us see that how we can have a discrete formulation of these derivations. So for discrete formulation what we have seen earlier that p r(r K) is given by n K divided by N where n K is the number of pixels having intensity value r K and n is the total number of pixels in the image. And a plot of this p r(r K) for all values of r K gives us the histogram of the image.

So the technique to obtain the histogram equalization by that the image enhancement will be; first we have to do the find out the Cumulative Distribution Function, the CDF of r K. And so, we will get s K which is given by T(r K) and this T(r K) now is the Cumulative

Distribution Function which is p r of say (r i) where i will vary from 0 to K and this is nothing but sum of n i by n where i will vary from 0 to K.

The inverse of this is obviously r K is equal to T inverse of (s K) for 0 less than or equal to s K less than or equal to 1. So if I use this as a transformation function, then the operation that we get is an histogram equalization and as we have said that this histogram equalization basically gives us a transformed image where the intensity values have an uniform distribution and because of this, the image, the processed image that we get appears to be a high contrast image.

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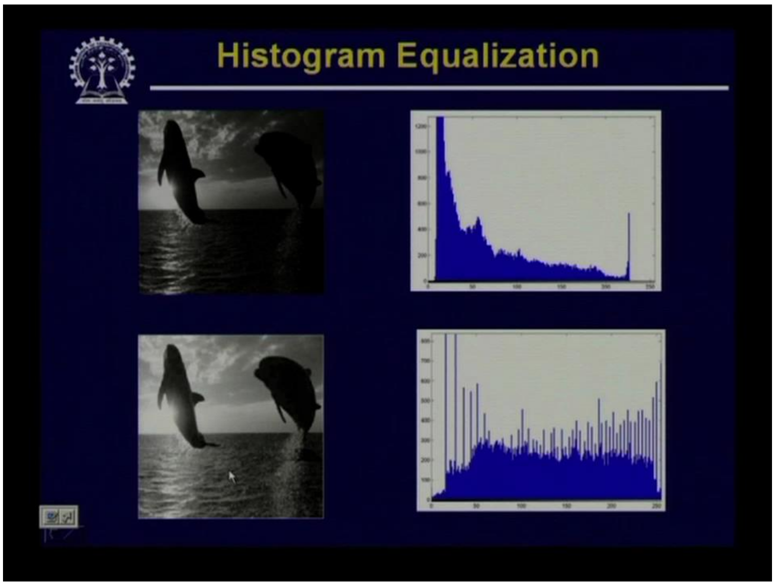

So let us see that what are results that we can get using such a kind of histogram equalisation operation. So here on the left hand side we have an image and it is obvious that the contrast of this image is very very poor. On the right hand side we have shown the histogram of this particular image and here again you find that from this histogram that most of the pixels in this particular image have intensities which very close to zero and there are very few pixels in this image which have intensities having higher values.

By this histogram equalisation the image that you get is shown on the bottom and here you find, that this image obviously has a contrast which is higher than the previous image because many of the details in the image are not very clear in the original image whereas those details are very clear in this second image. And on the right hand side, we have shown the histogram of this processed image.

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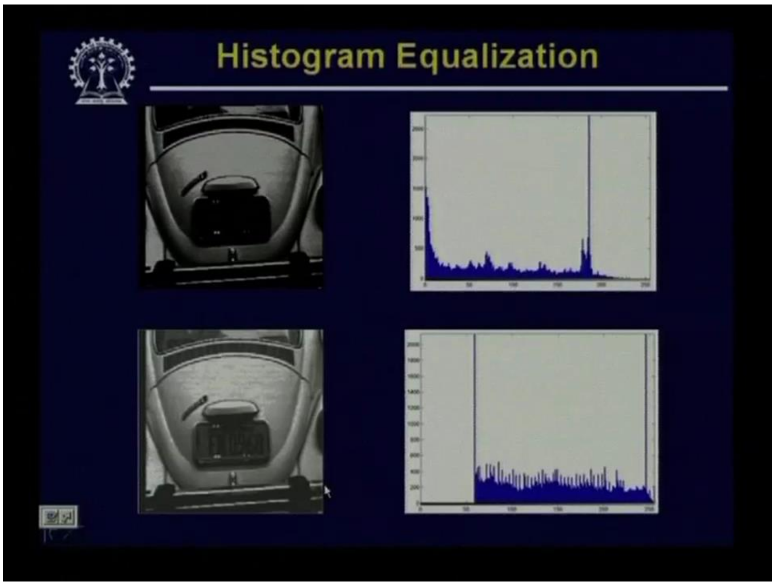

And if you compare these two histograms you will find that the histogram of this processed image is more or less uniform equalisation we can have such a kind of enhancement. This shows another image again processed by histogram equalization, so on the top you will find that the image of the of a part of a car and because of this enhancement not only the image appears to be better, but if you look at this number plate you will find that in the original image, the numbers are not readable.

Whereas in the processed image, I can easily read this number say something like F N 0 9 6 8. So this is not readable in the original image but it is readable in the processed image and the histogram that I get of this particular image which is almost which is near to be uniform. So this is one kind of histogram based processing technique that is histogram equalization which gives enhancement of the contrast of the image.

Now this though this enhancement, it gives contrast enhancement but histogram equalisation has got certain limitation. First of the limitation is using this histogram equalization whatever image you get, the equalized image you get that is fixed. I cannot have any interactive manipulation of the image. So it generates only a single processed image.

Now to overcome this limitation, if some of the applications, if some application demands that we want to enhance only certain region of the histogram. We want to have the details within certain region of the histogram not what is given by the histogram equalization process. Then, the kind of technique that should be used what is called histogram matching or histogram specification techniques.

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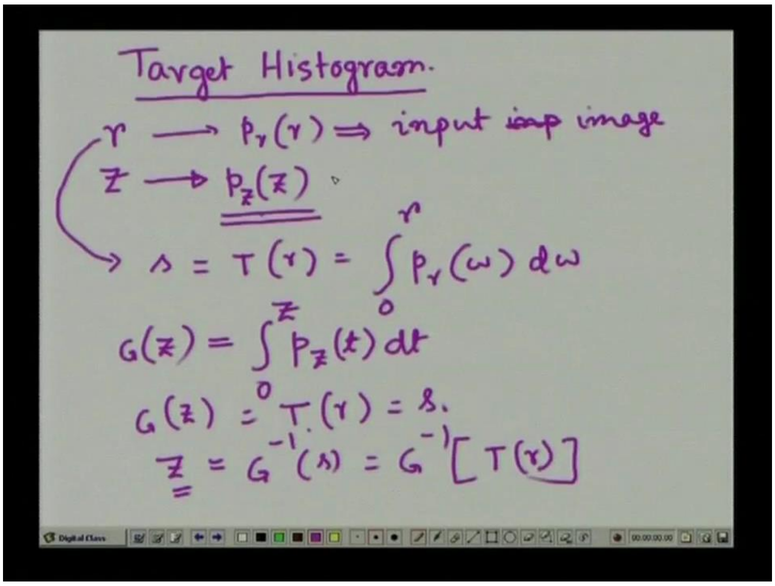

So in case of histogram specification techniques what we have to have is we have to have a target histogram. So we have to have a target histogram and the image has to be processed in such a way that the histogram of the processed image becomes same as that of the target histogram. Now to see how we can go for such a type of histogram specification or histogram matching or histogram modification.

Initially we assume that we again we have two variables. One is variable r representing the continuous gray levels in the given image and we assume a variable z representing intensities in the processed image or as the intensities in the original image and z represents the intensities in the processed image where this is specified in the form of the probability distribution function p z(z).

So this p z(z) specifies our target histogram. And from the given image r we can obtain p r(r) that is the histogram of the given image. This we can obtain from the input image whereas p z(z) that is target histogram is specified. Now for this histogram matching what we have to do is, if I equalize the given image using the transformation function s is equal to T(r) as we have seen earlier is equal to p r(w) dw within range 0 to r.

So if I equalize the image using this particular transformation function then what I get is an image having intensity values with Probability Distribution Function, probability density function which is uniform. Now using this p z(z), we compute the transformation function G(z). So this G(z) will be obtained as integration p z(z) sorry p z(t)into dt in the range 0 to z.

And then, from these two equations, what we can have is G(z) is equal to T(r) that is equal to s. Ok? And this gives z equal to G inverse(s) which is equal to G inverse T(r). So we find that the operations that we are doing is firstly we are equalizing the given image using histogram equalization techniques. We are finding out the transformation function G(z) from the histogram from the target histogram that has been specified.

Then this equalized image is inverse transformed using the inverse transformation G inverse(s). Ok? And the resultant image by doing this operation, the resultant image that we will get, that is likely to have an histogram which is given by this target histogram p z(z). So our procedure is, first equalize the original image obtaining the histogram from the given image.

Then find out the transformation function G(z) from the target histogram that has been specified,then do the inverse transformation of the equalised image using not T inverse but using the G inverse; and this G inverse has to be obtained from the target histogram that has been specified. And by doing this the image that you get becomes an histogram modified image, a processed modified image processed image whose histogram is likely to be same as the histogram that has been specified as the target histogram.

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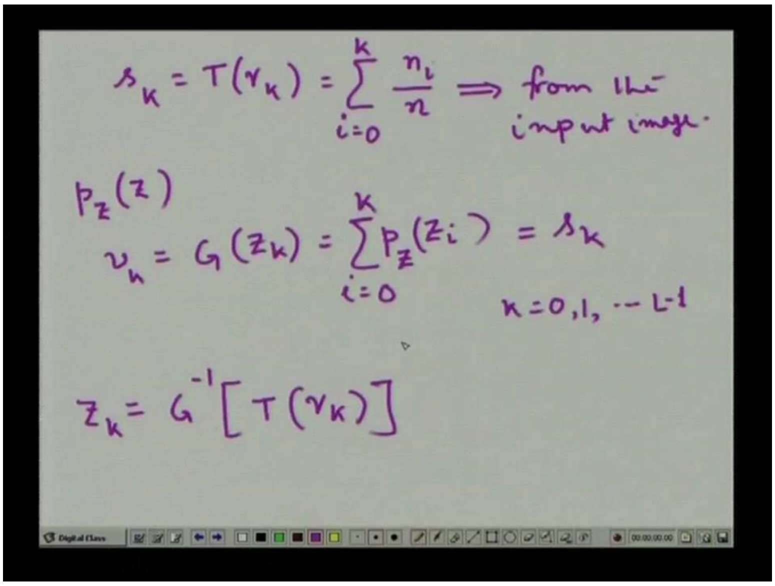

So again this is a continuous domain formulation but our images are digital so we have to go for discrete formulation of these derivations. So now let us see that how we can discretize this particular formulation. So again as before we can find out s K which is equal to T(r K) which is equal to sum of n i by n where n varies from sorry where r varies from, i varies from 0 to K.

And this we obtain from the given image, from the input image. And from the target histogram that is specified that is p z(z), we get a transformation function say v K equal to G(z K) which is equal to sum of p z(z i) where now i varies from 0 to K and we set this equal to K and this has to be for K equal to 0 1 up to L minus 1. And then finally we obtain the processed image as the inverse of or G inverse of T(r K).

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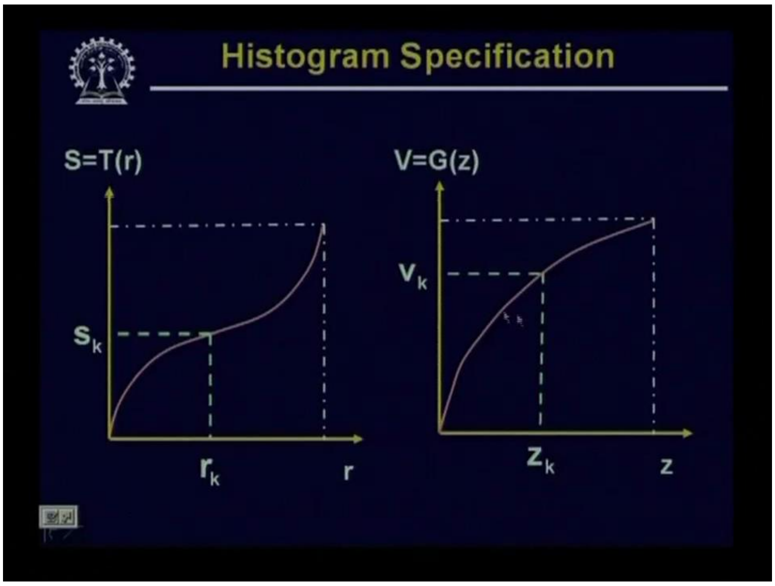

So this is the discrete formulation of the continuous discrete formulation of the continuous domain derivation that we have done earlier. Now let us see that using this, what kind of operations that we have. So here we it shows a transformation function T(r) is equal to T(r) on the left hand side which is obtained from the given image and using the target histogram we obtain the function G(z).

So this function T(r) gives the value s K for a particular intensity of value r K in the given image. The function G(z), it is supposed to give an output value V K for an input value Z K. Now coming to G(z) you find that Z K is the intensity value which is not known to us. We want to find out Z K from r K. So the operation that we will be doing for this is whatever s K that we get from r K we set that s K to this second transformation and now you do the inverse Transformation operation.

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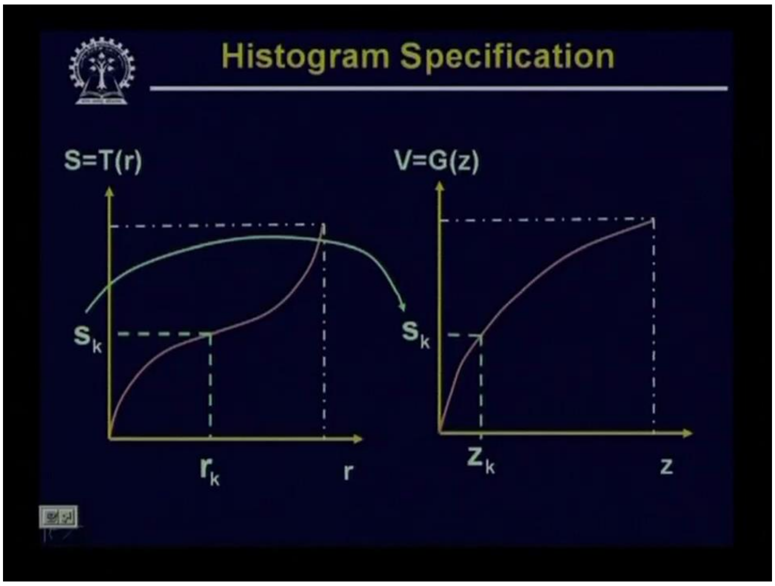

So as shown in the next slide, we set S K along the vertical axis of this v equal to G(z) transformation function. Then you do the inverse transformation that is from s K, you come to z K. So what we have to apply is an inverse transformation function to get the value z K for a given intensity of value r K in the original image. Ok? Now conceptually or graphically this is very simple but the question is how to implement it.

Here you find that in the continuous domain, we may not get analytical solution for G inverse but in the discrete domain, the problem becomes simpler because we are considering, we are dealing with only discrete values. So in case of discrete domain, this transformation function that is r K to s k or s equal to T of (r) or z K to or z K to v K that is v equal to G(z). These transformations can be implemented by simple look up tables.

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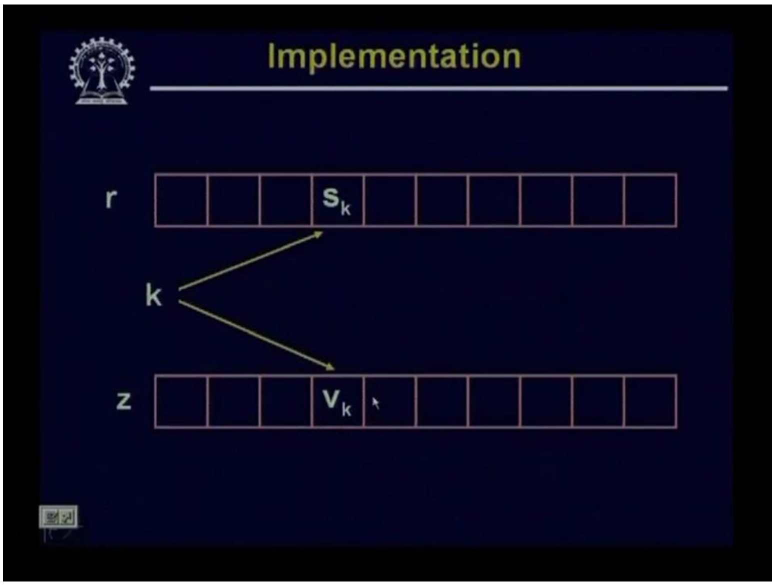

So by this what I mean is something like this. T(r) is represented by an array where for r K, the K indicates an index into an array and the element in that particular array location gives us the value s K. so whenever a value r K is specified using K immediately you will go to this particular array r and the content of that array location gives us what is the corresponding value s K.

Similarly, for v equal to G(z K), v K equal to G(z K), we have similar operation that if z K is known, I can use K as an index, go to the array z then I get the corresponding value v K. now the first case it is very simple. I know what is the value of r K so I can find out what is the corresponding value of s K from this array. But the second one is an inverse operation. I know s K or as we have equated s K to v K, I know what is v K.

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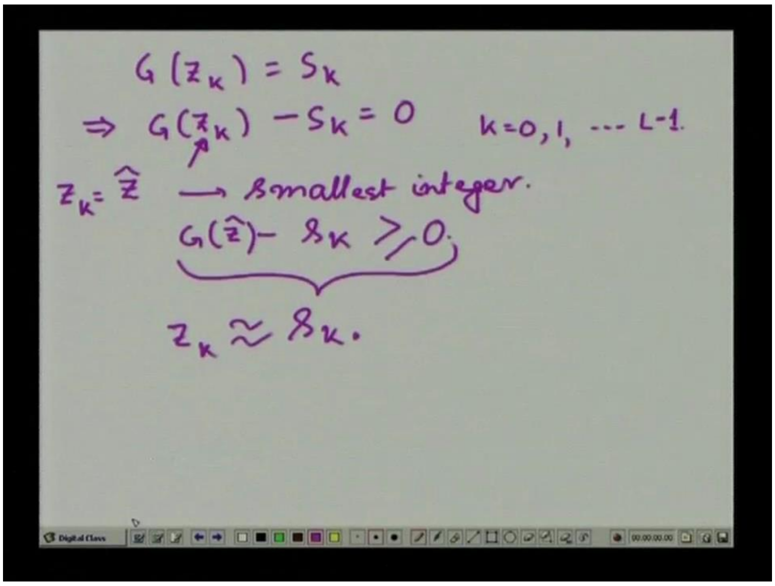

Now from this v K, I have to find out what is the corresponding value z K. So this is an inverse problem and to solve this problem, we have to go for an iterative solution approach. So iterative solution, we can obtain in this form. We know that G(z K), is equal to s K. so this gives G(z K) minus s K, this is equal to 0. So our approach will be to iterate on the values of z K to get a solution on this and this has to be done for K equal to 0,1 up to L minus 1.

So what we should do? The closest solution can be that we initialize z to a value say z hat. So we initialise z K is equal to z hat for every value of K where this z hat is the smallest integer z hat is the smallest integer which satisfies G(z hat) minus s K greater than or equal to 0. So our approach can be that we start with a very small value, the smallest integer of z hat then go on incrementing z hat by 1 at every step until and unless this condition is satisfied.

So when this condition is satisfied, then the value of z hat we get, that is the z K corresponding to this given value s K. So now let us stop our discussion today, we will continue with this topic in our next class. Thank you.