Digital Image Processing.
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Lecture-30.
Histogram Equalization and Specifications-1.

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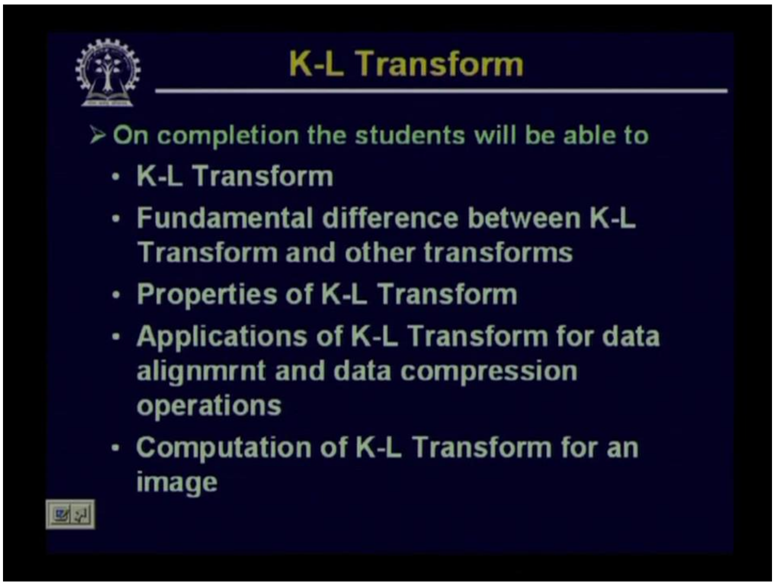

On Digital Image Processing. We will talk about another transform operation which is fundamentally different from the transformations that we have discussed in last few classes. So the transformation that we will discuss about today is called K-L transformation. We will see what is the fundamental difference between K-L Transform and other transformations. We will see the properties of K-L Transform. We will see the applications of K-L Transform for data alignment and data compression operations.

And we will also see the computation of K-L Transform for an image. Now as we said that K-L Transform is fundamentally different from other transformations. So before we start discussion on K-L Transform, let us see what is the difference. The basic difference is in all the previous transformations that we have discussed, that is whether it is the Fourier Transformation or Discrete cosine Transformation or Walsh transformation or Hadamard Transformation.

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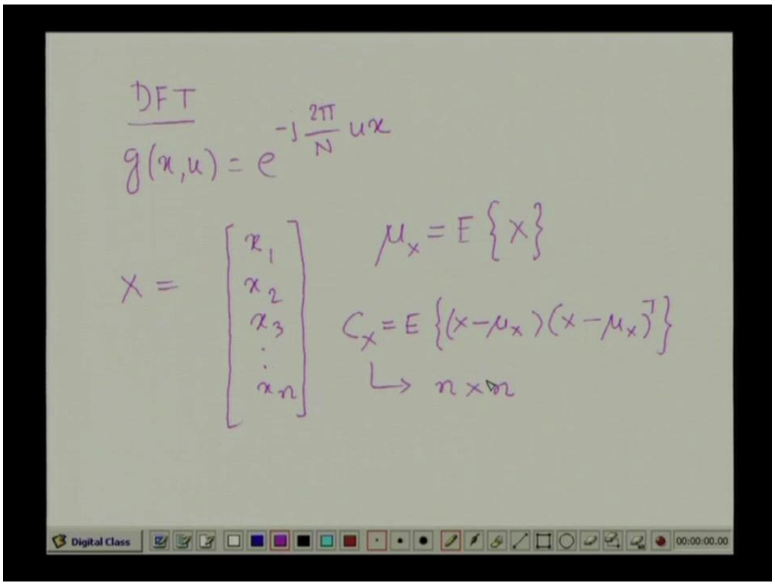

In all these cases, the transformation kernel whether it is forward transformation kernel or inverse transformation kernel, they are fixed. So for example in case of Discrete Fourier Transformation or DFT, we have seen that the transformation kernel is given by g(x,u) is equal to e to the power minus j 2 pi by N into ux.

Similarly for the Discrete Cosine Transformation as well as for other transformations like Walsh Transform or Hadamard Transform, in all those cases the transformation kernels are fixed. The values of the transformation kernel depend upon the locations x and the location u. The kernels are independent of the data over which the transformation has to be performed.

But unlike these transformations, in case of K-L Transformation, the transformation kernel is actually derived from the data. So in case of K-L Transform, it actually operates on the basis of statistical properties of vectored representation of the data. So let us see, how these transformations are obtained? So to go for K-L Transformation, our requirement is the data has to be represented in the form of vectors.

So let us assume,a population of vectors say X which are given like this. So we consider a vector population X which is given by say x 1, x 2, x 3 say upto x n. So these vectors X are actually vectors of dimension n. Now given such a set of vectors, population of vectors X, we can find out the mean vector given by mu X which is nothing but the expectation value of these vector population X.

And similarly, we can also find out the covariance matrix C X which is given by the expectation value of X minus the mean vector mu X into X minus mu X transpose. So here you find that X since X is of dimension n, this particular covariance matrix will be of dimension n by n. So this is the dimensionality of the covariance matrix CX and obviously the dimensionality of the mean vector mu X will be equal to n.

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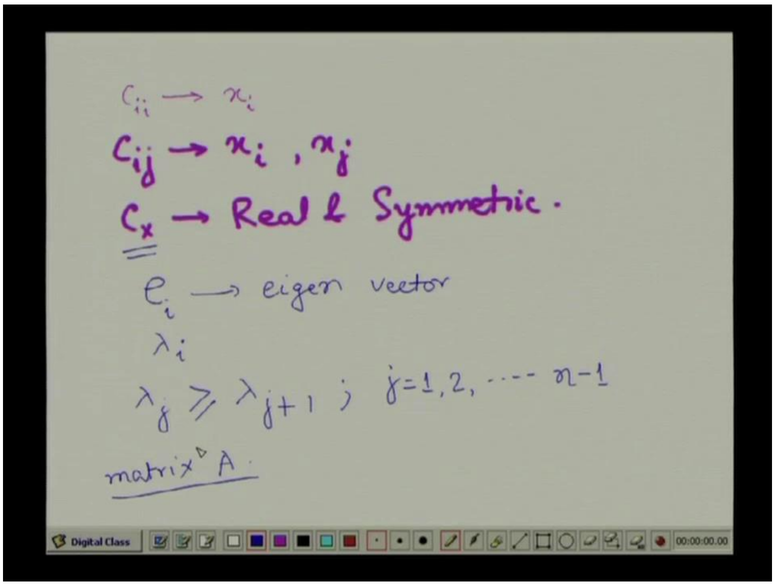

Now in this covariance matrix, C X, you will find that an element C ii, that is the element in the i th row and the i th column is nothing but the variance of the element x i of the vectors x. Similarly an element say C ij, this is nothing but the covariance of the elements x i and x j of the vectors x. And you find that this particular covariance matrix c x, it is real and symmetric.

So because this covariance matrix is real and symmetric, we can always find a set of n orthonormal eigenvectors. So because, this covariance matrix C X is real and symmetric, we can find out a set of orthonormal eigenvectors of this covariance matrix C X. Now if we assume that suppose e i is an eigenvector of this covariance matrix C X which corresponds to the eigenvalue lambda 1 lambda i.

So corresponding to the eigenvalue lambda i, we have the eigenvector say e i and we assume that these eigenvalues are arranged in descending order of magnitude of the eigenvalues. That is we assume that lambda j is greater than or equal to lambda j plus 1 for j varying from 1, 2 up to n minus 1. So what we are taking? We are taking the eigenvalues of the covariance matrix C X and we are taking the eigenvectors corresponding to every eigenvalue.

So corresponding to the eigenvalue lambda i we have this eigenvector e i and we also assume that this eigenvalues are arranged in descending order of magnitude that is lambda j is greater than or equal to lambda j plus 1 for j varying from 1 to n minus 1. Now from this set of eigenvectors, we form a matrix say A. So you form matrix A from this set of eigenvectors and this matrix A is formed in such a way that the first row of matrix A is the eigenvector corresponding to the largest eigenvalue.

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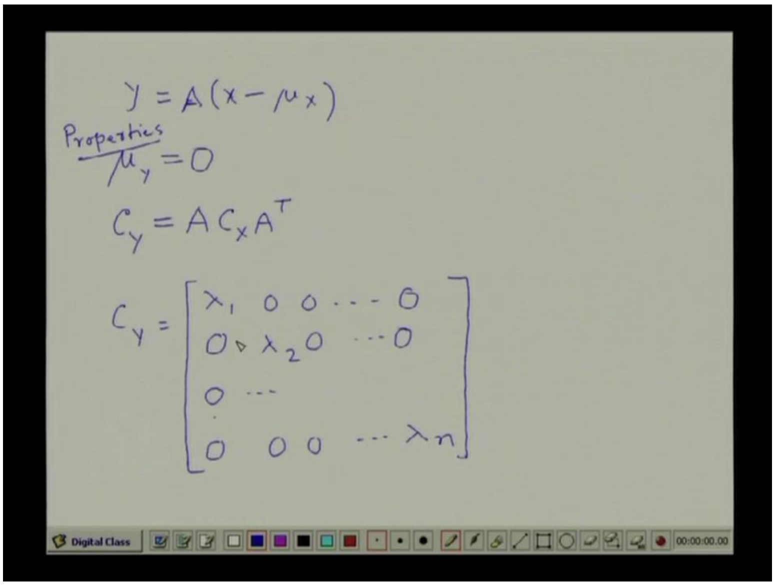

And similarly the last row of this matrix A, corresponds to the eigenvector is the eigenvector which corresponds to the smallest eigenvalue of the covariance matrix C X. Now if we use such a matrix A to obtain the transform operations, then what we get is, we get a transformation of the form say y equal to A into X minus mu X. So using this matrix A which has been so formed, we form a transformation like Y equal to X minus mu X where you find that X is a vector and mu X is the mean vector.

Now this particular transformation, the transformed output Y that you get that follows certain important relationship. The first relationship, the important property is that the mean of these vectors y or mu Y is equal to 0. So these are the properties of the vector Y that is obtained. So the first property is the mean of Y, mean of vectors Y, mu Y equal to 0.

Similarly the covariance matrix of Y given by C Y, this is also obtained from C X, the covariance matrix of X and the transformation matrix that we have generated A. And the relationship between the covariance matrix of Y is like this that C Y is given by A C X A transpose. Not only that, this covariance matrix C Y is a diagonal matrix whose elements along the main diagonal are the eigenvalues of C X.

So this C Y will be of the form lambda 1 0 0 so it continues like this. 0 then 0 lambda 2 0, it continues like this. Then finally we have 0 0 0 and up to this we have lambda 1. So this is the covariance matrix of Y that is C Y. And obviously, in this particular case, you find that the eigenvalues of C Y is same as the eigenvalues of C X which is nothing but lambda 1 lambda 2 upto lambda n.

And it is also a fact that the eigenvectors of C Y will also be same as the eigenvector of C X. And since in this case, we find that the off diagonal elements are always 0, that means the elements of Y vectors, they are uncorrelated. So the property of the vectors Y that we have got is the mean of the vectors equal to 0. We can obtain the covariance matrix C Y from the covariance matrix C X and the transformation matrices A.

The eigenvalues of C Y are same as the eigenvalues of C X and also as the off diagonal elements of C Y are equal to 0, that indicates that the elements of the vectors Y different elements of the vector Y are uncorrelated. Now let us see what is the implication of this. To see the implication of these observations let us come to the following figure. So in this figure we have, a binary image, 2 dimensional binary image.

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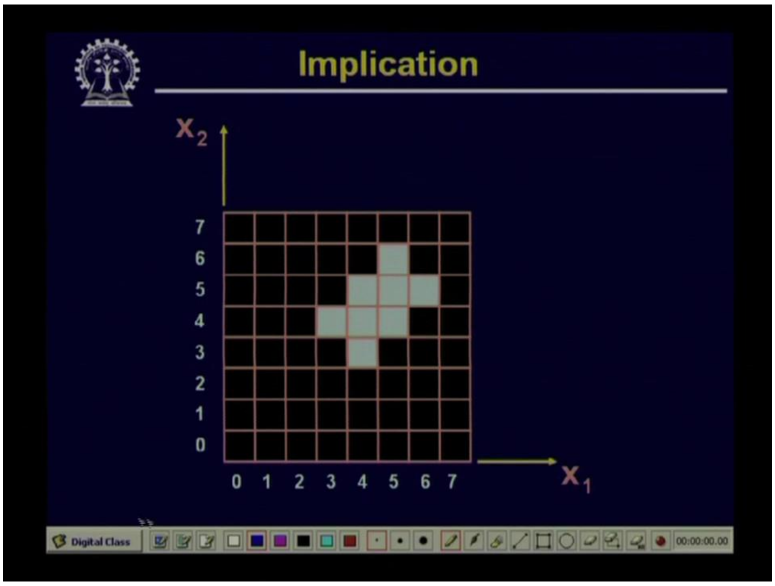

Here we assume, that all the pixel locations which are white, there an object is present an object element is present and wherever the pixel value is 0, there is no object element present.

So in this particular case, the object region consists of the pixels say 34, 43, 44 then 45 then 54 then 55 then 56 and 65. So these are the pixel locations which contains the objects and other pixel locations does not contain the object.

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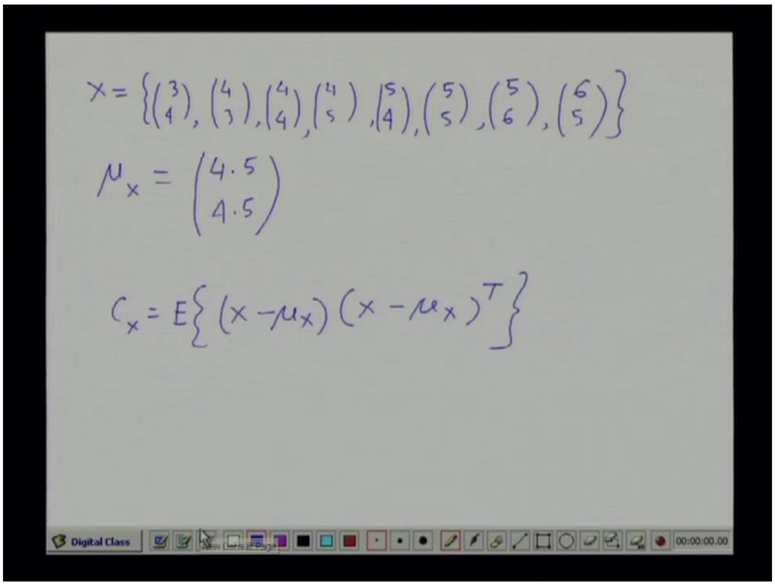

Now what we plan to do is we will find out the K-L Transform of those pixel locations where an object is present. So from this, we have the population of eigenvectors which is given by this. Just reconsider the locations of the pixels where an object is present that is the pixel is equal to white. And those locations are considered as vectors and so the population of vectors X is given by we have 3 4 because in location 34 we have an object present.

We have 4 3, here also an object is present. We have 4 4, here also an object is present. We have 4 5, we have 5 4 then 5 5 then 5 6 and then 6 5. So we have 1, 2, 3, 4, 5, 6, 7, 8 vectors, eight 2 dimensional vectors in this particular population. Now from these vectors it is quite easy to compute the mean vector mu X and you can easily compute that mean mune mean vector mu X in this particular case will be nothing but 4.5 4.5.

So this is the mean vector that we have got. So once we have the mean vector, now we can go for computing the covariance matrix and you will find that the covariance matrix C X was defined as the expectation value of X minus mu X into X minus mu X transpose.

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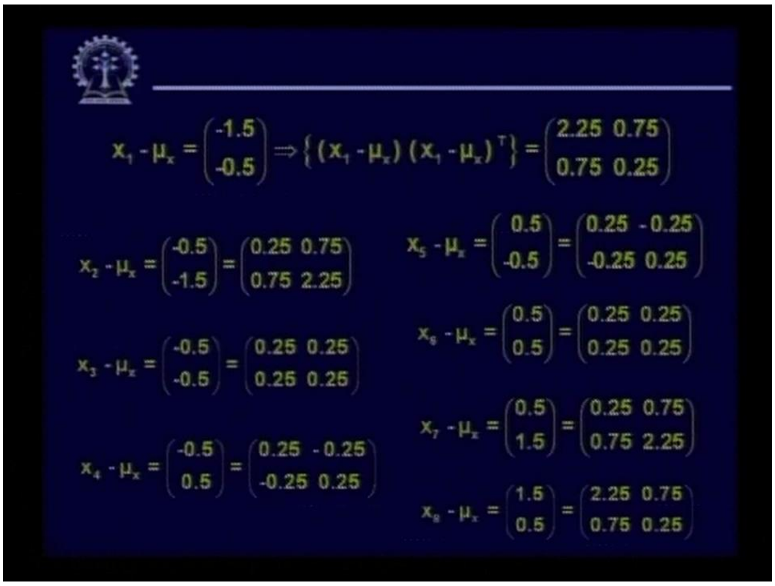

So finding out X minus mu X into X minus mu X transpose for all the vectors X and taking the average of them gives us the expectation value of X minus mu X into X minus mu X transpose which is nothing but the covariance matrix C X. So here for the first vector X 1, we can find out X 1 minus mu X as you find that X 1 is nothing but the vector 3 4.

So X 1 minus mu X will be equal to minus 1.5 and minus 0.5. So we can find out X 1 minus mu X into X 1 minus mu X transpose. If we compute this, this will be a value equal to 0.25, 0.75, 0.75, and 2.25. So similarly we find out X minus mu X into X minus mu X transpose for all other vectors in the population X.

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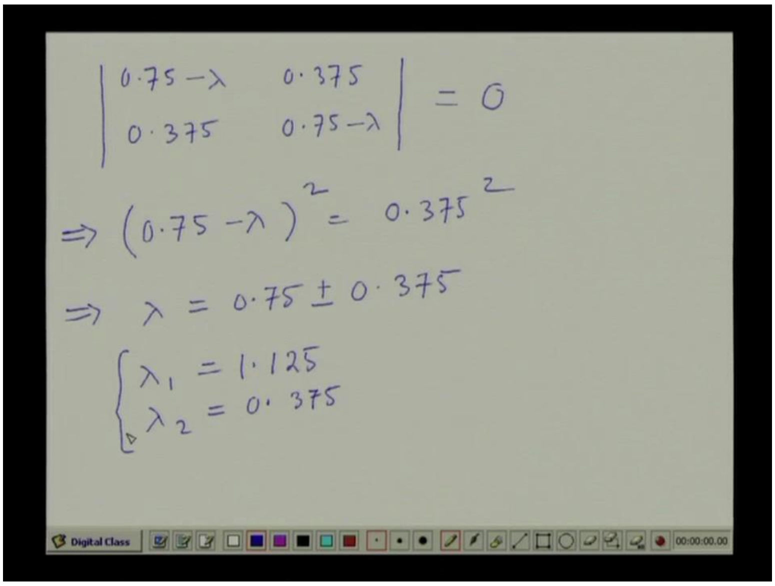

And finally, average of all of them gives us the covariance matrix C X and if you compute like this, you can easily obtain that covariance matrix C X will come out to be 0.75, 0.375 0.375 and 0.75. So this is the covariance matrix of the population of vectors X. Now once we have this covariance matrix, to find out the K-L Transformation, we have to find out what are the eigenvalues of this covariance matrix.

And to determine the eigenvalues of the covariance matrix, you all might be knowing that the operation is like this that given the covariance matrix, we simply perform 0.75 minus lambda 0.375 then 0.375, 0.75 minus lambda and set this determinant is equal to 0. And then you solve for the values of lambda. So if you do this, you will find that this simply gives an equation of the form 0.75 minus lambda square is equal to 0.375 square.

Now if you solve this, the solution is very simple. The lambda comes out to be 0.75 plus minus 0.375 whereby we get lambda 1 is equal to 1.125 and lambda 2 comes out as 0.375. So these are the two eigenvalues of the covariance matrix C X in this particular case and once we have these eigenvalues, we have to find out what are the eigenvectors corresponding to these eigenvalues.

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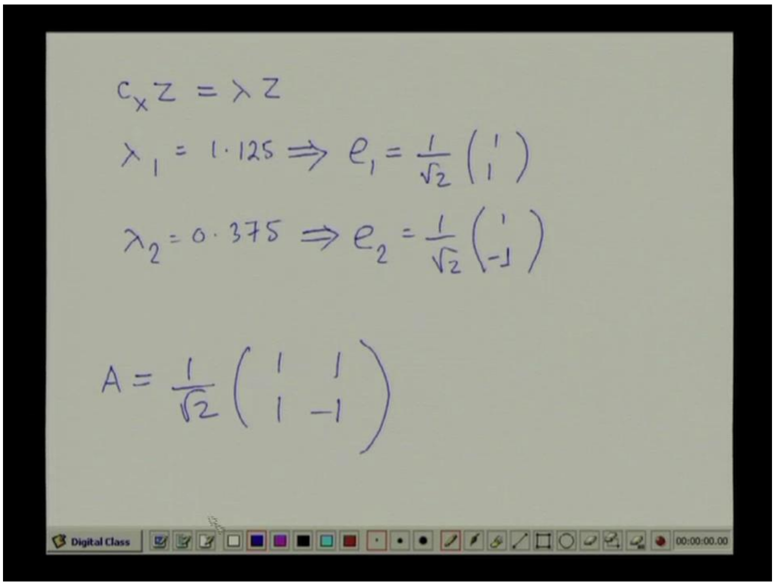

And to find out the eigenvectors, you know that the relation is for the given matrix, for a given matrix say A or in our particular case it is C X. So let us take C X. So C X into say vector Z has to be equal to lambda times Z if Z is the eigenvector corresponding to the eigenvalue lambda. And if we solve this, we find that we get 2 different eigenvectors corresponding to two different lambdas.

So corresponding to lambda 1 is equal to 1.125. We have the corresponding eigenvector e 1 which is given as 1 upon root 2 into 1, 1. So this will be the corresponding eigenvector. Similarly, corresponding to the eigenvalue lambda 2 equal to 0.375. This corresponds to the eigenvector e 2 which is equal to 1 upon root 2 into 1 minus 1.

So you find that once we get these eigenvectors, we can formulate the corresponding transformation matrix as we said we get the transformation matrix A from the eigenvectors of the covariance matrix C X where the rows of the of the transformation matrix A are the eigenvectors of C X such that the first row will correspond to the eigenvector will be the eigenvector corresponding to the maximum eigenvalue.

And the last row will be the eigenvector corresponding to the minimum eigenvalue. So in this case the transformation matrix A will be simply given by 1 upon root 2 into 1 1 1 minus 1. Now what is the implication of this? So you find that using this particular information, transformation matrix if I apply the K-L Transformation then the transformed output, the transform vector will be Y equal to A into X minus mu X.

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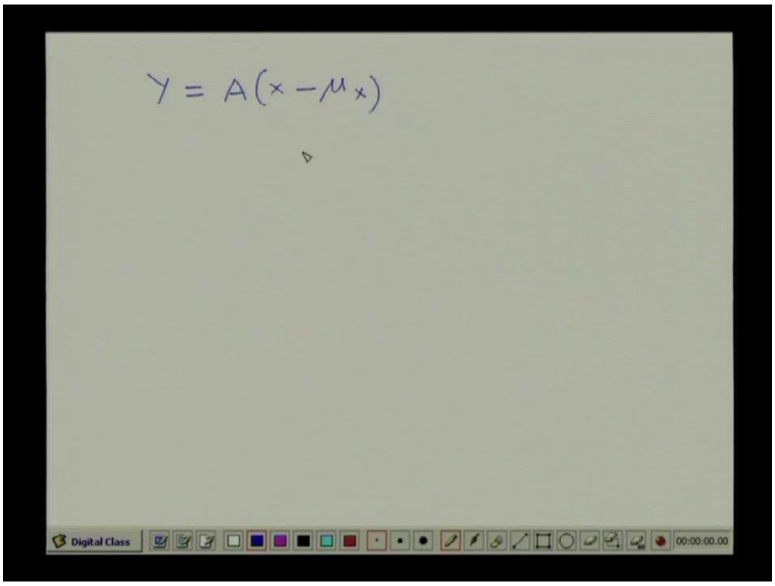

So you find that application of these particular transformation, this particular transformation amounts to establishing a new coordinate system whose origin is at the centroid of the object pixels. So this particular transformation K-L Transformation basically establishes a new coordinate system whose origin will be at the center of the object and the axis of this new coordinate system will be parallel to the direction of the eigenvectors.

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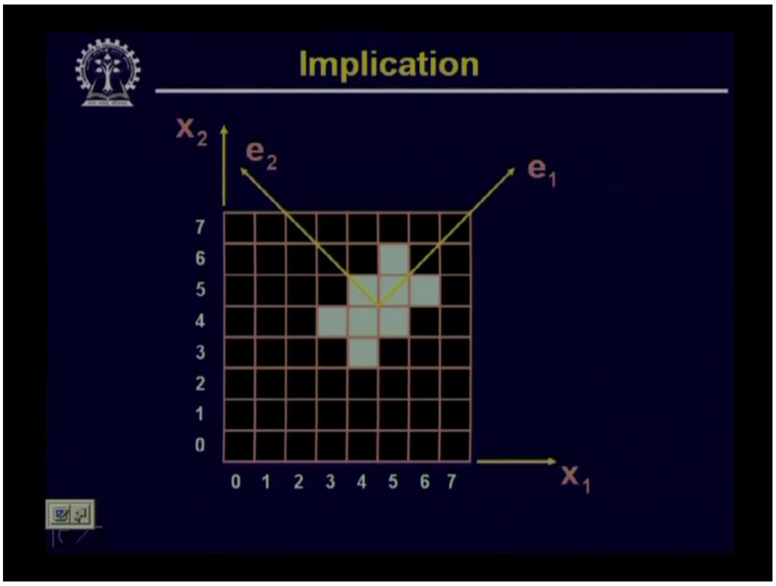

So by this what we mean is like this one. So this was our original figure where all the white pixels are the object pixels. Now by application of this transformation, this K-L Transform with transformation matrix A, we get two eigenvectors, the eigenvectors are this e 1 and e 2. So you find that this e 1 and e 2, it forms a new coordinate system and the origin of this coordinate system is located at the center of the object and the axes are parallel to the directions of the vectors e 1 and e 2.

And this figure also shows that this is basically a rotation transformation and these rotation transformation aligns the data with eigenvectors and because of this alignment different elements of the vector Y, they become uncorrelated. So it is only because of this alignment, the data becomes uncorrelated and also because the eigenvalues of lambda i appear along the main diagonal of C Y that we have seen earlier.

This lambda i basically tells the variance of the component Y i along the eigenvector e i. And later you will see the application of this kind of transformation to align the objects around the eigenvectors and this is very very important for object recognition purpose. Thank you.