Digital Image Processing.
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Lecture-29.
Hadamard Transformation.

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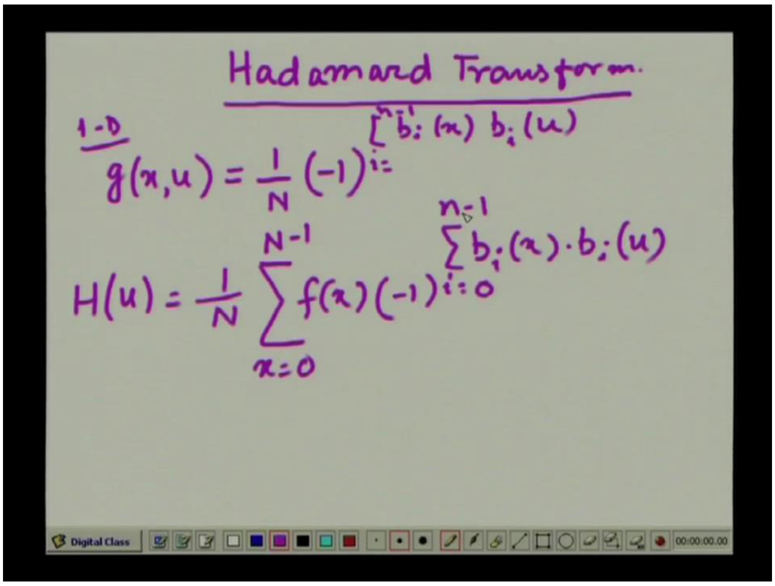

Hello, welcome to the video lecture series on Digital Image Processing. Next transformation which is called the Hadamard's transform. So the next transformation that we discuss is Hadamard transformation. In case of Hadamard transform, first let us consider the case in 1 dimension. The forward transformation kernel is given by g(x,u) equal to 1 upon capital N minus 1 to the power summation bi x into bi u where this i varies from 0 to lowercase n minus 1.

So again the capital N as well as lowercase n, they have the same interpretation as in the case of Walsh transformation. And using this forward transformation kernel, the forward Walsh transformation forward Hadamard transformation can be obtained as H(u) equal to 1 upon capital N summation x varies from 0 to capital N minus 1, f(x) into minus 1 to the power summation bi(x) into bi(u) where i varies from 0 to lowercase n minus 1.

And for Hadamard transformation also the forward transformation as well as the inverse transformation, they are identical. That is the forward transformation kernel and the inverse transformation kernel, they are identical. So here again the same algorithm can be used for forward transformation as well as the inverse transformation.

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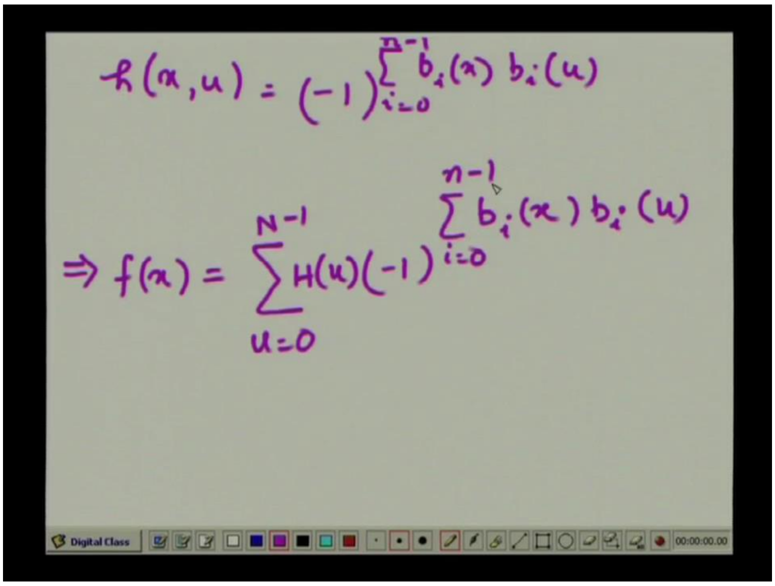

So here, the inverse transformation kernel is given by h(x,u) is equal to minus 1 to the power summation bi(x) into bi(u) where i varies 0 to lowercase n minus 1. And using this, the inverse Hadamard transformation is obtained as f(x)is equal to summation u varying from 0 to capital N minus 1 H(u), minus 1 to the power summation bi(x) into bi(u) where i varies from 0 to lowercase n minus 1. So these are the forward and inverse Hadamard transformation in case of 1 dimension.

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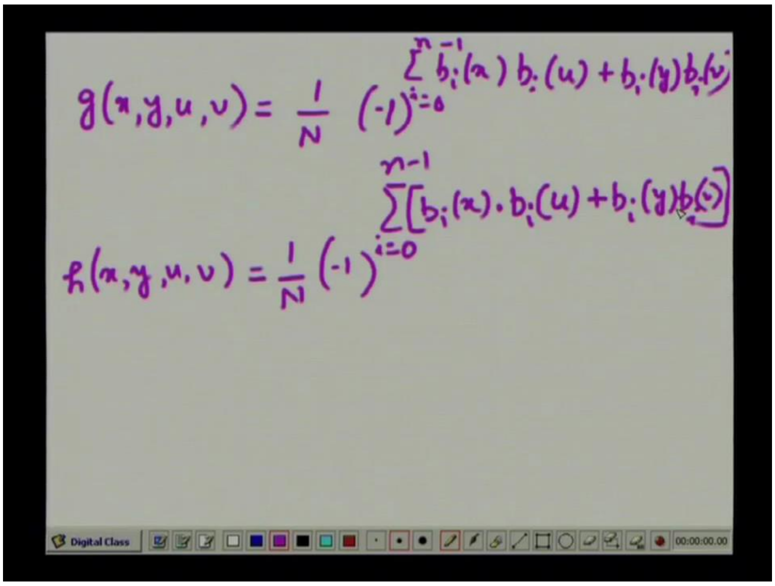

Obviously this can be easily extended into 2 dimension as in the other cases where (g-) the 2 dimensional forward and inverse transformation will be given by g(x,y,u,v) is equal to 1 upon capital N into minus 1 to the power, summation bi(x) into bi(u) plus bi(y) into bi(v) where the summation is taken from over i equal to 0 to lowercase n minus 1. And similarly, the inverse transformation kernel, is also given by h(x,y,u,v) which is same as g(x,y,u,v) that is 1 upon capital N, minus 1 to the power summation i equal to 0 to lowercase n minus 1, into bi(x) into bi(u) plus bi(y) into bi(v).

So we find that the forward transformation kernel and the inverse transformation kernel in case of 2 dimensional discrete Hadamard transformation are identical, so that gives us the forward transformation and the inverse transformation for the 2 dimensional discrete Hadamard transformation to be same which enables us to use the same algorithm or same program to compute the forward transformation as well as the inverse transformation.

And if you analyse this, you will find that this Hadamard transformations are also separable and symmetric. That means, in the same manner, this 2 dimensional Hadamard transformation can be implemented by using a sequence of 1 dimensional Hadamard transformation so for the image first we implement 1 dimensional Hadamard transformation over the rows of the image and then implement the 1 dimensional Hadamard transformation over the columns of this intermediate matrix. And that gives you the final Hadamard transformation output.

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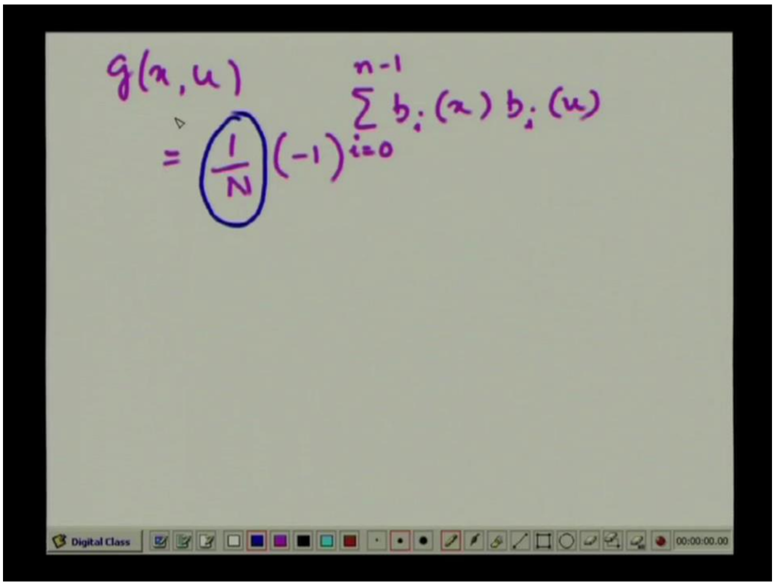

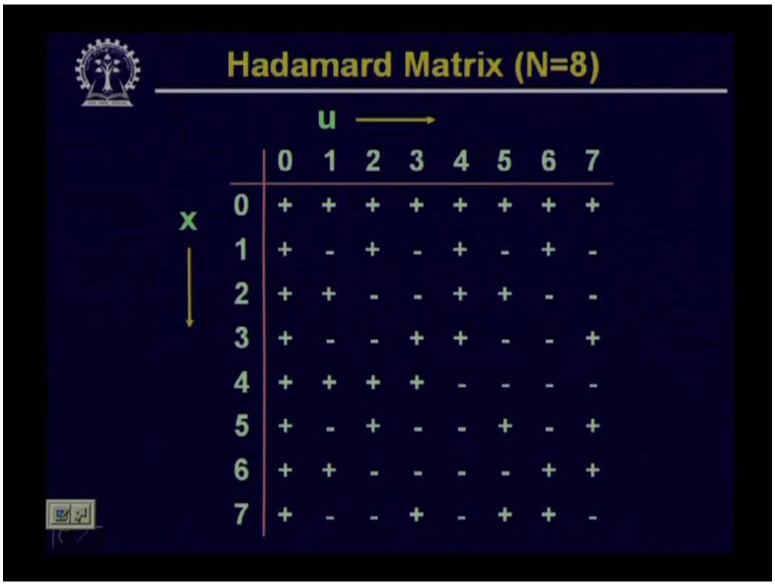

Now if I further analyse, the kernels of this Hadamard transformation and because we have said that 2 dimensional Hadamard transformation can now be implemented in the form of a sequence of 1 dimensional Hadamard transformations. So we analyse further with respect to an 1 dimensional Hadamard transformation. So as we, so as we have seen that 1 dimensional Hadamard transformation is given by g(x,u) is equal to 1 upon capital N, minus 1 to the power summation bi(x) into bi(u) where i varies from 0 to lowercase n minus 1.

And let me mention here that all these additions, that we are doing, these summations follow modular to arithmetic that means these summations are actually nothing but ORing operation of different bits. Now if I analyse this one dimensional forward Hadamard transformation kernel, you will find that if I omit the multiplicative term, 1 upon N.

So if I omit from this this multiplicative term, 1 upon capital N, then this forward transformation actually forms leads to a matrix which is known as Hadamard Matrix.

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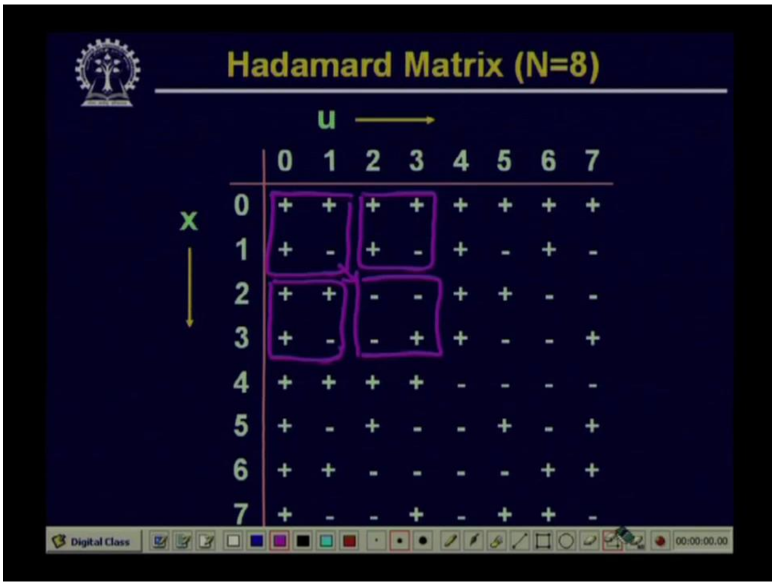

So to see that what is this Hadamard matrix, you find that for different values of x and u, the Hadamard matrix will look like this. So over here we have shown an hadamard matrix for Hadamard transformation of dimension 8.

So for N equal to 8, this Hadamard matrix has been formed and here, plus means it equals to plus 1 and minus means it is equal to minus 1. Now if you analyse this particular Hadamard matrix, you will find that it is possible to generate a recursive relation. It is possible to formulate a recursive relation to generate the transformation matrices. Now how that can be done?

You will find that these particular parts, if I consider say these four by four elements, these 4 by four elements these four by four elements, they are identical. Whereas these four by four elements of this matrix is just negative of this. And the same is followed, the same pattern can be observed in all other parts of this matrix. So by observing this, now we can formulate a recursive relation to generate these transformation matrices.

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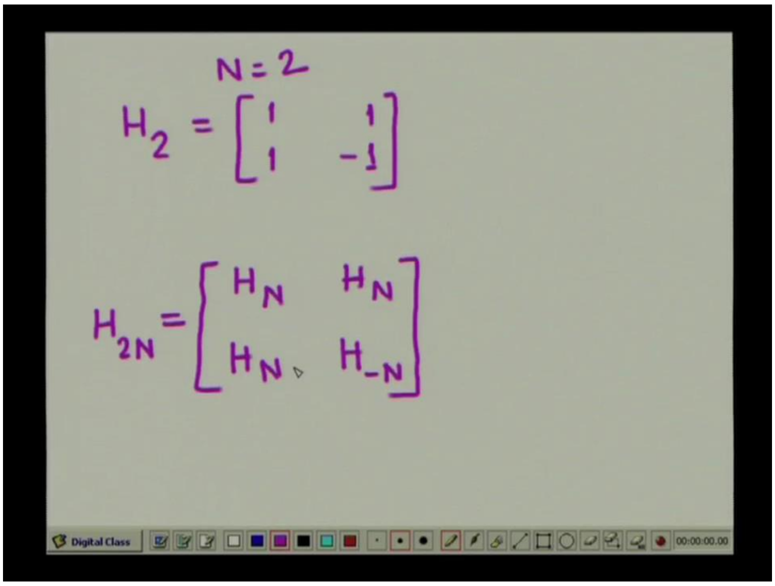

So to have that recursive relation, let us first have a Hadamard matrix of the lowest order, that is for value of N equal to 2. And for this lowest order, we have a Hadamard matrix H2 which is nothing but 1, 1, 1, minus 1. And then using this recursively a Hadamard matrix of dimension 2N can be obtained from a Hadamard matrix of dimension N which is given by the relation H N, H N, H N and H minus N.

So Hadamard matrix of higher dimension can recursively formed from a Hadamard matrix of lower dimension. So this is a very very important property of the Hadamard transformation. Now if we analyse, the Hadamard matrix further, let us see, suppose I want to analyse, I want to analyse this particular Hadamard matrix. Here you find, that if I consider the number of sign changes along a particular column.

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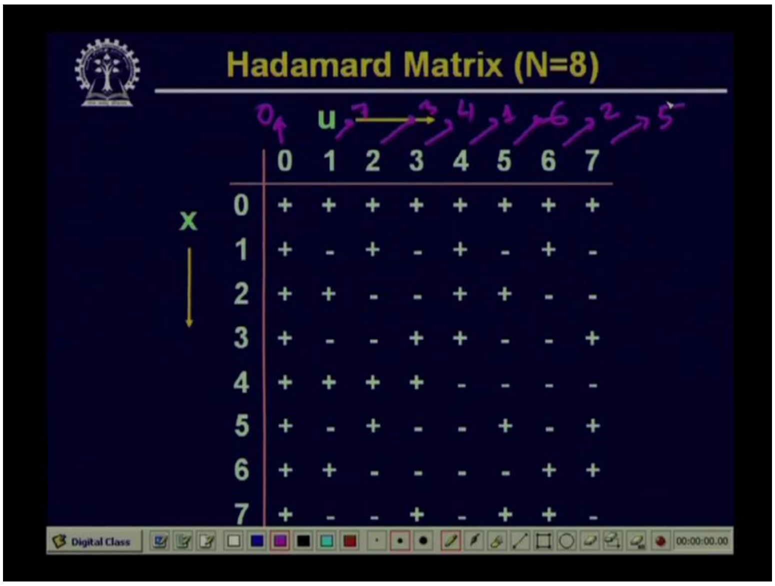

So you will find that the number of sign changes along column number zero is equal to zero. Number of sign changes, along column number one is equal to seven. Number of sign changes along column number two is equal to three. Along column number three the number of sign changes is equal to four. Along column four it is equal to one. Along column five, it is equal to six. Along column six it is equal to two. Along column seven it is equal to five.

So if I define the number of sign changes along a particular column as the sequency which is similar to the concept of frequency in case of discrete fourier transformation or in case of discrete cosine transformation. So in case of Hadamard matrix, we are defining the number of sign changes along a particular column for a particular value of u as the sequency. So here we find, that for value of u equal to 0, the sequency is equal to 0.

u equal to 1 the sequency equal to 7. u equal to 2, the sequency equal to 3. So there is no straightforward relation between the value of u and the corresponding sequency unlike in case of discrete fourier transform or in case of discrete cosine transform where we have seen that increasing values of the frequency variable u corresponds to increasing values of the frequency components.

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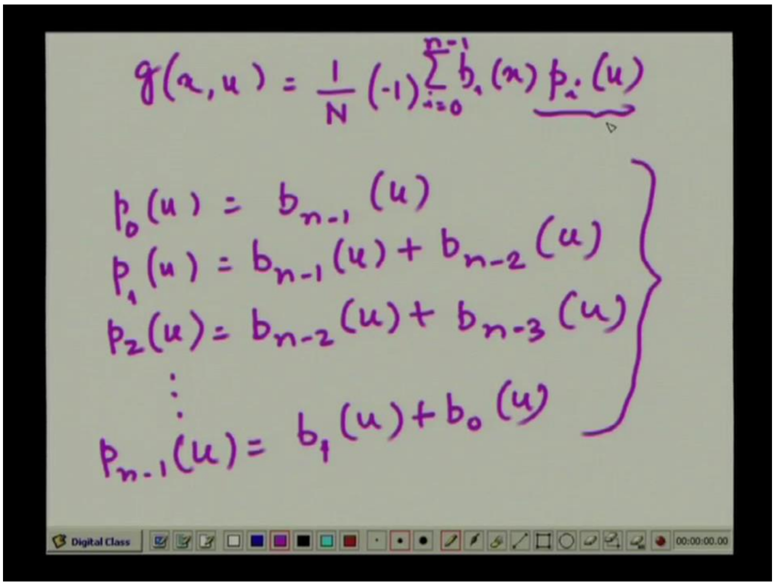

So if we want to have similar types of concepts in case of Hadamard transformation also then what we need is, we need some sort of reordering of this Hadamard matrix. And that kind of reordering can be obtained by another transformation where that particular transformation, the kernels of that particular transformation will be given by g(x,u) is equal to 1 upon capital N minus 1 to the power summation bi(x) into pi(u) now instead of bi(u) we are writing pi(u) where this summation is taken over i equal to 0 to lowercase n minus 1.

And this particular term pi(u) can be obtained from bi(u) using this relations. p0(u) will be given by b n minus 1(u). p1(u) will be given by b n minus 1(u) plus b n minus 2(u). p2(u) will be given by b n minus 2(u) plus b n minus 3(u) and continuing like this, p n minus 1(u)will be given by b1(u) plus b0(u). All these summations are again modular two summations, that is they can also be explained, they can be implemented using the binary or operations.

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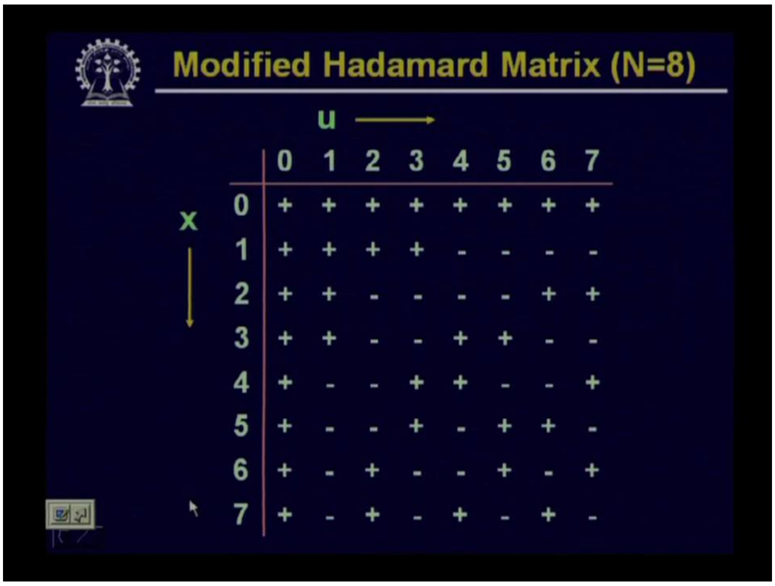

Now by using this modification, again this particular forward transformation kernel that you get, the modified forward transformation kernel that you get that leads to a modified Hadamard martix. So let us see what is this, modified Hadamard matrix. So the modified Hadamard matrix that you get is of this particular form. And if you look at to this particular modified Hadamard matrix, you will find that here sequency for u equal to 0 is again equal to zero.

Sequency for u is equal to 1 is equal to 1. Sequency for u equal to 2 is equal to 2 and now for increasing values of u, we have increasing values of sequency. So using this modified or ordered Hadamard matrix forward kernel in case of 2 dimension, the ordered Hadamard basis functions are obtained in this particular form.

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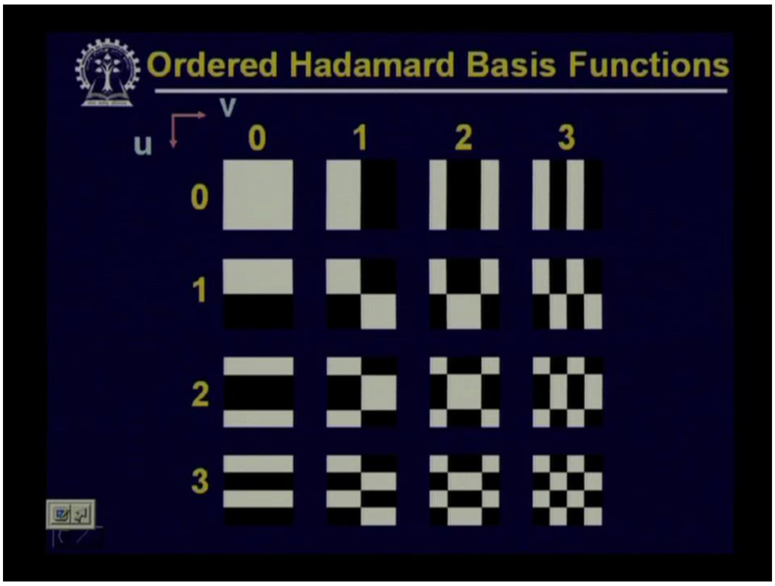

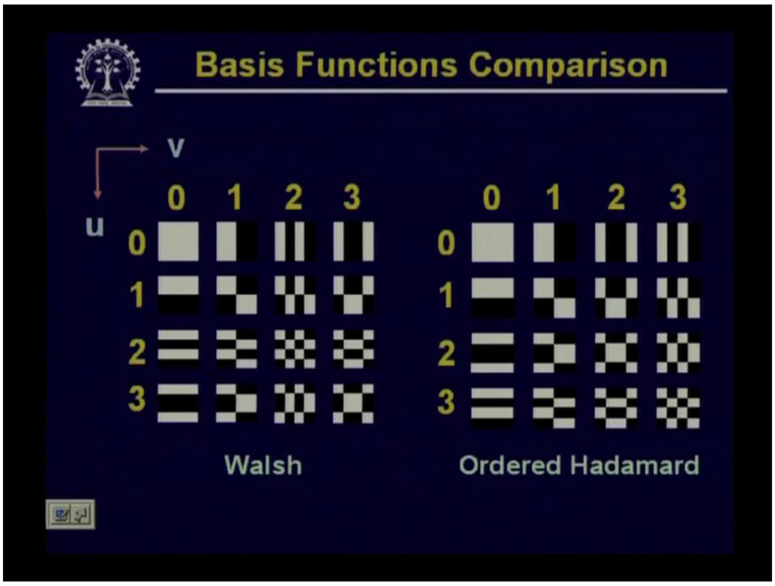

And using this ordered Hadamard basis functions, if I compare this with the Walsh basis functions, you will find that basis functions or basis images in case of Walsh transformation and the basis images in case of ordered Hadamard transformation. The basis functions are identical but there is a difference of ordering of the Walsh basis functions and the ordered Hadamard basis functions.

Otherwise, the basis functions for Walsh transformation and the ordered Hadamard transformation, they are identical and because of this in many cases, a term which is used is called Walsh Hadamard transformation. And this term Walsh Hadamard transformation is actually used to mean either Walsh transformation or Hadamard transformation. So this uses this means one of the two.

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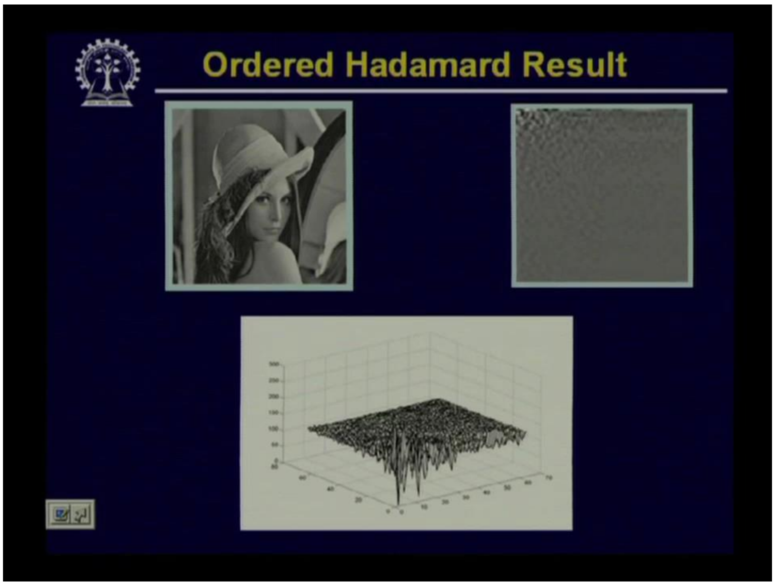

Now using this ordered Hadamard transformation, the results on the same image that you get is something like this. So here you find, again if I look at the energy distribution of different Hadamard coefficients. The ordered Hadamard coefficients, you will find that here the energy is concentrated more towards zero compared to the Walsh transformation. So the energy compaction property of the of the ordered Hadamard transformation is more than the energy compaction property of Walsh transformation.

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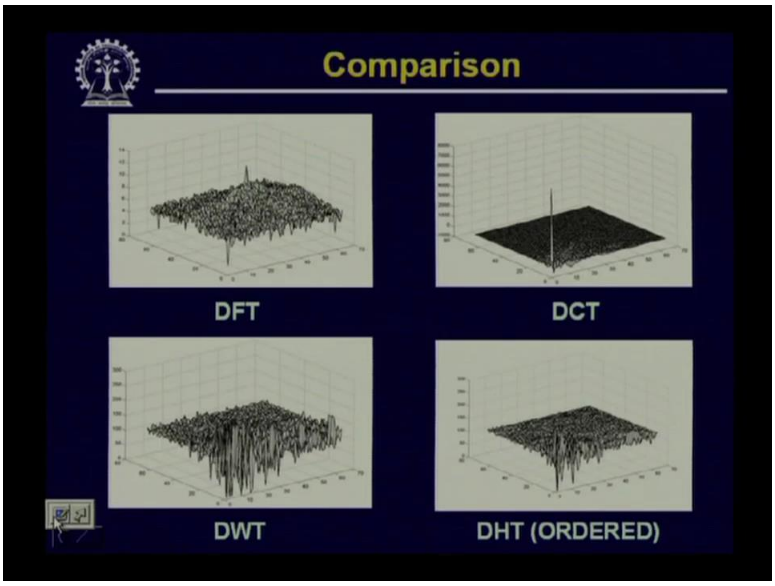

Now this slide shows, the comparison of the transformation coefficients of these different transformations. The first one shows the coefficient matrix for the Discrete Fourier Transformation. The second one shows the matrix for the Discrete Cosine Transformation. Third one is for Discrete Walsh Transformation and fourth one is for Discrete Hadamard Transformation. It is ordered Hadamard Transformation.

Now by comparing all these four different results, you find that in case of Discrete Cosine Transformation, the Discrete Cosine Transformation has the property of strongly concentrating the energy in very few number of coefficients. So the energy compaction property of Discrete Cosine Transformation is much more compared to the other transformations.

And that is why this Discrete Cosine Transformation is very popular for the data compression operations unlike the other cases. And in case of Discrete Fourier Transformation and Discrete Cosine Transformation, though we can associate, the frequency term with the transformation coefficients, it is not possible to have such a physical interpretation of the coefficients of the Discrete Walsh Transformation nor in case of Discrete Hadamard Transformation.

So though we cannot have such a kind of physical interpretation but still because of this energy compaction property, the Hadamard Transform as well as the Walsh transform can have some application in data compression operations. So with this we come to our end of our discussion on the Discrete Cosine Transformation, Discrete Walsh Transformation and Discrete Hadamard Transformation and we have also seen some comparison with the Discrete Fourier Transformation. Thank you.