Digital Image Processing.
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Lecture-26.
FT Result Display-2.

Hello, welcome to the video lecture series on digital image processing. In our last lecture, we have started discussion on the Fourier transformation and towards the end, we have seen some of the properties of fourier transformation.

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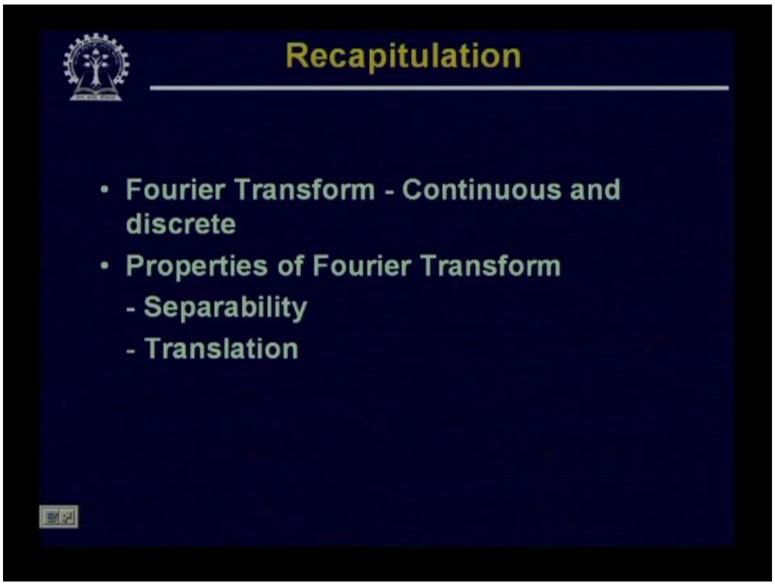

So what we have done in the last class is, we have talked about the fourier transformation both in the continuous and the discrete domain and we have talked about some of the properties of the fourier transformation like the separability property and the translation property.

Today we will continue with our lecture on the fourier transformation and we will see the other properties of the fourier transformation and we will talk about how to implement fourier transformation uhh in a faster way, that is we will talk about the fast fourier transformation algorithm.

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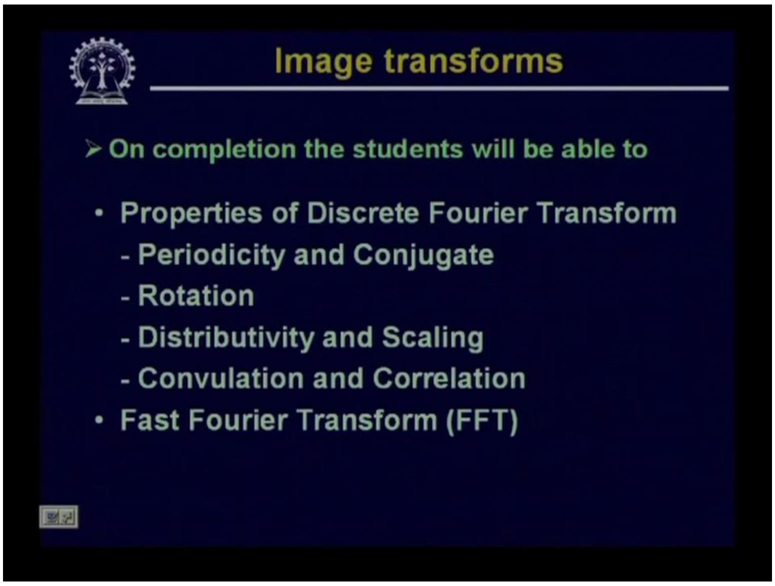

So in today's lecture, we will see the properties of the discrete fourier transformation, specifically the periodicity and conjugate property of the fourier transformation.

We will talk about the rotation property of the fourier transformation, we will see the distributivity and the scaling property of the fourier transformation followed by the convolution and the correlation property of the fourier transformation. And then we will talk about an implementation, a fast implementation of the fourier transformation which is called Fast Fourier Transform.

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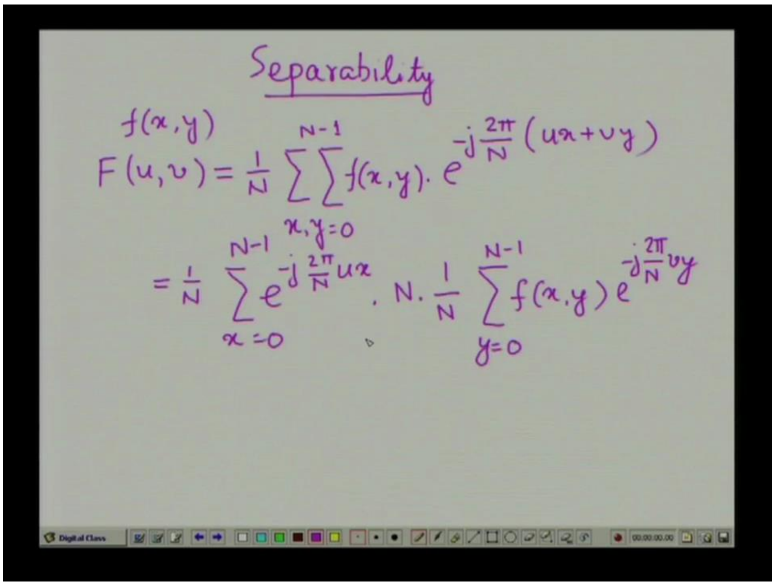

$$F(u,v) = \frac{1}{N}\sum_{x,y=0}^{N-1} f(x,y) \cdot e^{-j\frac{2\pi}{N}(ux+vy)}$$

$$= \frac{1}{N}\sum_{x=0}^{N-1} e^{-j\frac{2\pi}{N}ux} \cdot N \cdot \frac{1}{N}\sum_{y=0}^{N-1} f(x,y)\, e^{-j\frac{2\pi}{N}vy}$$

So first let us see, what uhh just try to repeat what we have done in the last class. So in the last class, we have talked about the separability. We have talked about the separability of the fourier transformation and here we have seen that given our 2 dimensional signal f(x,y) in the discrete domain, that is samples of this 2 dimensional signal f(x,y), we can compute the fourier transformation of f(x,y) as F(u,v) which is given by the expression 1 upon capital N where our original signal f(x,y) is of dimension capital N by capital N.

And the fourier transformation expression comes asf(x,y) into e to the power minus j 2 pi by N into ux plus vy where both x and y vary from 0 to capital N minus 1. And if I rearrange this particular expression, then this expression can be written in the form 1 upon N then summation e to the power minus j 2 pi by capital N ux and then multiply this quantity by capital N and then 1 upon N, again a summation f(x,y) e to the power minus j 2 pi by capital N vy.

So in the inner summation, it is taken from y equal to 0 to capital N minus 1 and the outer summation is taken from x equal to 0 to capital N minus 1 and here we have seen that this inner summation, this gives the fourier transformation of different rows of the input image f(x,y) and the outer summation, this outer summation gives the fourier transformation of different columns of the intermediate result that we have obtained.

So the advantage of the separability property that we have seen in the last classes because, because of this separability property we can uhh do the fourier transformation, 2 dimensional fourier transformation in two steps. In the first step we take the fourier transformation of every individual row of the input image array and in the second step we can take the fourier transformation of every column of the intermediate result that has been obtained in the first step.

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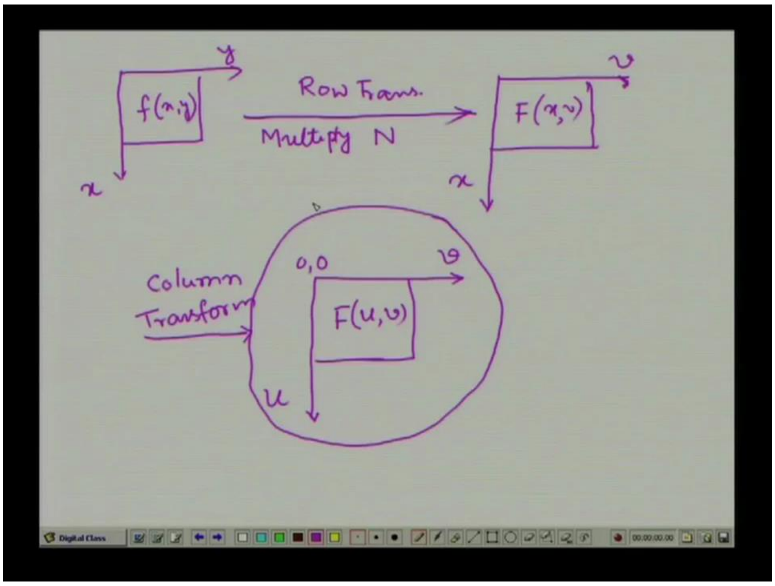

So now, the implementation of the 2 dimensional fourier transformation becomes very easy. So the scheme that we have said in the last class, is like this, if I have an input array given by f(x,y). Here this is the x-dimension, this is the y-dimension. So first what we do is, we do row transformation that is take fourier transformation of every row of the input image, multiply the result by by capital N.

So what I get is an intermediate result array and this intermediate result array gives fourier transformation of different rows of the input image. So this is represented as F(x,v) and this is my x-dimension and this becomes the v-dimension. And after getting this intermediate result, I take the second step of the fourier transformation and now the fourier transformation is taken for every column.

So we do column transformations and that gives us the final result of the 2 dimensional fourier transformation F(u,v). So this becomes my u-axis, the frequency axis u, this becomes the frequency axis v and of course this is the origin (0,0). So it shows that because of the separability property, now the implementation of the 2 dimensional fourier transformation has been simplified because the 2 dimensional fourier transformation can now be implemented as two step of 1 dimensional fourier transformation operations.

And that is how we get this final fourier transformation F(u,v) in the form of the sequence of 1 dimensional fourier transformations. And we have seen in the last class that the same is also true for inverse fourier transformation. Inverse fourier transformation is also separable. So given an array F(u,v), we can do first inverse fourier transformation of every row followed by inverse fourier transformation of every column and that gives us the final output in the form of f(x,y) which is the imagery.

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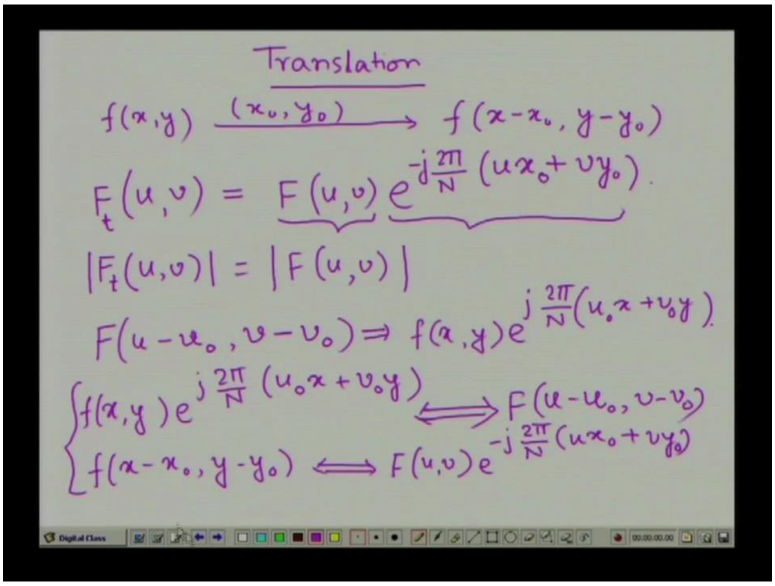

So this is the advantage that we get because of separability property of the fourier transformation. The second one, the second property that we have discussed in the last class is uhh the translation property. So this translation property says that if we have an input image f(x,y), input image that is f(x,y) then translate this input image by (x naught, y naught). So what we get is a translated image f(x minus x naught and y minus y naught).

So if we take the fourier transformation of this, we have found that the fourier transformation of this translated image which we had represented as Ft(u,v), this became equal to F(u,v) into e to the power minus j 2 pi by capital N ux naught plus vy naught. So if you find, uhh so in this case the fourier transformation of the translated image is F(u,v), that is the fourier transformation of the original image f(x,y) which is multiplied by e to the power minus j 2 pi by N ux naught plus vy naught.

So if we consider, the uhh fourier spectrum of this particular signal, you will find that the fourier spectrum that is f transpose uhh Ft(u,v) will be same as F(u,v). Now this term e to the power minus j 2 pi by N ux naught plus vy naught, this simply introduces an additional phase shift. But the fourier spectrum remains unchanged and in the same manner, if the fourier spectrum F(u,v) is translated by (u naught,v naught).

So instead of taking F(u,v), we take F(u minus u naught,v minus v naught) which obviously is the translated version of F(u,v)where F(u,v) has been translated by vector (u naught,v naught) in the frequency domain. And if I take the inverse fourier transform of this, the inverse fourier transform will be f(x,y) into e to the power j 2 pi by N into u naught x plus v naught y.

So this also can be derived in the same manner in which we have done the forward fourier transformation. So here we find, that if f(x,y) is multiplied by this exponential term e to the power j 2 pi by N u naught x plus v naught y, then the corresponding, in the frequency domain, its fourier transform is simply uhh translated by the vector (u naught,v naught). So you get is F(u minus u naught and v minus v naught).

So under this translation property, now the DFT pair becomes if we have f(x,y) e to the power j 2 pi by capital N uhh u naught x plus v naught y. The corresponding fourier transformation of this is F(u minus u naught, v minus v naught) and if we have the translated image, f(x minus x naught,y minus y naught), the corresponding fourier transformation will be F(u,v) e to the power minus j 2 pi by N u x naught plus v y naught.

So these are the fourier transform pairs under the translation. So this f(x,y) and uhh and f(x minus x naught, y minus y naught). So these two expressions gives you the fourier transform pairs, the DFT pairs under translation. So these are the two properties that we have discussed in the last lecture. Today let us talk about some other properties. So the third property that we will talk about today is the periodicity and conjugate property.

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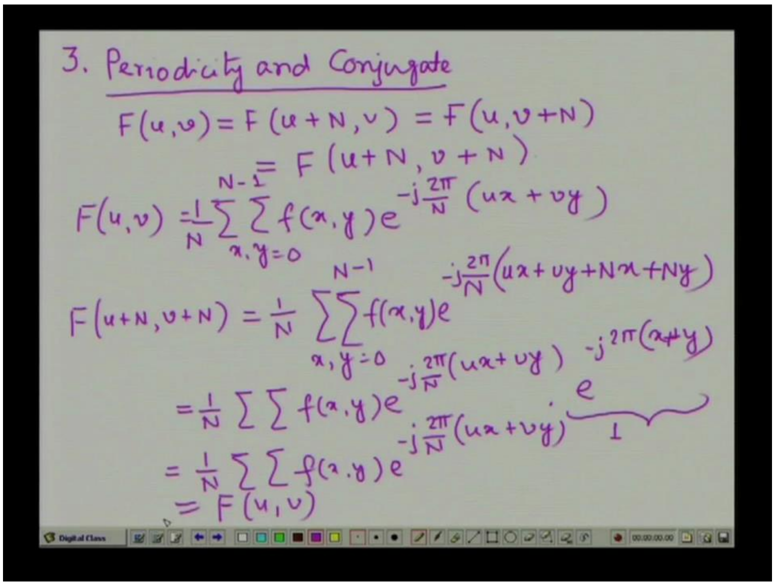

So the first one that we will discuss is the periodicity and the conjugate property. The periodicity property says that both the discrete fourier transform and the inverse discrete fourier transform that is DFT and IFT are periodic with a period capital L. So let us see, how this periodicity can be proved. So this periodicity property says that F(u,v), this is the fourier transform of our signal f(x,y) this is equal to F(u plus N, v) which is same as F(u,v plus N) which is same as F(u plus capital N,v plus capital N).

So this is what is meant by periodic. So if uhh find that the fourier transformation F(u,v) is periodic both in x direction and in y direction that give rise to F(u,v) is equal to F(u plun N) F(v plus N) which is same as F(u plus N, v) and which is also same as F(u, v plus N). Now let us see how we can derive or we can prove this particular property. So you have seen, the fourier transformation expression as we have discussed many times F(u,v) is equal to double summation f(x,y) e to the power minus j 2 pi by capital N u x plus v y.

Of course, we have to have the scaling factor 1 upon capital N where both x and y vary from 0 to capital N minus 1. Now if we try to compute F(u plus capital N,v pls capital N) then what do we get? Following the same expression, this will be nothing but 1 upon capital N then double summation f(x,y) e to the power minus j 2 pi upon capital N and now we will have ux plus vy plus capital N x because now u is replaced by u plus capital N so you will have capital N x plus capital N y. Here both x and y will vary from 0 to capital N mi s 1.

Now this same expression, if we take out this capital N x and capital N y uhh in a separate exponential then this will take the form 1 upon capital N, double summation f(x,y) e to the power minus j 2 pi upon capital N ux plus vy into e to the power minus j 2 pi into x plus y. Now if you look at the second exponential term, that is e to the power minus j 2 pi x plus y uhh you find that x and y are the integer values.

So x plus y will always be integer, so this will be the exponential e to the power minus j some k times uhh 2 pi. Ok? And because this is an exponentiation of some integer multiple of 2 pi so the value of this second exponential will always be equal to 1. So finally what we get is 1 upon capital N, double summation f(x,y) into e to the power minus j 2 pi upon capital N into ux plus vy.

And you find that this is exactly the expression of F(u and v). So as we said, that the discrete fourier transformation is periodic with period N capital N both in the u direction as well as the v direction and that is uhh that can be very easily be proved like this uhh by this mathematical derivation. We have found that F(u plus capital N, v plus capital N) is same as F(u,v).

And the same is true in case of inverse fourier transformation so if we derive the inverse fourier transformation then we will get uhh the similar result showing that the inverse fourier transformation is also periodic with period capital N. Now the other property that we said is the Conjugate property.

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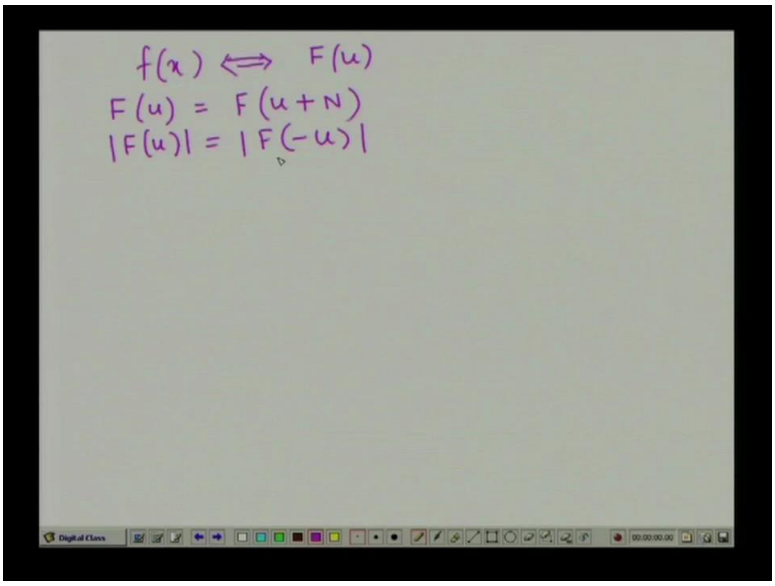

The conjugate property says that if f(x,y), this function, if this is a real valued function. f(x,y) if it is a real valued function, in that case the fourier transformation f(u,v) will be F*(minus u, minus v) where this F* indicates that it is complex conjugate. And obviously, because of this if I take the fourier spectrum, F(u,v) will be same as F of (minus u,minus v). So this is what is known as the uhh conjugate property of the discrete fourier transformation.

Now we find that using the periodicity property, uhh helps to visualise the fourier spectrum of a given signal. So let us see how this periodicity property helps us to properly visualise the fourier spectrum. So for this, we will consider a 1 dimensional signal obviously, this can very easily be extended to a 2 dimensional signal. So by this what you mean is if we have a 1 dimensional signal say f(x) whose fourier transform is given by F(u) then as we said, that the periodicity property says that F(u) is equal to F(u plus capital N).

And also the fourier spectrum F(u) is same as F(minus u) uhh. So this says that F(u) has a period of length capital N and because the spectrum, F(u) is same as F(minus u). So the magnitude of the fourier spectrum, of the fourier transform is centered at the origin. So by this what we mean is, let us consider a figure like this.

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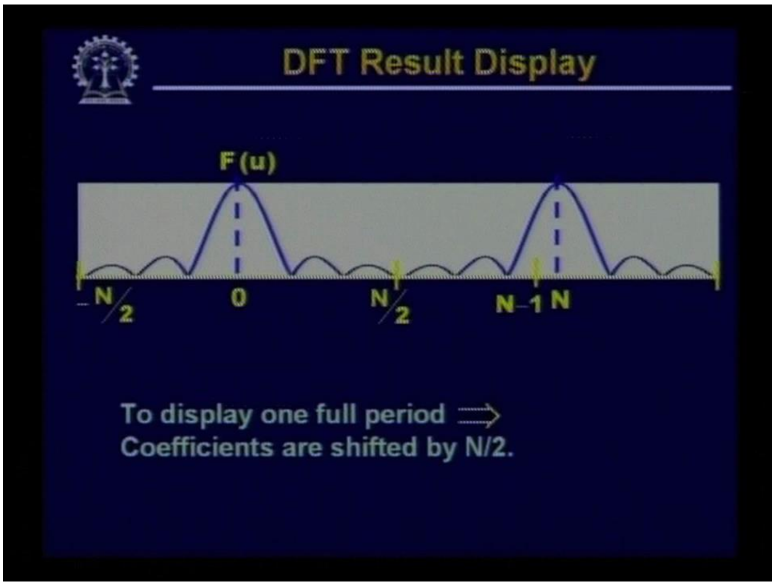

You will find that this the typical this is a typical fourier transform of a particular signal and here you find that uhh this fourier spectrum, the fourier uhh transform is centered at the origin and if you look at the frequency axis, so this is the u axis. If you look at this frequency axis, you will find that F of (minus u), the magnitude of F of (minus u) is same as the magnitude of the F of plus U.

So this figure shows, that the transform values, if we look at the transform values from N by 2 plus 1 so that is somewhere here. This is N by 2 plus 1, 2 N minus 1, so that is somewhere here. So find that the transform values, in the range N by 2 plus 1 to N minus 1, this is nothing but the transform values in the left uhh transform values of the half period in the left half of in the left of the origin.

So just by looking at this, the transform values from N plus (1) N by 2 plus 1 to N minus 1, you find that this values are nothing but the reflections of the half period to the left of the origin, 0. But what we have done is we have computed the fourier transformation in the range 0 to N minus 1. So you will get all the fourier coefficients in the range 0 to N minus 1 uhh so the fourier coefficients uhh ranging the values of u from 0 to N minus 1.

And because of this conjugate property, you will find uhh we find that in this range 0 to capital N minus 1, what we get is two back to back half periods of this interval. So this is nothing but two back to back half periods. So this is one half period, this is one half period and they are placed back to back. So to display this uhh fourier transformation coefficients in the proper manner, what we have to do is we have to displace the origin by a value, capital N by 2.

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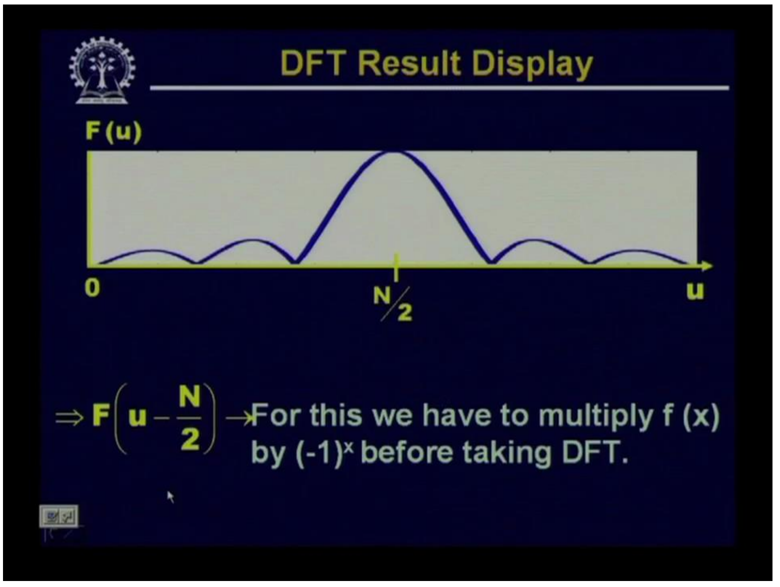

So by displacement, what we get is this. So here we find that in this particular case, the origin has been shifted to capital N by 2. So now what we are doing is instead of considering the fourier transformation F(u), we are considering, the fourier transformation F(u minus capital N by 2) and for this displacement, what we have to do is we have to multiply f(x) by minus N to the power x.

So every f(x) has to be multiplied by minus 1 to the power x and this result, if you take the DFT of this, then what we get uhh is the fourier transformation coefficients in this particular form. And this comes from the uhh shifting property of the inverse fourier transformation. So this operation we have to do if we want to go for the proper display of the fourier transformation coefficients. Thank you.