Digital Image Processing.
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Lecture-25.
Properties of Fourier Transform.

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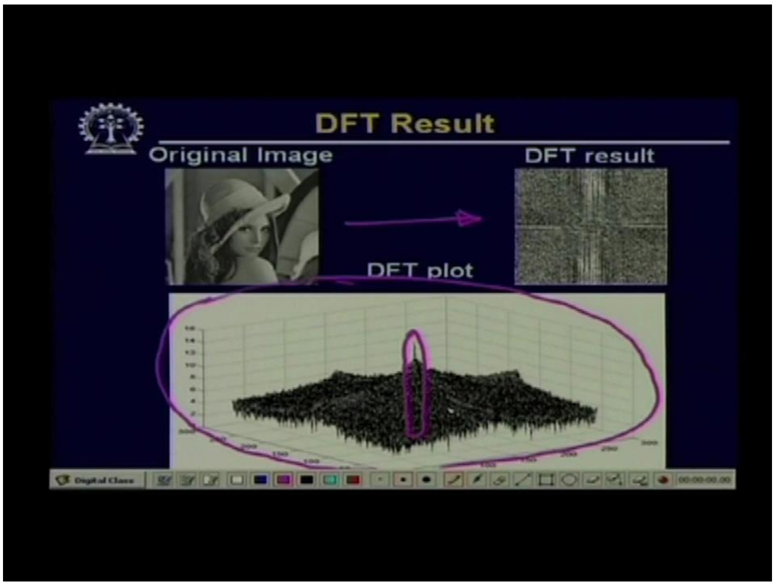

Hello, welcome to the video lecture series on digital image processing. So after discussing all these fourier transformation the inverse fourier transformation and looking at how the fourier coefficients look like, let us see some of the properties of these fourier transformation operations. So now, we will see some of the properties important properties of fourier transformation. So the first property that we will talk about is the separability.

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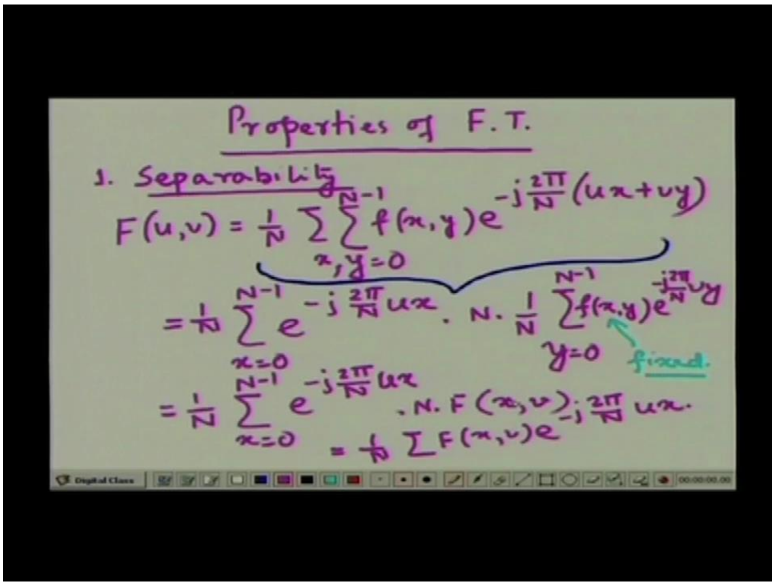

Now if you analyse the expression of the fourier transformation where you have said that the fourier transformation F(u,v) i s given by 1 upon N double summation f(x,y) e to the power minus j 2 pi by N, we are assuming a square image of size N by N into ux plus vy where both x and y varies from 0 to capital N minus 1.

Now find that this particular expression expression of the fourier transformation, this particular expression can be rewritten in the form 1 upon N into e to the power minus j 2 pi by capital N into ux where x varies from 0 to capital N minus 1 into N capital N into 1 upon capital N summation y varying from 0 to capital N minus 1, f(x,y)e to the power minus j 2 pi by capital N into vy.

So it is the same fourier expression but now we have uhh separated the variables x and y into two different summation of operations. So the first summation operation, you will find that it involves the variable x and the second summation operation involves the variable y. Now if you look at this function f(x,y) for which we are trying to find out the fourier transformations.

Now the second summation operation where the summation is taken over y where y varies from 0 to capital N minus 1, you will find that in this f(x,y) if we keep the value of x to be fixed, that is for a particular value of x, the defined values of f(x,y) that represents nothing but a particular row of the image. So in this particular case, for a particular value of x, if I keep x to be fixed, so for a fixed value of x, this f(x,y) represents a particular row of the image which is nothing but an 1 dimensional signal.

So by looking at that, what we are doing is, we are transforming the rows of image and different rows of the image for different values of x. So after expansion or elaboration of this particular expression, the same expression now gets converted to 1 upon capital N, x equal to 0 to capital N minus 1, e to the power minus j 2 pi by capital N ux into F of (x,v).

I represent this as F of (x,v) and of course there is a multiplication term which is capital N. Ok? And this is nothing but 1 upon capital N, summation F of (x,v) e to the power minus j 2 pi by capital N ux. So once if we look at these expressions, you find that the second summation, the second summation operation gives you the fourier transformation of the different rows of the image and that fourier transformation of the different rows which now we represent by F(x,v). Ok?

This x represents, the x is an index of a particular row and the second summation what it does is, it takes this intermediate fourier coefficients and on these fourier coefficients now it performs the fourier transformations over the columns to give us the complete fourier transformation operation or uhh F(u,v).

So the first operation that we are performing, is the fourier transformation over of different rows of the image multiplying this intermediate result by the factor of capital N and then this intermediate result intermediate fourier transformation matrix that we get, we further take the fourier transformation of different columns of this intermediate result to get the final fourier transformation.

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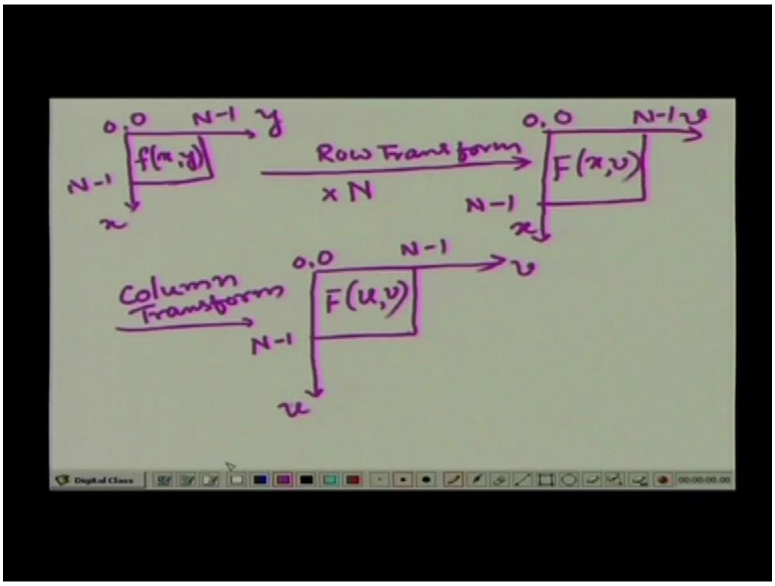

So graphically, we can represent this entire operation like this that this is our x-axis, this is our y-axis. I have an image f of (x,y). So first of all, what we are doing is we are taking the fourier transformation along the row. So we are doing row transformation. And after doing row transformation, we are multiplying all these intermediate values by a factor N. so you multiply by the by the capital by the factor capital N.

And this gives us, the intermediate fourier transformation coefficients which now we represent as capital F (x,v). So you get one of the uhh frequency components which is v. And then, what we do is we take this intermediate result and initially we had done row transformation and now we do column transformation. And after doing this column transformation what we get is, so here it will be x and it will be axis v and we get the final result as uv and our final transformation coefficients will be capital F u and v.

Of course, this is the origin (0,0). All these values are N minus 1 N minus 1. Here also it is (0,0). This is N minus 1, N minus 1. Here also it is (0,0). Here it is capital N minus 1, here it is capital N minus 1. So you find using this separability property, what we have done is, this 2 dimensional fourier operation is now converted into two 1 dimensional fourier transformation fourier operations.

So in the first case what we are doing is, we are doing the 1 dimensional fourier transformation operation over different rows of the image and the intermediate result that you get, that you multiply with the dimension of the image which is M and this intermediate result, you take and now you do again 1 dimensional fourier transformation across the different columns of this intermediate result and then you finally get the 2 dimensional fourier transformation coefficient.

So because of separability, this 2 dimensional fourier transformation has been converted to two 1 dimensional fourier transformation operations and obviously by using this your uhh operation will be much more simpler. So in the same manner as we have done in case of forward fourier transformation we can also have the inverse fourier transformation. We can also have the inverse fourier transformation.

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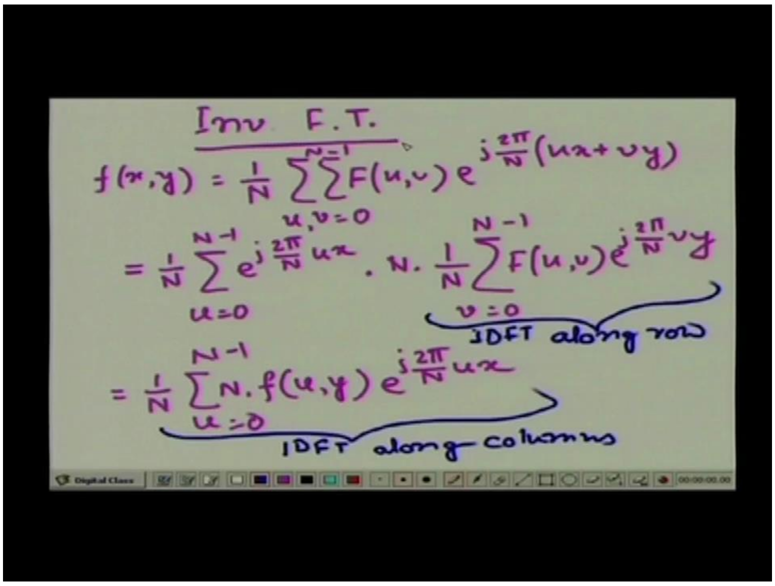

So in case of inverse fourier transformation, our expression was f(x,y) is equal to 1 upon capital N, double summation, F(u,v) e to the power j 2 pi upon N ux plus vy where both u and v varies from 0 to capital N minus 1. So in the same manner, I can also break this expression into two summations. So the first summation will be e to the power j 2 pi by N ux; here u will vary from 0 to capital N minus 1 multiplied by N into 1 upon capital N, F(u,v) e to the power J 2 pi upon N vy and now v will vary from 0 to capital N minus 1.

So again as before, you will find this second operation, this is nothing but inverse discrete fourier transformation along a row. Ok? So the second expression, this gives you the inverse fourier transformation along the row and when you finally convert this and get the final expression, this will be 1 upon capital N, summation N times f(u,y) into e to the power j 2 pi upon capital N ux and now u varies from 0 to capital N minus 1.

This particular expression is inverse discrete fourier transformation along columns. So as we have done in case of forward fourier transformation that is for a given image, you first take the fourier transformation of the different rows of the image to get the intermediate fourier transformation coefficient and then take the fourier transformation of different columns of that set of intermediate fourier coefficient to get the final fourier transformation.

In the same manner in the inverse fourier transformation, we can also take the fourier coefficient array, do the inverse fourier transformation alongs the rows and all those intermediate results that you get, for that you second step you do the inverse discrete fourier transformation along the columns. And these two operations completes the inverse fourier transformation of the 2 dimensional array to give you the uhh 2 dimensional signal F x of y (x and y).

So because of this separability property, we have been able to uhh convert the 2 dimensional fourier transformation operation into two 1 dimensional fourier transformation operations. And because now it has to be implemented as 1 dimensional fourier transformation operation, so the uhh operation is much more simple than in case of 2 dimensional fourier transformation operation.

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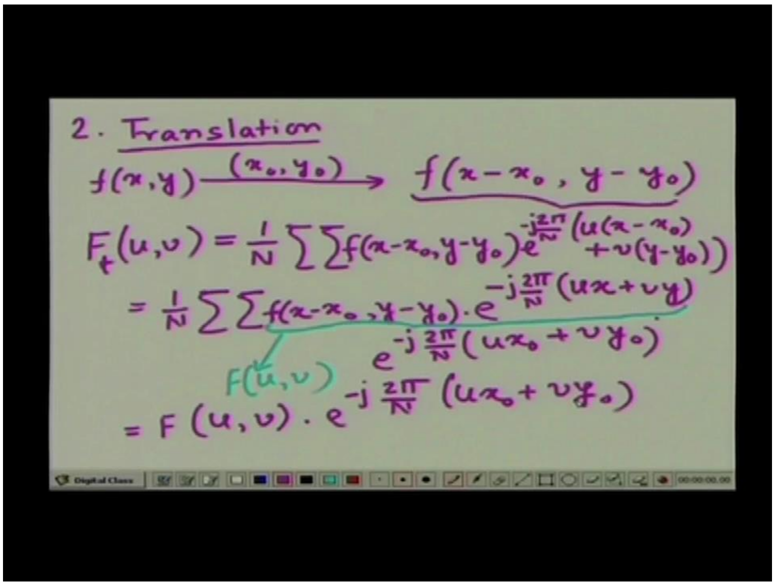

Now let us look at the second property of this fourier transformation. The second property that we will talk about is the translation property. Translation property says that if we have a 2 dimensional signal f(x,y) and translate this by a vector (x naught, y naught). So along x direction, you translate it by x naught and along y direction you translate it by y naught. So the function that you get is f(x minus x naught, y minus y naught).

So if I take the fourier transformation of this translated signal, if x minus x naught, y minus y naught, how the fourier transformation may look like. So you can find out the fourier transformation of this translated signal and let us call this fourier transformation as Ft (u,v), so I represent this as Ft(u,v). So going by the similar expression,this will be nothing but 1 upon capital N, f of (x minus x naught , y minus y naught) into e to the power minus j 2 pi by capital N into u x minus x naught plus v y minus y naught.

Ok? So if I expand this, what I will get is 1 upon capital N into double summation, f of (x minus x naught, y minus y naught) into e to the power minus j 2 pi by N ux plus vy into e to the power minus j 2 pi by N ux naught plus vy naught by simply expanding this particular expression. So here if you consider the first expression that is f(x minus x naught, y minus y naught) e to the power minus j 2 pi by N ux plus vy summation from uhh x equal to 0 to N minus 1, y equal to 0 to N minus 1.

This particular term is nothing but our fourier transformation F of (u,v). So by doing this translation, what you get is the final expression Ft of (u,v) will come in the form F of (u,v) into e to the power minus j 2 pi by capital N into ux naught plus vy naught. So this is the final expression of this translated signal that we get. So if I compare, if you compare these 2 expressions, F(u,v) and Ft(u,v) you will find that the fourier spectrum of the signal after translation does not change because the magnitude of this Ft(u,v) and the magnitude of F(u,v) will be the same.

So because of this translation, what you get is only it introduces some additional phase difference. So whenever f(x,y) is translated by x naught, y naught, the additional difference which is introduced by the e to the power minus j 2 pi by capital N u x naught plus v y naught. But otherwise, the magnitudes of the fourier spectrum or the magnitude of the fourier transformation that is the fourier spectrum, that remains unaltered.

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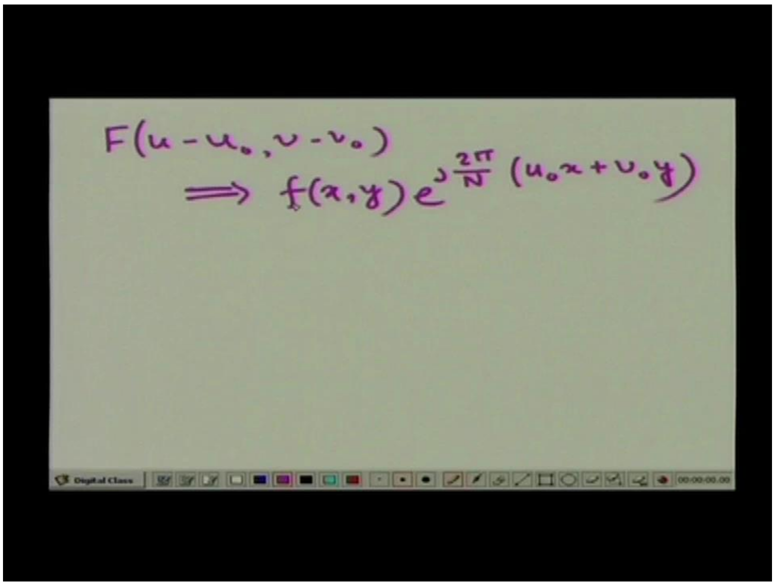

In the same manner, if we talk about the inverse fourier transformation, the inverse fourier transformation F of (u minus u naught, v minus v naught), this will give rise to f of (x,y) e to the power j 2 pi by capital N u naught x plus v naught y. So this says if f(x,y) is multiplied by this exponential term then its fourier transformation is going to be replaced is going to be displaced by the vector (u naught, v naught).

And this is the property which will we will use later on to find out that how the fourier transformation coefficients can be better visualised. So here in this case, we get the fourier transformation, the forward fourier transformation and the inverse fourier transformation uhh with translation and you will find that and we have found that a shift in f(x,y) say (x naught, y naught) does not change the fourier spectrum of the signal.

What we get is just an additional phase term gets introduced in the fourier spectrum. Thank you.