Digital Image Processing.
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Lecture-18.
Interpolation with Examples-I.

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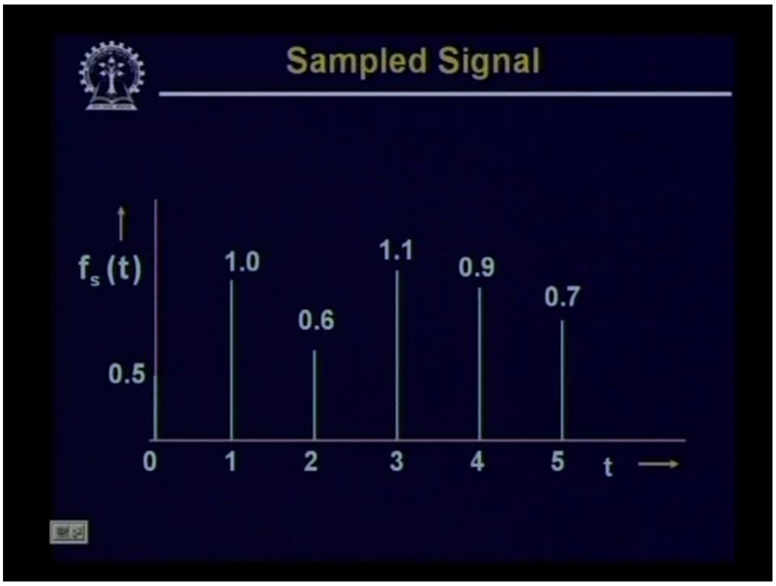

Lecture series on digital image processing. So let us see that what we mean by image interpolation operation. So here we have shown a diagram which shows the sample values of an one dimensional signal say f(t) which is a function of t. So we find that in this particular diagram we have given a number of samples and here the samples are present for t equal to 0, t equal to 1, t equal to 2, t equal to 3, t equal to 4 and t equal to 5.

The functions are given like them they sample data. You find that the sample values are available only at 0, 1, 2, 3, 4 and 5. But in some applications we may need to find out the approximate value of this function at say t equal to 2.3 or say t equal to 3.7 and so on. So again here in this diagram you find that at t equal to 2.3 say somewhere here I do not have any information.

Or say t equal to 3.7 somewhere here again I do not have any information. So the purpose of image interpolation is by making use or the signal interpolation is by using the sample values at these discrete locations, we have to reconstruct or we have to approximate the value of the function f(t) at any arbitrary point in the time access. So that is the basic purpose of the interpolation operation.

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So whenever we go for some interpolation we have to make use of certain interpolation functions. And this interpolation operation or the interpolation function should satisfy certain conditions. The conditions are the interpolation function should have a finite region of support. That means when we do the interpolation we should not consider the sample values from say - infinity to plus infinity.

Rather if I want to approximate the function value at location say t equal to 2.3, then the samples that should be considered are the samples which are nearer to t equal to 2.3. So I can consider a samples at t equal to 1, I can consider the samples at t equal to 2, I can consider the sample at t equal to 3, I can consider the sample at t equal to 4 and so on. But for if approximate the functional value to approximate the functional value at t equal to 2.3, I should not consider the sample value at say t equal to 50. So that is what is meant by finite region of support.

Then the second property which this interpolation operation must satisfy is it should be a smooth interpolation. That means by interpolation we should not introduce any discontinuity in the signal. Then the third operation the third condition that must be satisfied for this interpolation operation is that the interpolation must be shift invariant. That is if I shift the signal by say t equal to 5, even then the same interpolation operation the same interpolation function should give me the same result in the the same interval.

So these are what are known by the shift invariance property of the interpolation. And we have seen in the last class that B-Spline interpolation functions satisfy all these three properties which are desirable properties for interpolation.

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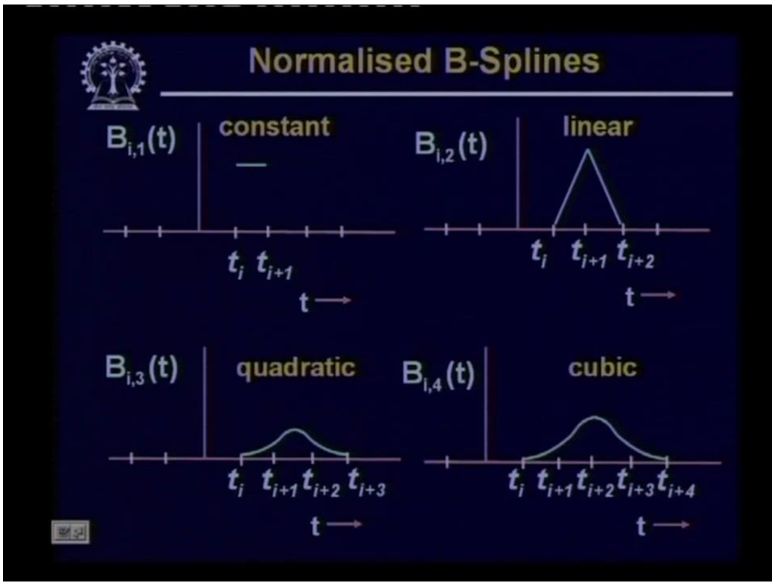

So these B-Spline functions are something like this. We have seen that for interpolation with the help of B-Spline function we use a B-Spline function which is given by Bi,k.

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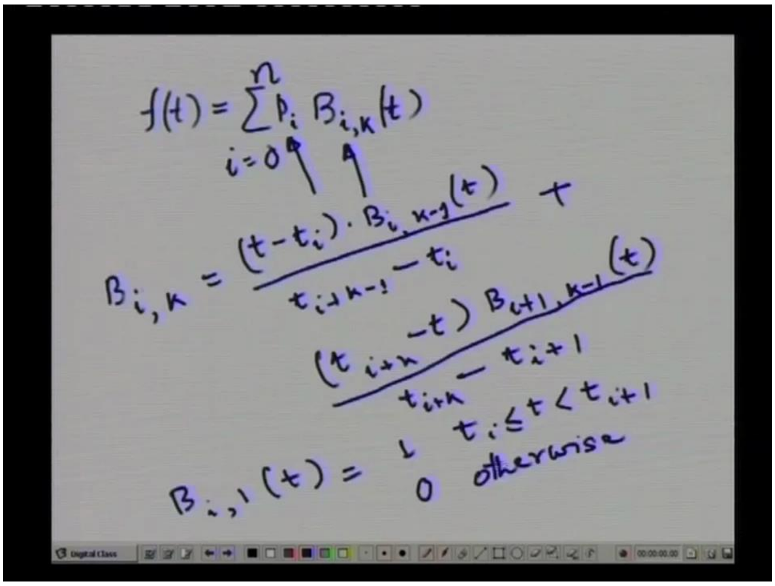

So let me just go to what is the interpolation operation that we have to do. So for interpolation what we use is say f(t) should be equal to sum pi into Bi,k(t). Where i varies from 0 to say n. Where pi indicates the ith sample and Bi,k is the interpolation function. And we have defined in the last class that this Bi,k can be defined as Bi,k it can be defined recursively as t - ti into Bi,k-1(t) upon ti+k-1-ti + (ti+k-t) into Bi+1,k-1(t) upon ti+k – ti+1 where Bi,1(t) is given by it is equal to 1 for ti less than or equal to t less than ti+1 and it is equal to 0 otherwise.

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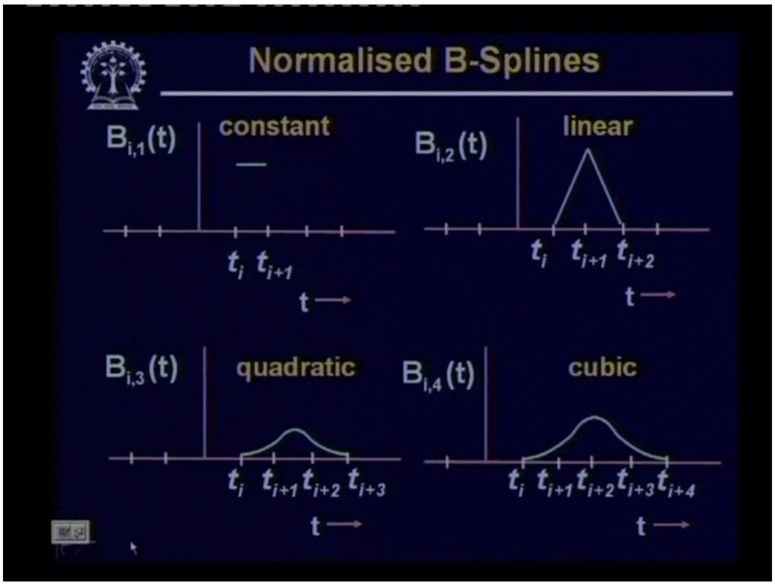

So you find that when we have defined Bi,1(t) to be 1 within certain region and it is equal to 0 beyond that region. Then using this Bi,1 I can estimate, I can calculate the values of other Bi,k by using the recursive relation. And pictorially these defined values of Bi,k for k equal to 1 it is a constant. For Bi,2 that is for k equal to 2, it is a linear operation, linear function. For k equal to 3, Bi,3 is a quadratic function. And for k equal to 4 that is Bi,4 it is a cubic equation.

And we have said in the last class, so here you find that the region of support for Bi,1 is just one sample interval, for Bi,2 the region the region of support is just two sample intervals. For Bi,3 it is three sample intervalsand for Bi,4 it is four sample intervals. And we have mentioned in the last class that out of this the quadratic one that is for the value k equal to three it is normally not used because this does not give a symmetric interpolation.

Whereas using the other three that is Bi,1, Bi,2 and Bi,4 we can get symmetric interpolation. So normally the functions the B-Spline functions which are used for interpolation purpose are the first order that is k equal to 1, the second order for linear that is k equal to 2 and the cubic interpolation that is for k equal to 4. And we have also said that these functions for k equal to

1, k equal to 2 and k equal to 4 can be approximated by B0,1(t) is equal to 1 for 0 less than or equal to t less than 1 and it is equal to 0 otherwise.

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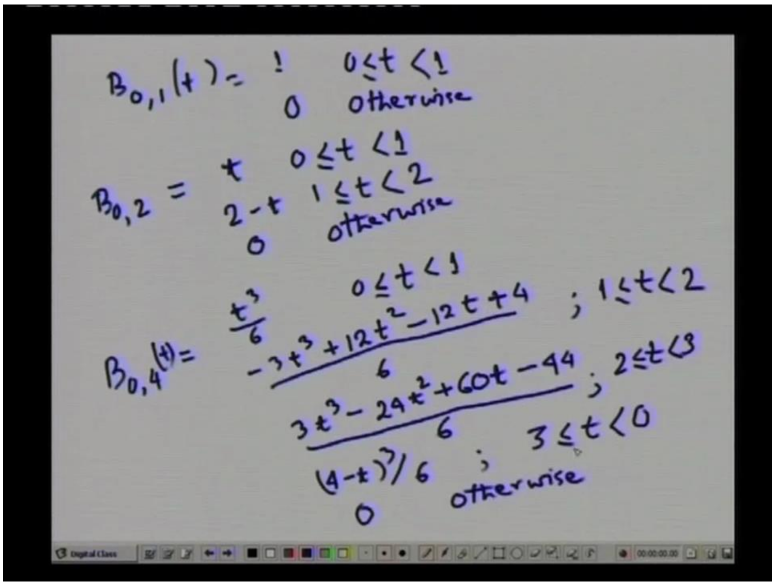

So only in the range 0 to 1 excluding t equal to 1, B0,1 equal to 1. And, beyond this range B0,1 is equal to 0. Then B0,2 is defined like this that it is equal to t for 0 less than or equal to t less than 1 and it is equal to 2 - t for 1 less than or equal to t less than 2 and it is equal to 0 otherwise.

So here again you find that for the values of t between 0 and 1 B0,2 (t) increases linearly. For t equal to 1 to 2 the value of B0,2 decreases linearly and beyond 0 and 2 that is for values of t less than 0 and for values of t greater than 2, the value of B0,2 is equal to 0. Similarly for the quadratic one sorry for the cubic one B0,4 is defined as B0,4 (t) will be defined as t cube by 6, for 0 less than or equal to t less than 1, it is defined as - 3t cube +12t square - 12t+4 divided by 6, for 1 less than or equal to t less than 2.

This is equal to 3t cube - 24t square +60t - 44 divided by 6, for 2 less than or equal to t less than 3. This is equal to (4 - t) whole cube divided by 6, for three less than or equal to t less than 4 and it is 0 otherwise.

So these are the different orders of B-Spline function, so here again you find that for value equal to value of t less than 0 B0,4(t) equal to 0. And similarly for value of t greater than 4 B0,4(t) is also equal to. So these are the B-Spline functions using which the interpolation operation can be done. Now let us see that how do we interpolate, again I take the example of this sample data where I have a number of samples of a function t and which is represented by fs,t.

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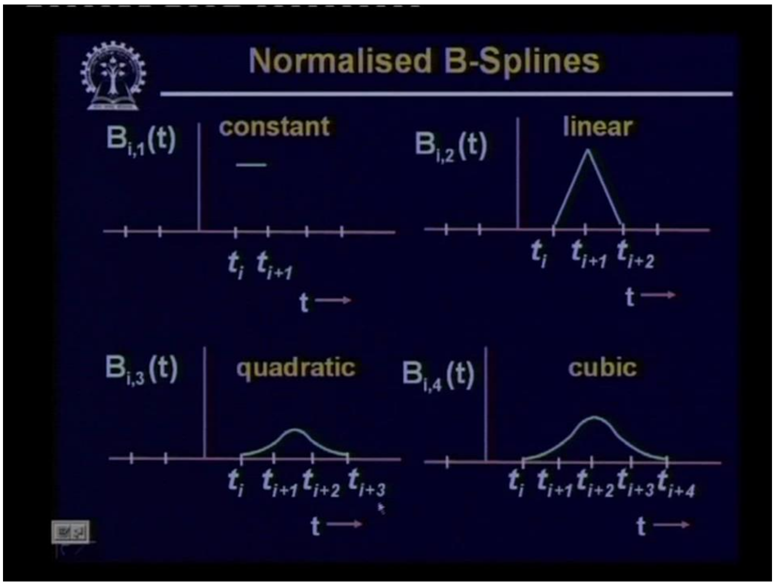

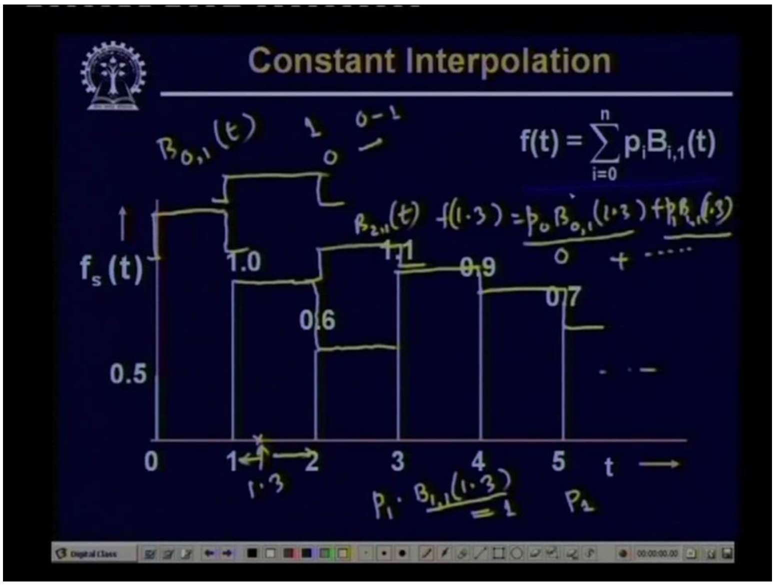

And the values of fs,t are present at t equal to 0, t equal to 1, t equal to 2, t equal to 3, t equal to 4 and t equal to 5. As we said that the interpolation function is given by this f(t) is equal to piBi,1(t) if I go for constant interpolation. Now here Bi,1(t), so when I had computed say B0,1(t) we have said that this value is equal to 1 for t lying between 0 and 1 and this is equal to 0 otherwise .

So if I interpolate this particular sample data say for example I want to find out what is the value of the signal at say 1.3, so this is the point 1. t equal to 1.3. So I want to find out the value of f(t) at t equal to 1.3. So to do this my interpolation formula says that this should be equal to f(1.3) this should be equal to p0 into B0,1(1.3)+p1B1,1(1.3) and so on. Now if I plot this B0,1(1.3) just super impose this on this particular sample datas sample data diagram.

You find that B0,1(1.3) or B0,1(t) the function is equal to 1 in the range 0 to 1 excluding 1. Similarly p1 B1,1(1.3) the value will be equal to 1 in the range 1 to 2 excluding 2 and it will be 0 beyond this. So when I try to compute the function value at t equal to 1.3, I have to compute p0 that is the sample value at t equal to 0 multiplied by this B0,1(t). Now because B0,1(t) is equal to 0 for values of t greater than or equal to 1. So this particular term p0 into B0,1(t) this term will be equal to 0.

Now, when I compute this p1B1,1 1.3 you find that this B1,1(1.3) is equal to 1 in the range 0 to 1 to 2, excluding 2. and beyond 2 the value of B1,1(t) is equal to 0. Similarly for values of t less than or (eq) less than 1, the value of B1,1(t) is also equal to 0. Similarly p2B2,1 that value is 1 within the range 2 to 3. So within this range B2,1(t) is equal to 1 and beyond this B2,1 equal to 0.

So, when I try to compute the value at point 1.3, you find that this will be nothing but P1 into B1,1(1.3) and this B1,1(1.3) is equal to 1 so the value at this point will be simply equal to P1, so in this case it is fs(1) and you find that for any other values of t within the range 1 to 2. The value of f(t) will be same as f(1) or p1. So I can approximate this or I can do this interpolation like this that is between 1 and 2, all the values the function values for all values of t between 1 and 2 will be equal to 1.

Following similar argument, you find that between 2 and 3 the function value will be equal to f(2), between 3 and 4 the function value will be equal to f(3). Between 4 and 5 the function value will be equal to f(4) and it will continue like this. So this is what I get if I use this simple interpolation formula that is f(t) equal to p1 piBi1,(t). Similar is also the situation if I go for linear interpolation.

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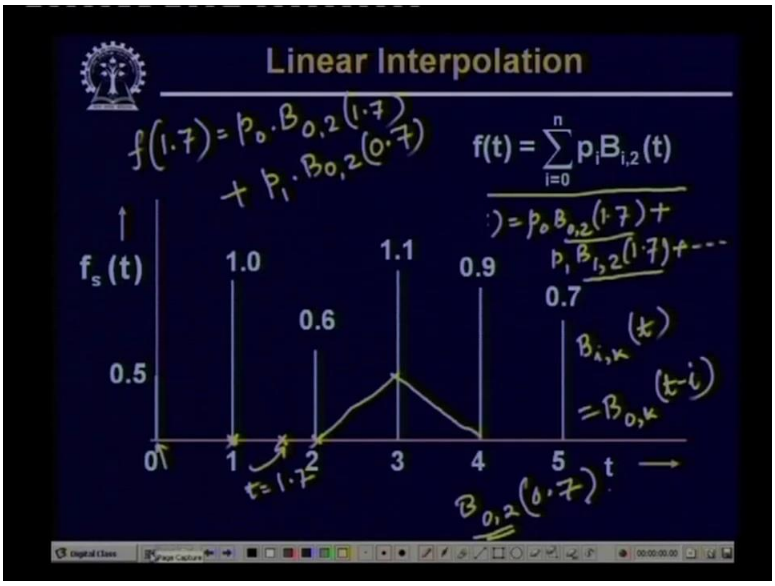

So what I get in case of linear interpolation. In case of linear interpolation f(t) is given by Bi,2 pi into Bi,2(t) where you have to take the summation of all these terms for values of i from i equal to 0 to n. So what we get in this case. You find that we have said that Bi,0(t) is nothing but a linearly increasing and decreasing function. So if I plot Bi, B0,2(t), B0,2(t) is a function like this. Which increases linearly between 0 and 1 at 1 this reaches a value 1 and then again from t equal to 1 to t equal to 2. The value of B0,2(t) decreases linearly and it becomes 0 at t equal to 2.

Similarly B1,2(t) will have a function value something like this it will increase linearly from 1 to 2, it will reach a value of 1 at t equal to 2 and again from t equal to 2 to t equal to 3 it decreases linearly then at t equal to 3 the value of B1,2(t) becomes equal to 0. So here again if I want to find out, say for example the value of function at say t equal to 1.7.

So I have the functional values at t equal to 1, I have the value of the function at t equal to 2. Now at t equal to 1.7, I have to approximate the value of this function using its neighboring pixels. Now if I try to approximate this you find that using this particular interpolation formula here again f(1.7) is to be computed as p0 B0,2(1.7) + p1 B1,2(1.7), so it will continue like this.

Now here you find that the contribution to this point by this sample p0 by this sample value p0 is given by this interpolation function B0,2(t) and by this the contribution to this point t equal to 1.7 by this sample f1 or p1 is given by B1,2(1.7), contribution to this point by the sample value f2 is given by B2,2(1.7). But B2,2(1.7) is equal to 0 at t equal to 1.7, because B2,2(1.7) is something like this so value of this function is 0 at t equal to 1.7.

So only contribution we get at t equal to 1.7 is from the sample f0 and from the sample f1, ok. So using this I can estimate what will be the value of B0,2at(1.7), I can also estimate what will be the value of B1,2(1.7). And in this particular case we have seen a property of this B-Spline function that is we have said earlier that Bi,k(t) is nothing but B0,k into (t - i), so that is a property of these B-Spline functions.

So when I do this you find that this B1,2(1.7) is nothing but B0,2 because this is t - i and value of i is equal to 1, so this will be B0,2(0.7). So if I simply calculate the value of B0,2 for different values of t, I can estimate that what will be B0,2(1.7). And in that case the value at this location that is f(1.7) will now be given by, if I simply calculate this. So in this case f of (1.7) will be given by p0 into B0,2(1.7)+ p1 into B0,2(0.7), because this is same as B1,2(1.7).

Where value of p0 is equal to 0.5 which is the sample value at location t is equal to 0 and value of p1 is equal to 1, that is the sample value at location t equal to 1. Now here you find that there is a problem that if when I am trying to compute the value at t equal to 1.7, the contribution only comes from the sample values at t equal to 0 and t equal to 1. But this interpolation of this approximate value does not have any contribution from t equal to 2 or t equal to 3.

So your interpolation or approximation that you are doing is very much worst because it is only considering the sample values to the left of this particular point we are not considering the sample values to the right of this particular point t equal to 1.7. So that is a problem with this basic interpolation formula.

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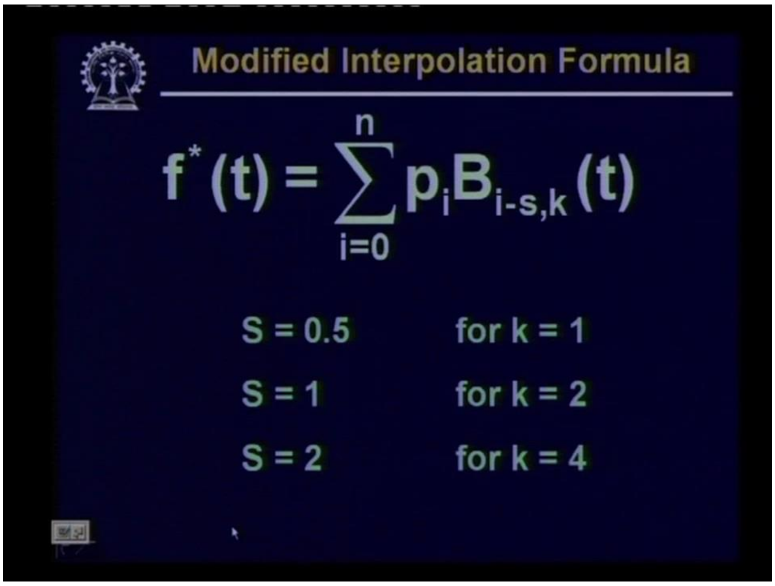

So to solve this problem what we do is instead of using the simple formula that is f(t) equal to summation to pi Bi,k(t), where i varies from 0 to n. We slightly modify this interpolation formula, thank you.