Digital Image Processing.
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Lecture-17.
Interpolation Techniques.

Now, whenever we interpolation operation should have certain desirable properties. Firstly the interpolation function that use that you use for interpolating the discrete values that should have a finite region of support. That means interpolation should be done based on the local information it should not be based on the global information or it should not take into consideration all the sample values of that particular digitized signal.

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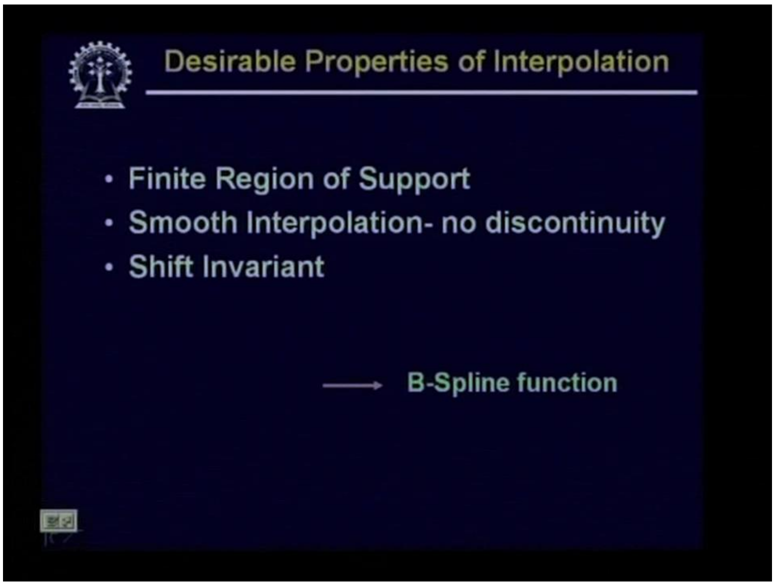

The second desirable property is the interpolation should be very smooth that is the interpolation should not introduce any discontinuity in the signal. And the third operation is the interpolation should be shifting variant, so if the signal is shifted or given some translation then also the same interpolation function should be available should be done. And the B-Spline function is one such function which which satisfies all these three desired properties.

Now let us see what is this B-Spline function, B-Spline function is a piecewise polynomial function that can be used to provide local approximations of curves using very small number of parameters. And because it is useful for local approximation of curves so it can be very very useful for smoothening operation of some discrete curves, it is also very very useful for interpolation for of a function from discrete number of samples.

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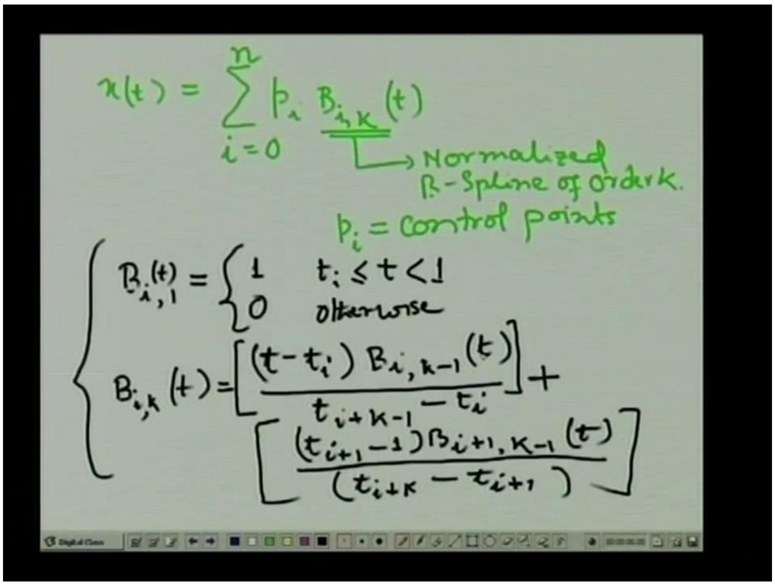

So let us see what is this B-Spline function, a B-Spline function is usually represented by say x(t) is equal to sum of pi into bi,k (t), where you take the summation from I equal to 0 to n, where, 0 to n that is n plus 1 is the number of samples which are to be approximated. Now these points pi they are called the control points and bi,k is the normalized B-Spline of order k. so, bi,k this is the normalized B-Spline B-Spline of order k and pi are known as the control points.

So, these control points actually decide that how the B-Spline functions should be guided to give you a smooth curve. Now this normalized B-Spline that is bik can be recursively defined as Bi,1 is equal to 1 whenever ti is less than or equal to t less than 1 so this is Bi,1 of t, it is equal to 1 whenever ti is less than or equal to t and which is less than 1 and Bi,1 t is equal to 0 whenever value of t takes other values.

So, this is equal to 1 for all values of t lying between ti and 1, ti is inclusive and bi,1t is equal to 0 for any other value of t. And then we can find out Bi of and k of t using the relation. So Bi,k(t) is equal to (t-ti) into Bi,k-1(t) upon ti+k-1-ti + (ti+k-1) into Bi+1,k-1(t) upon (ti+k-ti+1). So once we have bi,1 for different values of t than from this Bi,1 we can recursively compute the values of Bi,k using this relation.

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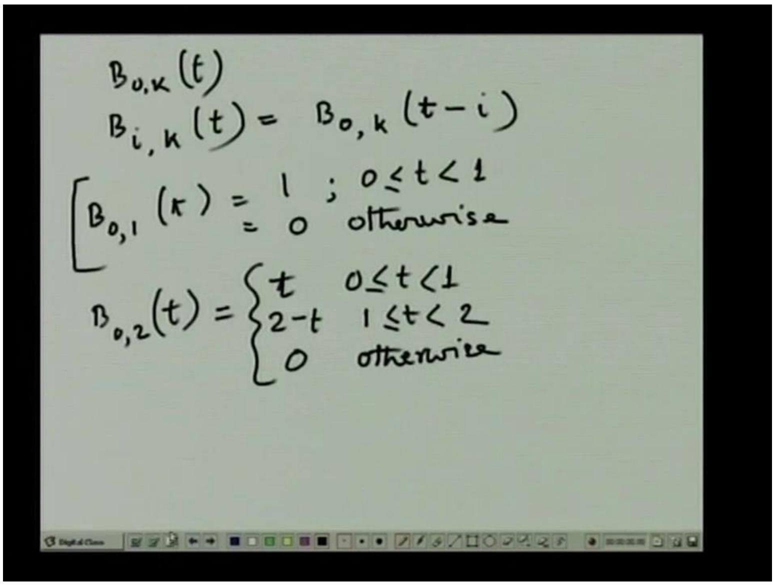

Now, we find that once we have this relation of $B_{,ik}(t)$, you can easily verify that once I have $B_{0,k}(t)$ then $B_{i,k}(t)$ is nothing but translates of $B_{0,k}(t)$. So this $B_{i,k}(t)$ can be written as this is nothing but $B_{0,k}(t-i)$. So, this can be easily verified from the way this B-Spline function is defined. Now, this B-Spline function for various values of i and k can be obtained like this you can easily get that $B_{0,1}(t)$ will be equal to 1 whenever 0 less than or equal to t less than 1.

Because, earlier we have said that $B_{i,1}(t)$ is equal to 1 whenever t_i is less than or equal to t which is less than 1 and it is 0 otherwise. So, just by extending this I can just write that $B_{0,1}(t)$ will be equal to 1 whenever t lies between 0 and 1, 0 inclusive and it will be equal to 0 otherwise. So we find that $B_{0,1}(t)$ is constant in the region 0 to 1. Similarly, we can find out $B_{0,2}(t)$ will be equal to t for 0 less than or equal to t less than 1 it will be equal to 2-t for 1 less than or equal to t less than 2 and it will be 0 otherwise.

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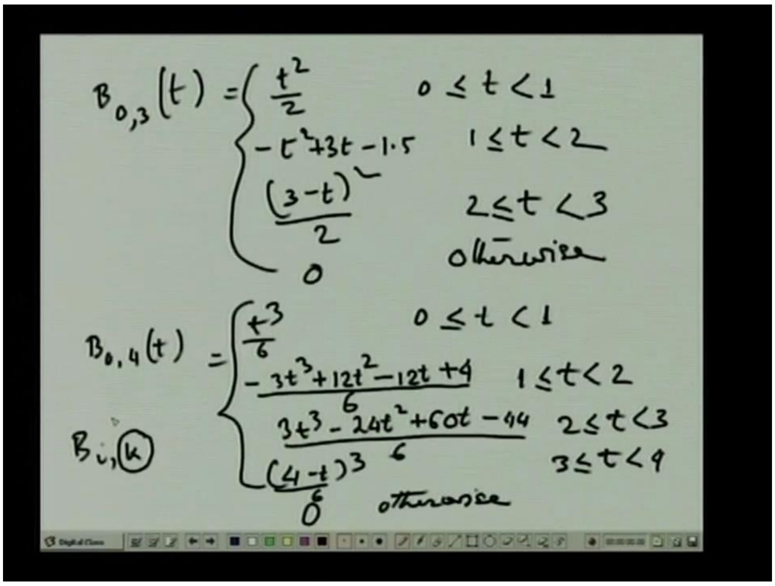

Similarly, you can find that B0,3(t) can be written as t square by 2 for 0 less than or equal to t less than 1 this will be equal to -t square +3t - 1.5 for 1 less than or equal to t less than 2 it will be 3 - t square upon 2 for 2 less than or equal to t less than 3 and it will be 0 otherwise. Similarly, we can also find out B0,4(t) will be equal to t cube by 6 for 0 less than or equal to t less than 1. It will be equal to - 3t cube +12t square - 12t+4 by 6 for 1 less than or equal to t less than 2.

It will be equal to 3t cube - 24t square +60t - 44 divided by 6 for 2 less than or equal to t less than 3. This will be equal to (4 - t)cube by 6 for 3 less than or equal to t less than 4 and it will be 0 otherwise. So, you can obtain all these values from the definition of the definition of the B-Spline function that we have defined earlier. Now, whenever I write this Bi,k this k is called the order of the Bezier of order of the B-Spline function.

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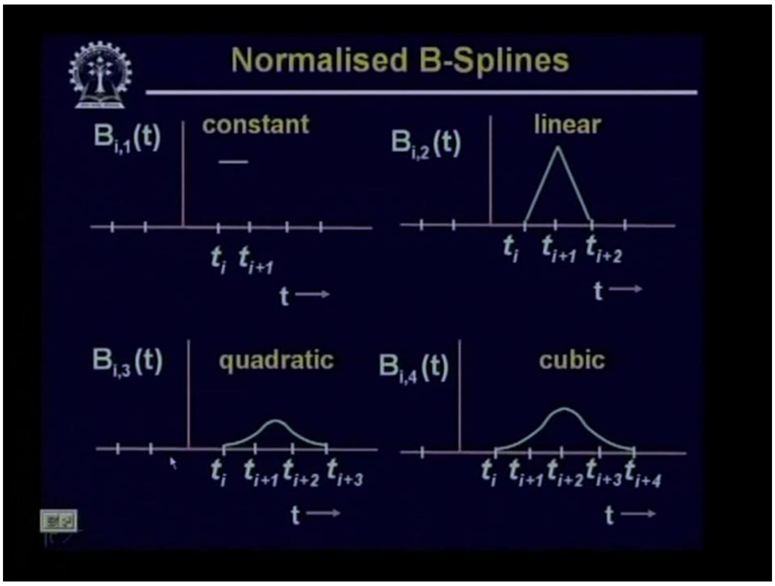

Now, after giving these equations let us try to see that what is the nature of this of this B-Spline functions. So we find that Bi,1(t) because it will be equal to 1 for i equal to 1 to 2, between i equal to 1 to 2, this Bi,1(t) will be equal to constant and it will be equal to 1. So, this first figure in this particular case tells you that what is the nature of this B1 Bi,1.

The second figure shows what is the nature of this B-Spline function if it is Bi,2(t). And it shows that it is a linear function. So, Bi,2(t) will lie between will have a support from i to i+2 and the points which will be supported by this Bi,2(t) are i, i+1 and i+2. Similarly, a quadratic function which is Bi,3(t) is given by this figure and a cubic function which is Bi,4(t) is given by this fourth figure.

And, here you find that the region of support for this cubic function is 5 points, the region of support for quadratic function is 4 points, the region of support for the linear B-Spline is 3 points whereas the region of support for B for the B-Spline of order 1 is only 2 points. So in all the cases the region of support is finite. Now, let us see that using these B-Splines how we can go for interpolation.

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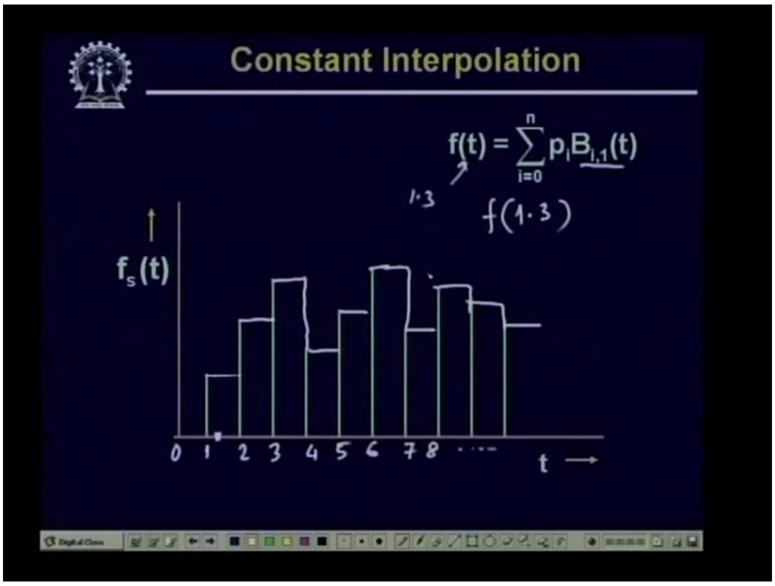

As, we have said earlier that using these B-Splines a function is approximated as f(t) equal to piBi,1(t) or Bi,k(t) where i varies from 0 to n. So, in this case let us take the value of k to be equal to 1 that means we have a B-Spline of order 1. And we have shown that if we have a B-Spline of order 1 then the nature of the B-Spline function is it is constant between i and i+1 and which is equal to 1.

So, using this if I try to interpolate this particular function as shown in this diagram which are nothing but some sample values. I take say t equal to 0 here, t equal to 1 here, t equal to 2 here, 3 here, 4 here, 5, 6, 7, 8 and so on. Now, if I want to find out say f of (1.3), so f of (1.3) should lie somewhere here. So to find out this f of (1.3) what I have to do is I have to evaluate this function f(t) where I have to put t is equal to 1.3 and this f(t) is given by this expression that is piBi,1(t) where I varies from 0 to n. Now we find that this Bi,t when if I take i equal to 1 so B1,1 is a function like this. So in between 1 and 2 this B1,1 is constant and that is equal to 1, ok.

So, if I find out the value at this particular point it will be pi that is fs,1 multiplied by B1,1 at t equal to 1.3 and which is equal to 1. And this B1,1 is equal to 1 for all values of t from t equal to 1 to t equal to 2 but excluding t equal to 2. So if I interpolate this function using this Bi,1(t) you will find that this interpolation will be of this form, sorry. This interpolation will now be of this form, so it goes like this. So all the values at all points between t equal to 1 and t equal to 2 the interpolated value is equal to fs,1.

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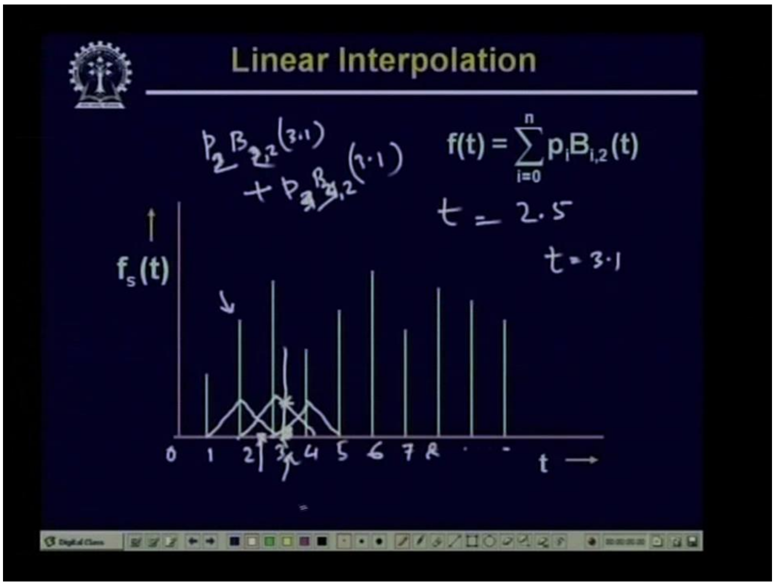

Similarly, between t equal to 2 and t equal to 3 the interpolated values is equal to fs,2 and so on. Similarly, if I go for interpolation using say linear interpolation where value of k is equal to 2 in that case again you find that if I put say t equal to 0 here, t equal to 1, t equal to 2, t equal to 3, 4, 5, 6, 7, 8, like this now B1,2 is a linear function between 1 and 3. So B1,2 is something like this. Similarly, B2,2 is something like this, B3,2 is something like this.

So if now I want to have I want to interpolate or I want to find out the value of this function at point say 2.5, at t equal to 2.5 say here. Then you find that the sample values which take part in this interpolation are f2 and f3 and by using this two sample values by linear interpolation I have to find out what is the interpolated value at t equal to 2.5. Now you take a case that I want to interpolate a interpolate this function value f(t) at t equal to say 3.1 that means somewhere here.

So if want to do this then what I have to do is this will be an interpolation of p3 so this will be nothing will be nothing but p3 into B3,2 at point t equal to 3.1 plus you can easily find out it will be p4 into B4,2 again at point 3.1. So, I want to interpolate this value here, now we find that weight of this p3 is given by only this much whereas, weight of sorry this will be p2 B2,2 and p3 B3,2.

So weight of p2 is given by this value whereas weight of p3 is given by this value. So when I am interpolating 3.1 I am giving less weightage to p3 which is nearer to this particular point t equal to 3.1 and I am giving more weightage to this particular point p2 which is away from

3.1, which is not very logical. So, in this case we have to go for some modification of this interpolation function.

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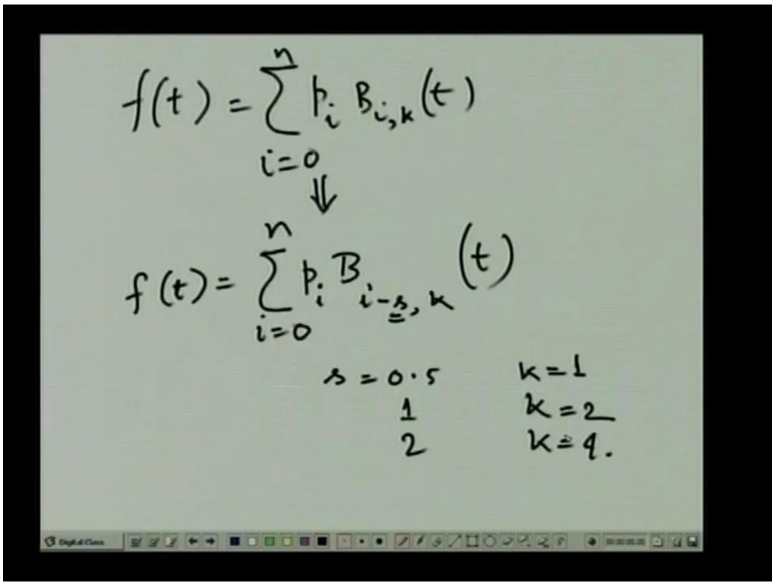

So, what is the modification that we can think of. The kind of modification that we can do in this case is instead of having the interpolation as say f(t) is equal to pi Bi,k(t) summation over i equal to 0 to n. I will just modify this expression as f(t) is equal to I will put same bi pi but B I will shift by some values, so I will take it Bi - s,k(t) again i will vary from 0 to n. And the value of this i will be value of this shift s will depend upon what is the value of k.

So, I will take s is equal to 0.5, if I have k equal to 1 that is constant interpolation I will take s equal to 1 if I have k equal to 2, that is linear interpolation and I will take s equal to 2 if k is equal to 4 that is I have by I have cubic interpolation. Now you find that I have not considered k equal to 3 which is a quadratic interpolation because quadratic interpolation leads to asymmetric interpolation.

If, I go for other interpolations like k equals to 1 with of course a shift of Bi,k by a value of 0.5, so s equals to 0.5 or if I take k equal to 2 with s equal to 1, or if I take k equal to 4 with s equal to 2 what I get is asymmetric interpolation. So, these are the different interpolation the B-Spline interpolation functions that we can use for interpolating the function from sample values.

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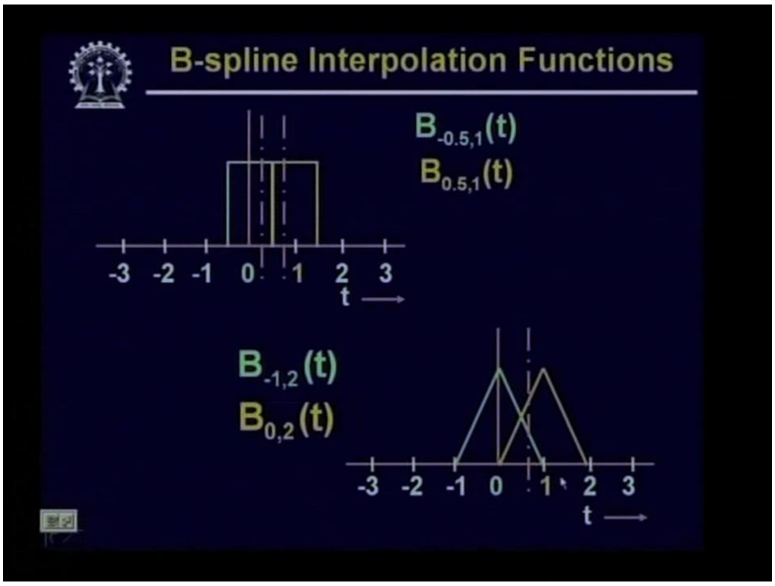

So this shows the situation when we have shifted this Bi,1 by a value 0.5, so here you find now that Bi,1 is constant from -0.5 to +0.5. Similarly, 0.5 to 1.5 it will be from 1.5 to 2.5 and so on. Similarly, Bi,2 now it is the regions of support for B0,2 is between -1 and 1, for B1,2 is from 0 to 2 and so on. So by using this similarly for cubic interpolation I do the corresponding shifting.

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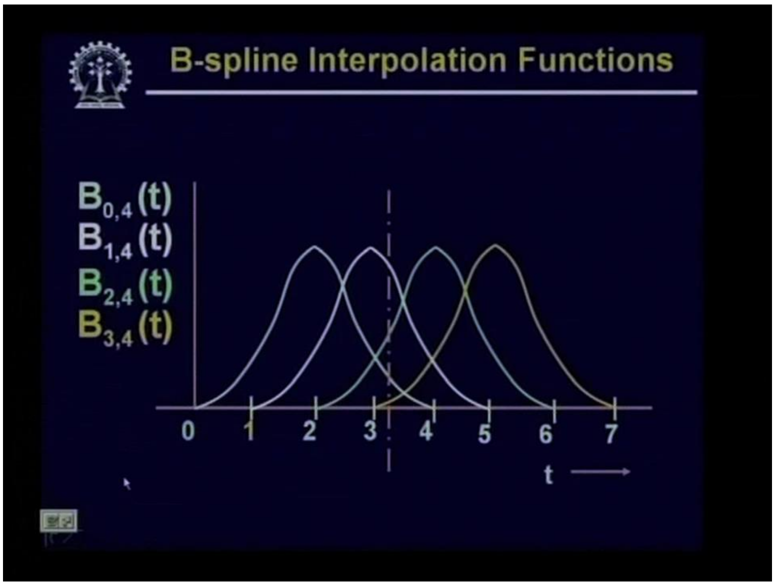

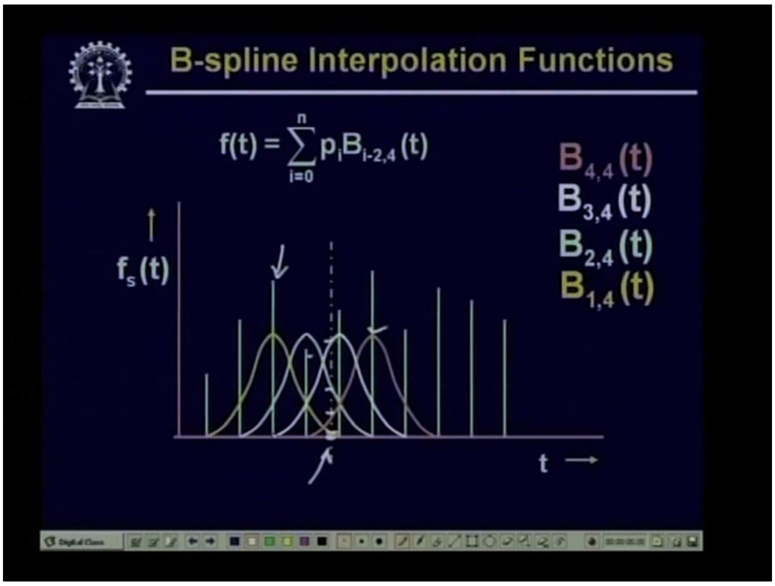

And by using this now the interpolation can be obtained by as if I go for cubic interpolation that f(t) is equal to piBi-2,4(t), where the summation has to be taken from i equal to 0 to i equal to n. And, here it gives that if I want to interpolate this function at this particular point, the weight given by this particular sample is only this much, the weight given by this particular sample is this much, the weight given by this particular sample is this much, the weight given by this particular sample this sample is weighted by this much.

So, by taking this weighted average the weights given by these B-Spline functions, I can find out what will be the interpolated value at this particular location.

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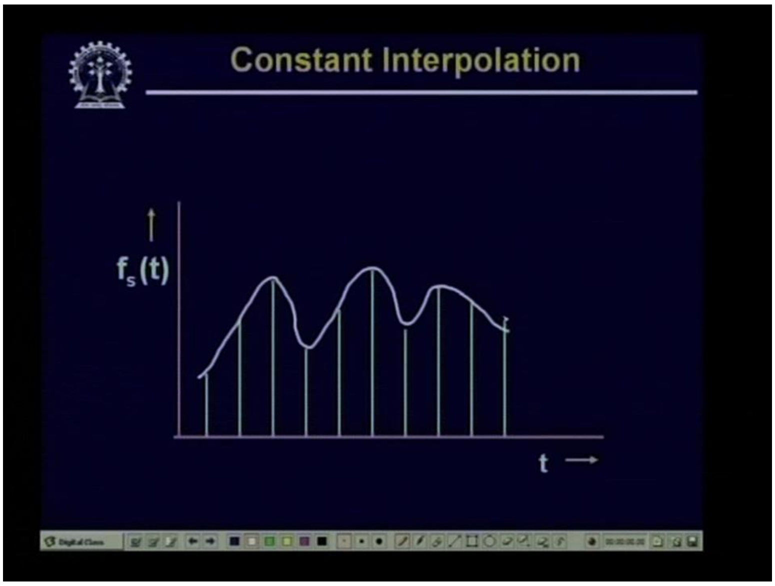

So if I use that cubic interpolation function possibly I will have for this state of sample values and interpolation is smooth interpolation like this. So may be this kind of smooth interpolation is possible using the cubic B-Spline function.

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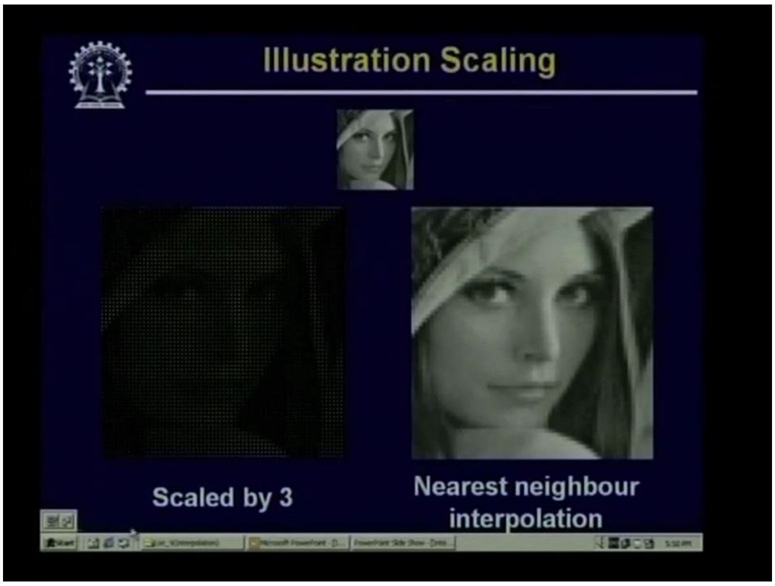

Now let us see some of the results on the images that we have obtained. So this is the example of a scaling operation I have a small image which is scaled up by factor 3 in both X direction and Y direction. You find that the image on the left side is obtained by scaling without applying any interpolation. So obviously you find that here the image appears to be a set of dots where many of the pixels in the image are not filled up.

If I go for nearest neighbour interpolation or constant interpolation this is the reconstructed image that I can get and you find that here I have the blocking artifacts and the image appears to be a collection of blocks.

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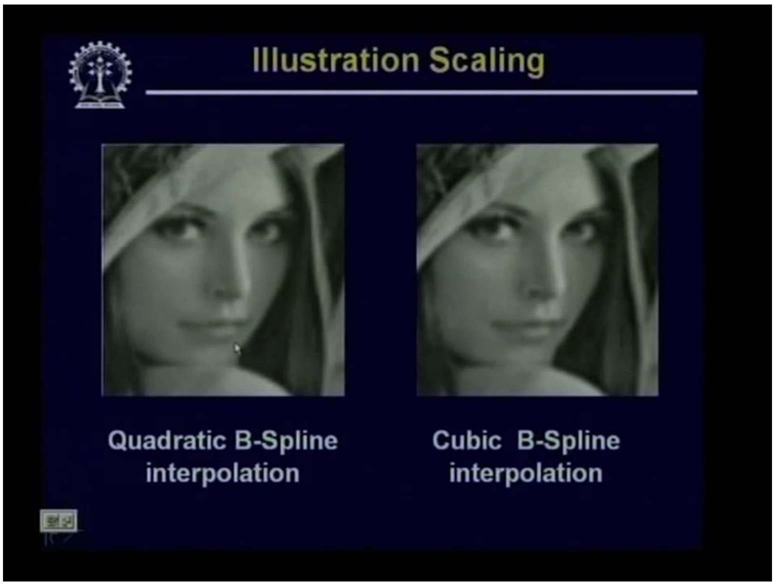

Similarly is I go for other interpolation techniques if I use quadratic B-Spline interpolation, then this is the quality of the image that we obtain. If, we go for cubic B-Spline interpolation then this is the quality of the image that we obtain.

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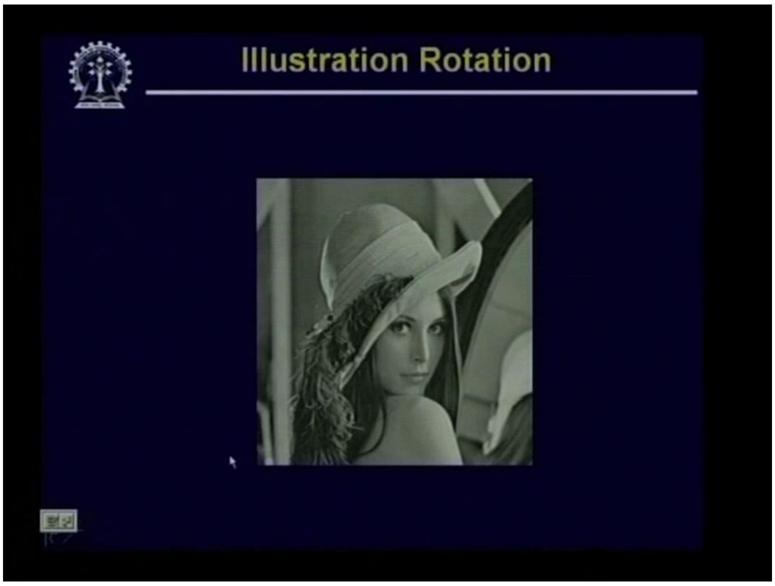

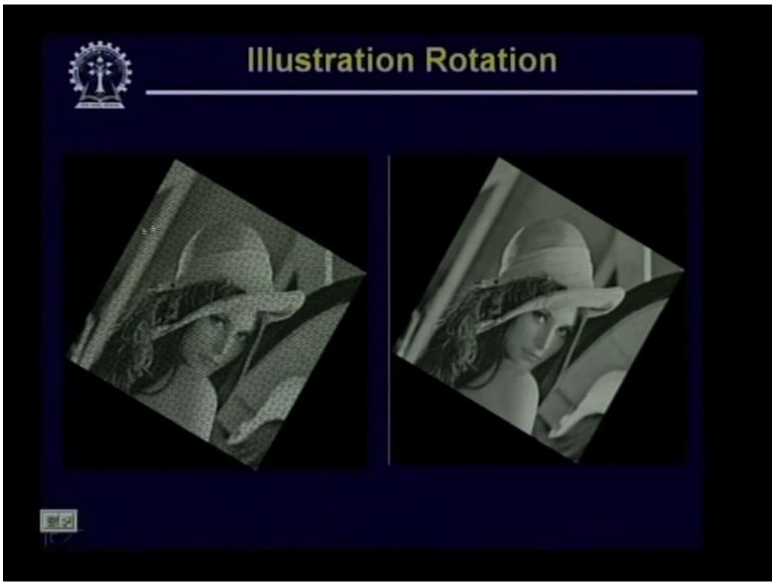

The same result is also the same experiment is also done in case of rotation. This particular image has been rotated by 30 degrees. And if I do not apply any interpolation so without interpolation the rotated image is shown on the left hand side, and here again you find that there are a number of black spots in this rotated image which cannot be filled up which could not be filled up, because no interpolation was used in this case. Where in the whereas in the right hand side this particular image after rotation has been interpolated using by cubic interpolation function. So you find that all those black spots in the digital image have been filled up.

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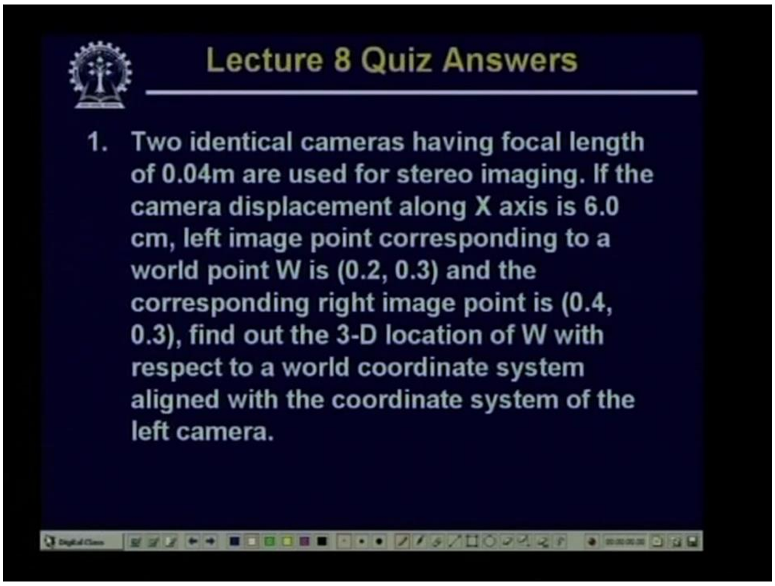

Now, let us see the answers of the quiz questions that we had given in the last class. In the last class we had given a quiz question for finding out the 3D coordinate point of a world point where its coordinate in the left camera and the coordinates in the right camera are given.

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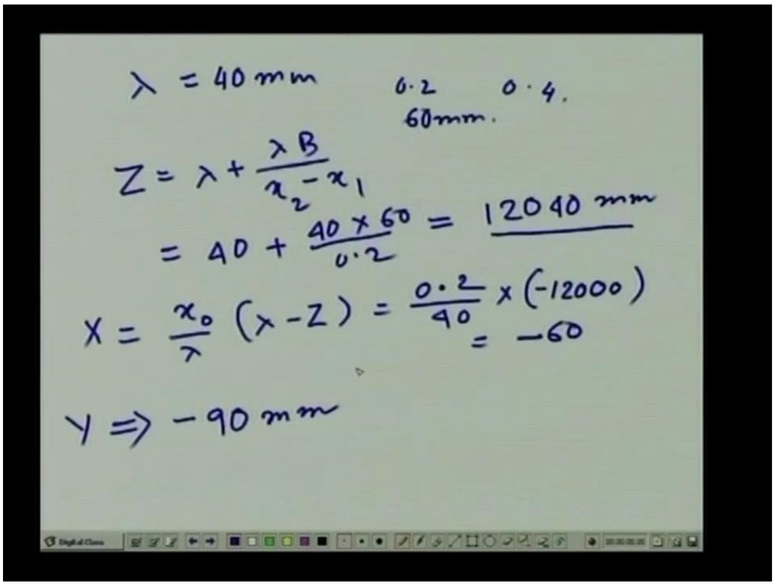

The value of the web the focal length lambda was given as 40 millimeter and the X coordinate in one case was given as 0.2 in the left camera and in the right camera it was given as 0.4.

And the camera separation was given as 6 centimeter that is 60 millimeter. So if I simply use the formula say Z equal to lambda + lambda B by x2 - x1, you find that this will be equal to 40+40 into 60 divided by0.2 because x2 - x1 in this case is 0.2 and which comes out to be 12040 so much millimeter, ok. So this is the value of Z that is the depth information.

And once I have the value of Z then the 3D coordinates X and Y can be computed from this value of Z, so X is nothing but x0by lambda into lambda - Z which is nothing but 0.2 upon 40 into -12000 which is equal to - 60 millimeter. And by apply applying the same procedure you can find out Y equal to - 90 millimeter. So this is about the first question.

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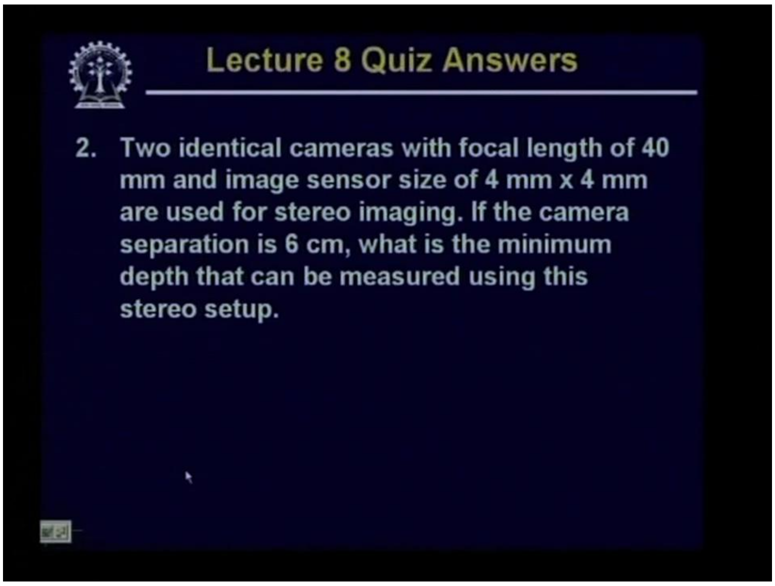

Second question was to find out what is the minimum depth that can be obtained by using the stereo camera where the the geometry of the stereo camera was specified.

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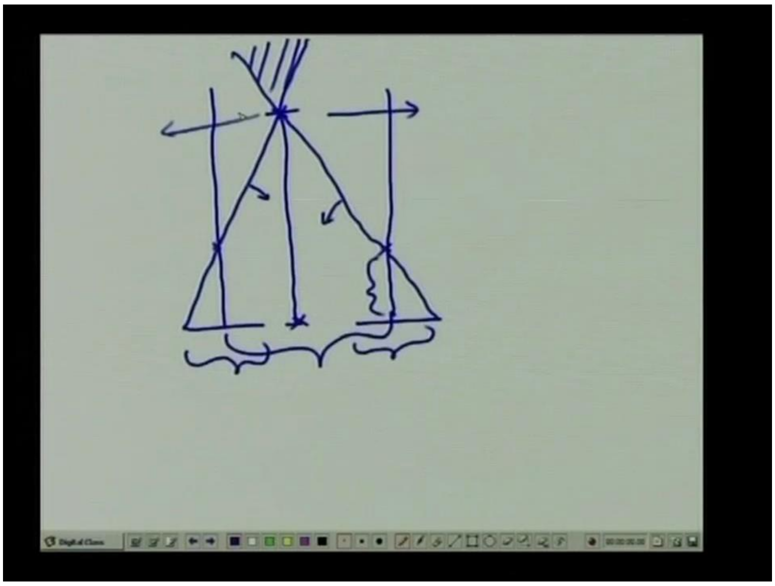

Now this is also very simple, what you can do is you find that we have 2 cameras with certain dimension of the imaging plate they have been specified certain focal length and if I just find out what is the limit of the imaging image points. So we find that if there is any point beyond this particular line. This point cannot be imaged by the left camera if there is any point in this direction that cannot be imaged by the right camera, because it goes beyond the imaging plate.

And also for finding out the depth information it is necessary that the same point should be imaged by both the cameras. So the points which can be imaged by both the cameras are only the points lying in this particular conical region. The points belonging to this region or the points belonging to this region cannot be imaged by both the cameras, so all the points must be lying within this.

So the minimum depth which can be computed is this particular depth. Now I know what is the separation between the cameras, I know what is the dimension of the imaging planes, ok. I also know what is the focal length. So, from these informations by using the concept of similar triangles you can easily find out what is the minimum depth that can be computed by using this stereo setup, thank you.