Digital Image Processing
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Module 03 Lecture Number 12
Image Formation - 2

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Hello, welcome to the video lecture series on Digital Image Processing and we have said that these transformations are very, very useful to understand the image formation process. So in the last class, what we had talked about is

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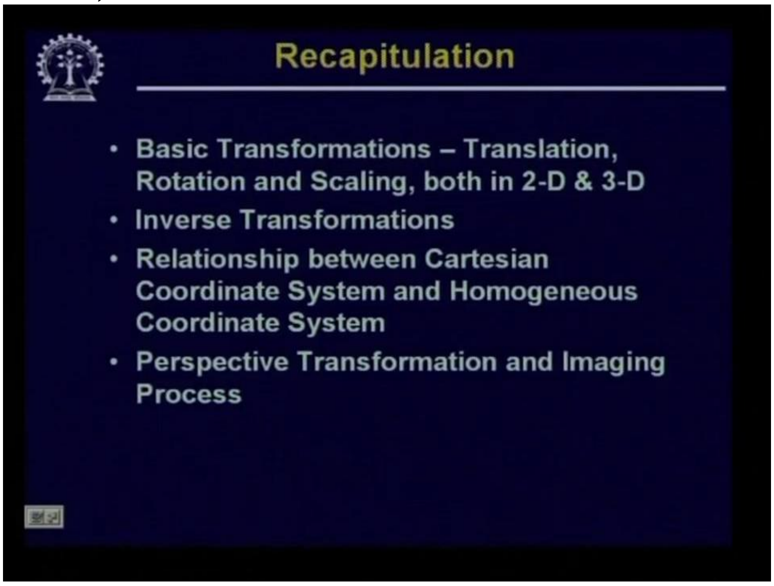

the basic transformations and we had talked about the transformations like translation, rotation and scaling and these transformations we have said both in the two-dimension and the three-dimensional cases. Then, for all these transformations we have also seen what is the corresponding inverse transformations. Then after that we had gone for the conversion from the Cartesian coordinate system to homogenous coordinate system and we have seen the use of homogenous coordinate system in perspective transformation where perspective transformation we have said is a approximation of the imaging process

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so that when a camera takes an image of a point in a three-dimensional world then imaging transformation can be approximated by the perspective transformation that we have discussed in the last class.

Today

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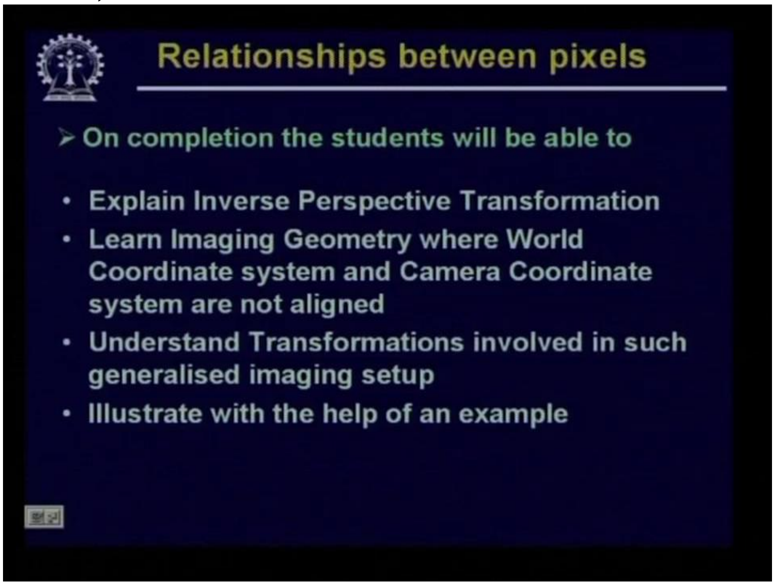

we will talk about the inverse perspective transformation. We have said

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that the perspective transformation takes an image of a point or a set of points in the three-dimensional world and these points are mapped to the imaging plane which is a two-dimensional plane. The inverse perspective transformation just does the reverse process that is given the point in the imaging plane; we will see that using this inverse perspective transformation whether it is possible to find out what is the point in the three-dimensional coordinate system to which this particular image point corresponds. Then we will also talk about the imaging

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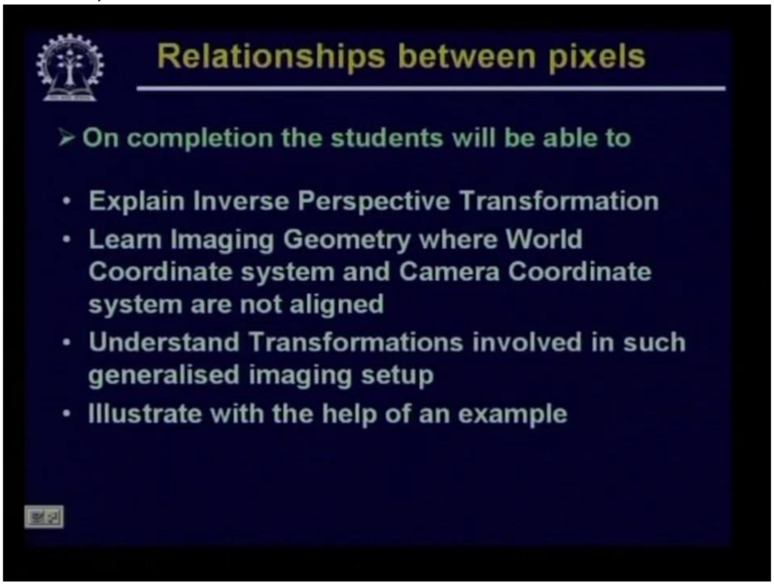

geometry where the world coordinate system and the camera coordinate system are not aligned You try to remember in the last class that the imaging

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geometry that we had considered, there we had assumed that the three dimensional world coordinate system is aligned with the camera coordinate system that is x axis of the camera is aligned with the x axis of the 3D world, y axis of the camera is aligned with the y axis of the 3D world and z axis of the camera is also aligned with the z axis of the 3D world. In addition to that the origin of the camera coordinate system also coincides with the origin of the image coordinate system. In today's lecture we will take a generalized imaging model where the camera coordinate and 3D the world coordinate system they are not aligned which is a general situation. Then we will try to see that, what are the transformations

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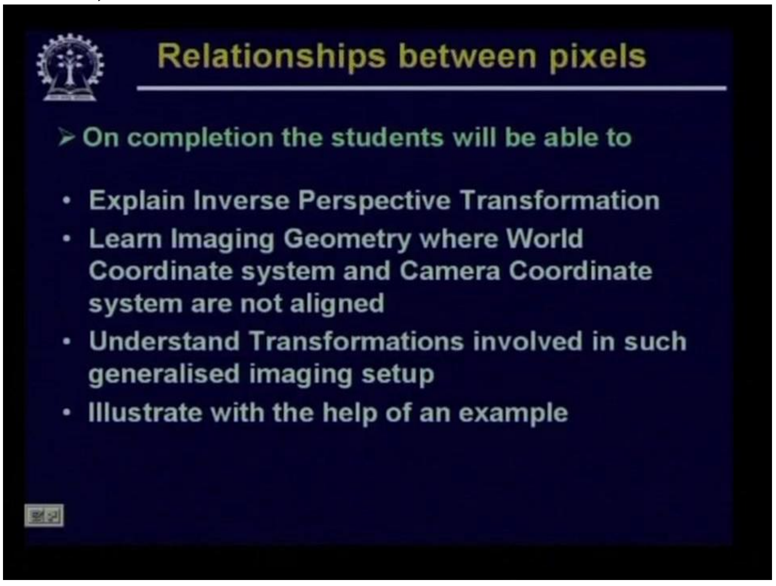

which are involved in such a generalized imaging set up, which will help us, understand the image formation process in a generalized set up Then we will illustrate this concept with the help of an example

Now let us briefly recapitulate

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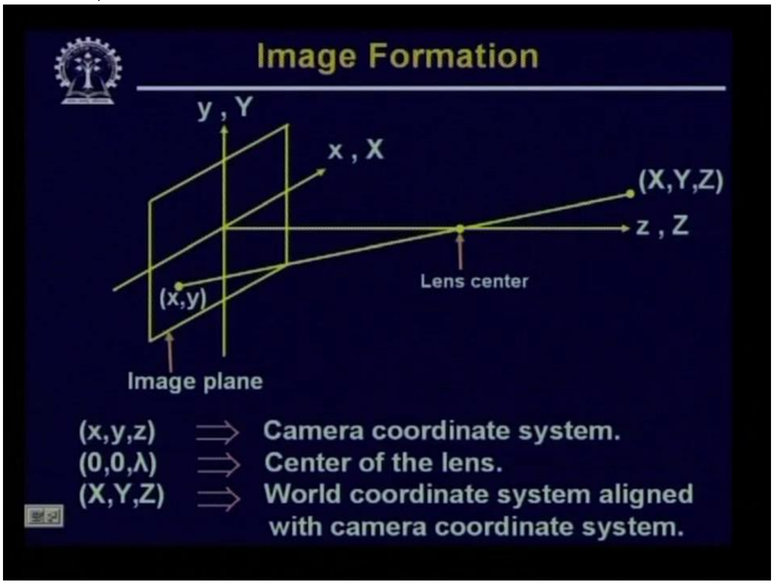

what we had done in the last class. This figure shows the image, the imaging geometry that you had considered where the 3D world coordinate system is aligned with the

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camera coordinate system There we have taken a 3D point whose coordinates are given by x, y z all in the capital and x, y lower case coordinates are the corresponding

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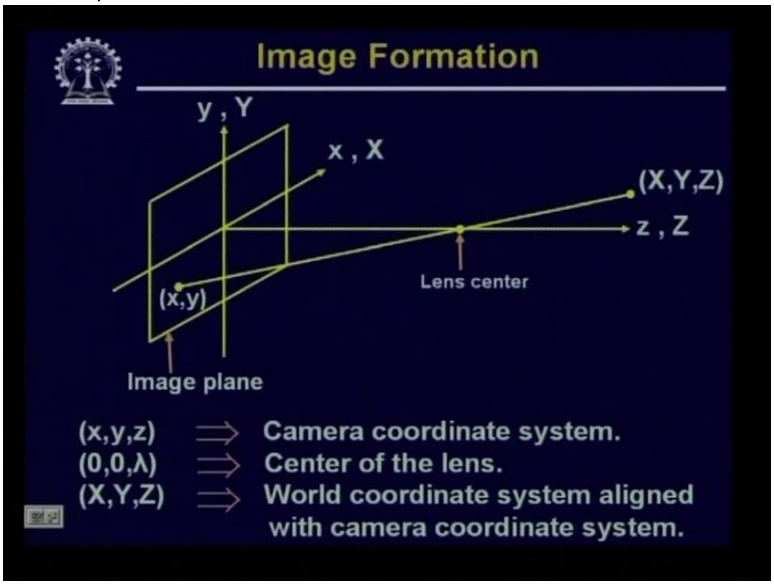

image points in the imaging plane. And we have assumed that the focal length of the camera is lambda. That means the coordinate of the focal point of the lens center is 0, 0, lambda. Now using this particular figure

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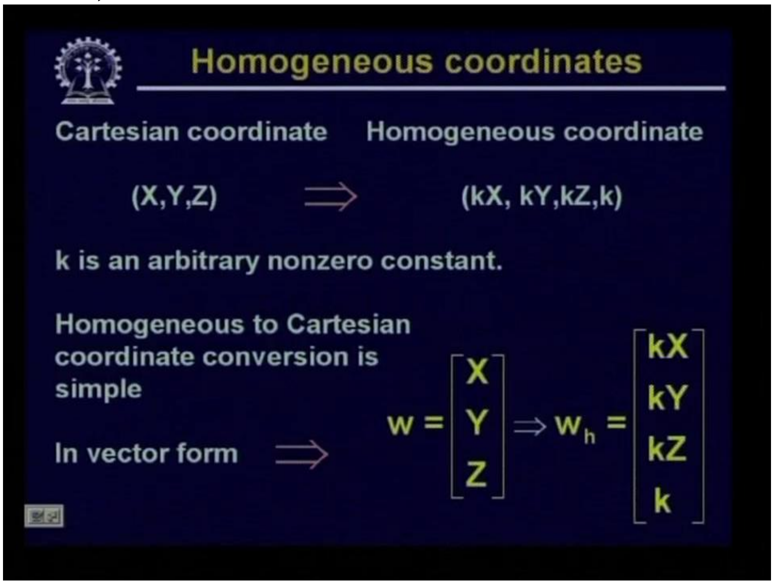

we have tried to find out a relation between the 3D world coordinate x, y, z and the corresponding

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image point which is x, y For that what we have done is we have

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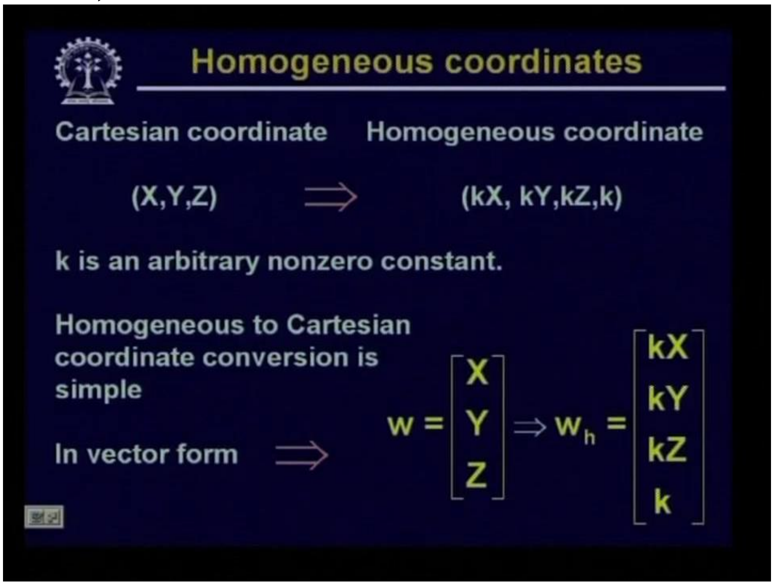

taken a conversion from the Cartesian coordinate system to the homogenous coordinate system So while doing this conversion what we have done is, every component of the coordinate, that is x, y z is multiplied by a non-zero arbitrary constant k and the same value is appended with the three components. For Cartesian coordinates x y z the corresponding homogenous coordinate is given

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by k x, k y, k z and z

So for a world coordinate point x, y, z once we have the corresponding homogenous coordinate k x, k y, k z and k then we found that after this conversion if we define a

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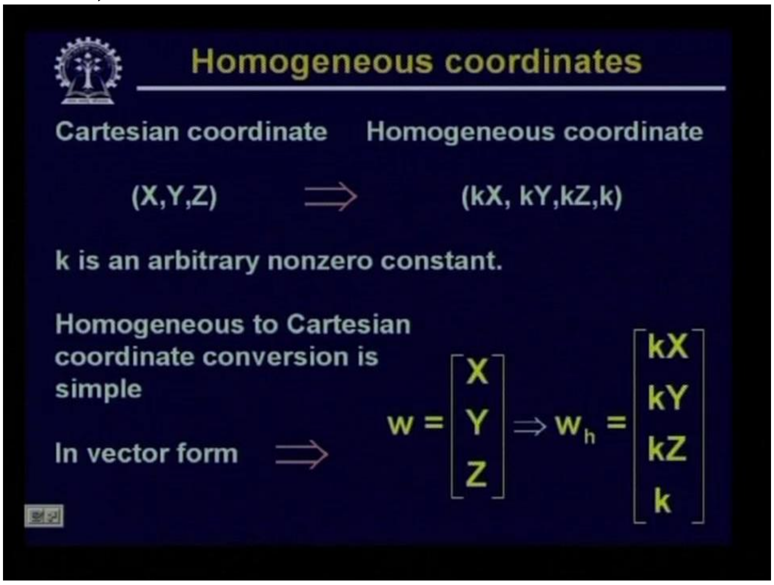

perspective transformation, so

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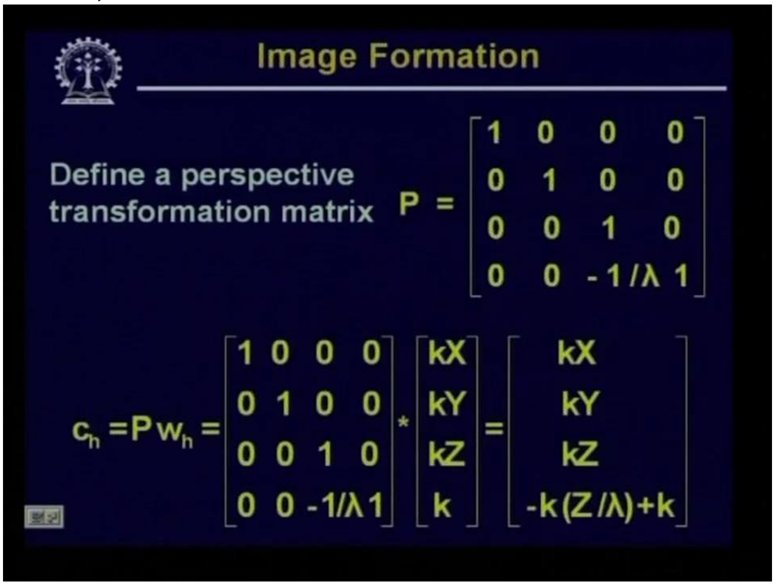

this perspective transformation matrix which in this case is 1 0 0 0, 0 1 0 0, 0 0 1 0 and 0 0 minus 1 upon lambda 1 and the homogenous coordinate w h is transformed with this perspective transformation matrix p then we get is the homogenous coordinate of the camera point to which this world point w will be mapped and the homogenous coordinate of the camera point of the image point after the perspective transformation is obtained as k x, k y, k z, minus k z by lambda plus k and you will see that if convert this homogenous camera point, the homogenous image point

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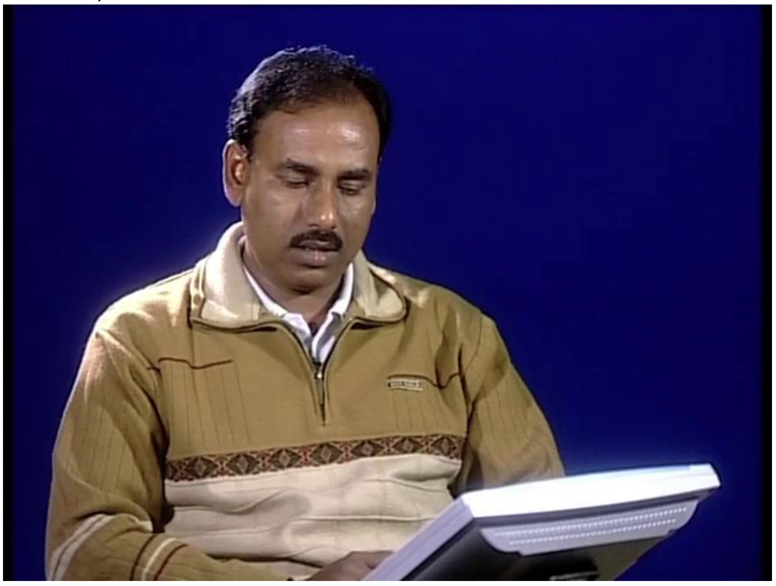

into the corresponding Cartesian coordinate then

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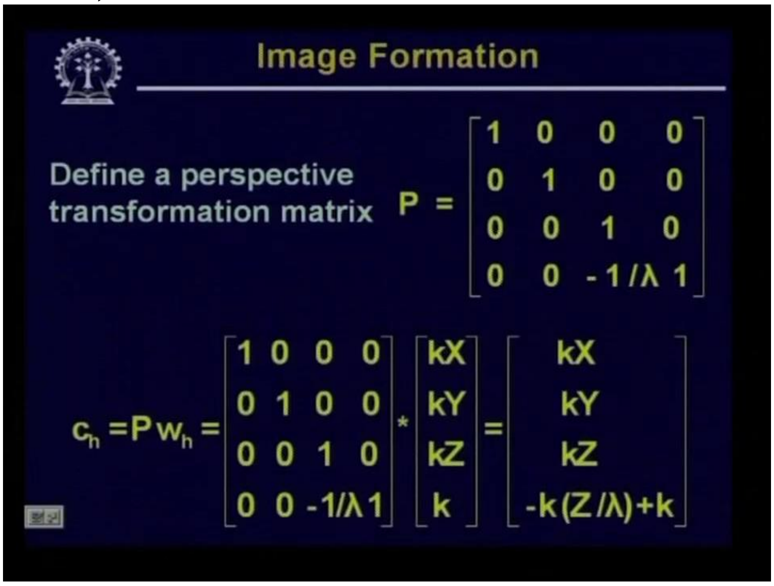

this conversion

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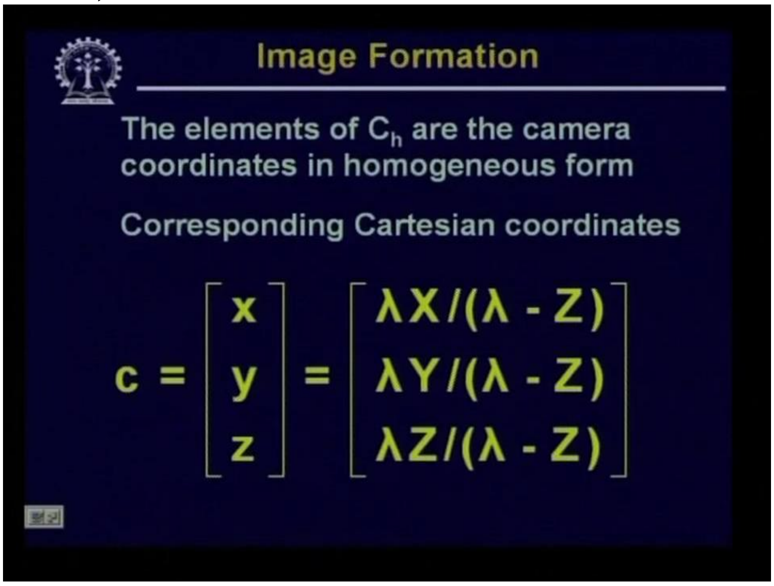

gives us the Cartesian coordinates of the image point as x y z equal to lambda x divided by lambda minus z lambda y divided by lambda minus z and lambda z divided by lambda minus z. So you just note that x y z in the lower case letters, these indicate the

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camera coordinate, the image coordinate where as x y z in the capital form, this represents the coordinate in the 3D world or the 3D coordinate of the world point w.

Now what we are interested in is the camera coordinate x and y at this moment we are not interested in the image coordinate z. So this can be obtained

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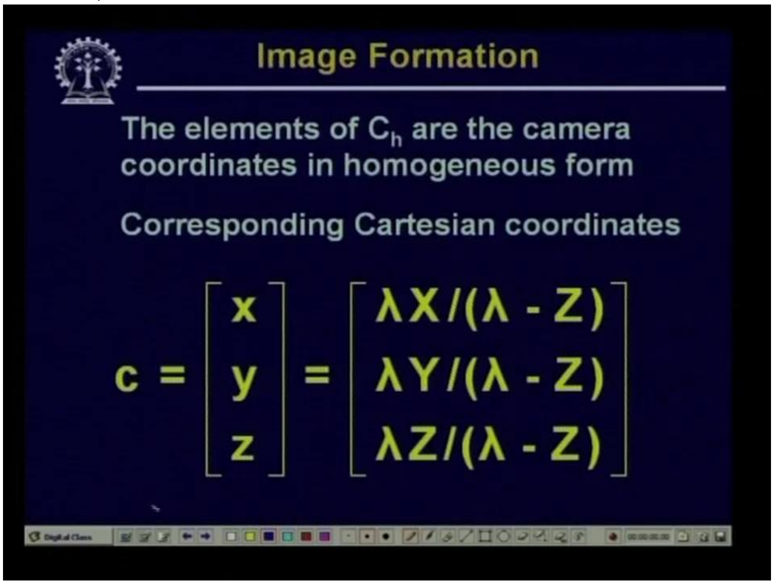

by simple conversion that if we find out the value of lower case z with respect to lambda and capital Z, then after solving the same equation here that is lower case x lower case y and lower case z. We find that the image coordinate x and y in terms of

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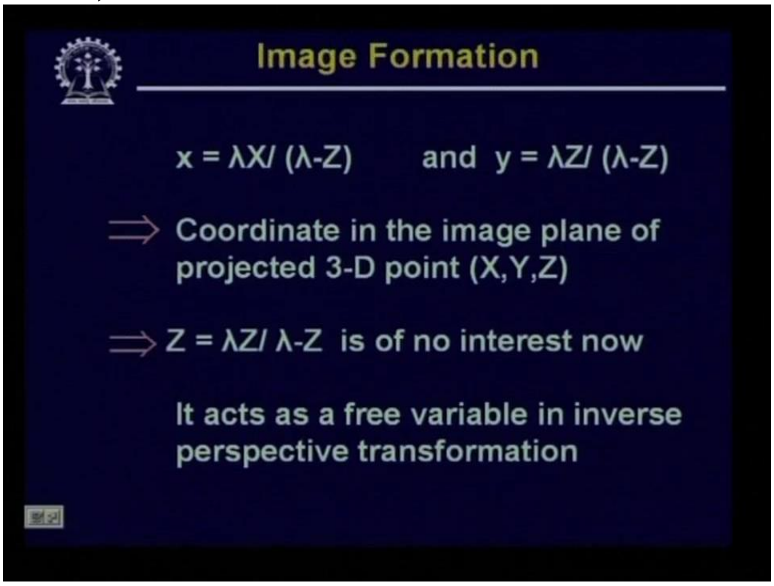

the 3D coordinate capital X and capital Z is given by x equal to lambda x divided by lambda minus capital Z and image coordinate y equal to lambda times capital Z divided by lambda minus capital Z.

So as we said the other value that is the z coordinate in the image plane is of no importance at this particular moment but we will see later that when we talk about inverse prospective transformation, when we try

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an image point to the corresponding 3D point in the 3D world then we will make use of this particular coordinate z in the image plane as a free variable so now let us see that

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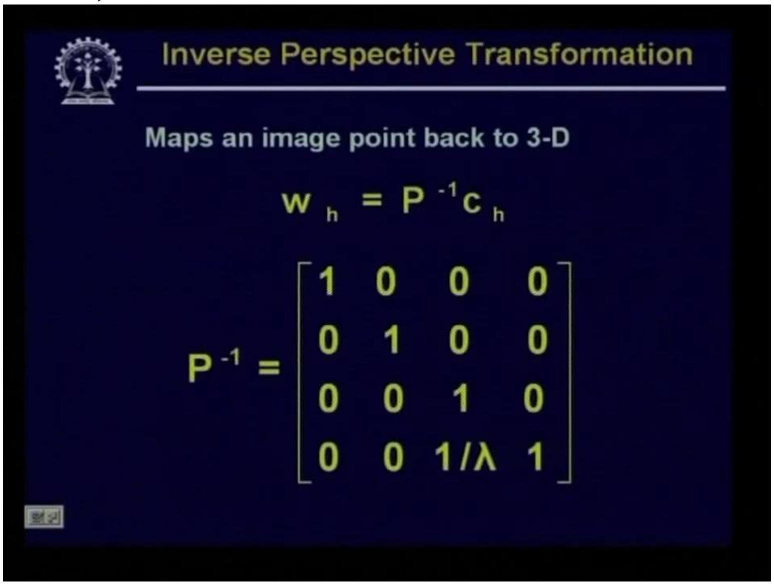

what is the corresponding inverse perspective transformation that we can have

So as we have said that a perspective transformation maps

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a 3D point on to a point in the image plane The purpose of inverse perspective transformation is just the reverse. That is, given the point in the image plane, the inverse perspective transformation or p inverse tries to find out the corresponding 3D point in the 3D world. So for doing that again we make use of the homogenous coordinate system that is the camera

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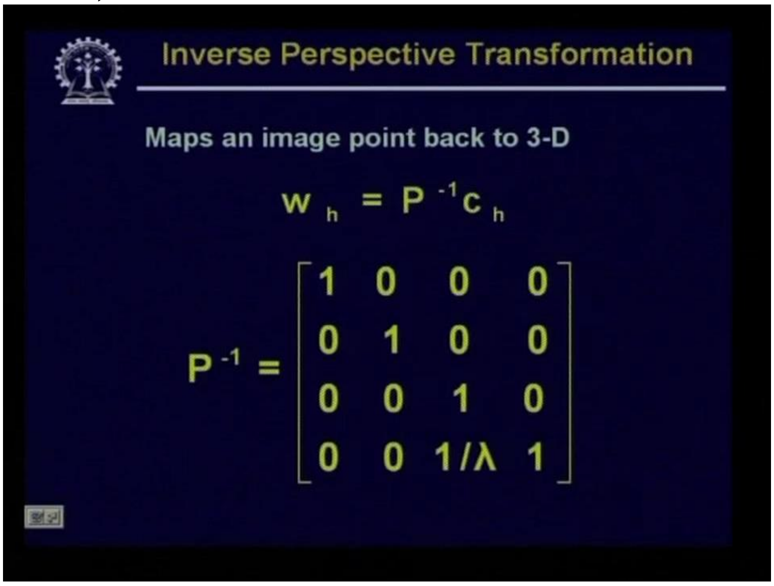

coordinate c of the image coordinate point c will be replaced, will be converted to the corresponding homogenous form which is given by c h and the world coordinate the world point w will also be obtained in the form in the homogenous coordinate form w h. And we define inverse perspective transformation p inverse which is given by 1 0 0 0, 0 1 0 0, 0 0 1 0 and 0 0 1 upon lambda 1 and you can easily verify

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that this matrix, this transformation matrix is really an inverse of the perspective transformation matrix p because if the if we multiply the perspective inf transformation matrix by this matrix p inverse what we get really is an unitary matrix

Now given this

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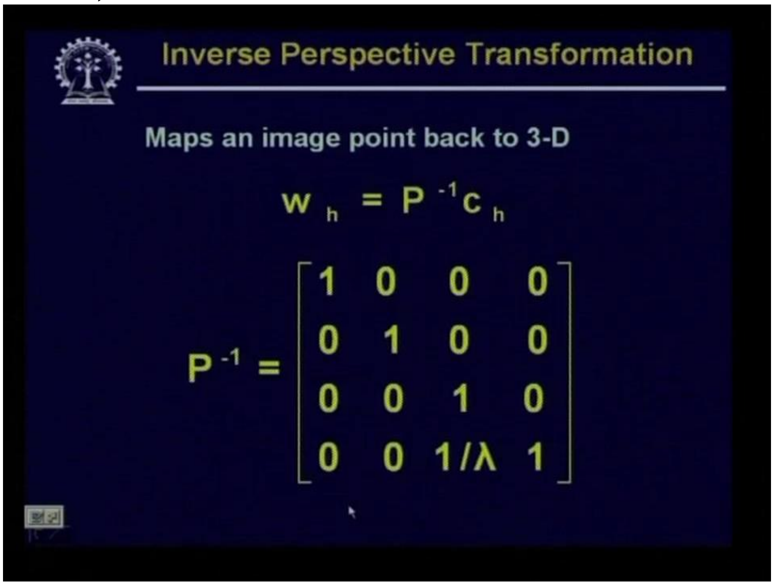

inverse perspective transformation matrix

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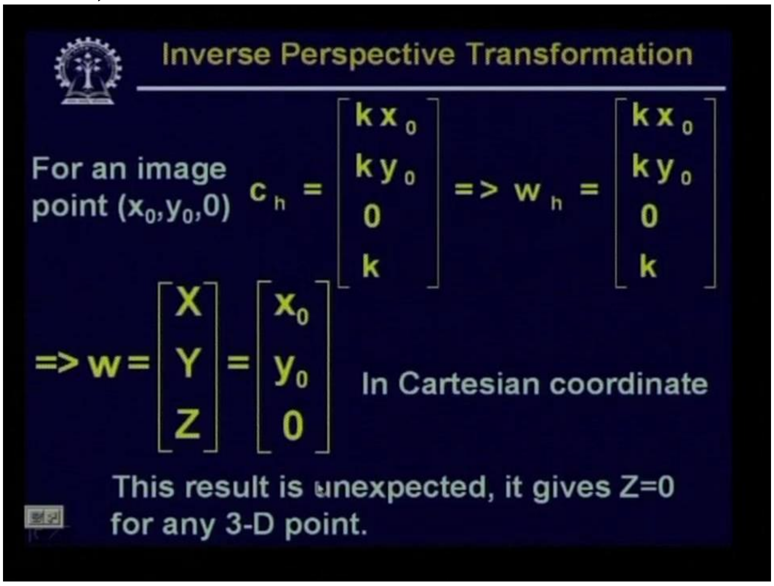

as we said that if we assume an image point say x naught y naught and we want to find out what is the corresponding 3D world point w to which this x naught y naught

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image point corresponds. So the first step that we will do is to convert this image point x naught y naught to the corresponding homogenous coordinate which will be obtained as k x naught k y naught and

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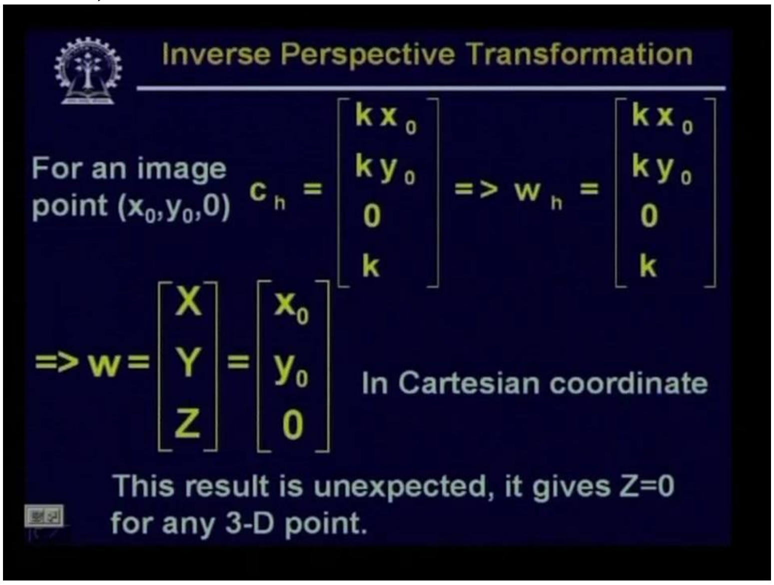

0 and the fourth component comes as k now you find that the third component or z coordinate we had taken as 0 because what we have is a point in two-dimension that is on the imaging plane so we have assumed the z coordinate to be 0.

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Now if we multiply or if we transform this homogenous coordinate k x naught k y naught 0 k with the inverse perspective transformation p inverse then what we get is the homogenous coordinate corresponding to the 3D world point which is obtained as w h as given in this equation w h is equal to k x naught k y naught 0 k

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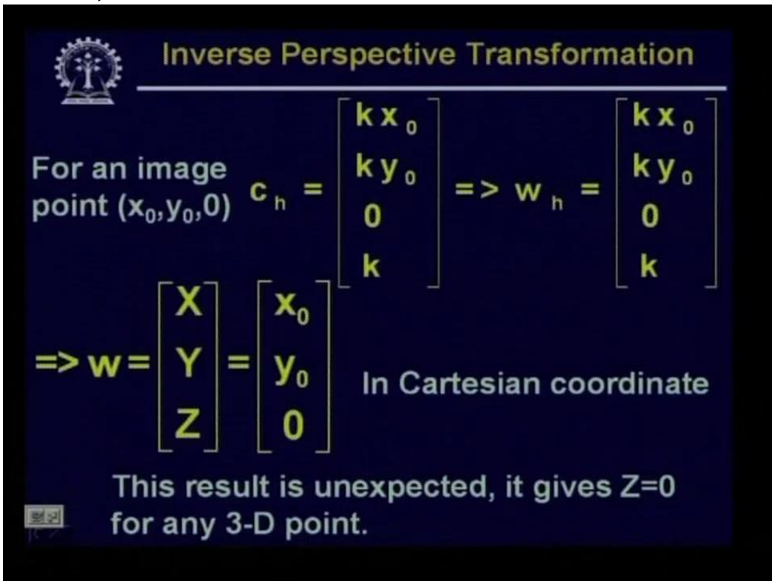

Now from this homogenous coordinate system, if I convert this to the Cartesian coordinate form then the Cartesian coordinate corresponding to this homogenous coordinate is obtained as w equal to capital X capital Y capital Z which is nothing but x naught y naught 0 so you find that in this particular case the 3D world coordinate is coming as x naught y naught 0 which is the same point from where we have started, that is the image point from where we had started.

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Moreover for all the 3D coordinate points, the z component always comes as 0.

Obviously this solution is not acceptable because for every coordinate or for every point in the three-dimensional world, the z coordinate cannot be 0. So what is the problem here? If you remember the figure of

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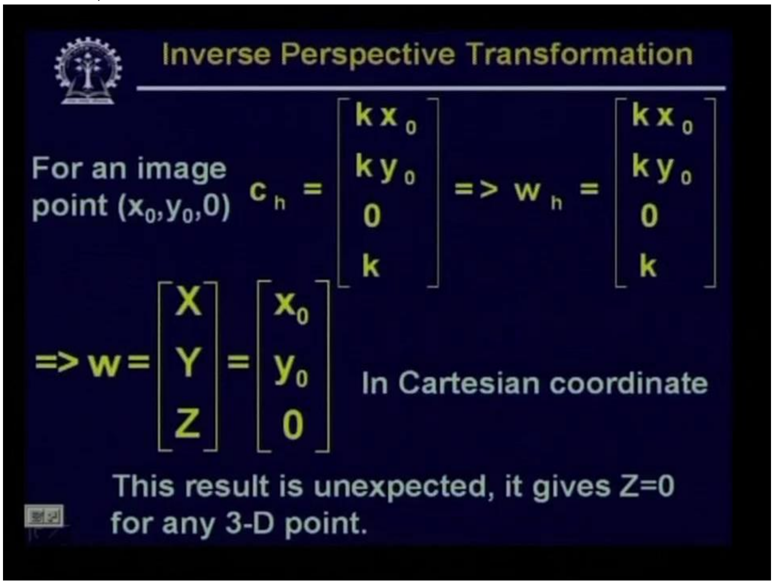

imaging system that we have used, let me just redraw this particular figure we had an imaging plane x y plane like this

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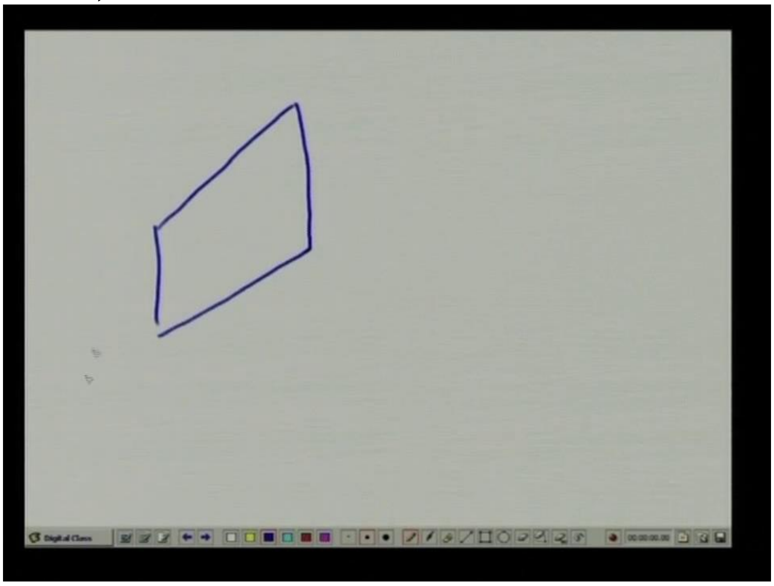

on which the camera coordinate system and the image coordinate system, camera coordinate system and the 3D world 3D coordinate system they are perfectly aligned. So we had this x same as capital X, we had this y same as capital Y, we had this z same as capital Z and this is the origin of both the coordinate systems and we had somewhere the optical center of the lens.

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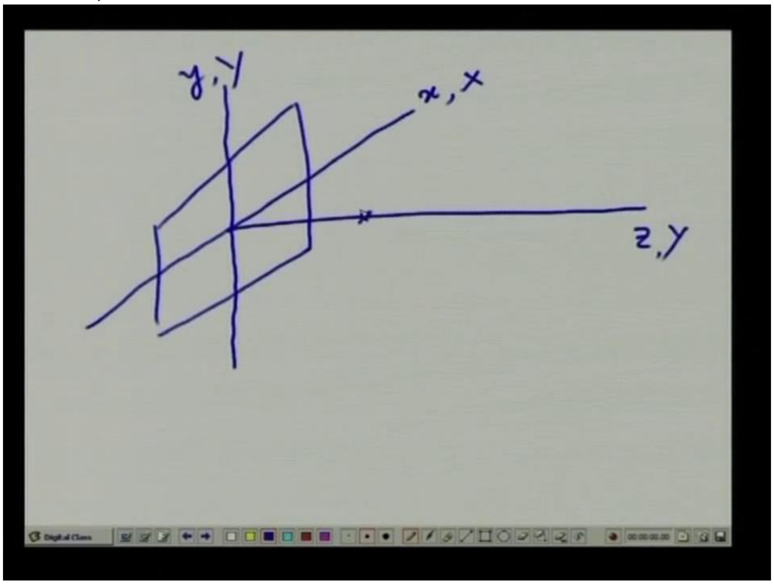

Now if I take some point here, some image point here and if I draw a line passing through this image point and the camera optical center and the world point w comes somewhere at this location. So we have seen in the previous figures that this point, if I call this point as c, this point c is the image point corresponding to this 3D world point w whose coordinate is given by capital X capital Y capital Z and this c in our case has a coordinate of x naught y naught and 0. And when we have

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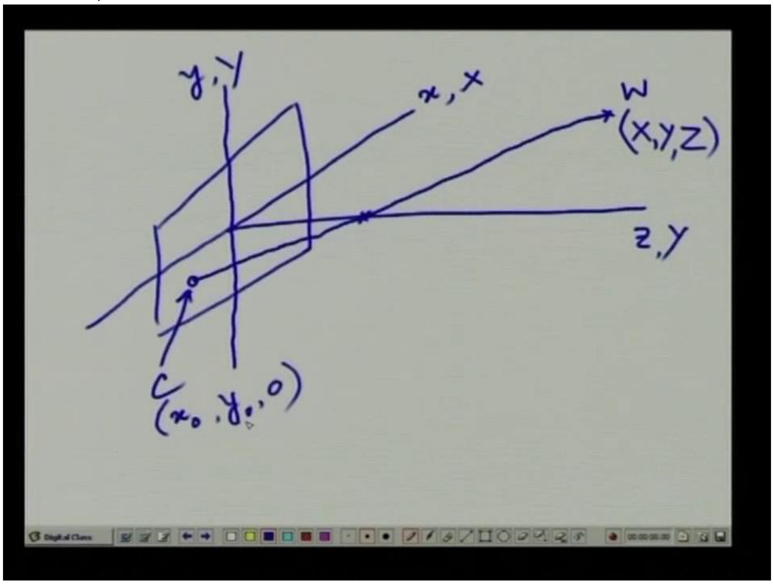

tried to map it back, this point c to the 3D world coordinate system what we have got is for every point w the value of z cannot be 0.

Now the problem comes here is because of the fact that if I analyze this particular mapping that is mapping of point w in the

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3D world to point c in the image plane this mapping is not a one to one mapping. Rather it is a many to one mapping. Say for example if I take any point

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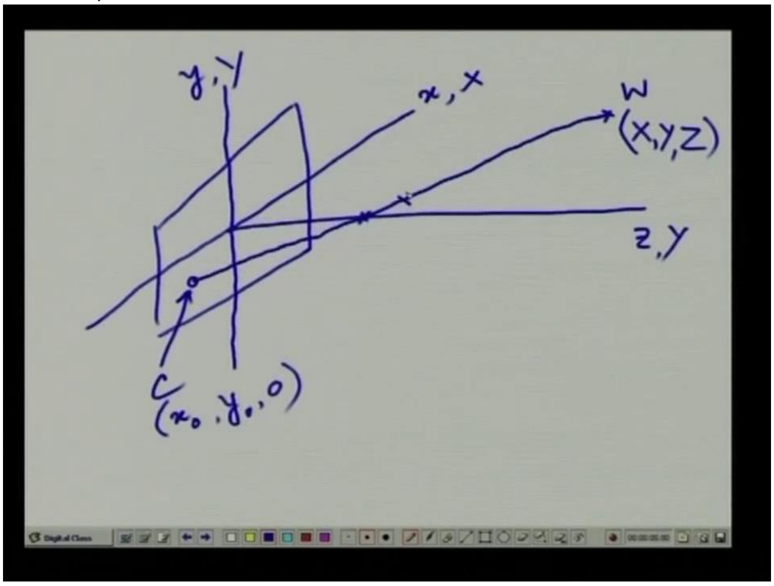

on this particular straight line passing through the point c and the point 0 0 lambda which is nothing but the optical center of the camera lens then all these points on this line will be mapped to the same point c in the image plane so

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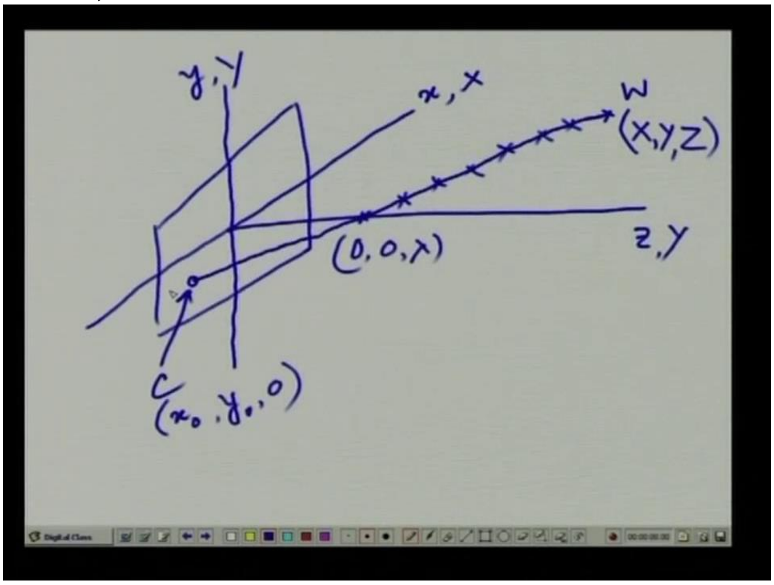

naturally this being a many to one mapping when I do the inverse transformation using the inverse perspective transformation matrix from image point c to the corresponding 3D world the solution

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that I get cannot be acceptable solution. So we have to have to have something more in this formulation and let us see what is that we can add

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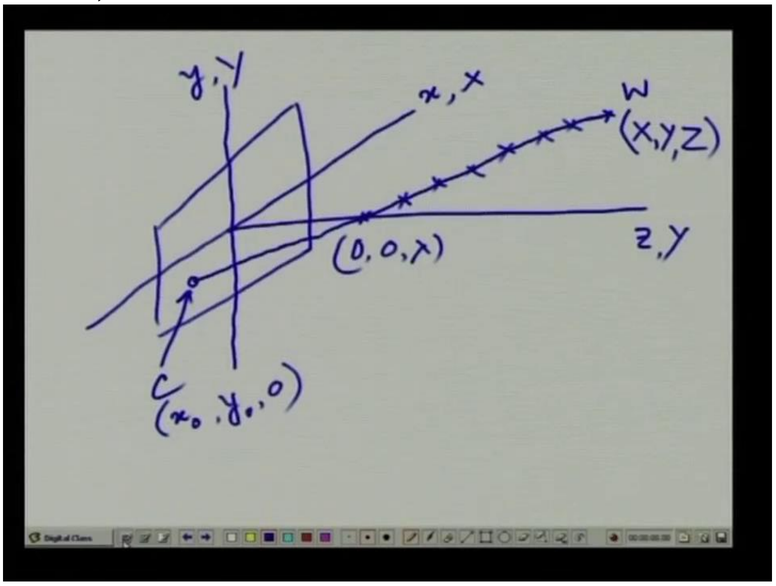

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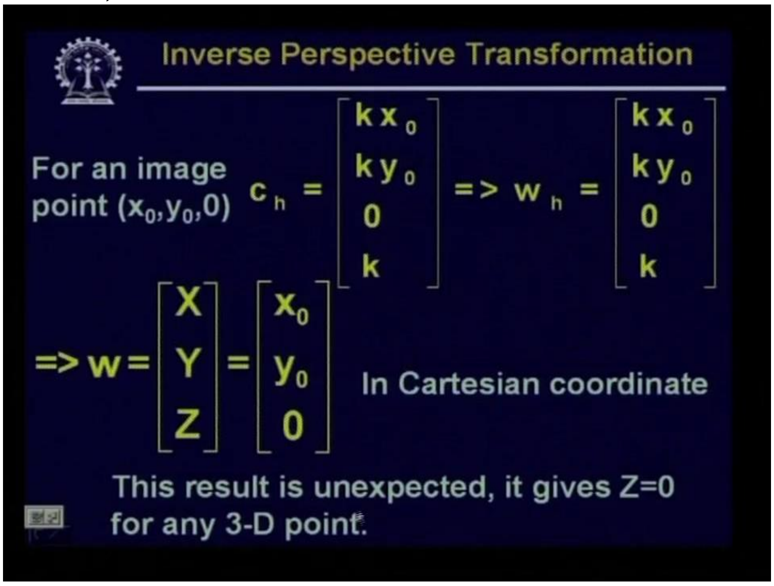

Now here

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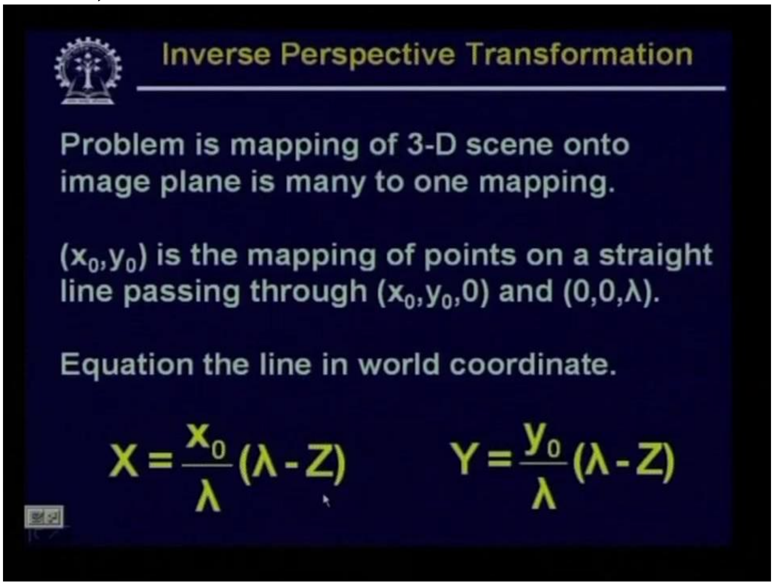

if I try to find out equation of the straight line which passes through the point x naught y naught

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that is the image point and the point 0 0 lambda that is the optical center of the camera lens the equation of the straight line will come of this form that is capital X equal to x naught by

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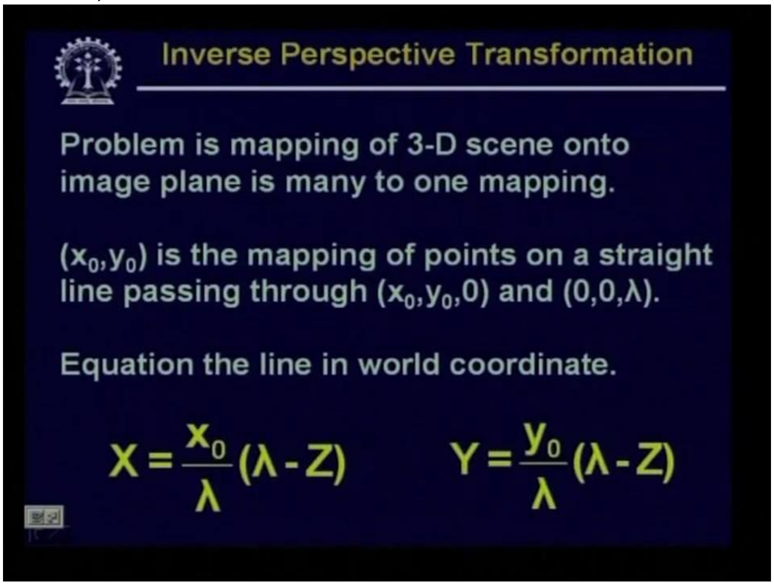

lambda into lambda minus capital Z and y equal to, capital Y equal to y naught by lambda into lambda minus capital Z. So this is the equation of the straight line so that

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every point in this straight line is mapped to the same point x naught y naught in the image plane.

So the inverse perspective transformation as we have said that it cannot give you a unique point in the 3D world because the mapping the perspective transformation was not an one to one mapping. So by using the inverse perspective transformation even if we can't get exactly the 3D point but at least the inverse transformation matrix should be able to tell me the points belonging to which particular line maps to this point x naught y naught in the image plane. So let us see if we can have this information at least.

So for doing this

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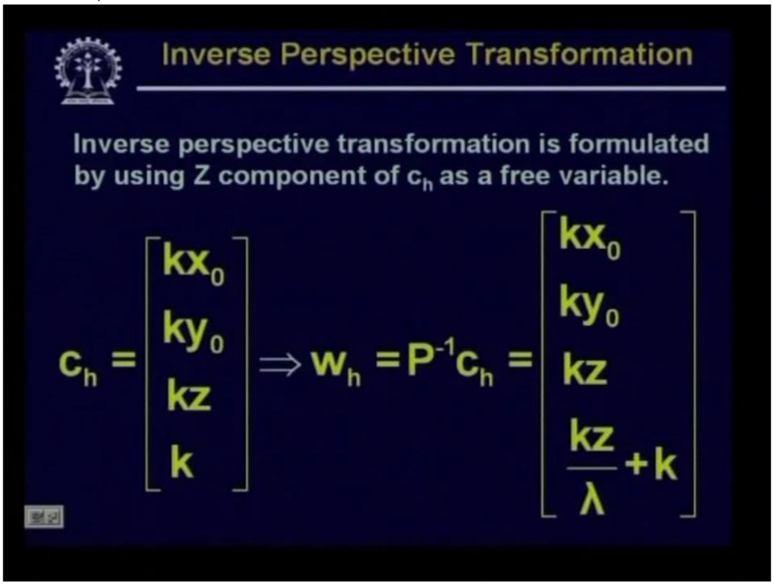

in earlier case when we have converted the image point x naught y naught to the homogenous coordinate then we had taken k x naught k y naught 0 and k. Now here the z coordinate what we will do is instead of assuming the z coordinate to be zero we will assume the z coordinate to be a free variable. So in our homogenous coordinate we will assume the homogenous coordinate to be k x naught k y naught k z and k. Now this point when it is inverse transformed using the inverse transformation matrix then what we get is the world coordinate the world point in homogenous coordinate system as w h equal to p inverse c h and in this particular case you find that this w h is obtained as k x naught k y naught k z k z plus lambda plus k so this w h we have got in the homogenous coordinate system.

Now what we have to do is this homogenous coordinate, we have to convert to the Cartesian coordinate system and as we have said earlier that for this conversion we have to divide all the components with the last components in this case k x naught k y naught k z all of them will be divided by k z by lambda plus k. So after doing this division operation

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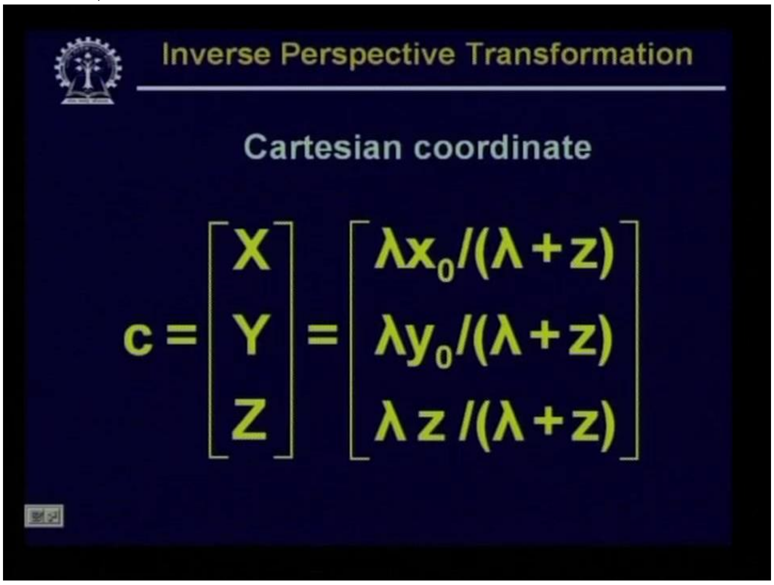

what I get in the Cartesian coordinate system is w equal to sorry, it is not c, it should be w, so w equal to x y z which is equal to so this point should be w, so what we get is w equal to x y z which is equal to lambda x naught divided by lambda plus z, lambda y naught divided by lambda plus z and lambda z divided by lambda plus z. So on the right hand side all the z's are the lower case letters which is the free variable that we had assumed that we had used for the image

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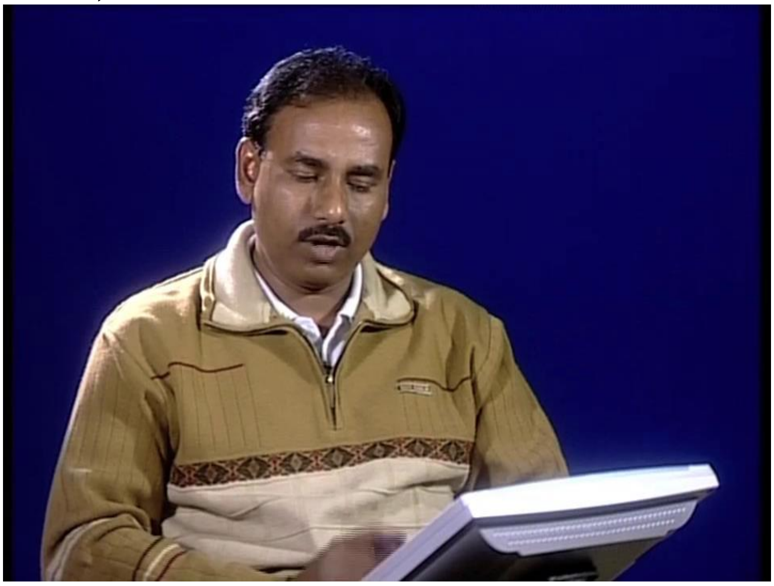

coordinate. And for the matrix, the column matrix on the left hand side, all x y z are in the upper case letters which indicates that these x y z are the 3D coordinates.

So now what we do is

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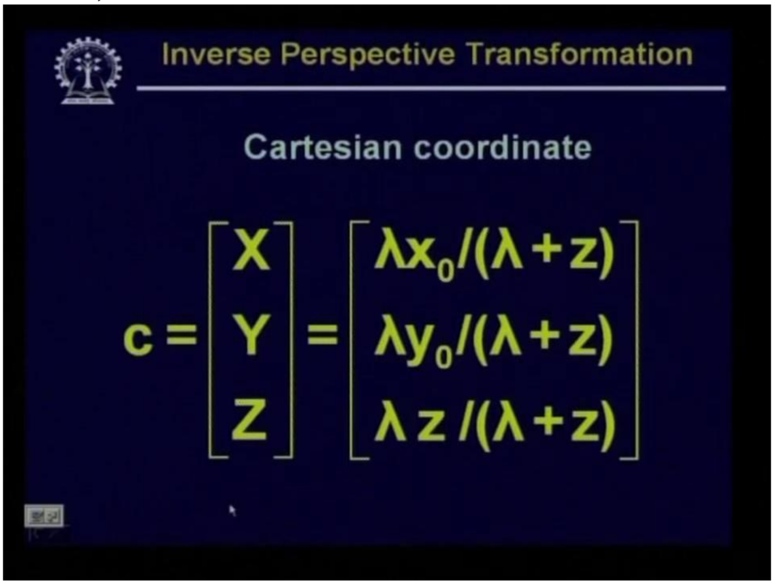

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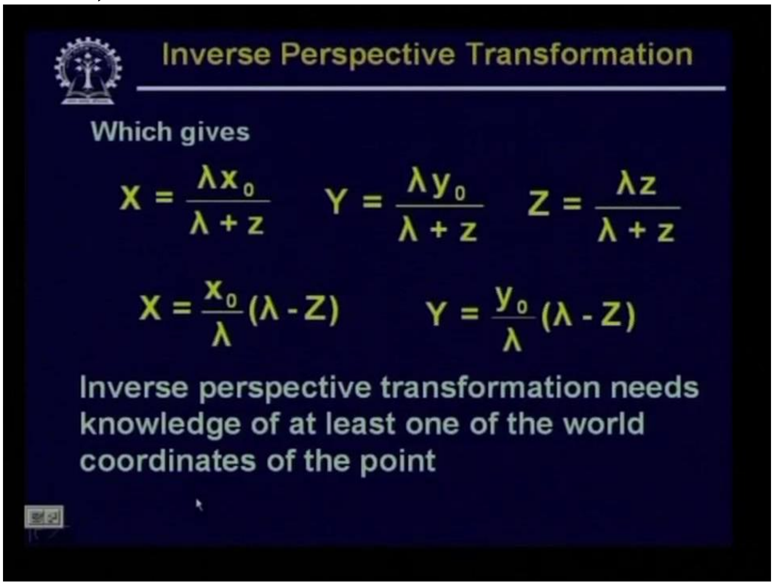

we try to solve the values, solve for the values of capital X and capital Y. So just from this previous matrix you find that capital X is given by lambda x naught divided by lambda plus lower case z, capital Y is given by lambda y naught divided by lambda plus lower case z and capital Z is equal to lambda lower case z divided by lambda plus lower case z. So from these three equations I can obtain capital X equal to x naught by lambda into lambda minus z and capital Y equal to y naught by lambda into lambda minus z.

So if you recall the equation of the straight line that passes through x naught y naught and 0 0 lambda you find that the equation of the straight line was exactly this that is capital X equal to x naught by lambda into lambda minus z, capital Z and capital Y equal to y naught by lambda into lambda minus capital Z. So using this inverse perspective transformation, we have not been able to identify the 3D world point which is of course not possible but we have been able to identify

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the equation of the straight line so the points on this straight line maps to the image point x naught y naught in the image plane And now if I want to exactly find out a particular 3D point to which this image point x naught y naught corresponds then I need some more information. Say for example I at least need to know what is the z coordinate value of the particular 3D point w and once we know this then using the perspective transformation along with this information of the z coordinate value we can exactly identify the point w which map to point x naught y naught in the image plane. Thank you.