Digital Image Processing
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Module 03 Lecture Number 11
Image Formation - 1

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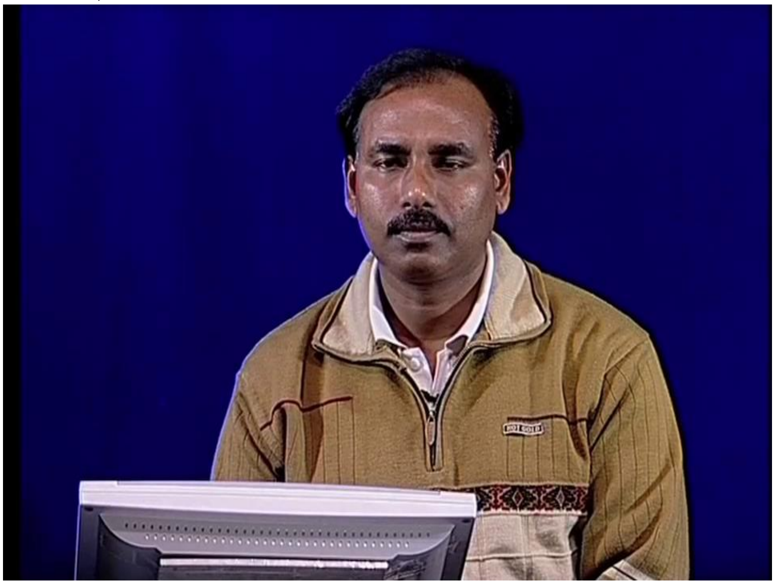

Hello, welcome to the video lecture series on Digital Image Processing. Now we will see another form of transformation which is called a perspective transformation. Now this perspective transformation is very, very important to understand how a point in three-dimension, in the 3D world is imaged by a camera. So this perspective transformation is also known as

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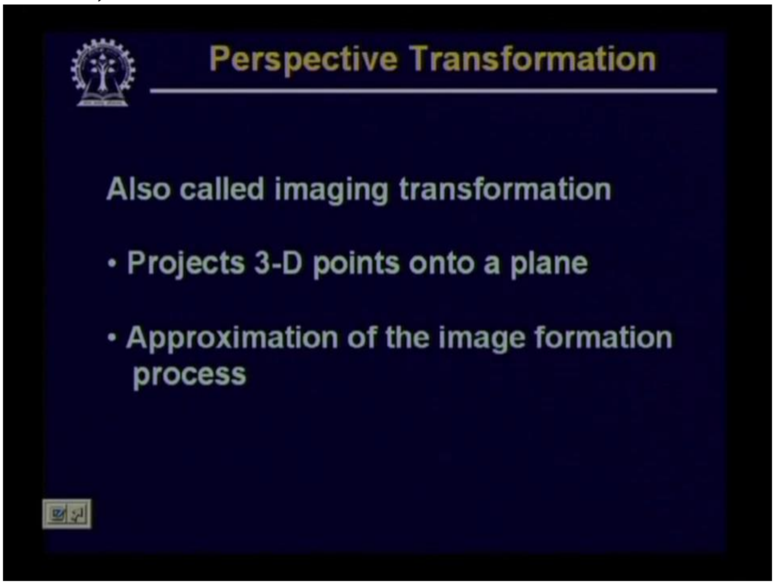

imaging transformation And the purpose of this imaging transformation is to project a 3D point, a 3D world point into an image plane. And this gives an approximation to the image formation process which is actually followed

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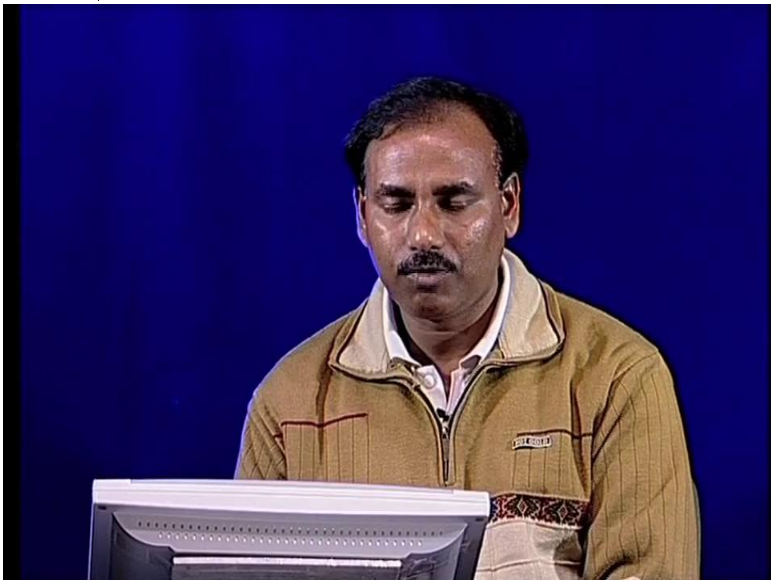

followed by a camera

Now let us see what is this perspective transformation.

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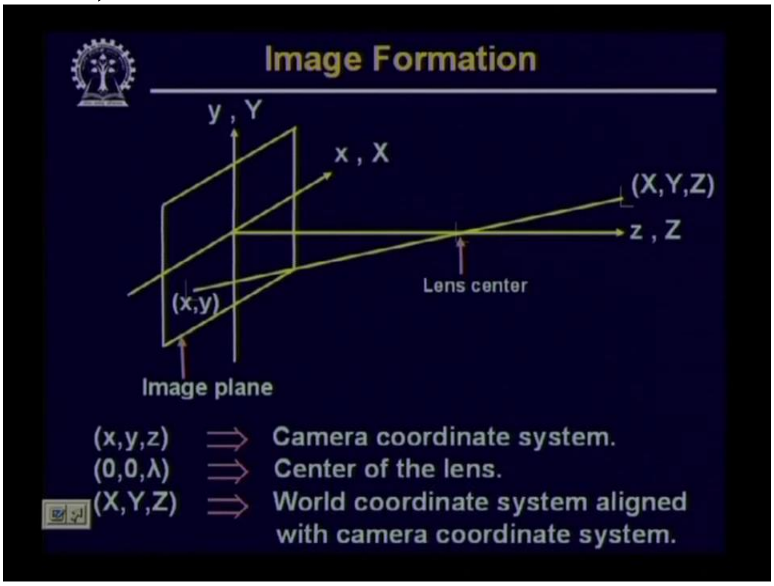

Here we have shown a figure which is an approximation of the image formation process. Here you find that we have two coordinate systems which are superimposed one over the other. One is the 3D world coordinate system represented by capital X, capital Y, capital Z so this is the 3D world coordinate system capital X, capital Y and capital Z and I also have the camera coordinate system which is given by lower case x, lower case y and lower case z.

Now here we have assumed that this camera coordinate system and the world coordinate system, they are perfectly aligned, that is, X axis of the 3D world coordinate system coincides with the x axis

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of the camera coordinate system, Y axis of the world coordinate system coincides with the y axis of the camera coordinate system; similarly the Z axis of the world coordinate system coincides with the z axis of the camera coordinate system. They have, both these coordinate systems have the same origin.

Now if I have a point x, y, z

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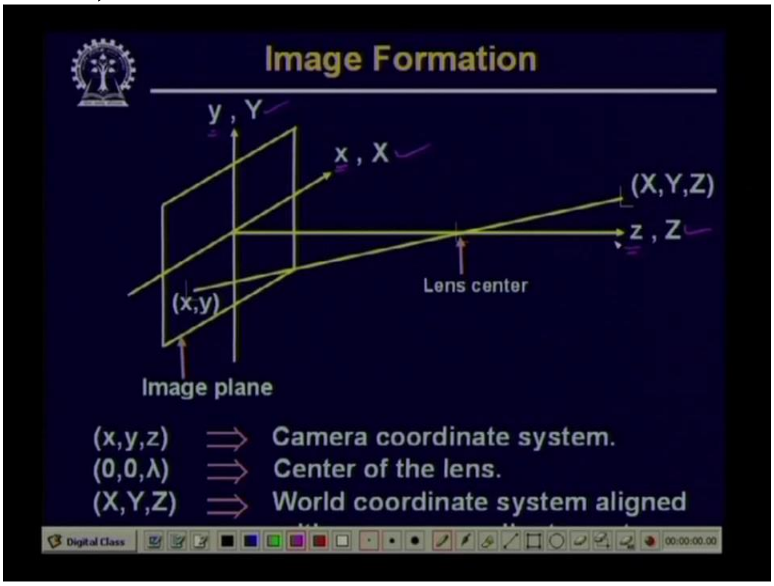

in 3D so this is the point x, y, z in three-dimension and I assume that the center of the lens is at the location 0, 0, lambda. So obviously the lambda which is the z coordinate of the lens center, this is also nothing but the focal length of the camera and this x, y, z this particular 3D point, we assume that it is mapped to the camera coordinate given by lower case x and lower case y. Now our purpose is that if I know this 3D coordinate system capital X, capital Y, capital Z and I know the value of lambda that is the focal length of the camera, whether it is possible to find out the coordinate, the image coordinate corresponding to this 3D world coordinate x y z.

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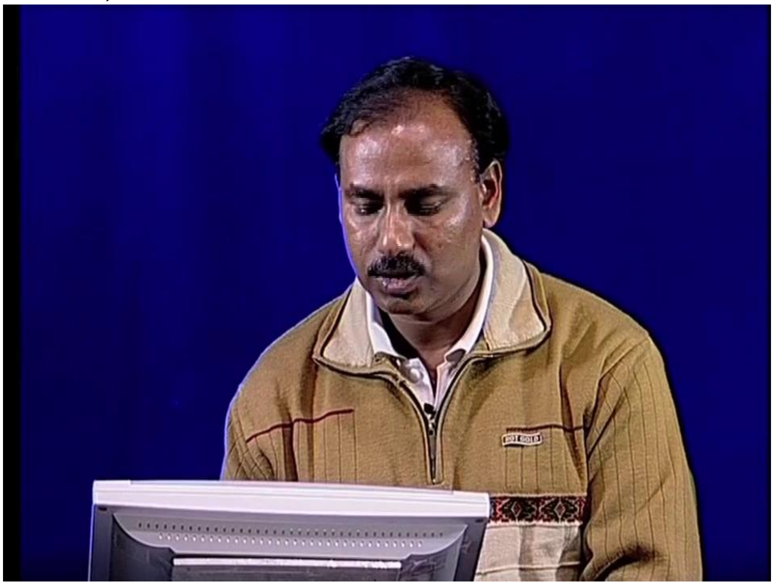

So to this, we apply the concept of

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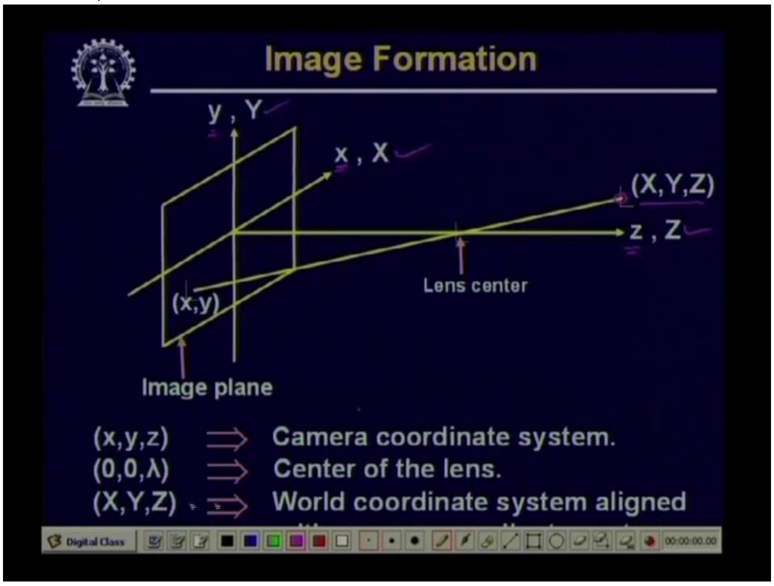

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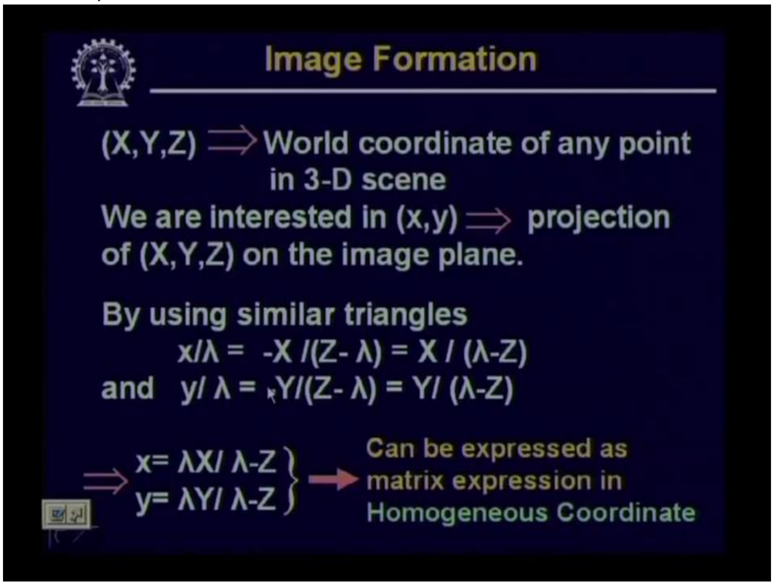

similar triangles Here what we do is, by using the similar triangles we can find out an expression that lower case x by lambda is equal to minus capital X by capital Z minus lambda which is nothing but capital X by lambda minus z and y by lambda is in the same manner given by capital Y by lambda minus capital Z. So from this I can find out that the image coordinates of the 3D world coordinate capital X, capital Y, capital Z is given by x coordinate, image coordinate x is given by lambda x by lambda minus capital Z, similarly the image coordinate y is given by lambda capital Y divided by lambda minus capital Z.

Now these expressions can also be represented in the form of a matrix and here we will find that if I go for homogenous coordinate system then this matrix expression is even simpler.

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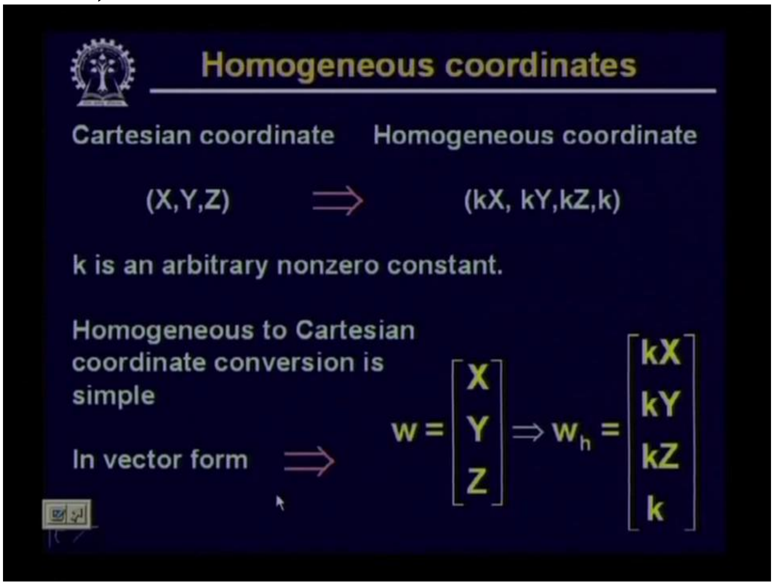

So let us see what is this homogenous coordinate system? Homogenous coordinate system is if I have the Cartesian coordinate capital X, capital Y, capital Z then we have said that in unified coordinate system we just append a value 1 as an additional component. Homogenous coordinate system is instead of simply adding 1, we add an arbitrary non-zero constant say k and multiply all the coordinates x, y and z by the same value k. So given the Cartesian coordinate capital X, capital Y, capital Z I can convert this to homogenous

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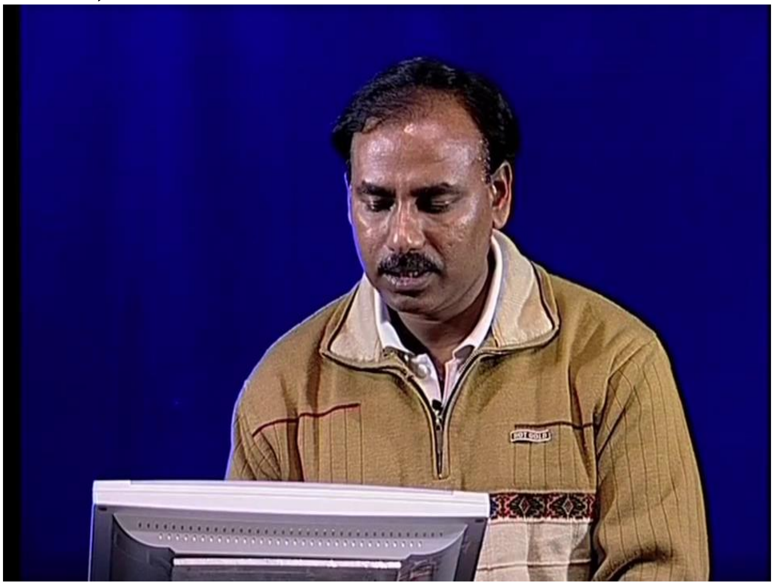

coordinate by k times capital X, k times capital Y and k times capital Z.

The inverse process is also very simple. That if I have a homogenous coordinate then what I have to do is I have to divide all the components of this homogenous coordinate by the fourth term. In this case the fourth term is k and all other terms were k x, k y and k z. So if I divide all these three

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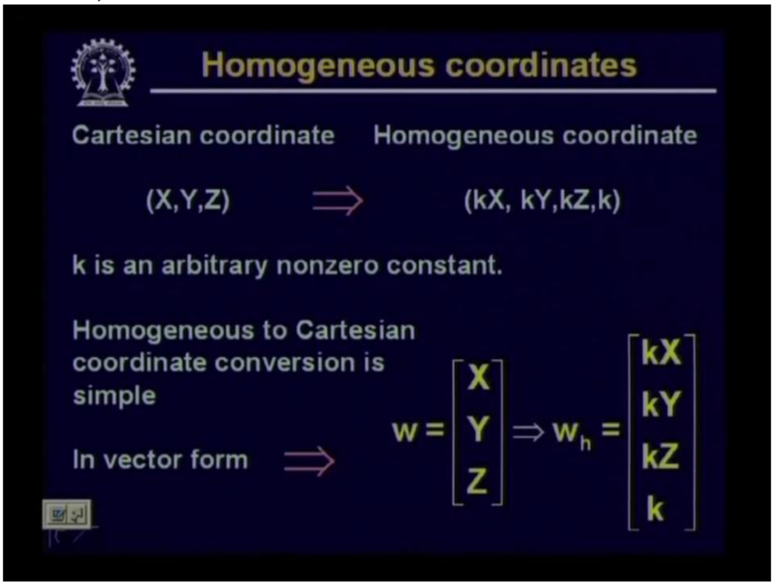

terms by the fourth component k I get

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the Cartesian coordinate x, y, z So I can convert

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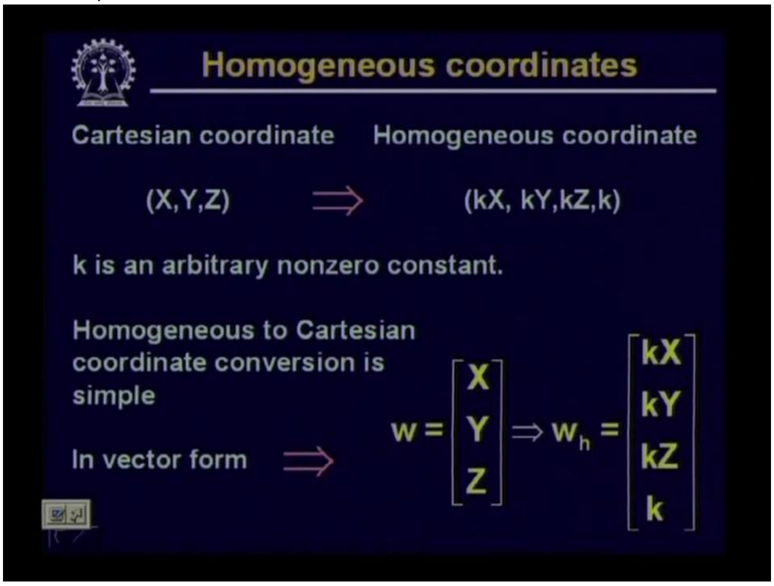

the 3D point, the coordinates from the Cartesian coordinate system to the

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homogenous coordinate system and I can also very easily convert from homogenous coordinate, from homogenous coordinate system to Cartesian coordinate system.

Now to understand the imaging process

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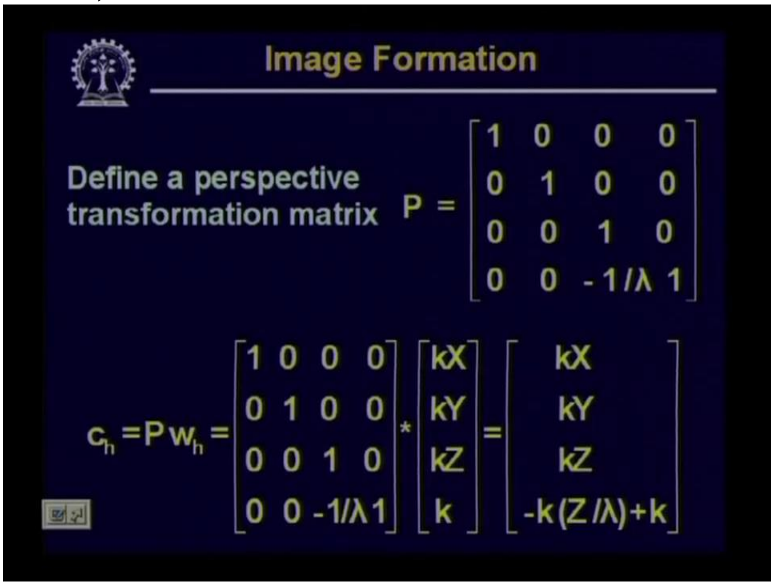

let us define a perspective transformation which is given by p equal to 1 0 0 0, 0 1 0 0 then 0 0 1 0 then 0 0 minus 1 upon lambda you remember this lambda is the focal length of the camera and then 1. And we translate our world coordinate W to the homogenous coordinate so it becomes k x, k y, k z and k. Now I translate, if I transform this homogenous world coordinate by this prospective transformation matrix p then I get the homogenous camera coordinate c h which is given by k x, k y, k z then minus k into z minus lambda plus k. So this is the homogenous camera coordinate.

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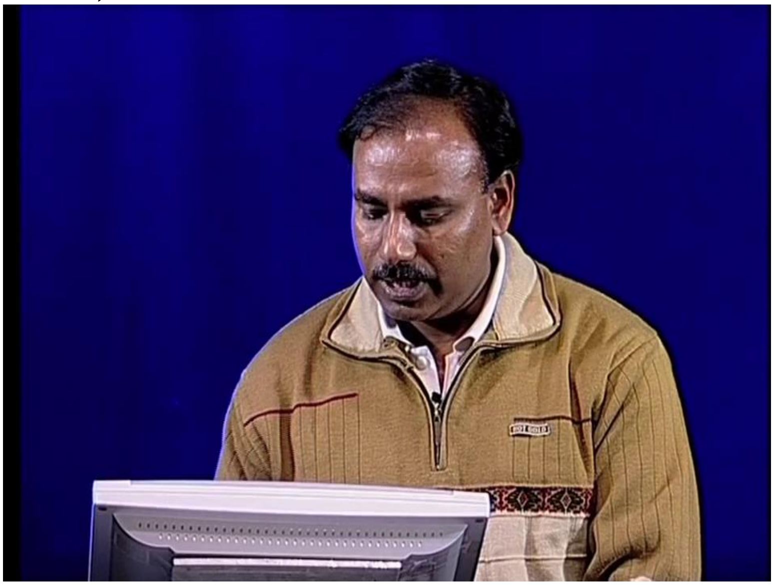

Now if I just convert this homogenous coordinate

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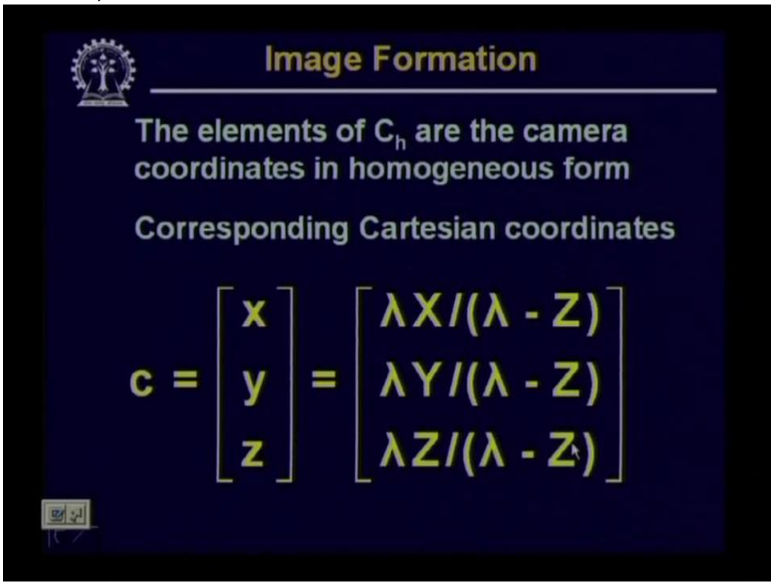

camera coordinate to the Cartesian camera coordinate I find that the Cartesian camera coordinate is given by c equal to small x, small y, small z which is nothing but lambda x divided by lambda minus z, lambda y divided by lambda minus z and lambda z divided by lambda minus z. So, on the right hand side this x, y and z they are all in upper case indicating those are the coordinates of the world point.

Now if I compare these expressions

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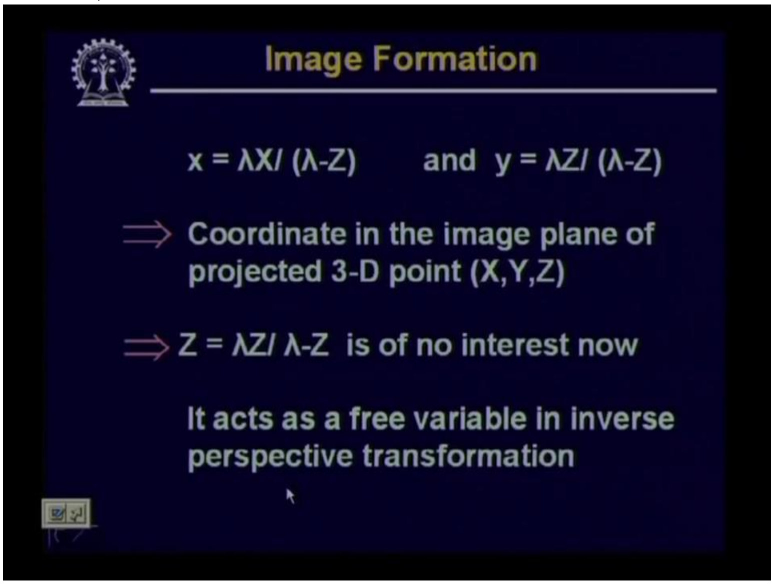

with the camera coordinates that we have obtained with respect to our previous diagram you find that here we get lower case x is nothing but lambda capital X divided by lambda minus Z, similarly lower case y is also nothing but lambda capital Z by lambda minus Z. So using our previous diagram we have also seen that these lower case x and lower case y, they are the image points on the image plane of the world coordinate capital X, capital Y and capital Z. So this shows clearly that using the perspective transformation that we have defined as the matrix

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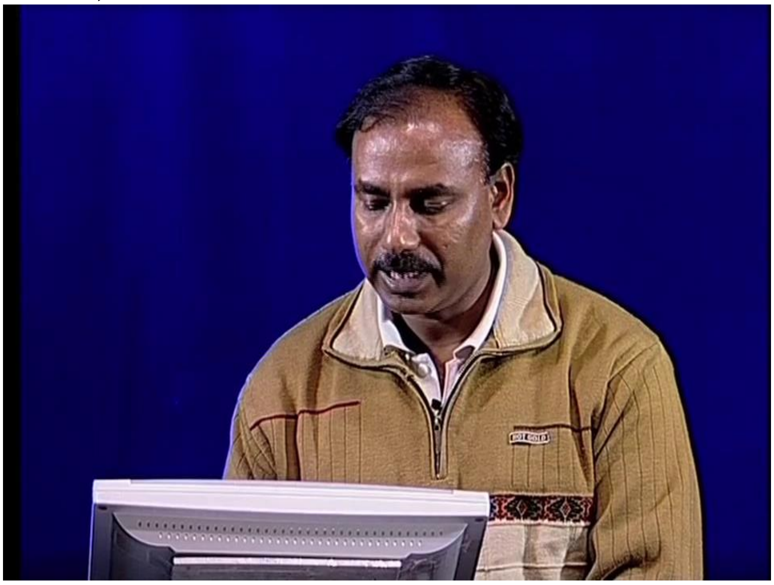

p, I can find out the image coordinates of world coordinate point

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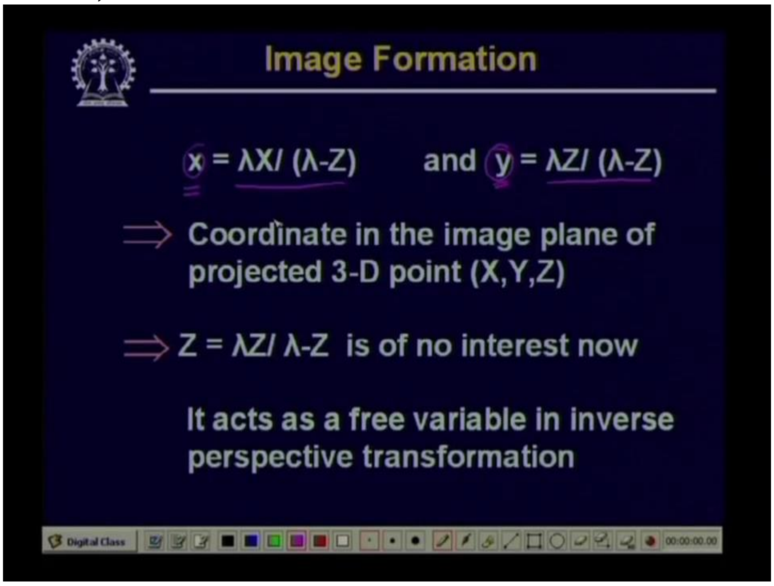

capital X, capital Y, capital Z following this particular transformation p.

Now here in this particular case, the third component that we have obtained

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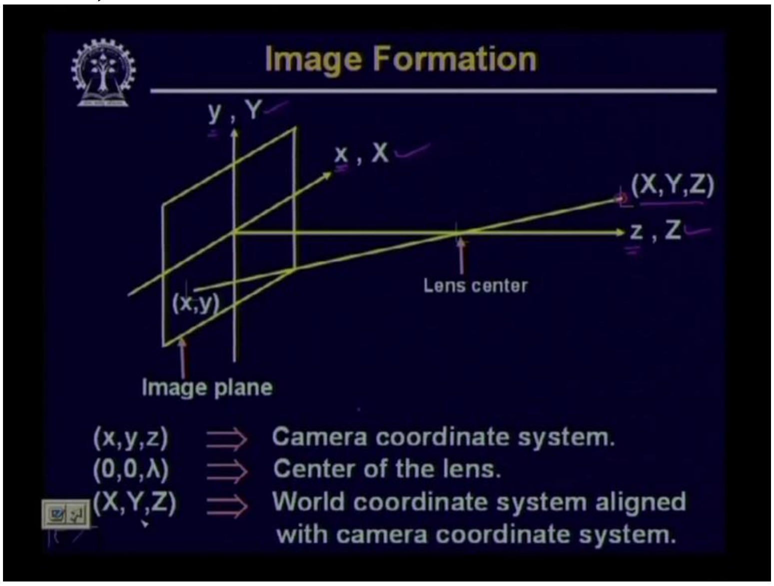

that is

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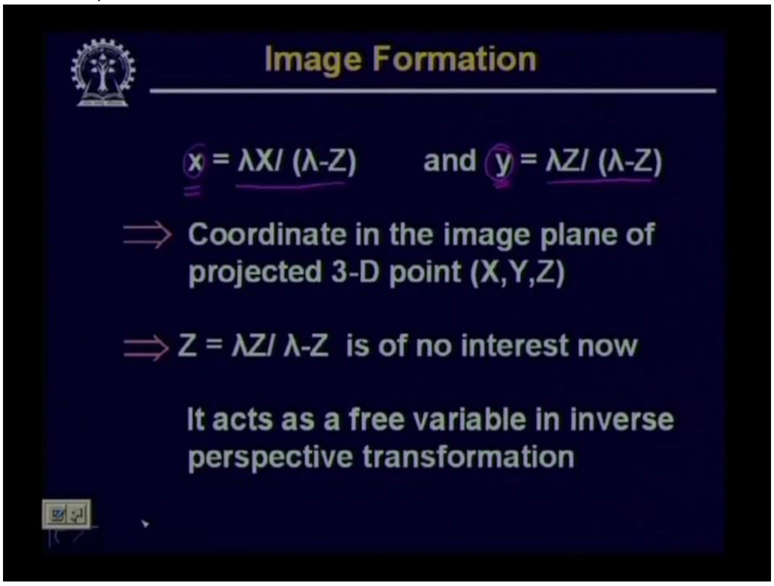

the value of z which is not of importance in our case because in the camera coordinate the value of z is always equal to 0 because we are assuming that

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the imaging plane is the x y plane of the world coordinate system as well as the camera coordinate system. So we will stop our discussion here today and in the next class we will see that as we have seen with the perspective transformation we can transform a world coordinate, we can project a world point, a 3D world point on to a imaging plane, similarly using the inverse perspective transformation, whether it is possible that given a point in a image plane, whether we can find out the corresponding 3D point in the 3D world coordinate system. Thank you