Discrete Time Signal Processing

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Lecture – 02
Discrete Time Signals and Systems

In the previous lecture, we are considering shifting, shifting to the right shifting to the left. If the sequence is actually, a real time sequence, that is obtained by sampling, an analog signal and therefore, this is the 0-th sampling point, this is the first sampling point, second sampling point, then this shifting actually, amounts to delaying of the signal, because, delaying or advancing, because suppose, this is your analog signal, you obtained a, you obtained b, you obtained c, dot, dot, dot, dot, a prime, you obtained them.

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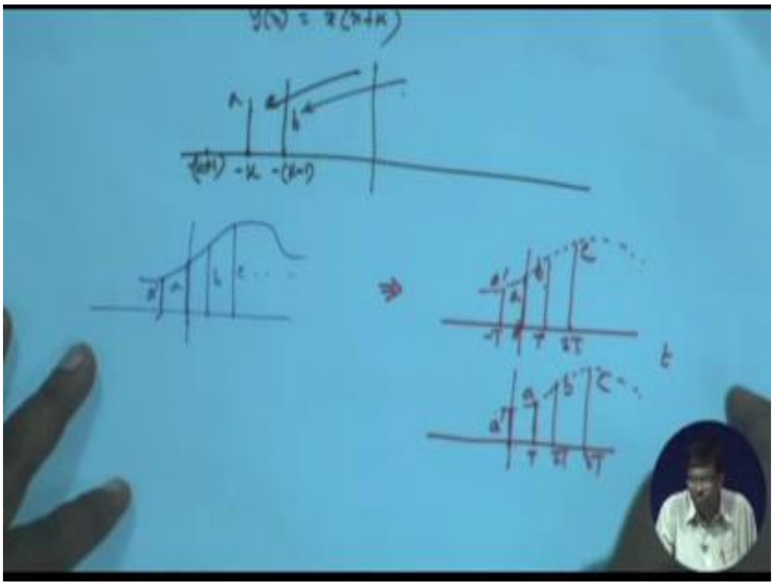

So, you got a sequence, a prime, a, b, c like that, and, if it is actually time, if I do not write as an index. So, this is at 0, this is at T, this is at 2T this is at minus T, like that.

Now, if I shift it to the right by1. So, a will go here, a will go here, a prime will come here, b will go here, c will go here. So, this is T, a will come to T, b will come to 2T, c will come. So, this sequence actually, if it is a plotted as a function of time, this is delayed. Because c was coming at 2T now, it is coming at 3T, b was coming at T, it is now coming at 2T, a was coming at 0, now it is coming at T, a prime was coming at minus, it is coming at a prime at 0. So, it is shifted, it is delayed by an amount capital T.

So, shifting by, shifting the sequence by1 point, means this time signal, if you make it as a function of time, this is delaying by1 sampling point, and therefore, if you shift that original sequence by k indices, like yn minus k, it will amount to delaying it by k times t, that is from 0, a will move to k t, from capital T, b will move to k plus 1T, from 2T, c will move to k plus 2T.

So, k times the sampling period, that many clock it will be delayed. So, if it is a pure time signal, shifting in sequence, actually means, in time domain, the sequence is, or the signal is delayed. Or if it is shifted to the left; that means, I am moving it back so, it is advanced in time. So, either delayed in time, or advanced within time. But that is when; the sequence is actually obtained from a, temporal sequence that is a function of time. Sequence actually this is, this axis corresponds to time. The sampling points are nothing, but, this, this index points are nothing, but sampling points. So, 0, 1 means 1 into T, 2 means 2 into T, 3 means 3 into T you can plot it as a function of T. This is obtained possibly, by sampling this, like this, this was the envelope or sampling this.

Now, the envelope is shifting. So, analog function is delayed, by capital T. And then if I sample, this a prime does not occur here, it occurs here a does not occur here, a occurs here. The analog signal is delayed by capital T times. So, actually, it amounts to delaying time, for the analog signal, or for that sampled train, as a function of time, when plotted as function of time, which gets delayed by capital T for every shift, all right? Having said this, there are some important sequences, basic sequences, which you should know one is called delta n.

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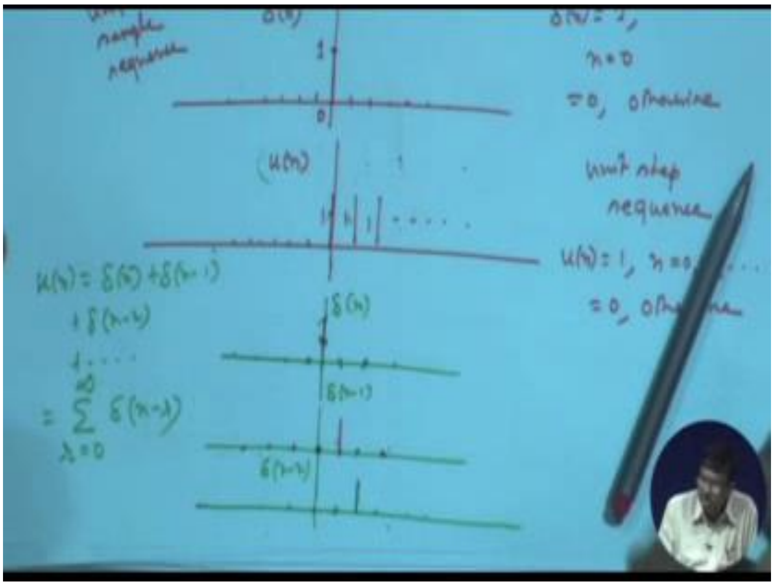

Now, this delta has nothing to do with (Refer Time: 04:31) delta function, in analog signal processing. We borrow the same symbol, by it is definition it is purely on it is own here. It means it is a sequence which has exact value equal to 1; it has value equal to 1 at n equal to 0. Otherwise it has all 0 values, everywhere. So, delta n is equal to 1, for n equal to 0, and equal to 0 otherwise, all right? There is another sequence u n, it means it is a unit step sequence; this is called unit sample sequence, not unit impulse. We call it unit sample sequence, but I will tell you one thing, because we have been using this delta function, or you know impulse, unit impulse function, in analog.

So, often with this symbol delta sometimes, we also call it unit impulse sequence by mistake, but that mistake is well accepted. There is nothing impulsive about it, where, if we say unit impulse sequence, it basically needs unit sample sequence it is value is 1. It has got nothing to which actual impulse function that is delta function. If I even call it impulse sequence, unit impulse sequence, basically, that is a mistake, that, though, acceptable mistake, and that actually would mean this. This is not that original; say impulse function that is derived delta function. This is a word of caution. u n is called unit step sequence, 1, all values 1 on this side, 0 on this side. So, u n is equal to 1, for n, 0, 1, 2, dot, dot, dot, and 0, otherwise.

Then, if I ask you, can I write u n in terms of delta n? And can I write delta n in terms of u n? Now you see, u n, you can give as though, that is 1 impulse, and 0, 0, 0, everywhere, delta n. Then, another one, if I take delta n minus 1, it will be shifted, so, this fellow will come here, and then 0s, 0s, 0s as before. If I add these 2, I will get 1 coming here, 1 plus 0. So, 1, 1 plus 0, 1, and all other points 0 plus 0 0, 0 plus 0 0, 0 plus 0 0. So, I will get 1 and 1, this part will come, other parts will be 0, but if I have another say delta 1, delta say minus 2, n minus 2. So, delta n means this 1 will be shifted to location 2, and 0 here, because this 0 will move here, this 0 will move here, this 0 will move here. So, all other points are 0s. if I add the 3, 1 plus 0 plus 0, 1, 0 plus 1 plus 0, 1, 0 plus 0 plus 1, 1. All other points will be 0s, because 0 plus 0 plus 0, 0 plus 0 plus 0. But this way if I go on adding, delta n, plus delta n minus 1, plus delta n minus 2, plus dot, dot, dot, dot, then I get back u n.

So, u n, easily you can see, delta n, then delta n minus 1, then delta n minus 2, dot, dot, dot, dot. So, compactly you can write, r, 0 to infinity, delta n minus r. For equal to 0 means delta n, or equal to 1, minus delta n minus 1, you will get minus 2 dot, dot, dot, and dot.

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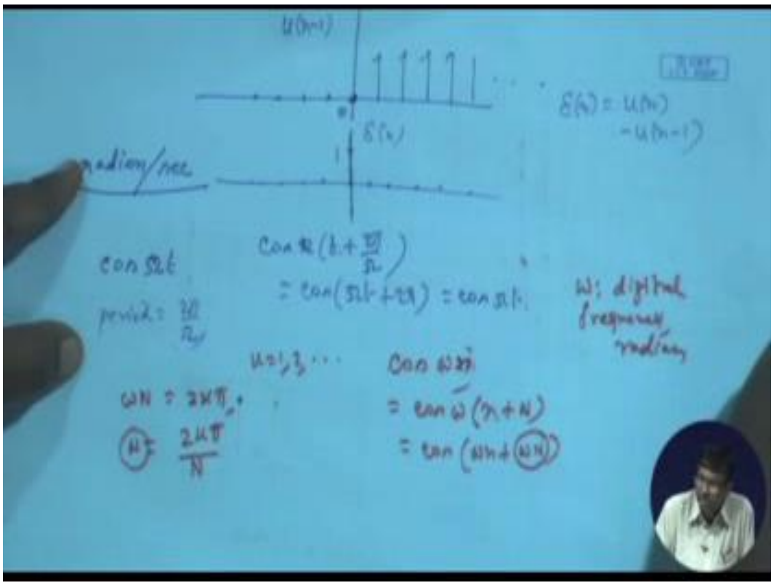

So, this is how you write u n, in terms of delta n. And delta n can be written in terms of u n in this way. This is our u n. And what is u n minus 1? This is nothing, but this sequence to be shifted to the right by1. So, earlier this 1 was occurring at 0, this will occur at 1, next 1 was occurring at 1, this will occur at 2, this 0 was occurring at minus 1, this will move here. So, this will be 0, and then all others are 0s and the 1s will continue. Then if I subtract u n minus 1, from u n, what will I get? If I subtract u n minus 1, from u n, these 2 will cancel, these 2 will cancel, and these 2 will cancel. They will all cancel, only this 1 minus 0 that will remain. This side is 0, minus 0, 0 minus, those are all 0s.

So, what will happen is, you will get a 1 here, 1, and then 0, 0, 0, 0, 0, 0, 0, 0, 0, but this is delta n; That means, delta n is u n. One more interesting thing in analog, so you take cosine, capital Omega T. Remember, in analog, frequency, we denote by, frequency as radian per second. Radian per second, we denote by capital Omega, radian per second, into time second. So, second, second cancel, because this whole argument has to be an angle - radian per second, into second, so that it becomes radian, cos of radian. So, this is analog frequency. But for any capital Omega, we know this will be periodic. Over a period, 2 pi by capital Omega. This is because, if, instead of trying T, you go further up, by this amount 2 pi by Omega, this Omega, Omega, cancels. So, you get basically cosine, Omega T plus 2 pi, Omega, Omega, cancels, which is cosine Omega T it is periodic.

For any capital Omega, this periodic over this period, for any period is this. If it is analog frequency, is capital Omega radian per second, then we know, it has to be a periodic function, we know how it looks like, sinusoidal form, and period is this. But if I construct, in an analogous manner, some small Omega n, instead of capital Omega, in digital case, we use small Omega, and hereby again, this has to be radian, but there is no time involved in n, it is dimensionless. Therefore, small Omega must be just radian, not radian per second, because there is no second, involved in n, n is just a dimensionless integer.

So, small Omega is called digital frequency, it is unit is radian. But you will see that this is not necessarily periodic, for any small Omega. Why? Because suppose it is periodic, for any small Omega, over a period capital N. So, whatever I observed at small n, at small n plus capital N also, you should observe the same thing, it should equal, if it is periodic; That means, cosine, this you write Omega small n, plus Omega capital N. So, this is possible only if this is, either 2 pi, or 4 pi, or 6 pi, like that. That is 2 k pi, where k can be 1, 2, dot, dot, dot, dot which means, Omega is 2 k pi, by n. So, not for any arbitrary Omega, this relation will be valid. This will be valid, that is it will be periodic, over a period capital N, for specific choice of Omega. That this is, 2 k times pi, it can be 1, 2, 3 anything divided by capital N, for those specific, discrete, omegas only, this will be periodic, but in the case of analog, this periodic for any capital Omega. This is a very big difference.

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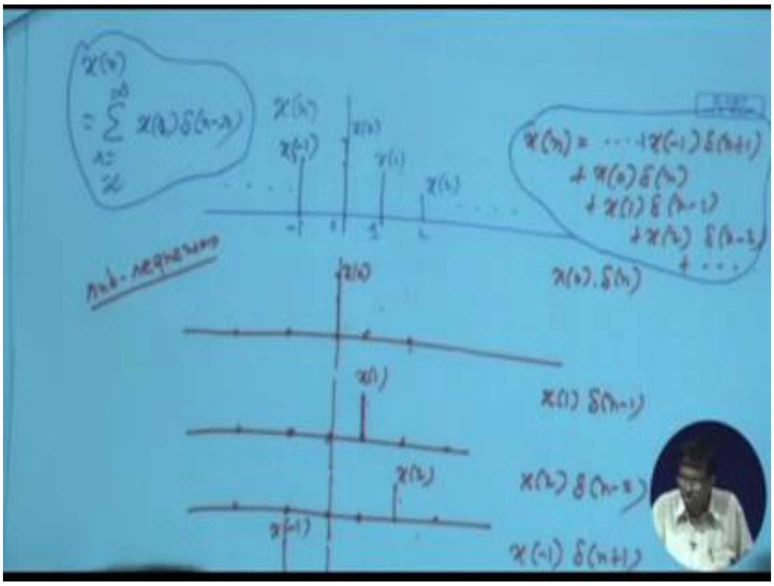

Now, I persuade to general sequence xn, I persuade a general sequence xn. Say x of n, it is x0 at 0th point, it is x1 at point number 1, x2, at point number 2, dot, dot, dot, dot. May be, it is x minus 1, at point minus 1, dot, dot, dot, dot. Now this I can write, in this way, is a summation of again various sub sequences. Say one, which is x0 here, but 0 everywhere. Then another one, which is x1 here, but 0 everywhere; then another one, x2 here, but 0 everywhere else. Another one, if I go to the minus side, x minus 1, this value at minus 1, but 0 everywhere else. Like this, if I you see, add all of them, x0, plus 0, plus 0, plus 0, plus 0, you get x0, x, 0 plus x1, plus 0, plus, 0 plus, 0, you get x1, 0 plus, 0 plus x2 plus 0 plus dot, dot, dot, dot you get x2. 0 plus, 0 plus, 0 plus x minus 1, plus 0 you get x minus 1. So, by adding them, that is super imposing them, you get back your original sequence. But what is this? This is nothing, but x0 times delta n. Why? Because you remember what was delta n? What was delta n? This is delta n, 1 here, at 0-th point, 0 everywhere.

So, what is some constant x0 times delta n? So, this 1 will be multiplied by x0. So, height will be x0, 0s will be multiplied by x0, but that value will be 0. So, it will be x0 and all 0s. Next, this will be x1 into delta n minus 1. What is delta n minus 1? This sequence will be shifted to the right by1, this 1 will move here, and this will become 0.

This is 0. If you multiply this, by x1, so this height will become x1, all other 0s will remain 0, so it will become x1, 0. Then by a same logic, it will be x2, delta n minus 2, it will be x minus 1, now this is shifted to the left, so n plus 1, if delta is n plus 1, this will be shifted to the left, 1 will move here, and 0, 0, 0. That will be delta n plus 1, and that if you multiplied by x minus 1, this height will be x minus 1, others will remain 0, you get this. So, if you add them, you get xn, which means, xn is a summation of some sub-sequences.

One sub-sequence, another sub-sequence, so various sub-sequences are superimposed on each other and you will get your xn. So, you can write like this, you know, x minus 1, dot, dot, dot, dot, x minus 1, delta n plus 1, from here. Then x0, delta n. Then x1, delta n minus 1, x2, delta n minus 2, and dot dot dot dot. This, this entire thing, if you have to write, in a compact form, like this, you start with xr, at delta n minus r, and r will be from minus infinity to infinity. Then see if r equal to 0, x of 0 delta n minus 1, you have got x of 0, sorry, x of 0, delta n minus 0, if r is 0, so, x0, delta n - x0, delta n. If r is 1, x1, delta n minus 1, x1, delta n minus 1, r is 2, x2 delta n minus 2, x2 delta n minus 2. So on and so forth. On the other hand, if r is minus 1, x minus 1, delta n plus 1, x minus 1, delta n plus 1 and so and so forth on the back. This is a very important result this we will be using. So, any sequence, it means, is a sum of various sub-sequences, alright? This is very important. Now, we will be considering processing of sequences.

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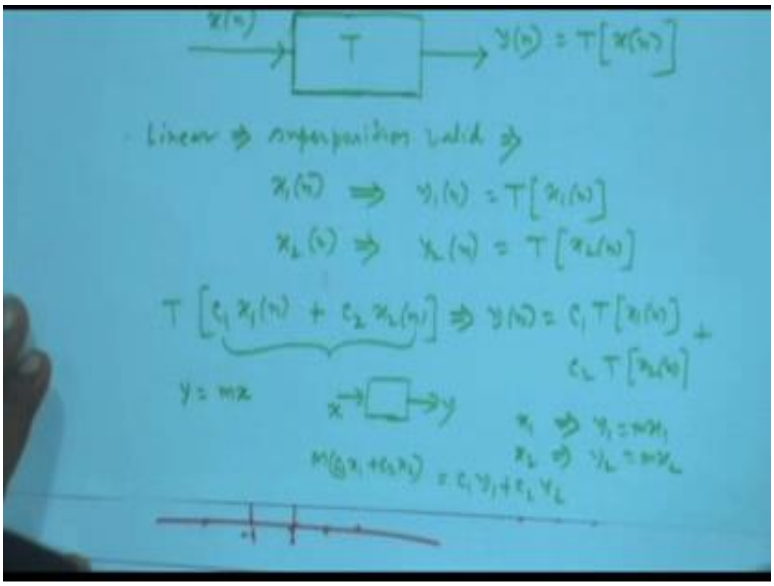

That is, there is one system, what inside is there, what there is, there inside, I do not know, I do not care, but it takes one sequence, may be xn, as a input, does something on it, it gives an output, y n, which gives you an output, y n.

Now, this system will be called linear system, if it follows superposition principle. Means, suppose I give this system you know I denote by an operator T. This is not time, this is an operator T. That is y n I write as T working on an input xn. So, system yn is T working on xn. Now is it, if it is linear, superposition will be valid, that is, suppose I give 1 input, x1n, and output is y1n, which is nothing but, T working on x1n, fine? Suppose I give another input x2n, and find out the output and I call it y2n, this is nothing, but T, x2n. In that case, now, if I make one linear combination, some constant times c1 x1n means every sample multiplied by some constant c1, as I told you, and c2 x2n that is every sample here multiplied by c 2, and these 2 sequences are added, these I give to the input.

So, T of that that is the new input, obtained by combining them - c1x1n, one sequence c2 x2n another sequence, adding this is the total sequence which I give at the input a linear combination of 2. Then output, yn, should be, if it is linear, should be, c1 should go out, T should work on this only, again c2 should go out, T should work on this only, these and these 2 components will get added, because this was added, if this is true it is linear, otherwise not. It means, total output will be, total output of the system will be, response, summation of response due to this component, and response due to this component that is this part, and this part and response due to this component means, response due to this basic component x1n, multiplied by the constant c1. This T, that is first you multiplied by c1, and then process through T, you obtain the same first process by T, and then again, multiply by the same number, that should be valid, and individual responses that is, response due to this part here, response due to this part here, they should get added, if this is true, it is linear, otherwise, now, this is called principle of superposition.

Very interestingly, an equation like this, y equal to mx, if x is an input to a system, forget about sequence, there is no sequence. x is one variable which is input to a system, output is y, and output and input is related, right? Suppose there is a system, there is no sequence, it is just a variable x output is variable y, and they are related by mx, then it is linear, because if you give x1, output is y1, say mx1. If you go to x2, output is mx2, if you keep c1 x1 plus c2 x2, output will be m times this, which you can write as c1 into mx1, but mx1 is y1, plus c2 into, mx2, mx2 is y2. So, this is satisfying superposition principle. So, y equal to mx is linear, but if I give you, if I give you y equal to, mx plus c, then it is not linear, though the plot will be laying straight line, but technically it is not linear.

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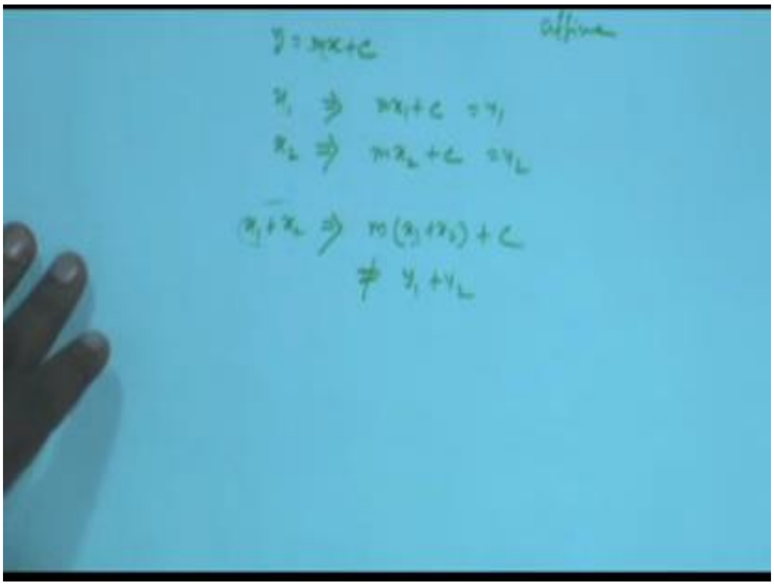

It is called actually affine. It is not linear, because suppose x1 you give, you get mx1, plus, you get x2 mx2, plus, if I give just a combination, say x1, c 1 is 1, this is my y1, this is my y2. So, x1 gives us to y1, x2 gives us to y2. So, if I give x1, plus x2 output should be y1, plus y2, if linear, but now output will be, m into, this is my x now. So, x1 plus x2, plus c. m into input, input is x1, plus x2, this is x1, plus x2m into that plus c, but this is mx1 plus mx2 plus c. This is not same as y1 plus y2, because if you have y1 plus y2, you have m x1 m x2. So, m x1 m x2 present, but you have got 2 c, here you have got only 1 c.

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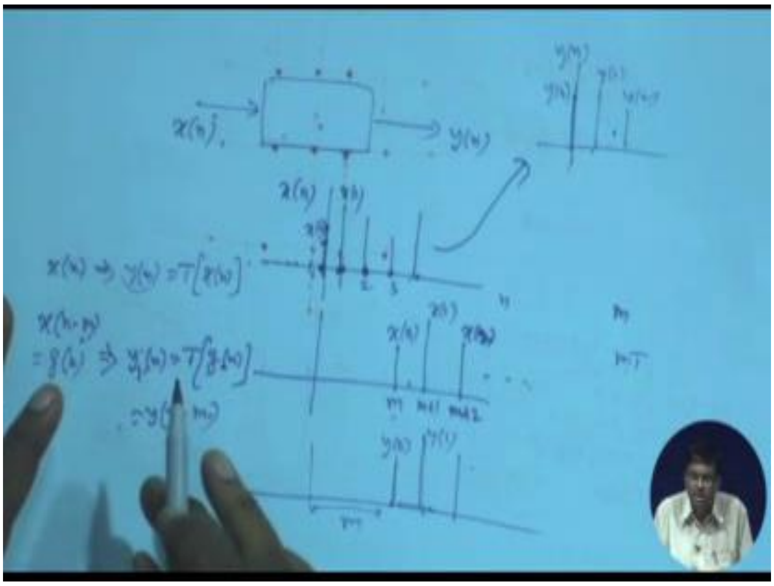

So, it is not strictly linear, this is very interesting. So, with this, this for linearity and there is another thing, called shift invariant, which comes from time invariance, that is, suppose, you are giving xn, there is a yn. Now say xn, as a matter of fact, suppose xn is obtained by, indeed sampling some analog signals. So, these are this sampling, this index points, 1 2 3 they actually correspond to sampling point, this corresponds to capital T, this is 2 T, this is 3 T, like that as a we are taking for example, suppose this is like this. So, actually I am writing as a function, as a sequence. So, this is 0 1 2 like that, but is actually a time sequence. So, this will be in terms of time, it is 0 capital T 2 T 3 T like that and this sequence is given here, we are getting an output.

Now, suppose that original time sequence is delayed. Original time sequence is delayed; that means this will be shifted. Suppose it is shifted by some amount, say m; that means, this guy, x0, will come to m. x1 will come to m plus 1, and like that x3, sorry, x2 will come to m plus 2 dot dot dot dot. Assume that this left side was 0, I started from this index, so, that means, this is delayed by this, in terms of actual time, this is delayed by m into T, that much of time, but T is every sampling period. If I give these to the input now, what will happen? If the system is such, this property does not change, then what will happen, instead of giving the input this sequence now, if I delay it by some amount, new output will be nothing, but, of the same type as yn, because system properties has not changed, but that output will, will be placed here. Whatever earlier was getting, y 0, now y 0 will also occur, here y1 will occur here, y2 will occur here.

So, output also gets delayed, our earlier output may be was like this, y0, y1, y2 like that, earlier. When I gave these, I got that. If I shift it, that means, actual time signal is delayed by mT. These outputs, if the system does not change the property, (Refer Time: 27:42) then, delayed input, so what? It has come down, my processing will remain same. So, I will get the same similar output, only it will be located from here onwards. So, output also will get shifted, this will form will just get shifted by the same amount m, so; that means, if input is shifted by any amount m, output also should be shifted by any amount of m.

If the system property does not change with time; this, if this happens, then it is called time invariant, or shift invariant. That is, if xn gives rise to yn, as T of xn, and suppose I give a delayed sequence n minus m, it is a delayed sequence, you can call it new sequence g n, which is nothing, but delayed version of this. Then new output may be y1 n, which is T, working on this g n, will be same as previous output, but that was also delayed by the same amount. That is I give xn, what yn, xn I now delay, shift, and this can be positive negative both.

So, if it is shifted, as a shifted person, I give it a new name g n, that I give here, like this. I gave here corresponding output y1 n, that will be that is T, working on g n, that is if it is g n here, T working on gn, that is the new output, y1n, that should be nothing but, whole output only, but delayed by the same amount, if input is delayed by m, input is delayed by m so here, input shifted here, output also shifted by the same amount m, if this happens, then I say the system is shift invariant. If it is coming from a time signal, I also call it time invariant.

So, I am stopping now, from these we will now build on, systems, linear and shift invariant systems, and calculate the output in terms of input.

Thank you very much.