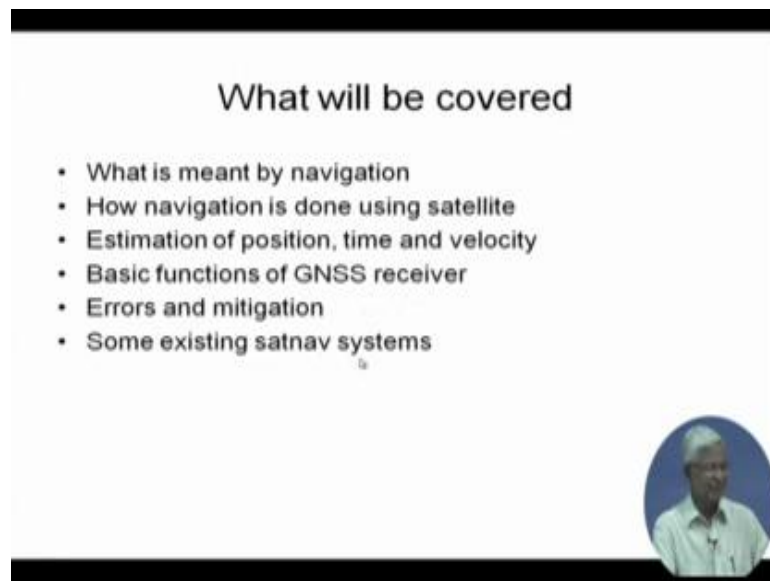


**Satellite Communication Systems**  
**Prof. Kalyan Kumar Bandyopadhyay**  
**Department of Electronics and Electrical Communication Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 39**  
**Satellite Navigation – I**


Welcome. So, we have learnt about the satellite communication the code part of it as well as certain issues that can develop out of that now, what we will discuss is one of the applications which are very interesting application and most of us are using it daily. So, let us look at satellite navigation satellite navigation is an application of satellite communication. So, what will we cover today is what is meant by navigation as it is and then how navigation is done using satellite.

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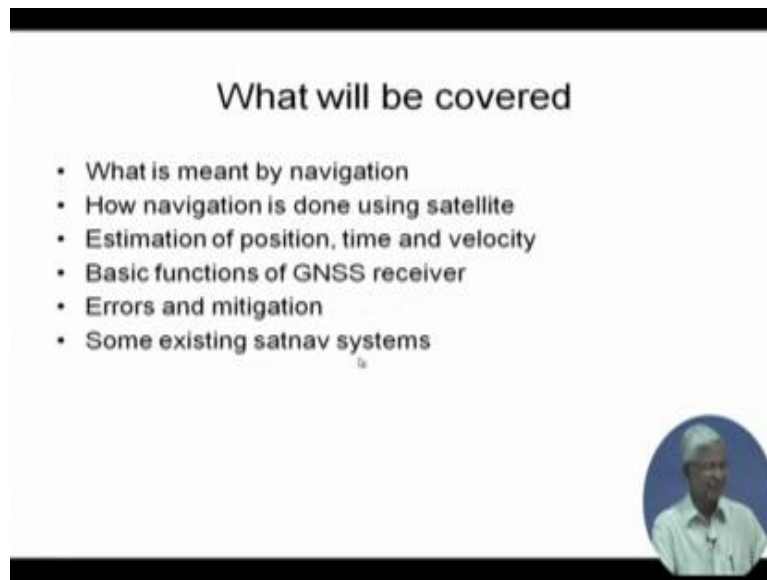
**What will be covered**

- What is meant by navigation
- How navigation is done using satellite
- Estimation of position, time and velocity
- Basic functions of GNSS receiver
- Errors and mitigation
- Some existing satnav systems



And estimation of position time velocity, how do you estimate that basic functions of a global navigation satellite system receiver GNSS and error and mitigation for the errors and some existing satnav systems some description of that.

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So, first let us get into the definition navigation what is meant by navigation, it is determination of position velocity and time three parameters three parameters are required. So, position is a values in preferred coordinate system you can have a simple condition coordinate system  $x y z$  you can have ECI or ECEF coordinate system at the beginning of the satellite communication course we briefly mentioned coordinate system they are Keplerian coordinate systems, so many coordinate systems.

So, when a preferred coordinate is normally latitude longitude altitude is our preferred coordinate system. So, it will convert into that, but it could be done in any other coordinate system. So, there is a value of the position of the receiver velocity is velocity along the chosen coordinate whatever coordinate, we have chosen in the velocity vector in that and time is the universal time coordinate  $u t c$  time now in short it is definition determination of PVT the term what we use is PVT position velocity and time.

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## Satellite Navigation System

- What is Satellite Navigation?

Any type of Navigation aided by Satellite that sends reference information to users in coded form from which the PVT estimations can be derived

- Advantages :

Wide Coverage

- Disadvantages :

Very weak signals, Propagation Impairments



Now, how do we do it with satellite, What is satellite navigation that is a any type of navigation that can be aided by satellite and that satellite sends reference information to the user in coded form from which the PVC estimations can be derived. So, satellites helps you to get the PVT estimation a satellite gives you certain information and from that that information can be coded from that you derive the position velocity and time. So, that is the advantage of satellite navigations; obviously, wide coverage with few satellites you can cover whole world and there is disadvantage we know it is a satellite is quite high above the sky. So, the signals which will be received at the receiver are very weak and of course, it is passing the signal is passing through ionosphere troposphere in this propagation impairments that can happen. So, these are the two disadvantages of that.

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**Satellite Navigation System**

There are four major Global Satellite Navigation Systems

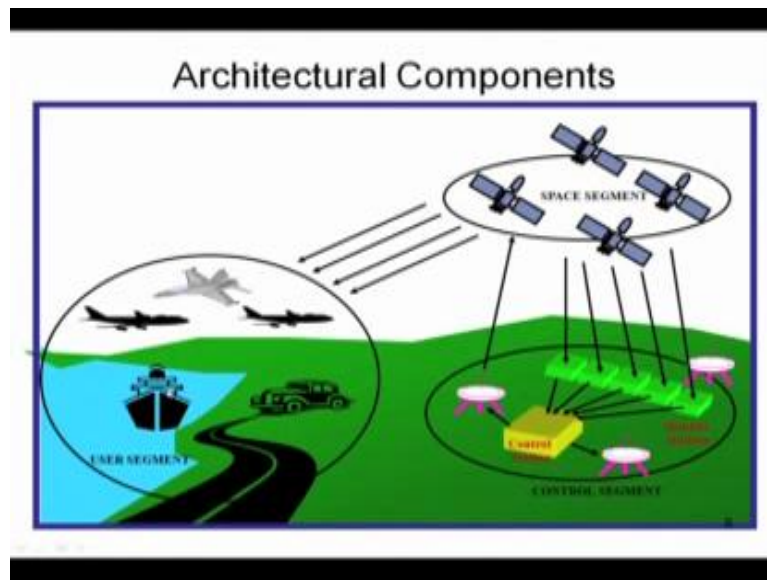
- 1. The Global Positioning System (GPS)**
  - US system, Declared operational in 1995
  - Declared by ICAO, as an alternative for civil aviation.
- 2. The Global Navigation Satellite System (GLONASS)**
  - Russian system, operational
- 3. GALILEO**
  - European System, under development
- 4. COMPASS**
  - Chinese system, under development



So, there are four global satellite navigation systems popularly known there are many some of them are operational some of them are just starting to operate or some of them are just upgrading themselves and the most commonly used and most commonly used as well as very popular system on satellite navigation is global positioning system. Which is GPS it is a u s system which is declared operational in nineteen ninety five before that some experiments were there and international civil aviation authority ICAO has declared that this could be alternate this could be an alternate civil aviation system. That is while aircraft landing or taking off the GPS system can be taken help of in addition to the other systems what they have I l s and d o r v o r etcetera.

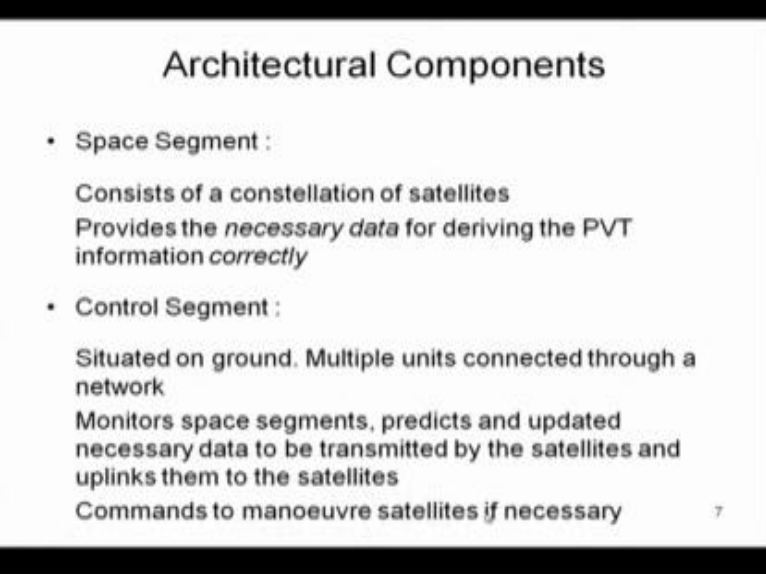
There is other one which is a Russian system global navigation satellite system called GLONASS. It is operational, but not fully they are upgrading it and there is European system which is called Galileo. It is under development couple of lounges are taking place it will take few more years to start operation china has started launching some with the satellite for their system which is called compass that is under development which is not fully operational these are all global systems there are regional systems also we will discuss about that later.

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Now, the architectural conference of this satellite navigation system is three main components one is the space segment; obviously, the satellites and the space segment has to be controlled by a segment which is called controlled segment, which are many earth stations which does certain measurements and upgrades the information to the satellite and then satellite broadcast those information about their location and that is used by a user community, which is user segment that can be because its position velocity and time it will be mostly mobile the pictures are shown all mobile systems it can be used for static purpose also.

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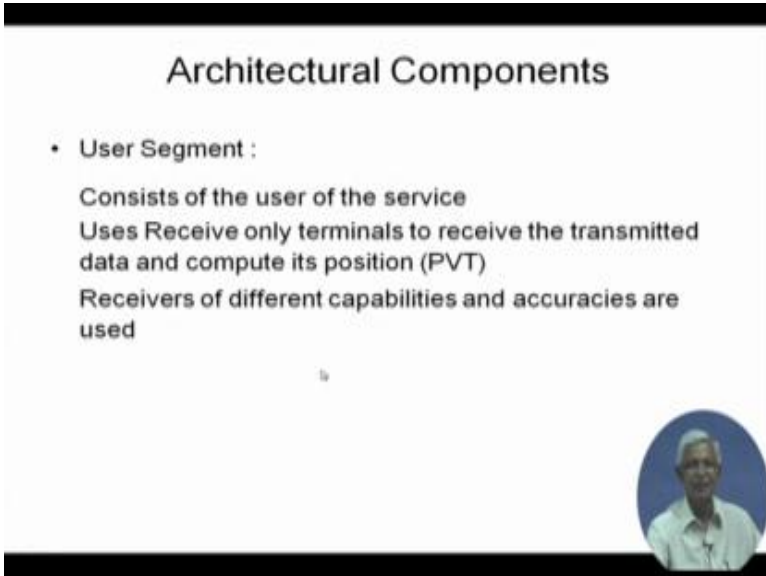
**Architectural Components**

- **Space Segment :**  
Consists of a constellation of satellites  
Provides the *necessary data* for deriving the PVT information *correctly*
- **Control Segment :**  
Situated on ground. Multiple units connected through a network  
Monitors space segments, predicts and updated necessary data to be transmitted by the satellites and uplinks them to the satellites  
Commands to manoeuvre satellites *if necessary*

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So, space segment consist of a constellation of satellites single satellite because it is a global system it is a constellation of satellite and provides necessary data for deriving the PVT information correctly and control segment situated on the ground multiple units are connected that takes certain measurements through a network and then, the monitors space segment is monitored by the control segment it predicts the ephemeris are it predicts the orbit and update the necessary data to be transmitted by the satellite the data is updated to the satellite by control segment and also commands to Manoeuvre the satellite if necessary to keep it in the orbit.


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**Architectural Components**

- **User Segment :**  
Consists of the user of the service  
Uses Receive only terminals to receive the transmitted data and compute its position (PVT)  
Receivers of different capabilities and accuracies are used

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The user segment consist of the users of the service users you have to remember that this is a receive only terminal there is some misconception many times people have you would know yourself only your position velocity time oh it is a receiver only it does not transmit. So, use is receiving only terminals to receive the transmitted data from the satellite and compute its position velocity and time. So, receivers are of different capabilities and different accuracies are available and used all over the world.

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**Estimation of Parameters**

- **Position:**  
3 Reference positions for 3-dimensional system  
derived using distance from reference locations
- **Time:**  
Reference time  
derived using elapsed time from reference instant
- **Velocity:**  
Derived using Doppler, or Position & Time

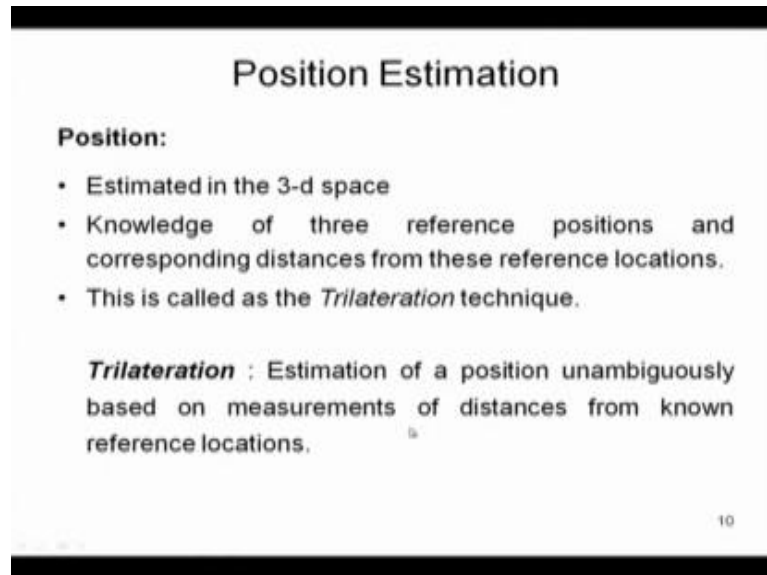


Now, how do we estimate this parameter that is interesting we should know that how this parameters estimate position velocity and time at position is estimated using a three reference positions in a three dimensional system  $x y z$ . Let us take  $x y z$  coordinate with a center point at center of the earth. So, three reference positions that are reference positions are three satellites they at that instant of time it is reference. So, three reference position for a three dimensional system and derived using distance from the reference location the user is receiving these reference position, how far away from each reference position is.

So, that is distance once it is known once it is measured observed and then position can be derived time is a reference time which is  $u t c$  and that is derived using the elapsed time from the reference instant at the user. When receives it can estimate what is the time when it has received in terms of  $u t c$  and converted into the local time and velocity is derived from the position velocity and time a this is a sorry velocity is derived from the

position and time that is derivative of that or using a Doppler technique. So, this and the other three methods we will go into a little more detail of estimation and position.

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**Position Estimation**

**Position:**

- Estimated in the 3-d space
- Knowledge of three reference positions and corresponding distances from these reference locations.
- This is called as the *Trilateration* technique.

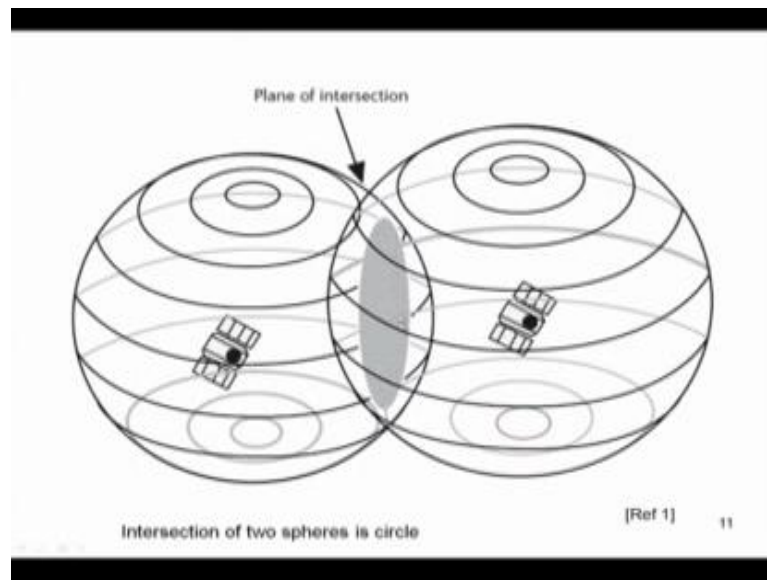
**Trilateration** : Estimation of a position unambiguously based on measurements of distances from known reference locations.

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Position estimation it is estimated in the three dimensional space three coordinates that is x y z knowledge of three reference position and the corresponding distance from the reference location. That is that is important. So, this technique is called trilateration trilateration not triangulation which is done in the normally survival here it is trilateration what is trilateration it is the estimation of a position unambiguously based on measurement of distance from known reference locations. So, for three dimensional system this three known reference locations and if you know the distance between them, you get a trilateration.

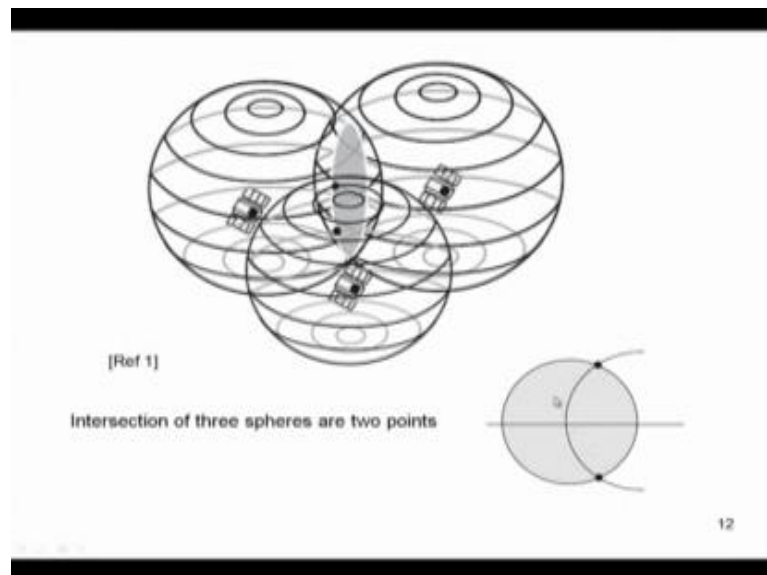


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Let us see a try to understand easily let us assume that satellite each satellite is as isotropically radiating radiations of signal is going spreading like sphere and another satellite is also at the same time is radiating is spreading like a sphere. So, these two spheres will cut or cross themselves in a circle plane in a circular plane.

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And then if there is a third satellite it is also isotropically radiating at the same time. So, that it will cut into two points. So, intersection of the spheres are at 2 points, if you know the positions of these you will really position of these reference satellites you know

really your position can be one of these two cut points, whichever is nearer to earth that is taken that is one of the methods.

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## Position Estimation

3 equations required to solve for 3 unknowns.  
 3 quadratic equations formed using 3 reference positions and corresponding ranges.  
 3 observation equations for 3 satellites are:

$$r_1 = \sqrt{(X_{s_1} - X_u)^2 + (Y_{s_1} - Y_u)^2 + (Z_{s_1} - Z_u)^2}$$

$$r_2 = \sqrt{(X_{s_2} - X_u)^2 + (Y_{s_2} - Y_u)^2 + (Z_{s_2} - Z_u)^2}$$

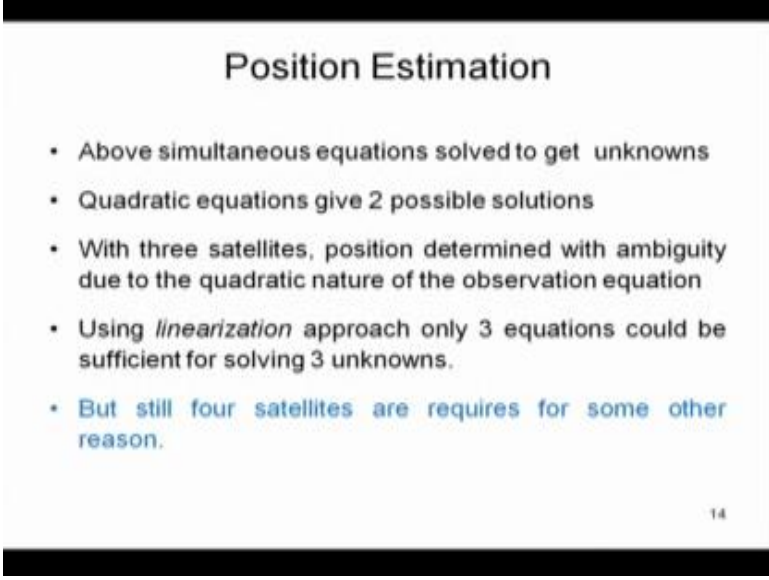
$$r_3 = \sqrt{(X_{s_3} - X_u)^2 + (Y_{s_3} - Y_u)^2 + (Z_{s_3} - Z_u)^2}$$

$X_{s_i}, Y_{s_i} & Z_{s_i}$  are the  $i^{\text{th}}$  satellite coordinates ( $i = 1, 2, 3$ )  
 $X_u, Y_u & Z_u$  are the unknown user position

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So, three equations are required to solve three unknowns its known and three quadratic equations formed using three reference positions and corresponding ranges and these quadratic equations are based on measured distance which is called observed distance or it is called observable equation or observation equation. So, three observation equations for three satellites could be simple our geometry r1 is that observable that is the distance is root over of x s 1 minus x u whole square plus y s one minus y u whole square plus z s one minus z u whole square here s stands for the satellite 1 2 3 are three satellites and u is the user positions that is x s I y s I and z s I are ith satellite coordinate that is 1 2 3 and x u y u z u are unknown user positions.

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### Position Estimation

- Above simultaneous equations solved to get unknowns
- Quadratic equations give 2 possible solutions
- With three satellites, position determined with ambiguity due to the quadratic nature of the observation equation
- Using *linearization* approach only 3 equations could be sufficient for solving 3 unknowns.
- But still four satellites are required for some other reason.

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So, if the  $x$ ,  $y$ ,  $z$  are known which is broadcast from the satellites individual satellites and if we can observe or measure this  $R_1$ ,  $R_2$  and  $R_3$ . Then our unknowns are  $x$ ,  $y$ ,  $z$ . Three equations it can be solved easily. So, above simultaneous equations are solved to get the unknowns which are  $x$ ,  $y$ ,  $z$ , but it is quadratic equation remember, there are two possible solutions that will be coming. So, with three satellites position determined with ambiguity, where is ambiguity due to quadratic nature of the observation equation now using a linearization approach only three equations could be sufficient for solving three unknowns this is some approximation approach then three equations can be used to solve 3 unknowns.

But is a, but we will see later there are four satellites and four equations required for some other reason we will see that first.

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**Position Estimation**

*Reference Positioning* : Locating position of reference satellites.

Done using the ephemeris of satellites. Ephemerides are Keplerian parameters required for position estimation.

Satellite Ephemerides are estimated at control segment and are uploaded to satellites. Satellites broadcast them to users.

User derives satellite Ephemerides from corresponding navigation message broadcast from each satellite.

Using the satellite position at any instant receiver can calculate the range to that satellite.

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Let us see how we do the linearization. So, reference positioning locating the position of the reference satellite it is done using ephemeris of the satellite now this ephemerides are keplarian parameters, which are you? If you remember at the beginning of our course we talked about eccentricity and then inclination then the semi major axis like that there are 6 keplarian parameters that defines a satellite in a orbit satellite position in a orbit it can be translated or transformed into other coordinate system. So, satellite transmits keplarian parameters of their position at a particular time instant these satellite ephemerides are estimated at the control segment. Why it is estimated because satellites might be drifting from its orbit a little out because of sun moon drag earth drag, we have discussed this one earlier.

So, this ephemeris at a particular instant when it is visible to the control segment the control segment calculates its exact ephemeris at that instant uploads and the next time when it visits the control segment it will give the correction to be in between there will be little more drift. So, that has to model. So, let us for the time being assume that ephemeris at each instant are available at the satellite which is uploaded by the control segment or every orbit it is corrected satellite broadcast this ephemeris to the user at a particular time at that instant of time so; that means, satellite send certain message down to the earth each satellite.

Now, user derives these satellite ephemerides from the corresponding navigation

message. So, that navigation message which is broadcast from the satellite from each satellite its broadcast identify and. So, user try to derive the satellite ephemeris the broadcast message contains many other things which we will see later. So, it will take out the ephemerides and then at then using the satellite position at any instant the receiver can calculate the range up to that satellite at that instant, what was the range from the satellite to the user how do you do the range estimate because it is the only one way if you remember if you recollect in our telemetry telecommand discussion, we did some ranging and that ranging was a two way ranging its full loop a tone was sent or a p r b r sequence was sent to the satellite down links them. So, a two way ranging in that we estimate the range, but here it is only one way because the receivers user receiver GPS receiver. GNSS receiver, it is receive; only it does not transmit. So, it has to be a one way ranging how does it how we do it.

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### Position Estimation

**Ranging** : Measure of distance of user from each reference point, i.e. satellite

Range is obtained using Time of Arrival (TOA) technique or the total time taken by the signal to travel from the satellite to the user

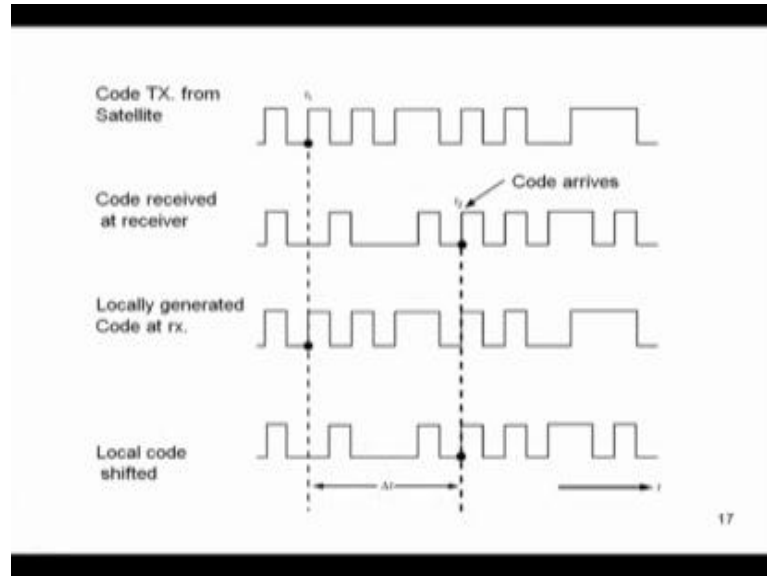
Total traveling time of signal is obtained from difference between time of transmission and time of reception

$t_1$  = Time of transmission of data  
 $t_2$  = Time of reception of data  
 $(t_2 - t_1)$  = Time taken to propagate  
 $R = c \times (t_2 - t_1) = \text{Estimated Range}$

So, this ranging is a measure of distance of the user from each reference points that is the satellite range is obtained using time of arrival technique of the or the total time taken by the signal to travel from the satellite to the user total traveling, time of the signal is obtained from the difference between the time of transmission and time of reception its simple pictorially say at  $t$  equal to  $T_1$  the satellite transmits and at  $t$  equal to  $T_2$  the receiver receives it assuming the a signal is traveling with velocity of right  $c$   $r$  is equal to  $c$  into  $T_2$  minus  $T_1$ .  $T_1$  is a time of transmission from of the data from the satellite  $T_2$  is the time of reception of the same data  $T_2$  minus  $T_1$  is the time taken to propagate and

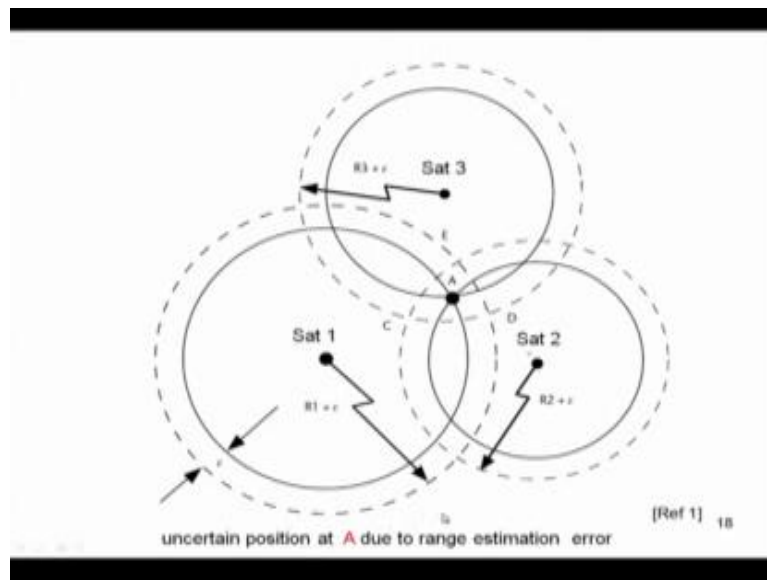
assuming  $c$  is constant during propagation  $c$  is equal to  $T_2$  minus  $T_1$  which is the estimated range.

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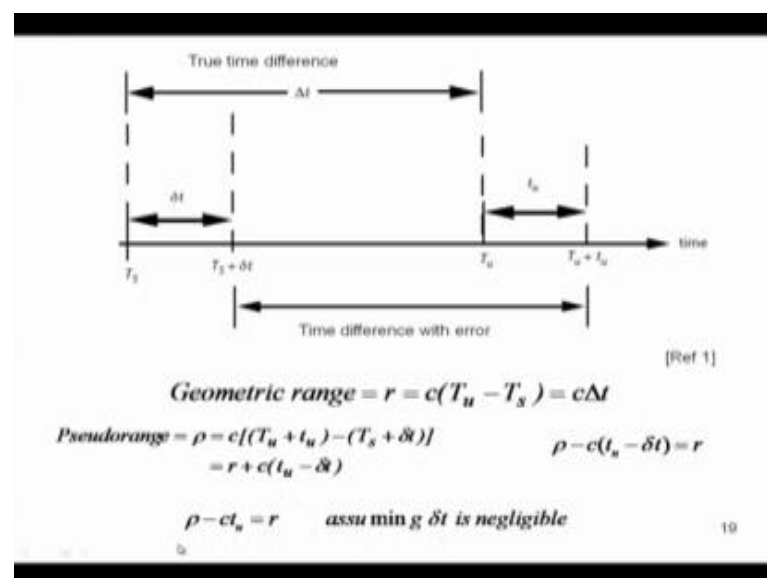
Now, how do you know exactly what time it has transmitted? So, therefore, digital codes are used with a sequence unique type of sequence. Let us say appearance sequence the code is transmitted from the satellite. let us say this code is 1 0 1 0 1 1 0 like that a code. Now that is transmitted at a instant  $T_1$  and this code is received after certain delay code has arrived that is at this point  $T_2$ ,  $T_1$  it was transmitted at  $T_2$ . It is received that is 1 0 1 0 1 1 0, like now receiver locally generated the same code and that is 1 0 1 0 locally generated and that code is shifted and compared with that. So, when the auto correlation works. When it is properly correlated to find out the shift in time or shift in number of bits. So, that is  $\Delta t$ .

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So, once you know the delta t that is T2 minus T1 you know the range assuming velocity of light it is traveled. Now, there may be some errors. So, there if there is a reference station, reference satellite at 3 has error epsilon each of them, I assumed error epsilon same. So, the position will be with some inaccurate position with some region, it may be anywhere the, is may be anywhere from three satellites it is transmitted the dotted lines say certain errors are there, uncertain position at a due to range estimation error why this uncertainty suddenly I talked about.

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Let us see the  $t_s$ , let us I am slightly changing  $t_s$  is the capital T  $s$  is the time, when the satellite is transmitted and capital T  $u$ , when the signal is received at the receiver. So,  $\Delta t$  that is  $t_s$  minus  $t_u$  is the true time difference from that you can find out the range, but these times are derived from some clock by cycle counting and that clock may have some error some instability we have talked about it earlier. So, that let us assume satellite clock error is  $\Delta t$ . So,  $t_s$  plus  $\Delta t$  actually it was transmitted it may be minus also and at user also has a clock which is a different clock physically it is a different clock and different stability.

So, it let us have a small  $t_u$  its stability its error. So, it is received at capital T  $u$  plus small  $t_u$ . So, you get a time difference which is not same as  $\Delta T^2$  time difference. So, it is with the error. So, geometric range the correct range should have been  $r$  is equal to  $c$  into  $t_u$  minus  $t_s$  is  $c \Delta t$  where as we get a range which may not be correct which is called pseudorange. Now what is pseudorange this we call it  $\rho$ ,  $\rho$  is  $c$  into capital T  $u$  plus small  $t_u$  that is the user clock drift minus capital T  $s$  plus  $\Delta t$  that is the  $\Delta t$  is the satellite clock. So, the times shift, because of these two you get a true range plus  $c$  into  $t_u$  minus  $\Delta t$  which is that is you can change the sign and you can find out what is the true range  $r$  is equal to  $\rho$  minus that is should the range be minus. Into the clock differences between the two stations one is user receiver another is satellite.

So, but at the satellite people use atomic lock. So,  $\Delta t$  is negligible  $\Delta t$  is negligible. So, therefore,  $\rho$  minus  $c$  into  $t_u$  that is user clock error that is the time error is equal to the region  $\Delta t$  is put as a negligible. So, all this type of navigation satellite carry atomic clock that is complexity increases in the satellite, but your calculation becomes much easier calculation of the range becomes easier.



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### Position Estimation


$T_s$  : Derived from satellite clock  
It is an Atomic clock of very high stability ( $10^{-13}$ )

$T_u$  : Obtained from receiver clock  
Receiver clock is cheap and of stability  $\sim 10^{-6}$  to  $10^{-9}$   
Receiver clock drifts with satellite clock  
Causes clock-offset error leading to erroneous estimations in the delay, hence in erroneous ranging

Receiver clock offset also taken as unknown in solving for position, thus requires an extra observation

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u$$

$j = 1 \text{ to } 4$



Now, say I am telling the same thing again  $t_s$  is derived from the satellite clock, it is an atomic clock of the very high stability cesium clock ten to the power minus 13, oh we know what is stability of clock we have discussed earlier and  $t_u$  that is capital  $T_u$  is obtained from the receiver clock its stability is much lower of the order of 10 to the power minus 6 to 10 to the power minus 9, depending what type of receiver it is and the receiver clock drifts with respect to satellite clock. So, causes clock offset error leading to erroneous estimation of the delay. So, the erroneous ranging once the delay estimation error ranging estimation also will be error so, this is a variable thing this difference of the clock offset and this is a unknown also.


So, therefore, we can use another equation that is receiver clock offset also taken as unknown for solving the position thus requires an extra observation. Now you know why four satellite observations are required or 4 equations are required. So, we can write it as a  $\rho_j$  which is the  $j$  is one to four that is four satellites that is the pseudorange is root over of  $x_j$  minus  $x_u$  whole square  $y_j$  minus  $y_u$ , whole square plus  $z_j$  minus  $z_u$  whole square plus  $c$  into  $t_u$ .

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## Position Estimation

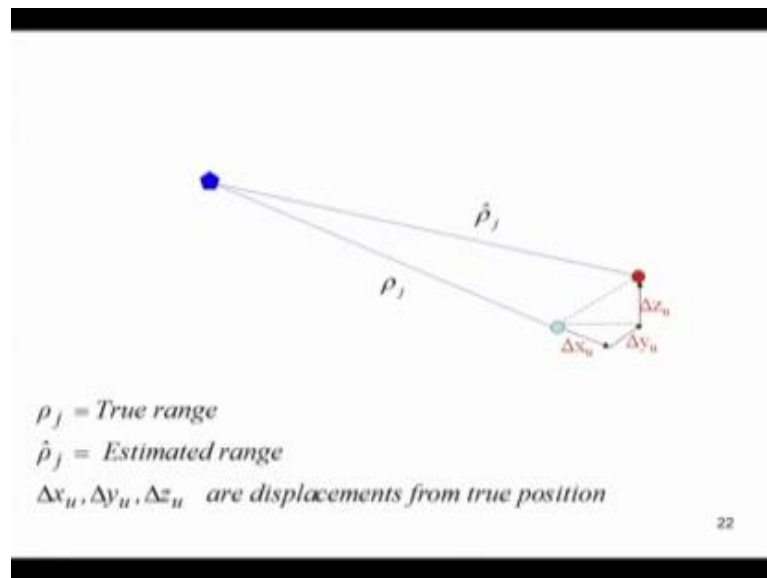
*Solution for the unknowns* : obtained from four observation equations, solved using *Linearization*

- *Linearization*: Technique to linearise equations about approximate position (initial guess) and solve iteratively
- Method uses an a-priori approximate knowledge of the solution and then solves iteratively
- Reduces quadratic equations into linear equations
- Reduces problems arising from non-linearity
- Less computational intensive than its quadratic for



Now, the how do you find out the solution solutions for the unknowns obtained from the four observation equation solved using a technique which is called linearization most of you know about it. So, linearization is a technique to linearize equations about approximate position that is initial guess you guess that this would be my position and solve it iteratively. So, if you go on improving the guess you remove the higher order terms. So, method uses a approximate a priori approximate knowledge of the solution and then solves it iteratively, it reduces the quadratic equation into linear equation and it reduces the problem arising of the non-linearity and it is a less computational intensive because quadratic part is not there.

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Let us say how do we do it let us say this is a satellite this is a user position and this is the pseudorange and we estimate at rho cap j which is the estimated range and that is a red dot.

So, it is away from the true range by del x u del y u and delta z u this by these three vectors it is away. So, true range and estimated range and these are the displacement from the true position del x u del y u delta z u is displacement from the true position remember this figure.

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Let pseudorange is

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u$$

$$= f(x_u, y_u, z_u, t_u)$$

Using approximate location  $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$  and time bias  $\hat{t}_u$  an approximate pseudorange  $\hat{\rho}$  can be calculated as

$$\hat{\rho}_j = \sqrt{(x_j - \hat{x}_u)^2 + (y_j - \hat{y}_u)^2 + (z_j - \hat{z}_u)^2} + c\hat{t}_u$$

$$= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)$$



Now, let the pseudorange is equal to we have seen this equation. Now it can be written as a function of  $x_u$   $y_u$   $z_u$  plus  $t_u$  using approximate location  $\hat{x}_u$   $\hat{y}_u$   $\hat{z}_u$  and time bias also approximate  $\hat{t}_u$  cap, you can put  $x_j$  minus  $\hat{x}_u$  cap, whole square plus  $y_j$  minus  $\hat{y}_u$  cap whole square plus  $z_j$  minus  $\hat{z}_u$  cap whole square root over of that plus  $c t_u$  cap which could be function written, as function of  $\hat{x}_u$   $\hat{y}_u$   $\hat{z}_u$  cap and  $\hat{t}_u$  cap.

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$$\begin{aligned}
 & \text{As} \\
 & x_u = \hat{x}_u + \Delta x_u, \quad y_u = \hat{y}_u + \Delta y_u, \quad z_u = \hat{z}_u + \Delta z_u \\
 & \text{and} \quad t_u = \hat{t}_u + \Delta t_u
 \end{aligned}$$

$$\begin{aligned}
 f(x_u, y_u, z_u, t_u) &= f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u) \\
 &= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u} \Delta x_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u} \Delta y_u \\
 &+ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u} \Delta z_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u} \Delta t_u + \text{higher order terms}
 \end{aligned}$$

Higher order (nonlinear) terms are neglected




So, as this you try to remember that vector. So, the  $x_u$  is  $\hat{x}_u$  plus  $\Delta x_u$  this is a true position this is approximate position and this is displacement similarly  $y_u$   $z_u$  and  $t_u$  also. So, the equation can be written as a function of  $\hat{x}_u$   $\hat{y}_u$   $\hat{z}_u$   $\hat{t}_u$  is function of  $\hat{x}_u$  cap plus  $\Delta x_u$   $\hat{y}_u$  cap plus  $\Delta y_u$  like that and differential equations, can be formed out of it and there will be higher order terms. Now you can neglect this higher order term because they are very small. So, how do we derive this you see this is a true range this is a true pseudorange will be called true pseudorange a pseudorange and this is approximate pseudorange. So, differences of these two are all these differential terms.

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The difference between pseudorange and approximate pseudorange can be expressed as

$$\Delta\rho_j = a_{xj}\Delta x_u + a_{yj}\Delta y_u + a_{zj}\Delta z_u - c\Delta t_u$$

Assuming coefficients as H matrix this can be expressed in matrix form

$$\Delta\rho = H \times \Delta x$$
$$\Delta x = H^{-1} \times \Delta\rho$$



Now, this difference between the pseudorange and approximate pseudorange can be expressed as  $\Delta\rho_j$  as coefficient  $a_{xj}$   $\Delta x_u$   $a_{yj}$   $\Delta y_u$   $a_{zj}$   $\Delta z_u$  minus  $c$  and  $\Delta t_u$  assuming coefficients of this  $a_{xj}$   $a_{yj}$   $a_{zj}$  etcetera as  $H$  matrix this can be expressed in the form of matrix form  $\Delta\rho = H \times \Delta x$ . So,  $\Delta x$  can be found out by  $H$  inverse matrix solution.

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Once unknown  $\Delta x$  are calculated, User position and time can be calculated using

$$x_u = \hat{x}_u + \Delta x_u, \quad y_u = \hat{y}_u + \Delta y_u, \quad z_u = \hat{z}_u + \Delta z_u$$

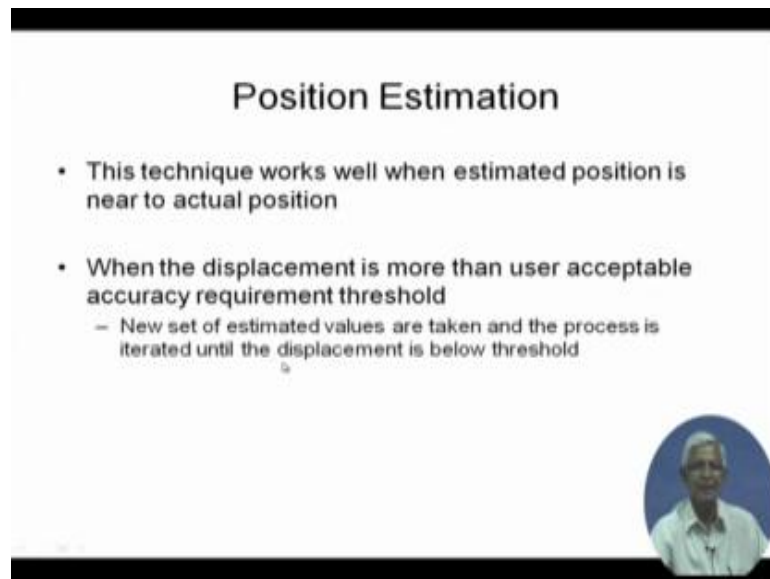
and  $t_u = \hat{t}_u + \Delta t_u$



Now, once you know that the  $\Delta x$  are calculated user position and time can be calculated using the word original expression that is  $x_u$  is approximate plus, now you


have found out  $\Delta x$   $\Delta y$   $\Delta z$   $\Delta t$ . So, you find out this.

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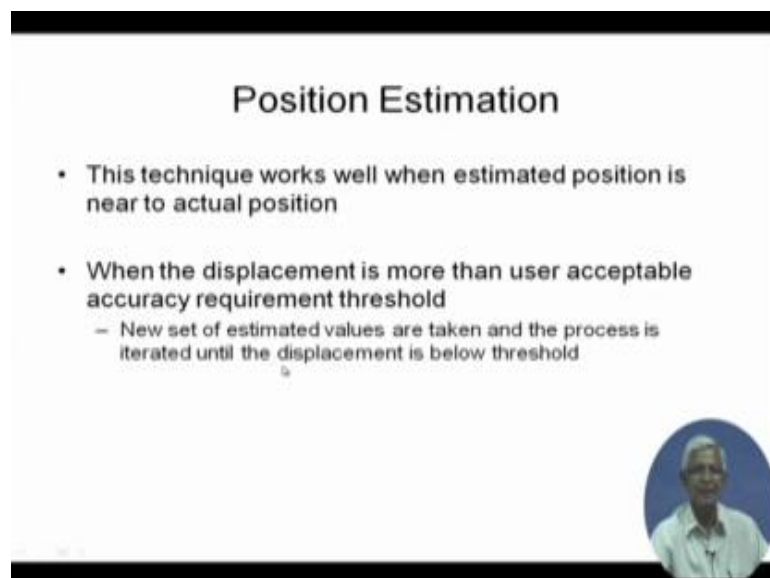
**Position Estimation**

- This technique works well when estimated position is near to actual position
- When the displacement is more than user acceptable accuracy requirement threshold
  - New set of estimated values are taken and the process is iterated until the displacement is below threshold




And then this technique works well when the estimated position is near the actual position. When the displacement is more user acceptable accuracy requirement thresholds then new set of estimated value are taken and the process is repeated.

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**Position Estimation**

- This technique works well when estimated position is near to actual position
- When the displacement is more than user acceptable accuracy requirement threshold
  - New set of estimated values are taken and the process is iterated until the displacement is below threshold



So, since the time is up we will continue this discussion in the next session.

Thank you very much.