Satellite Communication Systems
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Lecture – 13
Link Budget-3

Welcome back. We were discussing about the noise and noise power how it can be calculated.

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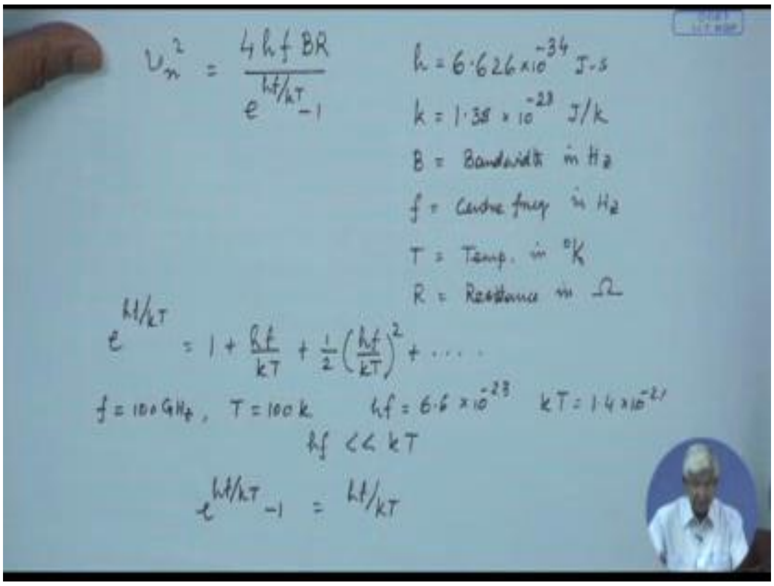

$$v_n^2 = \frac{4hfBR}{e^{hf/kT} - 1}$$

$h = 6.626 \times 10^{-34}$ J-s

$k = 1.38 \times 10^{-23}$ J/k

B = Bandwidth in Hz

f = Centre freq in Hz

T = Temp. in °K

R = Resistance in Ω

$$e^{hf/kT} = 1 + \frac{hf}{kT} + \frac{1}{2}\left(\frac{hf}{kT}\right)^2 + \dots$$

$f = 100$ GHz, $T = 100$ k $\quad hf = 6.6 \times 10^{-23} \quad kT = 1.4 \times 10^{-21}$

$$hf << kT$$

$$e^{hf/kT} - 1 = hf/kT$$

So, let us go back to our last page where we have said that the noise voltage square is vn square it can be expressed as by Planck's black body radiation 4 hf br by e to the power hf by kt minus 1. Where, h is Planck's constant k is (Refer Time: 00:52) constant bandwidth B, f is center frequency, T is temperature and R is the resistance. Now in the denominator we have e to the power hf by kt which in Taylor series can be expanded and by putting some value of frequency which is quite high then micro frequency and nominal value of temperature 100 k and f is equal to 100 Giga hertz we can put into this constants and we can find out hf is much smaller then kt.

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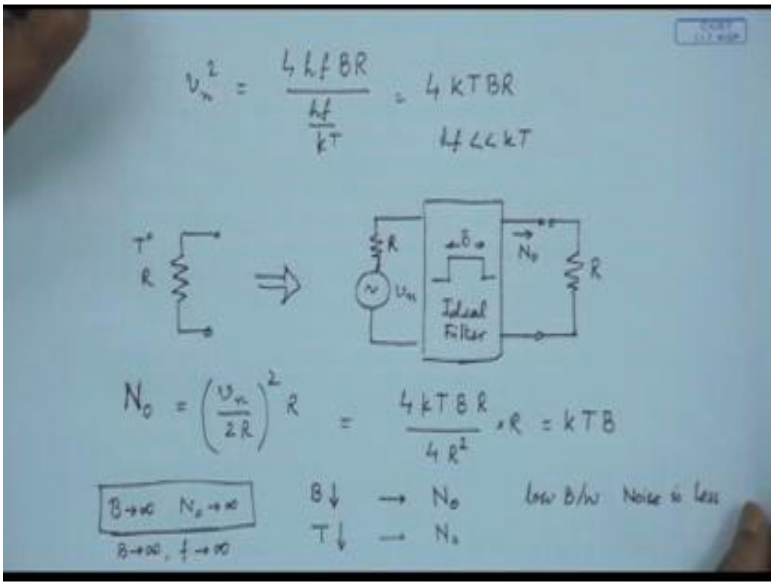

$$v_n^2 = \frac{4hfBR}{\frac{hf}{kT}} = 4kTBR$$

$$hf \ll kT$$

$$N_0 = \left(\frac{v_n}{2R}\right)^2 R = \frac{4kTBR}{4R^2} \times R = kTB$$

$$\boxed{B \to \infty \quad N_0 \to \infty}$$

$B \to \infty,\ f \to \infty$

$B \downarrow \ \to \ N_0$ low B/W Noise is less

$T \downarrow \ \to \ N_0$

So therefore, higher order terms in the Taylor series hf by kt square hf by kt cube etcetera can be neglected and we can take e to the power hf by kt minus 1 which is the denominator is itself is equal to hf by kt. Therefore, that vn square now stands as 4 h f B R by hf by kt that is 4 k T B R. Now look at it very carefully that is now vn square does not have any term of frequency, so its frequency independence that means, if it is a noise voltage it is a white noise. But, obviously there is an assumption the assumption is it is true for hf smaller than kt. Otherwise we have to go back to the blanks original formula.

Now, let us see there is a that resistance which is their which is R which is at temperature T degree which equivalent could be drawn as a noise voltage source vn and there is loss less or noise less I should not say loss less noise less register which is the voltage is filtered by a ideal band pass filter of a bandwidth B hertz, this is ideal filter. Then the output is the noise power n out, that noise power can be transferred maximally if the load resistance is same as R.

So, now we can write the noise power N 0 has the current which is vn by 2 R current square into R, so that is the noise power. And if we put the vn value here it is 4 k T B R divided by 4 R square into R that gives us k T B. This always we remember that noise power is equal to k T B noise power is equal to k T B, how it is derived we see. If you look at it more carefully you just reduce the bandwidth B and then obviously the noise power will be reduced, if the B is reduced the noise power is reduced. That means, for low bandwidth noise is less. We talked about the residual noise power has to be reduced, so low bandwidth noise is less. Similarly look at T if I reduce T then also noise power is less. So, as the temperature of the system subsystem is lower the noise is less. If you recollect that our space segment discussion we said that the hotspot or heat generated by the component has to be dissipated, so the dissipation was taken by heat pipe or it is exposed to the cooler temperature and the temperature of the circuit or that particular component it reduced it will generate less noise.

But then from these also you can see if B is made large that is when B is made infinity, the noise power will be infinity; no this is not true because, B infinity means f is infinity and your basic thing we said that hf is much less then kt. As you go on increasing f this is not valid this assumption is not valid. So in those cases we have to go back to that original blank formula. So, now from these we understand that noise power is equal to k T B and that is in a resistance with a particular temperature T.

Now let us see we have said that this is a resistance, but any of the devices that can have similar characteristics.

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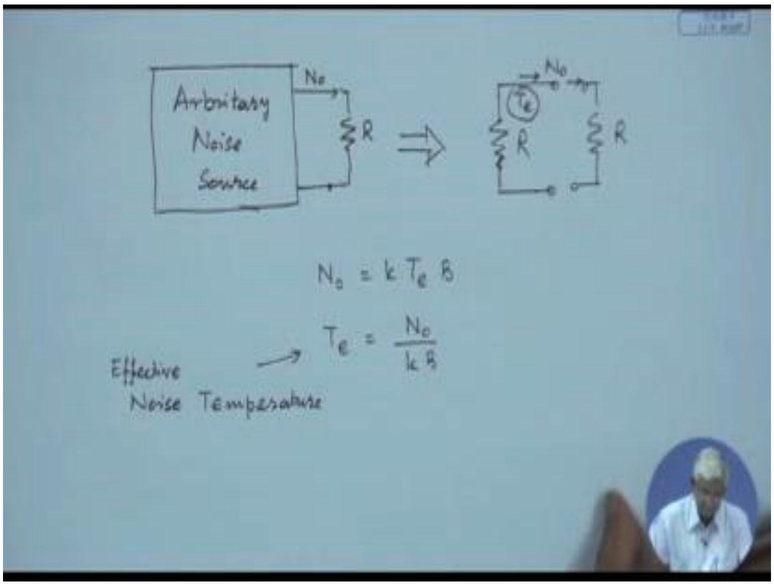

$$N_o = k T_e B$$

$$T_e = \frac{N_o}{k B}$$

Let us say there is an arbitrary noise source which generates the noise power N 0 and with the maximum power you can transferred when their loads are matched. So, these can be equivalently said that as if there is a resistance at a temperature which is effective temperature of (Refer Time: 07:54) not physical temperature. Effective temperature of the resistance which generates these noise power and maximum can be transferred when the source and loads resistance are equal.

So, this is an effective temperature of the resistance that means, N 0 now can be said as k Te B. Any device which generate certain noise thermal noise or any noise can be put effectively as if it is a resistive source with a effective temperature, it is not a physical temperature remember Te as a, that is that Te is equal to N 0 by k into B. So, most of the components are characterized by these temperature which is effective noise temperature; is called Effective Noise Temperature that is Te which N 0 by kB.

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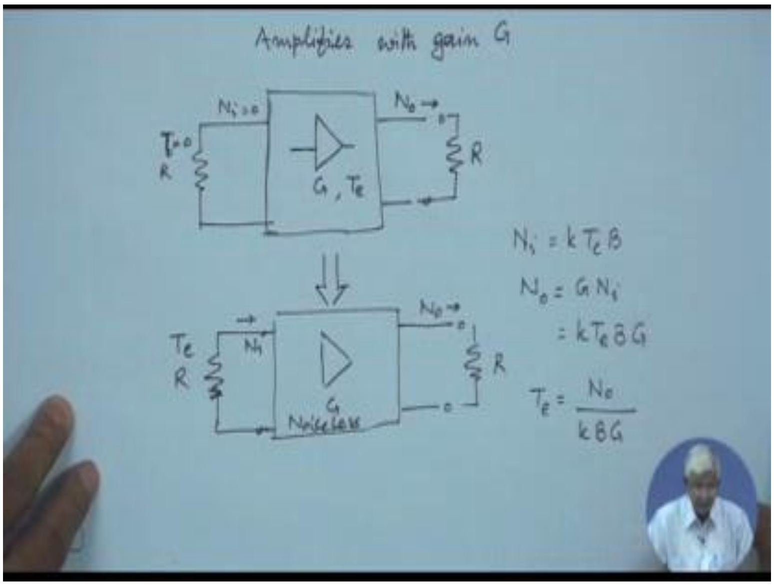

Amplifier with gain G

$$N_i = kT_eB$$

$$N_o = GN_i$$

$$= kT_eBG$$

$$T_e = \frac{N_o}{kBG}$$

Now, let us look at one of the type of circuits which often we come across that is amplifier with gain is equal to G. So that can be modeled as there is a amplifier which has a gain G and it has the effective noise temperature, because it is generating its noise by itself inside so it is the effective temperature Te.

Now, this can be modeled as if this particular unit is having input power noise in I which is equal to 0 which is generated by a resistance at T is equal to 0, so you call it T 0 or let us say T equal to 0 whichever. So, input is 0 and it is giving out put as N 0 which is the noise it is generating to a load resistance R. This can be draw as equivalent as a noise less amplifier with a gain G; is a noise less amplifier and as if it has a input resistance R with effective temperature Te which is generating this Ni.

Since this is noise less G times that input will come so output will N 0 will come out of this which is connected to the load. We can write Ni is equal to k Te B that is Ni and No is G times Ni which is k Te B into G or that effective noise temperature of this amplifier is N 0 by k B into G. So, the amplifier which is having a physical temperature generates its own noise and the effective noise temperature is estimated as N 0 the noise power which is generated divided by k into the bandwidth and into the gain of that.

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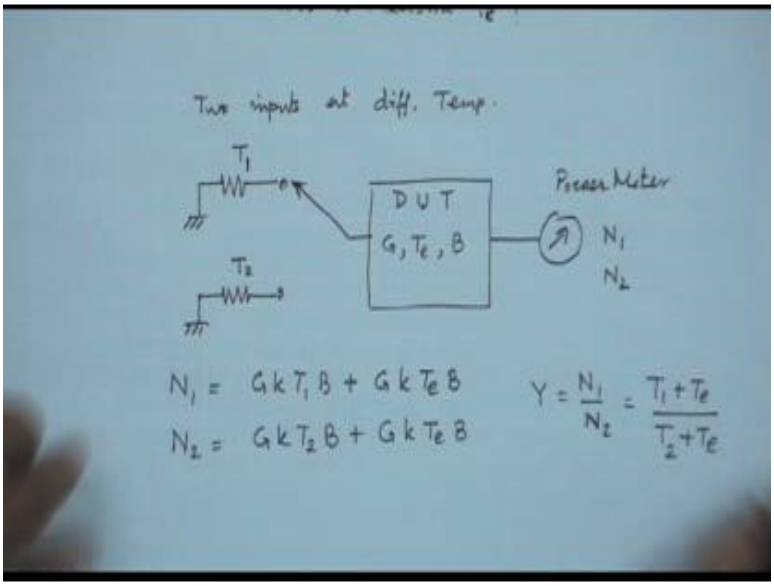

Two inputs at diff. Temp.

$$N_1 = GkT_1B + GkT_eB$$
$$N_2 = GkT_2B + GkT_eB$$
$$Y = \frac{N_1}{N_2} = \frac{T_1+T_e}{T_2+T_e}$$

Now, how do we measure this Te? There is a simple method which has two inputs at different temperature and measure the power. Let us us draw it. Let us see this is a device which is at a temperature T1 and this is another device which is temperature at T2. These devices are different temperature sources or noise sources. And then we put the device under test which has a gain, which has an effective noise temperature, which has a bandwidth. Then there is input noise power coming, this device under test based on its gain effective noise temperature and bandwidth generates some noise. So, the total noise is measured in a power meter.

So, when the switch is towards the T1 source the measured output power is N1 when the switch is towards T2 source the measured output is at N2. You can write N1 is the input noise that is multiply input noise multiplied by the gain of this that is G k T1 B and the noise generated by itself that is G k Te B. Input is having a temperature T1 and the device has its own effective noise temperature Te, so input is multiplied by the gain and the device itself generates G k Te B. Similarly, N2 will be G k T2 B and its own G k Te B.

Now if you take the ratio these two and we call a factor called Y is equal to N1 by N2. If you take the ratio of these two many of the things like a gain k B will get canceled and you will left with T1 plus Te by T2 plus Te. From these we can find out what is Te in terms of Y.

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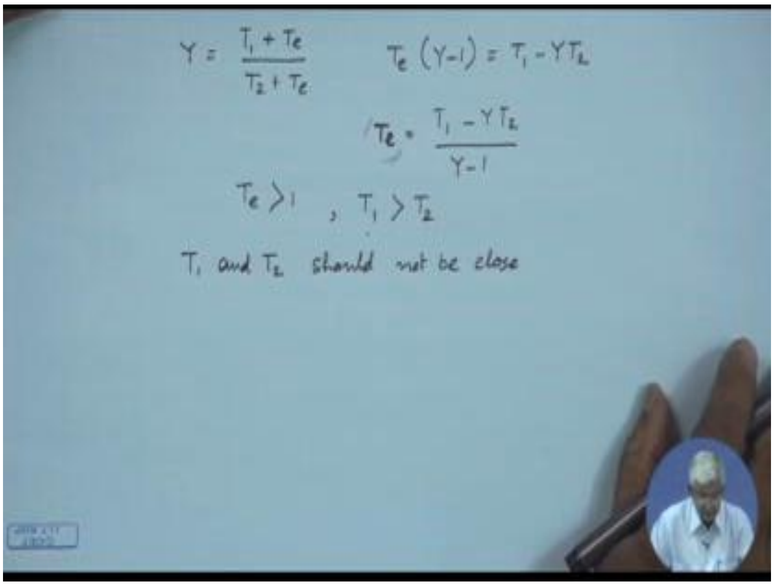

$$Y = \frac{T_1 + T_e}{T_2 + T_e} \qquad T_e (Y-1) = T_1 - Y T_2$$

$$T_e = \frac{T_1 - Y T_2}{Y-1}$$

$$T_e > 1 \quad , \quad T_1 > T_2$$

T_1 and T_2 should not be close

So, by chaining the side that is let us say - Y is equal to T1 plus Te by T2 plus Te. So, we can say Te into Y minus 1 is equal to T1 minus Y into T2, so Te is equal to T1 minus Y into T2 by Y minus 1. Since, these effective noise temperature is suppose to be larger than 1 we have to make T1 larger then T2 and also T1 and T2 should not be close. In that case the Y will be very near to 1 and the denominator will be very small so the accuracy of Te will be lost.

So therefore, the T1 and T2 these two are kept for a part. Now this effective noise temperature is we have measured this is one way characterization is of a component. There are other ways of characterization.

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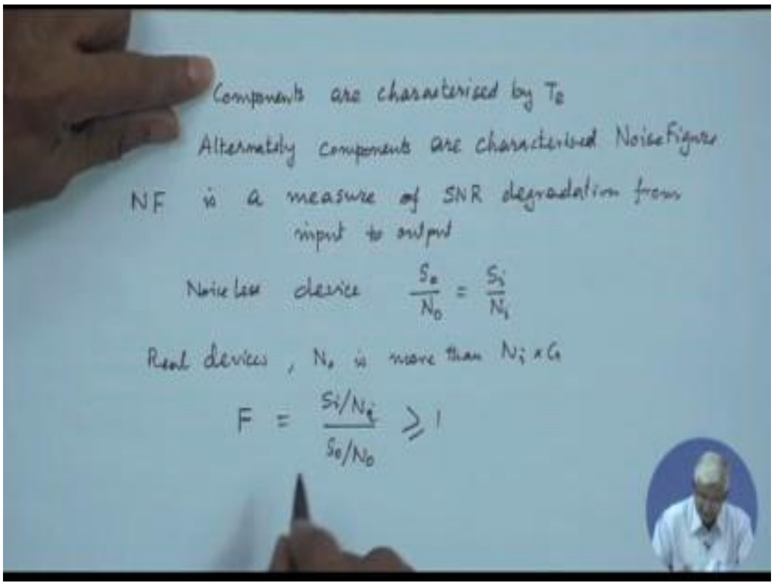

Components are characterised by T_e

Alternately components are characterised Noise Figure

NF is a measure of SNR degradation from input to output

Noise less device $\frac{S_o}{N_o} = \frac{S_i}{N_i}$

Real devices, N_o is more than $N_i \times G$

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

So, normally components or subsystem are characterized by Te or the noise temperature. It is also alternatively components are characterized by another parameter which is called noise figure. Now the noise figure is a measure of SNR degradation from input to output. By definition that noise figure is the ratio of input signal to noise ratio to the output signal to noise ratio.

So, if it is a noise less device or component the SNR of output is same as SNR of input. That means, if the device is having some gain the signal is elevated by the same gain and noise is also elevated by the same gain and it is a noise less device it does not introduce anything so signal to noise ratio at the input is same as signal to noise ratio as the output. But, for real device that N 0 is more than Ni into if there is a gain.

So, signal to noise ratio is less at the output, so that is the noise figure if we denoted by F is signal to noise ratio at the input sorry, and signal to noise ratio at the output is greater than or in case of noise less it is equal to 1; this is what is the noise figure definition. Now let us try to calculate this noise figure in terms of noise temperature.

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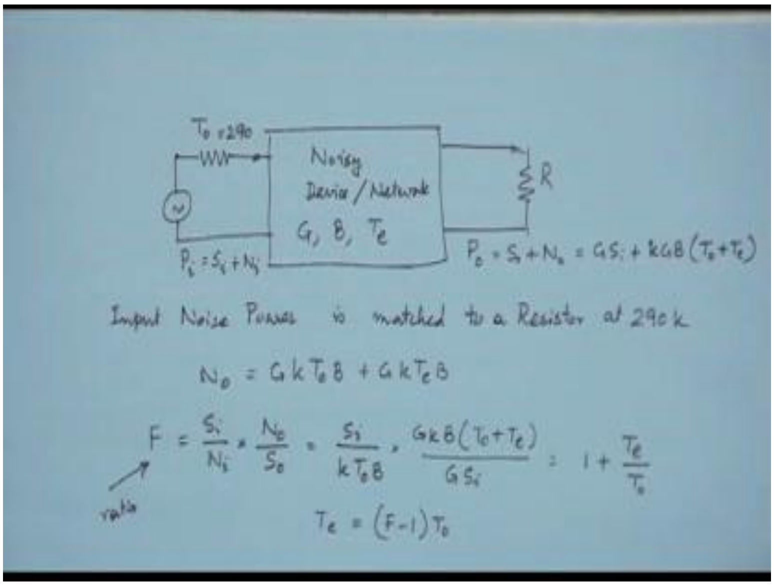

$P_i = S_i + N_i$

$P_o = S_o + N_o = GS_i + kGB(T_0 + T_e)$

Input Noise Power is matched to a Resistor at 290 K

$N_o = GkT_0B + GkT_eB$

$$F = \frac{S_i}{N_i} \cdot \frac{N_o}{S_o} = \frac{S_i}{kT_0B} \cdot \frac{GkB(T_0+T_e)}{GS_i} = 1 + \frac{T_e}{T_0}$$

$$T_e = (F-1)T_0$$

So, there is a noisy device or network which is having a gain G and B and effective noise temperature Te. It is assumed this input to this is a source which is at normal temperature T 0 290 degree and it is matched to this source. So, the power input to this noise device is the signal power and the noise power is coming as an input noise power is generated because of this T 0 but remember it is matched to this. So, the assumption is input noise power is matched to a resister assuming resistor at which can be taken as 290 Kelvin for the timing temperature that is normal temperature.

And then the noise power output to the load is output signal power and noise power that is total power output is there is a gain G, so G times the input signal power plus k G B times that input temperature plus effective noise temperature of this device or the noisy network. That means, the N 0 is equal to G into k T0 B plus G into k Te b. So, noise figure F is Si by Ni divided by S 0 by N 0, so it can be said N 0 by S 0 here or O means output I should not say 0 it is N 0 by S 0, so Si by Ni is k T 0 B and N 0 is G k B T 0 plus Te divided by G into Si that is S 0.

So, all these things get canceled and resultant is 1 plus effective noise temperature by the T 0 or the effective noise temperature of the device can be put in terms of F minus 1 into T 0. Remember for all these calculations this term F is a ratio not in dB we should write differently because this (Refer Time: 25:18) can confuse.

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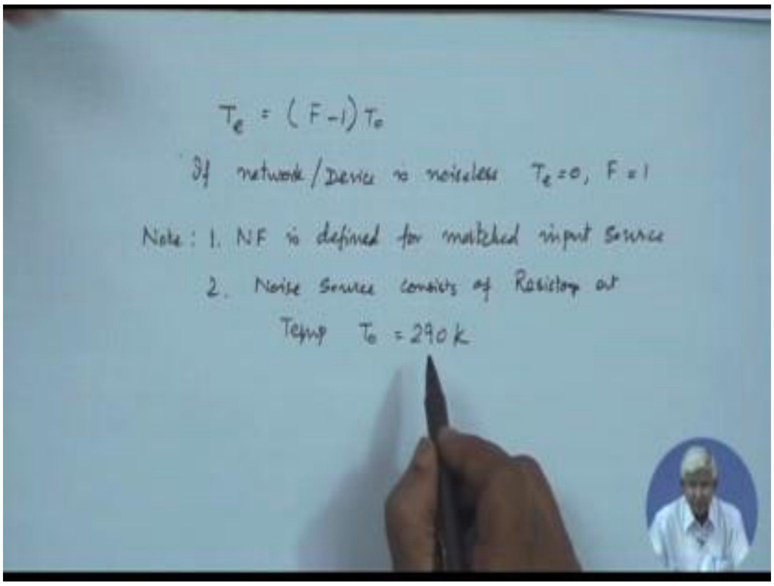

$T_e = (F-1)T_0$

If network / Device is noiseless $T_e = 0$, $F = 1$

Note : 1. NF is defined for matched input source

2. Noise source consists of Resistor at

Temp $T_0 = 290\,k$

So, we should write Te is equal to F minus 1 into T 0. If network or device is noise less then Te is equal to 0 and F is equal to 1. This note I am repeating that is noise figure is defined for matched input source that is number 1 point. And second is the noise source which is at the input consists of an equivalent resistor at temperature T 0 is equal to 290 k, some people may take it 390 k normal temperature say as if it is equivalent to a matched source with a resistance with the temperature of 290. If there is a different in noise temperature coming in then it is to be calculated with that, need to remember this.

So, we continue our discussion for the next class.

Thank you very much.