Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 35

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Okay so let us continue with where we left in the previous video so now we in the last video what we had done was we are looked into when t passes = 0 and we have understood the all three transitions matrices because we have broken a time interval in three different steps and then attractively how we can actually can keep on doing computation till we find the stability in the packet flow at every at the output of every stage and they all once they are become same we just take whatever is our computed values and then use that to essentially measure the through put performance of the switch okay so.

That was basically the method but that is one of the extremes that when t passes 0 and for selecting the packet to an outgoing port we consume the complete slot there is another extreme when t select a 0.

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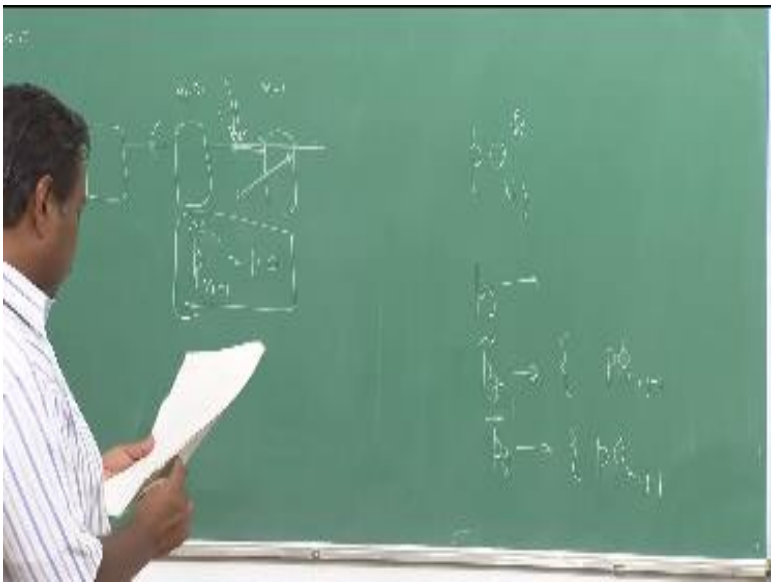

So selection that which packet is going to go to which outgoing port happens almost instantaneously but packet transmission does stay require complete full slot now consequence of this is that has we have discussed earlier that their two packets and they want go to the next in this current slot this scene is almost instantaneously only one of them will be moved out and this will require complete slot when the next time you will in the next slot you going to take a decision we can decide this.

Packet can go here this we can decide this packet can go here but I cannot move this packet here because it will still require one full slot to transfer this packet to this outgoing port so we will not be able to transfer this packet here but we will be transferring first this and when this will be empty then only I will do that action now because of this I need not spilt my one time slot into three separate steps.

So earlier I was doing separate steps because if these two packets would have been there when t pass was 0 this packet could have move and this packet could also have move so I was actually factiously created three steps there by first this packet will be exiting second packet will be coming and the next will be happening so this event was happening sing cross in every time step

for all stages so we were permitting that both of these packets actually can move so this can come here and this can go here and that is why those three steps for there but now we actually do not need them.

So as a consequence I do not talk about the steps anymore so which actually means that lot of our computation for example what is this p0 ij kv actually had computed this we had computed this we had computed this thing now remember when I was doing it I was looking into what happens after step one in the next stage there is no steps now so in fact in the equation which we had use earlier in same equation instead of using p1 whatever was the state some j + 1 I simply replace it by 0 same I am going to do for pj bar so I will just replace all once by 0 so and then we will have same terms which will be coming in denominator.

And numerator and we will get the same expressions accept one is going to be replace by 0 this was any was having the 0 this terms remain same so we will be only handling these four elements in this case and of course our boundary conditions now will slightly change in that case we have to only worry about n- second stage the movement the packet is there this is going to surely going to go out even if to or contending because it is a instantaneous transmission they will all go out surely.

So pn – in fact PJ p(n- 2) ~ was 1.0 that was mu boundary condition there but now it is no more true so the boundary condition which we have to use now in computation is not this but I will be using PJ p9 n- 1~) = 1.0 as my boundary condition okay so once we do it we can now build up this state transition table correspondingly so first one is we call it t04 so I can write it down so this is t04.

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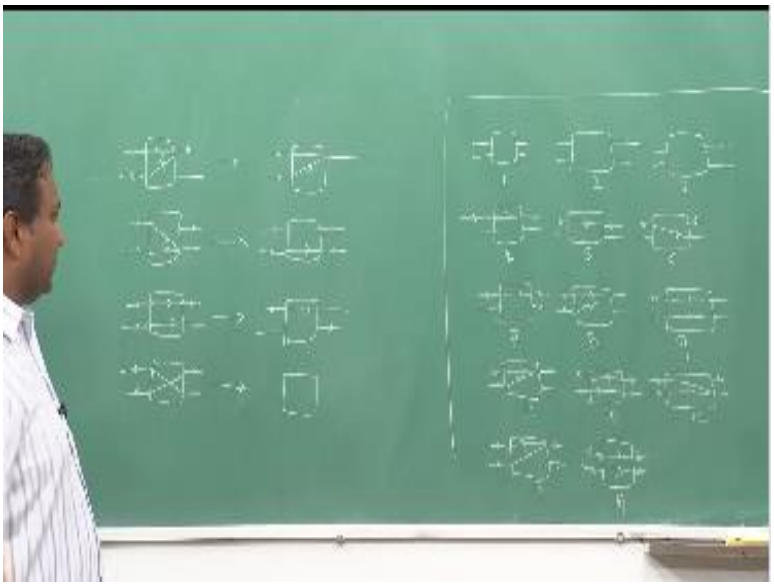

So earlier remember what I had done was I have done TJ after step one Tj after step two Tj after step three they where three transition tables which were done and I am now not mentioning j this is for only stage 0 so I am actually going to have three transition table one for a stage 0 one for stage n- 1 and other for any one of the intermediate stages okay so in this case now we have to understand when I am looking into the first stage my arrival probability that PJ Po bar is always 1.0 that is my boundary condition I am going to use that okay.

So once I have this so let me what is going to come so if you are in a state one and I am talking about this stage 0 the first stage of the network so if you are switch is in a stage one what is going to happen we are bound to get two packets in the slot you are bound to get two packets with probability one you will end up in a situation which is either nine or eight okay so this will ne happening with equal probability so it is half so I can define one okay now there is another thing because in the current slot when the packet will be coming I will be taking decision almost instantaneously at the beginning of the time slot.

And I can push those two packets also to the output so these two actually can move to this particular output that is one possibility or I can actually get into a situation only one of them goes

out and other one has to be held up here with thy because of the conflict because both of them are want to go to the same port so between three and five it will be happening with half of probability so you can go from one to three actually with half probability and you can go to five with half probability now this is surely going to happen there is no estimation which is required because this is the stage.

0 which I am talking about now if you I look into state two then what is going to happen so this is a special case the movement of a packet is available a state two again there can be two packets which can come in so one of the packet and both packets wants to go to the upper port they will not be able to go because in this current slot this is also been transmitted out this buffer is not empty so we will end in a situation it okay.

And so let me see what all things can happen so two packets will surely will come so you have a packet here so you may end up in after the end of the slot you will get into situation where these both are directed on this side then packet will be go read out and I will end up in a situation of this kind or if these two packets are directed both of them are being directed towards bottom so in that case one of the packet will be transferred here in this current slot this will be read out.

And I will be in this situation okay it is possible that I can have a scenario like this so this will lead to a situation that this packet will still remain here want to go to the free port and I will have a packet here there is a possibility that there cross over so this will lead to a situation this is at the beginning of the slot and this is at the end of the slot so I have to worry about what is going to be at the end of the slot these packets anyway is going to come instantaneously and then they can be transmitted out, so if this is the situation you will have this so these two are actually same this one and this one are two different case.

So this one belongs to let me check what is this case 8 this one is state 5 this one is going to be state 6 this is also state 6, so 8 will be happening with probability ¼ this will be having with ¼ this will be happening with ½, okay. And of course this will depend on whether this packet is going to go out or not, okay. So I am assuming this packet is going to go out so I have to write 1 over this will be Po, okay.

In state 0 actually ~ so this is what is going to happen is the packet does not go out then where you will actually so let us see, if the packet does not go out you will end up in the same situation so none of them so they will be as contending you are here so this will be happening with ¼ Q0~ this will be converted to one packet here this packet remains as it is one of the packet will get transferred, okay.

So you will come to this situation this will be also happening with ¼ Q0~ and then of course with this situation you will have one packet block here other one will be here this is the same scenario so this also happens with ¼ Q0~ in this scenario you will be having so this is also same ¼ so these three actually I can combine together so this will be ¾ Q0~ ¼ Q0~ so these are 8, 5, 6 and now this particular state is state 10.

Then we have next state this one is 11 so 11 goes by this so I can write down here just a minute, Oh! this is not 11 this is 7 actually sorry this 7, so let us put the values so 2 will correspondingly will have at 5 P0/4 which is 2 the we have 6 SP0/2, 7 I left 3 Q0/4 8 you will have ¼ P0~/4 then 9 and then 10^{th} you will have Q0~/4, okay. Let us complete the remaining once correspondingly.

So the third one in this case in the same manner so third one is we have this scenario there two packets at the output so they both can go out they both may not go out and one of them may go out actually so with that probability and then there is surely going to be two packets which are here so this is going to C at the beginning of the time slot so with probability QJ_2 none of them are going out you will end up in the same state, okay.

This one actually can go therefore possibility the both packets are staying so they can be in this fashion so this 4 possibilities are there with QJ bar square so which one of them so these two are same and these two are same so this is QJ ~ ½ QJ $\sim^2$ / ½ so this actually corresponds to 14 and 13 so this one is 14^{th} state and this one is 13 state so once you understand let me just put it down all the values.

So from 3rd I will be getting so 8th state will be there ½ PJ ~2 9th will be there with ½ PJ ~2 10th will be there with ½ of PJ ~ QJ ~ 11th will be there with PJ ~ QJ ~ 12th will be there with ½ PJ ~ QJ ~ in fact all should be 0 not J, okay and then we have 13 and 14 so this is ½ 2J ~ 2 ½ 2J~2 okay. So we can actually keep on putting it so the 4th one will come to 3 with half probability and 5 with half probability.

5th will be at the 6th ½ P0~ at 7th ½ Q0~ 8th ½ P0 ~ and 10th ½ Q0~ okay. Then 6th will be at the 5th location of P_0 ½ P_0 $Q_0^{\sim}$ seventh this actually has many terms 7th will have with 8th state $1/2P_0^{\sim 2}$ next will ne ½ $P_0^{\sim 2}$ $1/2P_jQ_j^{\sim}$ 11th P_b $P_jQ_j^{\sim}$ 12th will be ½ $P_jQ_j^{\sim}$ 13th will be ½ P_j^2 ½ P_j^2, okay so 8th will be 5 with probability 1, 9th will become 3 with probability 1, 10th will become 8 with $P_0^{\sim}$ and 10th with $Q_0^{\sim}$ so in fact wherever I put j you just simply put 0, so this is j=0, okay sp 10th and then 11th has 6th and 7th, 12th will 5 and 7, 13th you will have go to 8 and 12th and 13th term will be Q_0^2. So 14th similarly I can put down for this particular matrix as go to the 9th one, yeah.

11th and 14th term q_0^2 $q_0^{2\sim}$ so that will be the complete transition matrix for the stage 0, and correspondingly the equation which will be there it will be this at the step k+1 being in state i you will get based on earlier calculations m=1 to 14 so p_0 m stays 0 in k T_0^4 (m,i) so this will be the equation where m will go from 1 to 14. So for a tpass=0 to generic equation this is for stage 0. So similarly we can actually write down the equations for state 6 transition matrix, sorry we write for n- first stage, so we call a transition matrix 6 then we can write down 1 for in general and we call a transition matrix 5 so I am not writing down those detail value.

So I can giving a reference and the URL from where you can download those two matrixes in fact you can derive on the on your own. So what I will do is I will drive only certain cases for n-1 6 and Tj^5 so you understand the principle remaining thing either you can download or you can build up a matrix on your own. So what to build up for example $Tn\text{-}1^6$ then what is going to happen let us see.

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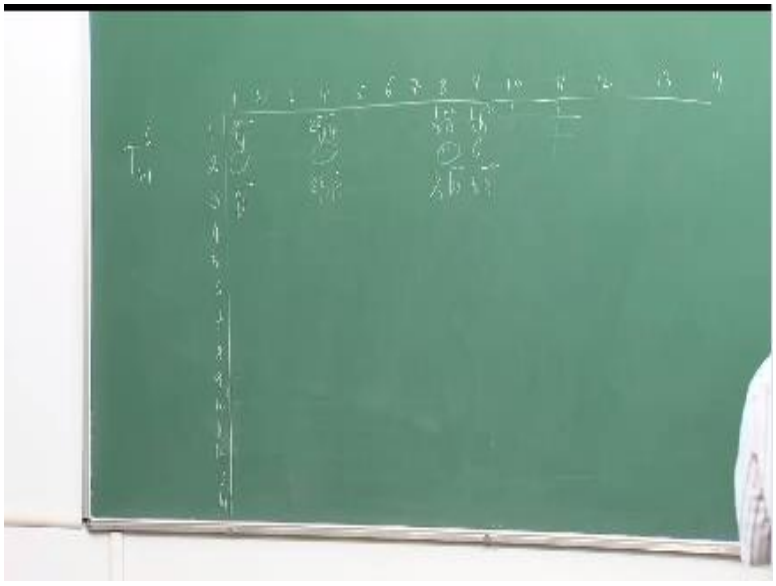

Okay, so now important thing is when you are in stage n-1 with probability 1 the packet will be going out if it is there at the outgoing port, okay. So we can take up certain cases so if you are in for example state 1 then what is going to happen, so you can actually get a packet or you may not get a packet, okay so that is the only possibility which can happen there is no packet at the outgoing port. So you will remain state 1 if there is no packet arrival which will happen, so with Qj^2 bar you will remain in state 1, if there is only one packet which arrives other one does not arrive in that case you will end up in state 4, okay.

So 4 will be 2 Qj bar Pj bar so j will be is equal to n-1 here, for this matrix, okay and then of course if both the packets will arrive then you can get into situation of either 8 or 9 with equal probability, so 8 or 9 will be ½ of Pj^2 bar ½ of Pj^2 bar. Remember there is no ~ probability is coming here because this n-1 stage. So let me take up a case where the packet is there at the output then what is going to happen.

So take a case of 3, what is going to happen here, surely these two packets is going to go out, okay in the current slot but the two packets which will be coming in they cannot be pushed out, they will be only coming in this particular slot, okay so that is the only possibility so they will be

coming in and they cannot be transferred out so these buffers are empty at the end of this kind slot you will have only two packets here. They can be directed for the same port or they can be directed for the different port, okay.

So let us look at that case, this will be very similar case if both the packets will come, if there no packets which are coming in and these two packets go out you remain in state 1 that happens with same probabilities or third one will actually the Pj^2 bar, if only one of the packers will come the both the packets which are at the output port will be moving out, so you will end up in only this situation and the same thing actually will repeat here, the second also column the same thing will be repeating the same thing will come, so we can try out for something else.

For example, you can say 5 let us see what will happen to the 5 so 5 is this so there is a packet here which is are directed here in the current times lot the packet this packet will be red out shoddily there is no blocking is so at this probability 1.0 pn-1 ~ is 1.0 okay so only one packet will come and me it not come if it is not coming you cannot transfer this packet to the outgoing side.

So with probability qj you will end up in a situation which is 4 because this packet will read out in the current times lot if the packet arrives this can be directed to this as well as to the next one, okay so will be coming in this current lot so this cannot be transferred at the same time in to the outgoing side so you will end up in a situation so if the packet does come so this can be this packet will be going out.

So you may actually get with pj bar there is possibility of having these both coming on this side or this both going on different sides okay, so you will have only 3 possibilities so from 5 you can come to 2 in this passion so this will be qj bar and these 2 possibilities which corresponds to 8 and 9 no 5 and 6 sorry so at the beginning of the slot this probability of the packet that will be arriving.

So in the current times lot packet can actually move the out so one of the packet will be moved here in other case this packet will be moved here and moved so you will be in one of these two

states, so 5 or 6 will be there so with half of pj bar and half of pj bar this is what is going to happen, okay so you can actually build up this matrix you can actually see this on page 280 of an article which came in I triply transitions on computers.

Volume c 30 number 4 April 1981 so in this journal on page 280 this actually 3 matrixes are given so this particular paper is also available on sorry this is wrong arrival so you can actually go here so go to this particular you are l go to the apathy 3 which is mention there go to the open axis courses or you can actually registered yourself as a stinting course and digital switching and go to digital switching and in the course content you will actually find out I have actually applauded this paper.

And you can down load and look at this particular page for the table okay now just quickly for when table is for middle stage it is not the last one in the first one then you will be having both pj bar and qj bar and pj~ and qj~ all for variable which is actually there in lot of expressions. So we I need actually case were the packet it here it is the output so I'm will only look for the second one so this is I am talking about t5 j I will be looking in to this particular expect if I am in a state 2 where I am going to go.

So I am giving an example this also given in the same paper so this one is situation is this is the packet situation which were there in the current times lot at the beginning the 2 packets will arrive okay so once the 2 packets will arrive they will be selected and they have to be pushed out so 2 packets may arrive may not arrive actually, so you are in a state 2 so with probability qj^2 you will get this one only and with probability pj now on this probability of pj~ the packet will go out you will end up in this state with qj ~.

The packet will not go out you will end up in this particular state oaky there is a possibility that one packet will come so one packet which can come is here and in the current times lot this will be a red out and this will also can be transferred depending on if there is no conflict, so if this packet is directed to this side so this can be directed to both up and down okay so this is pj bar and qj bar okay with half probability it can be down and with half probability it will be up okay.

So if that happens so if it is down then this packet will be red out and this packet will be there so you will end up in and this packet maybe red out may not be red out so with probability sorry this should not be yeah this is square so with this probability pj bar qj bar and if this packet move out pj~ you will end up in a situation where there is no packet you have a packet at this side you may actually the packet does not go out with this probability you will have a situation when they are 2 packets of the outgoing side okay.

So with this half this will be happening with half of pj, qj bar you might be having a packet directed towards the packets which is already there so now this also will have 2 possibilities if the packets is goes out you will have this situation if packet does not go out you will have this situation okay, if both the packets will come so I am actually numerating all possible cases if both the packets are coming okay they both can be directed to the different ports.

So cross and bar not matter so this will happening with probability pj bar half and there is going to be a case when both are directed to this, this will be happening with 1/4 pj^2 and that is going to be a case so when it is directed to like this 1/4 pj/2 okay so in this scenario the packet this packet can go out at the end of times lot and this packet can come on this side so packet will go out may not go out this situation okay if both the packets will come so I am actually numerating all possible cases if both the packets are coming okay so they both can be directed to the different ports cross and bar does not matter.

So this will be happening with probability pj bar1/2 and there is going to be a case when both are directed to this ,this will be happening with ¼ pj^2 and there is going to be a case ,so when it is directed to like this ¼ $pjbar^2$,so in this scenario the packet ,this packet can go out at the end of time slot ,and this packet can come on this side .

So packet will go out may not go out if packet does not go out you will end up in having this situation ,okay so if packet does go out you will end up in having there is no packet on this side and there is a packet on this side directed, this scenario in this case again the same way if the packet does not go out you will end up in getting ,so there is packet has not gone out both of them are directed to the same port, so they will remain there ,you will end up i this.

If the packet goes out you will still have both the packets with stuck and same in this case so this packets will not go out it will remain there one if the packets will come here, other one will be contenting with pj~ if the packet goes out, okay so this there will not be any packet this packet will be contenting, i have to just now numerate by actually multiplying the probabilities.

So this is state1 this will happen with pj~ qj, things let me build up a ,so this one we have been doing it for 2 ,okay, so we have been doing it for 2 so from 2 I will I will come to 1 with this probability qj bar square pj~ so I can write ,qj bar square pj~ okay ,now from here if I want to come to this state from ,I will come back to 2 so anywhere else I am repeating to know I am not having 2 anywhere else ,so this will have a probability of qj bar square qj~ .

Now look at this particular one so I have to keep on adding this state 2's so this one is state 2 ,this comes out be this is also state 2 ,there is no other state 2 so I have to add these 2 ,so this will become $qj\sim^2$ qj bar square qj~ +1/2pj bar sorry this one the packet has to arrive in this slot ,this whole calculation which means wrong in the sense.

In the same j^{th} slot this packet has to arrive this cannot be moved out on this side ,so this has to end up in this particular state ,okay so it does not matter whether it's going on which side so i have to just redo this. so this one is correct, this one is correct right, so this one the packet will be arriving here, it's not already present here, so packet will be arriving in this current slot so this can only be directed to either the packets which is going out it does not matter where it is directed, we will end up in this state.

And with probability qj either packet does not go you will actually either have this scenario or you will have this scenario ,okay, only these two scenario will be there right ,so the next one a packet arrives here with the ½ probability so this packet can go on this side or it can go on this side these are actually same right ,so this state 1 this is state 2 this is state 4 this one is state 5 this one is state 6,this one is state 4 sorry and this one is state 6.

So this is for this particular combination from here so this will not be going out this will be remaining here ,right, so this packet will not be there ,so this one is state having 2 going on different side ,this eleventh the next one is ninth, next one is tenth this one is tenth ,this one is twelfth ,and this one is tenth again ,so i have to just add up all the values and then put in .

So state1 corresponds to qj bar square pj~ which is perfect , I have put in there this on turns out to be state 2 is not repeating any where its qj bar square qj~ ,which is perfect, okay, now look at state 4 these two terms have to be added ,so 1t is ½ pj bar qj bar pj~ ,from this side if I go 1/2pj bar qj bar and with qj~ ,oh sorry ! This is also qj~, okay, so you have to keep on solving in this fashion, ultimately put up the values of the matrix.

So once you have this you have to again do the same iteration until you get the steady state flow and based on that you can find out what is going to be the throughput performance, so this actually complicated but so I am leaving it to you to offensively doing at on your own ,so once you have understand the concept you can always start analyzing buffered system .so infect there is no close form solutions we actually do the computational procedure to estimate it .

And of course the through put is going to be better then what you're going to go with the un buffered ~ and as you keep on increasing buffers it keeps on improving but the incremental improvement reduces with that, so, with that we come to the end of the course on the first one these are basically theoretical basics on digital switching, we probably will have a digital switching 2 also where will be talking about actual implementations of the system, basically voice over IP telephonic.

So there architecture, so see you next time in the second course so which it will be offered very soon thank you very much

Acknowledgement

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