Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 27

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So in the earlier video we have looked at the output queued system the we have estimated the waiting time at the output q by taken in two different components and analyzing it thoroughly we ultimately figured ultimately figured out that when the size other switch becomes infinite thr output q will behave as if it is a Marcoviean arrival process and is a dimes departure one packet going per time slot there is only server in this because I am talking about every output port separately and the buffer sizes infinite okay.

But if it is a infinite size switches then n- 1/ n will also get multiplied in the delay so that actually kind of gives an understanding or insight into the output queued system but remember you have operate the switch at n times the speed of factor okay n times is speed of factor we have to use so in one slot when a packet will arrive at the input or one slot when packet will go out in the same time period you will be able to transfer n packets from input to output port so switches operating at n times the speed so now we will talk about when my value of k the speed of factor is one it is not n.

Which actually means packets cannot pass through the input q if there is a contains only one of them will be able to go to the output port so what does actually I mean.

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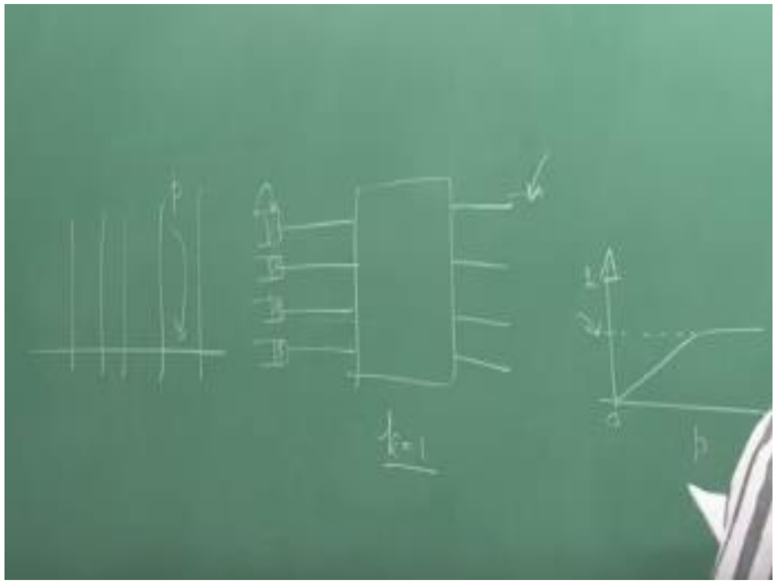

By this is if they are for example a situation like this I am taking 4/4 now remember you have to understand now that k = 1 the speed of factor is unity so one slot one packet comes one slot only one packet can go from input to output not multiple of them so if there is a buffer here is important thing here you do not require buffering here you require buffering in the output queued system only output side buffering will be required okay.

So if there is a packet like this there is a front packet so I have 1 1 1 1all four of them want to go to the outgoing port one what will happen only one of them have to be selected and it will go out and will be transmitted so the next time slot some new packet which will be jumping to the next thing and these packets will still remain there okay and these are containing packets now let us try to analyze this particular switch so first question is what is a maximum possible through put which I can get in this particular switch.

What is a maximum possible through put so when this is going to happen this will happen when I am going to operate the switch in saturation so the control variable for this witches the probability that a packet is going to arrive in a remember to a time is slotted they have many slot every slot a packet will come or may not come for an input so I define that packet is going to arrive in a time slot for an input this probability is given by p okay so this p is the load on the line as this space increasing.

I keep on increasing p so my output packets which are going out will also start increasing at some point I will not be able to increase it any further in case of a output queued system when p will become one output will also will turn to become one okay so it does not saturate output queued system does not saturate you can operate the switch even with when the probability of arrival is going from 0 to 1 but here even before I actually look at the output utilization and if I change my p from 0 to 1 after some value of p.

You will no more be able to go to one so output will be having less number of packets per slot coming out and it will saturated should saturate after this you cannot do much now the question is what is this value how to find it out for a input queued system so let us investigate this particular case so now important thing I have to now draw a timing diagram usual clarify the whole issue.

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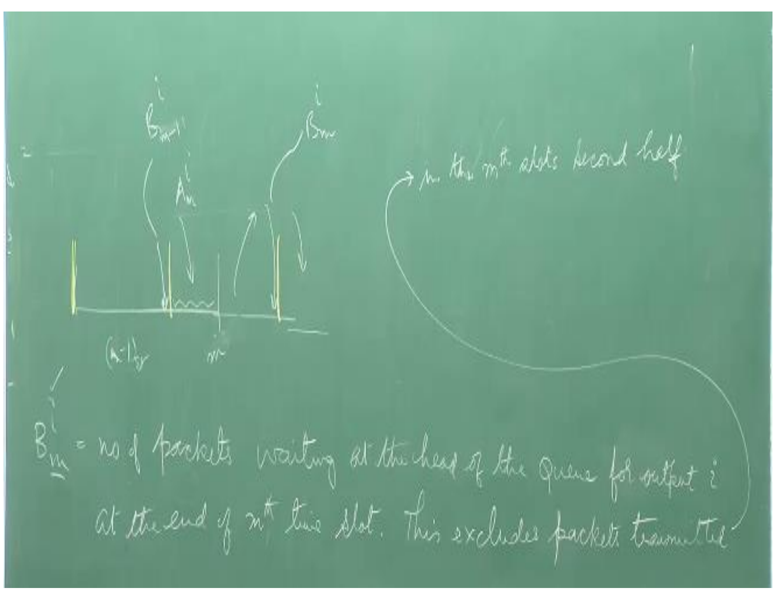

So everything is happening in terms of time slots the way actually I am assuming that in these just before this, this is the I – first slot this is the i^{th} slot the way it is actually operating at the end of this slot just before that time period ends how many packets are there I only looking at the packets in the front of the q I am also assuming that the movement you transfer a packet to the outgoing thing immediately there is a packet always available behind it which will come to the front.

They will never ever be a case when a packet is pushed out from input to output and there is no packet available packet is also available it will be coming back to the form of the q so at any point of time you will never see the front of q in any of the inputs to be free they will always be occupied it will be happen immediately but the way it will be happening is this will be the arrivals, this is all these packets which will be coming from second position to the first position will be happening here I call it Ami sorry this is m slot not i and this is the so I will actually use a marker for time with the yellow one okay and then what will happen is the packets will they allow to move out, out of all contenting packets so they will be going out.

And at the end of it, then there will be some input slots which will be available because the packets are moved out in this half part okay and then what will happen is in the next time slots half part the packets on second position will move to first position so switches always in saturation so okay this is what we call operate the operating the switch in the saturation resign and basically what I want is what is going to my number of packets per unit slot in the outgoing port so we have to define certain variables.

So I will define them like this first fall I will define a variable called bmi okay let me write it down here so what I am actually doing is I am looking at the add of the q's and this is happening just closer after the packets have been transmitted out okay here so the number of packets so some slots will be vacant whose packets have moved out to the outgoing port the arrival from second to one that movement has not happen I will look at these packets which are back logged for that outgoing port.

So for one this and this for example is back logged for to this is the back logged so I will sum these up and that is what is going to happen this is the end of the m slot how many packets are their which are a still pending at the add of the q for port number i that is what it actually means so I write it down number of packets waiting at the head of q for output i at the end of m^{th} time slot so you have to understand n terms of this timing diagram so BMI is actually at this point is BM -1 because this is m-1 slot, okay. So EMI is a new arrivals than departures and this value will be now BMI and of course this will exclude because packets are going out before you are estimating this, so these packets which are moving out to the outgoing port are excluded this excludes.

Now this is a kind of a convenient for my own convenience I assume this assumption actually this happens simultaneously but this actually makes it much more easily method degrade track table thing that within a time slot different events, the sequence of those how they are happening I have actually identified in the market, so now what is going to be ai? I also need to define this so let me write it down here.

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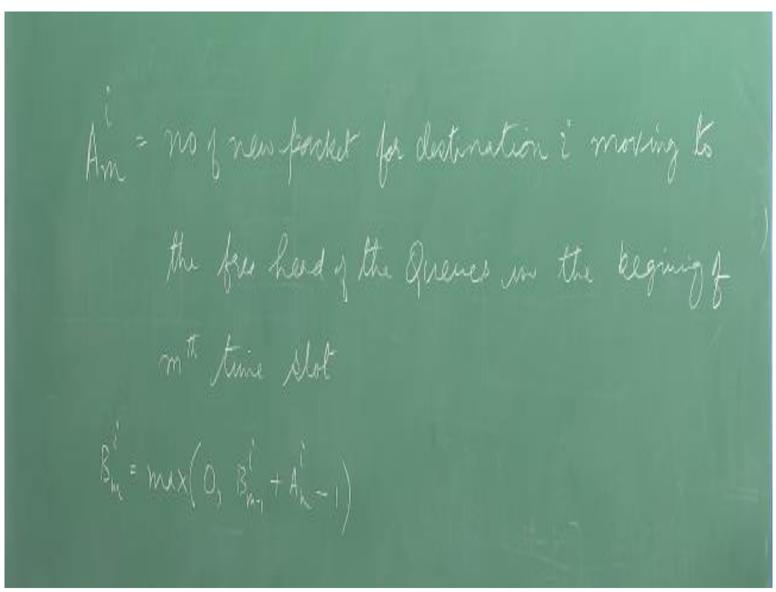

These are number of new packets foreign destination I coming in the n^{th} time slot, so for different destination different arrival total number of arrivals which are moving from second position to the head of the queue will be some of these over all possible I is actually, okay. So number of new packets for destination I moving to the free head of the queue in the beginning of m^{th} time slot.

So that is what is AMI, now one you have this I can very well write that whatever is my BMI this should be equal to maximum of zero whatever was there before at the end of the m-1 slot, okay. Plus there where new arrivals from second to second position to the first position in the to the head of the queues -1 because for an output only one can go out the maximum of zero these two that is what should be new value of BMI.

Now this equation is very similar to what we had used in the output queues also, okay. That is the kind of batch arrival, okay. Here also batch arrival but coming to the output of the head of the queue, there it was a batch arrival going to the output queue. So I can very well write down if this is a expression I can actually use the same value same methods, same expression which I had use earlier, so kind refer to the earlier video for this particular thing, so we will end up in getting.

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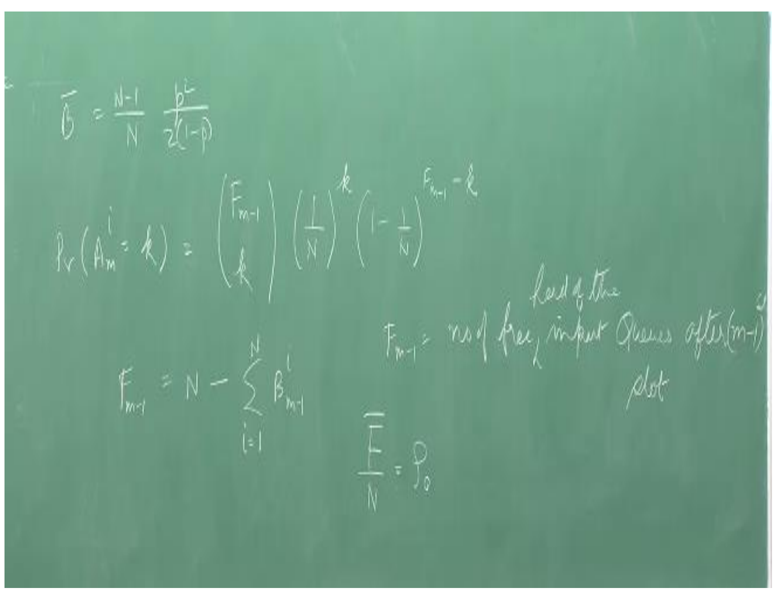

B bar will be given by N-1/N, okay. Into $b^2 / 2(1-p)$ okay so that actually essentially depend on this statistics so we will find out what is going to be my statistics which has to be used here, so what is p and what is N that we need to identify. And especially when N will go to infinity you will only have this particular thing, so we have to also look into what is a probability that A_{mi} = k.

Now I do not have a fixed number of input terminals where packets are coming in they are call going out and out of that only I will be going to an outgoing port, a number of packets which are coming to the input to head of the queue that itself is changing it is N every time, okay. Is changing every time depending on how many free out of the queues actually where available. So based on that then estimating what is the value of how many number of packets which are going to be directed to an outgoing port, now it is a tricky business.

So I have to use another variable here, I call it at the earlier slot how many slots which where free, how many head of the queue which where free that is F_{m-1} out of that k packets where directed to an output port I and hence with equal probability the packets can be directed it is 1/N which I am taking, so that is what will be the PDF, now what is F_{m-1}? These are total number of free slots at the end of the m-1 slot, so this should be N- N is total number of incoming ports minus whatever which are already occupied so B_{m-1} I that is the occupied slots occupied head of the buffers at the input side for an output port I.

So if I submit over all outgoing ports I will get and take a difference I will get this value so this should be B_{m-1i} I goes from 1 to N, okay. So this is some of the packets which are blocked for an outgoing port I and summed over all possible outgoing ports so whatever you remove this thing out of this those are the those many number of packets she will be coming to the front of the queue, so F_{m-1} is now is going to be written as, so let me formally write it number of head of the input queues after m-1 first slot, okay.

And these are the also queue which will have the new packets these many number of things so you can actually write F – bar by N that should be the utilization of the line at any point utilization is technically how many packets so for any outgoing line this should be the load these many packets which should be going out, I need to find out what is this ρ_o okay so if I can find out the average value of this then divide by N that is a fractional utilization for N line for the whole switch, okay. That will become ρ_o so you can the F bar will be can be expressed as let me write it here.

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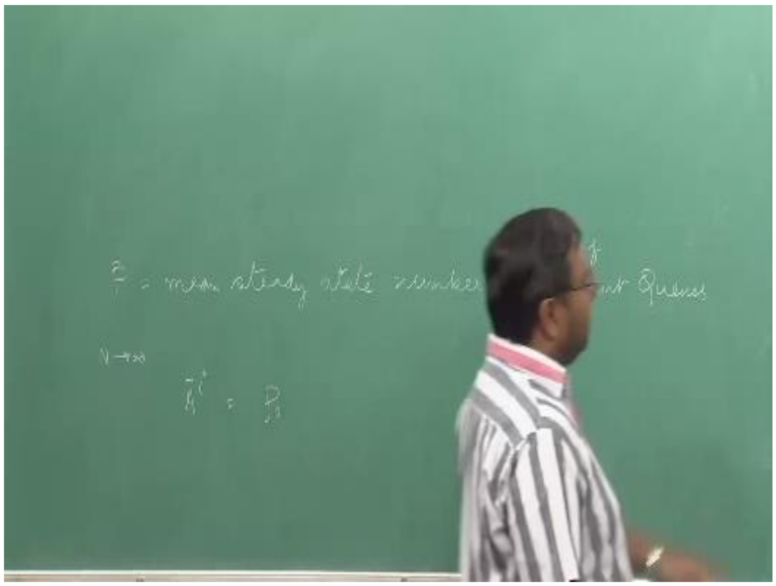

This is a mean a steady state number of free input queues , free I think heads of input queues so that is the F bar, so ρ_0 will actually will be given by this. Now in limit when n actually goes to infinity then what will happen. So a steady state number of packets which are moving to the head of the free input queues and distant for output i, okay so that will be A^i so that is a average value essentially over time this is a mean steady state value, this will become Poisson with value ρ_0, okay so that what the utilization which I am talking about this is the ρ_0, okay. so once this is happening and when N limit when I am also taking N goes to infinity.

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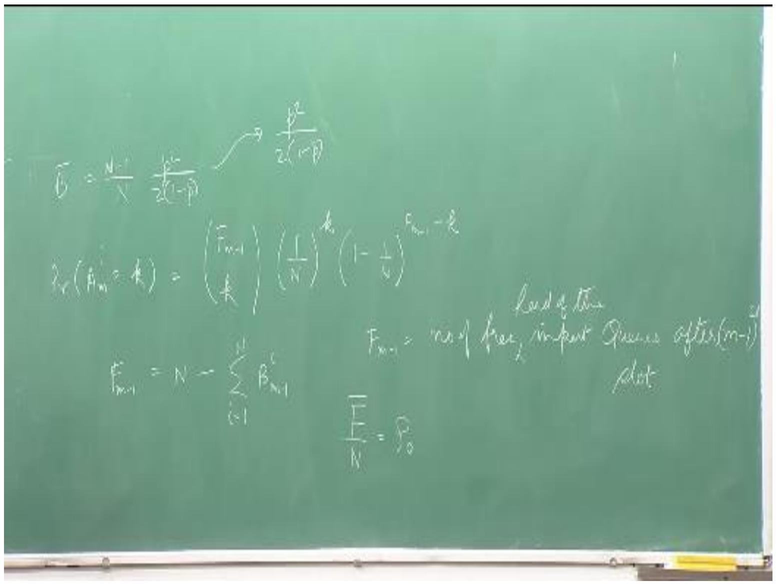

This term will now get converted to $p^2/2(1/p)$ and of course interestingly also I can actually to use this expression and get.

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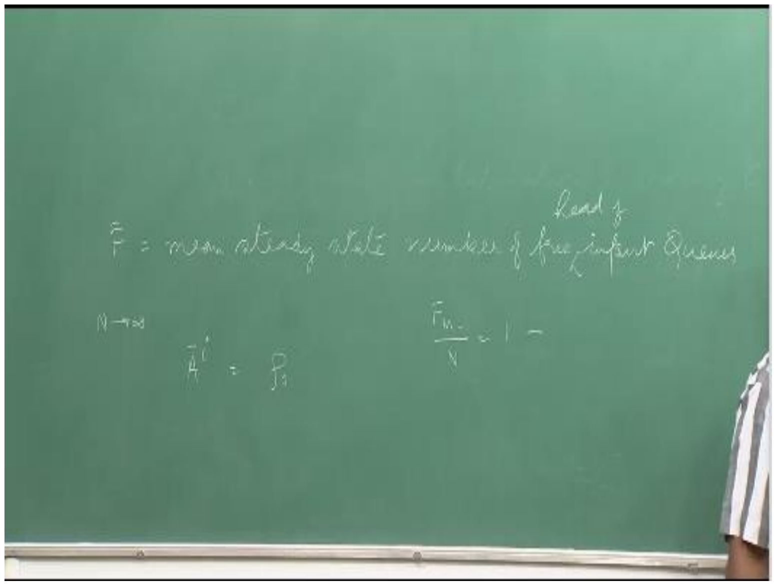

F_m-1/N 1- this summation which I am doing for N if I take knows the average value steady state average of this multiplied by N should be equal to this summation.

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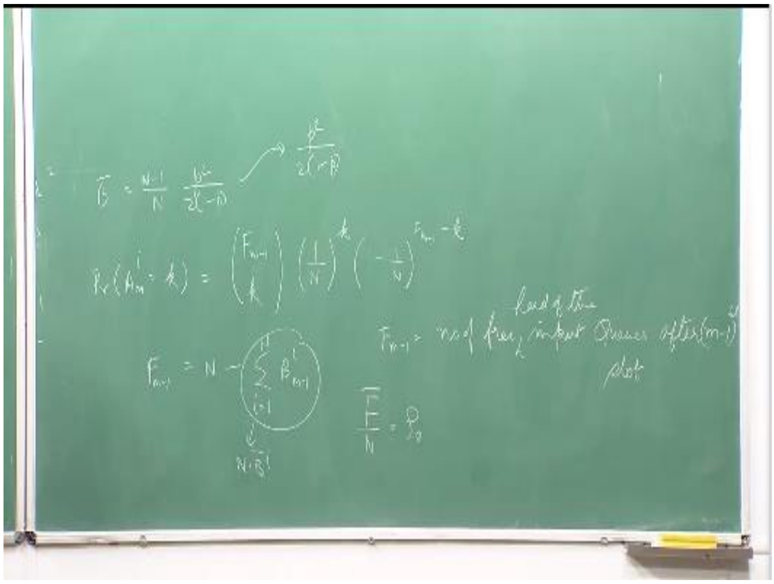

So this should be N.B bar, okay. So N.B_i bar so I can write this as.

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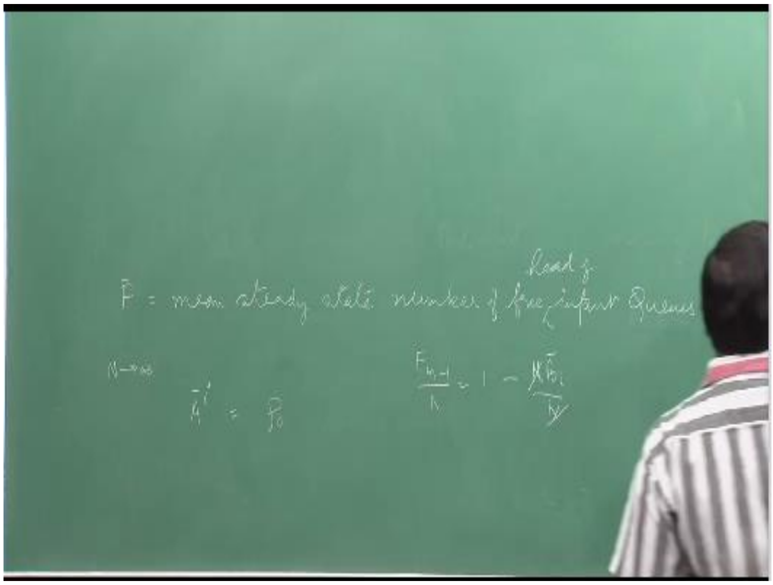

N.B_i bar/N for this what it will be, okay and we have already figured out that my B_i bar.

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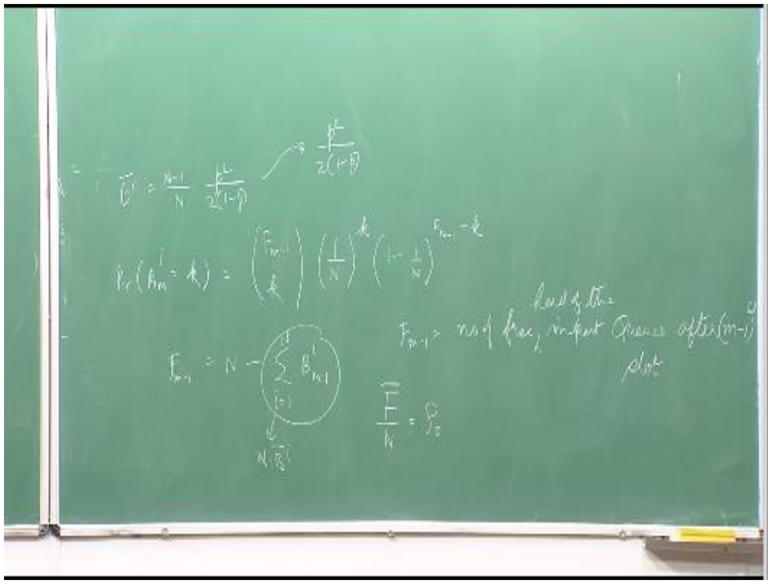

Is this value n limit when N goes to infinity where p is nothing but technically the utilization of the line.

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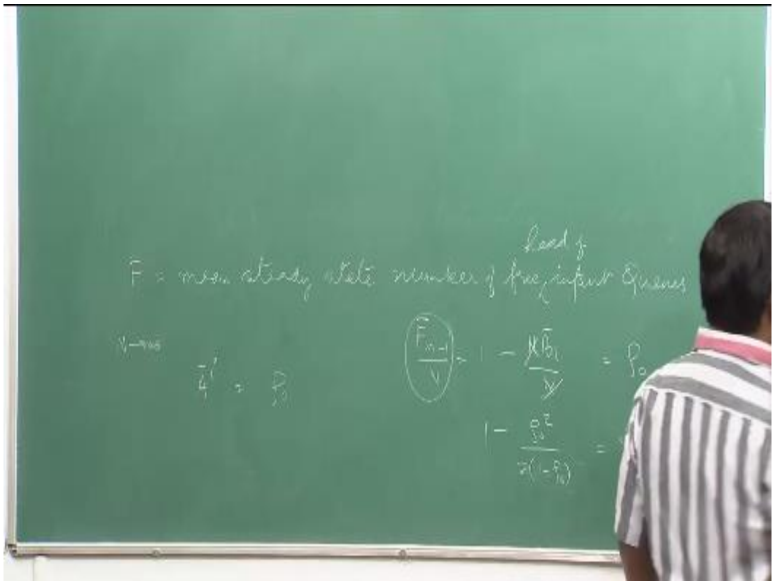

So that p I can replace by ρ_0 so I can write down the expression. I can write down and of course this on an average bases when I am doing averaging F bar by N this value turned out to be ρ_0 F bar/ N this ρ_0 I have already estimated so this will be ρ_0 and 1- I am taking the queue from it will be p^2 by, so now when I am making p=ρ_0 p is the general expression ρ_0 is the value or the loading of value of p at the saturation condition.

So I am just replacing p_0 by ρ_0 I am looking this is the condition to saturation so this should be equal to ρ_0, so this will give me the value of mu output utilization of the links under saturation condition that is the maximum throughput which you can achieve in a input queued system. So this value once we estimate.

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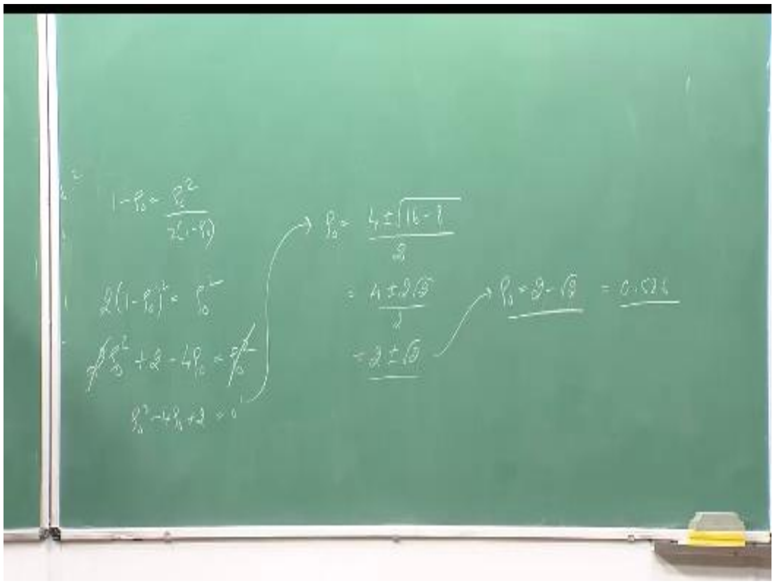

This value will be, so I can just actually use this $1-\rho_0^2/2(1-\rho_0)$ this 2 into, so $2\rho_0^2+2-4\rho_0$ so this what will be the expression, so this one goes out with this so you have $\rho_0^2-4\rho_0-+2=0$ so this will give you ρ_0 will be equal to $4\pm\sqrt{16-x}/2$ so this will be, now ρ_0 actually cannot never ever be greater than 1, ρ_0 is the probability so the second solution actually does not make sense, so the solution of ρ_0 will be $2-\sqrt{2}$ this is the maximum throughput which is achievable in an input queued system.

And this is roughly the value is 0.586 so maximum utilization can go to up to which you can actually go in an input queued system is 58.6%, okay and remember this is the condition when switch is of infinite size and by when N goes to infinity, so that is the maximum which you can get. Now the question is can you get a throughput which is higher than this, this is what you can get under saturation you keep on increasing your p as I told in the graph.

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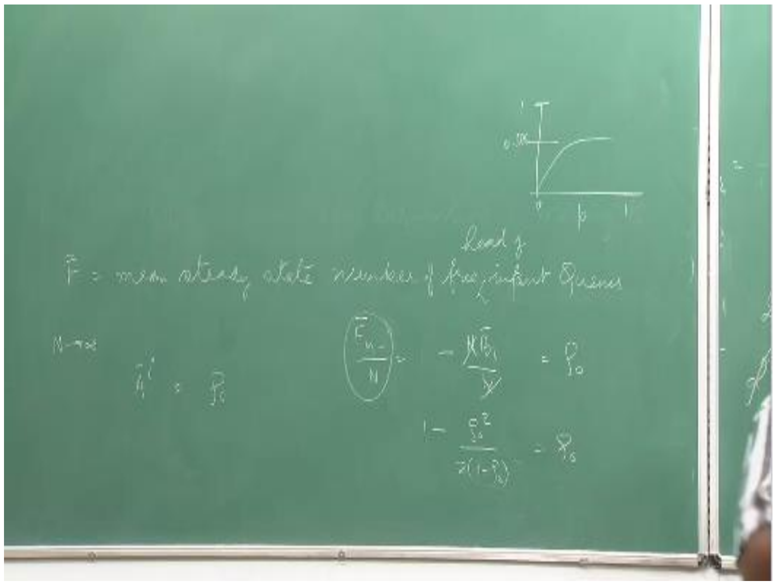

You keep on increasing your p it increases then latterly it limits at 0.586 it never be which is to 1, so you are actually bombarding so there is no use of pumping in more and more traffic into the input queued system, once you have reached to 0.586 so roughly about 0.586 after that even if you try you would not be able to go through, your queue will essential get clocked, they will build up to infinity your outgoing maximum possible throughput is 0.586 so your input has to lower than that for a stable input queue.

So input queue will become unstable, and now we should actually ask a question can I do even better than this, is it possible. There is a way now if you look into input queue can I relax some condition what I am actually currently doing is if there is a hat of the line blocking and just letting the packet stay and then I am trying to ask them to go out in the next round, so some of them are coming in so that is actually leads to limitation of number of packets which can come to the hat of the queue at every time slot.

And that is 0.586 that is ρ_0 and that is what lead to your limitation, so can I do something that if they are many packets which are containing for an output, one of them will be pushed to output let the remaining one will be dropped. So what will be the throughput in that condition, so let me make an estimate of that.

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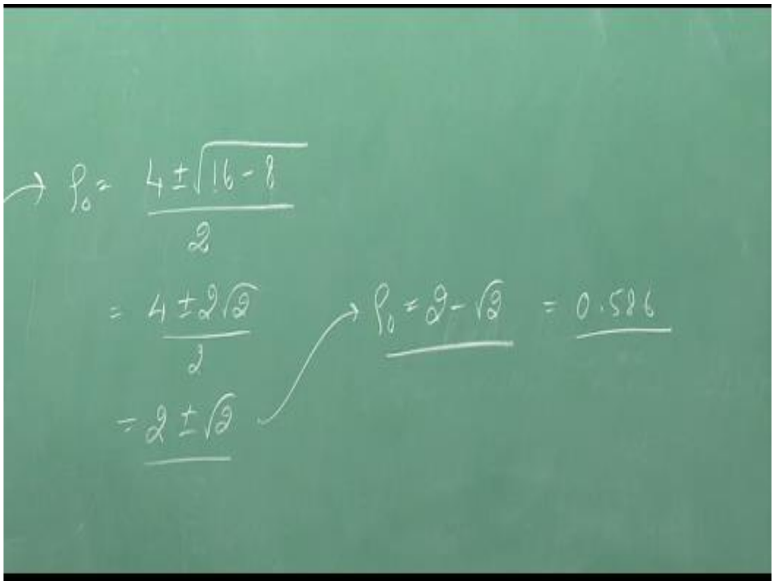

So in that condition and then will compare the situation figure out if I can operate the switch even better.

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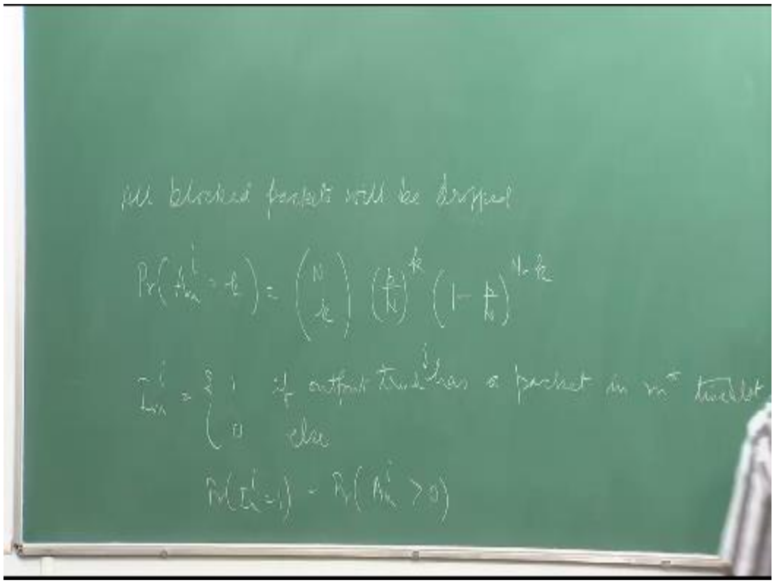

So all blocked packets will be dropped that is our strategy which will take up, so I will define. In this case every time the number of free head of the queues will be always equal to N, it will not be changing so which actually implies that probability that you will get in after in the nth time slot for an output i=k I will not be using F_m-1 I will using now N because all blocked packet actually have been dropped, so I will get N k this is actually NCk just writing in a different way P/N probability the packets are coming in their are been directed to one of the outgoing port.

That is 1/N probability so P/N is getting a packet at an outgoing port k $(1-P/N)^{N-k}$ so is arrival probability we define a variable call I_m^i for an input this will be , if output trunk I has a packet in m times lock else it will be 0 so you have to just find out what is a probability that mi =1 so that is a through put of an output link okay so that is a through put so this is nothing but the probability that mi is > or = to in fact it should be > 0 that is good enough. So this value will be nothing but.

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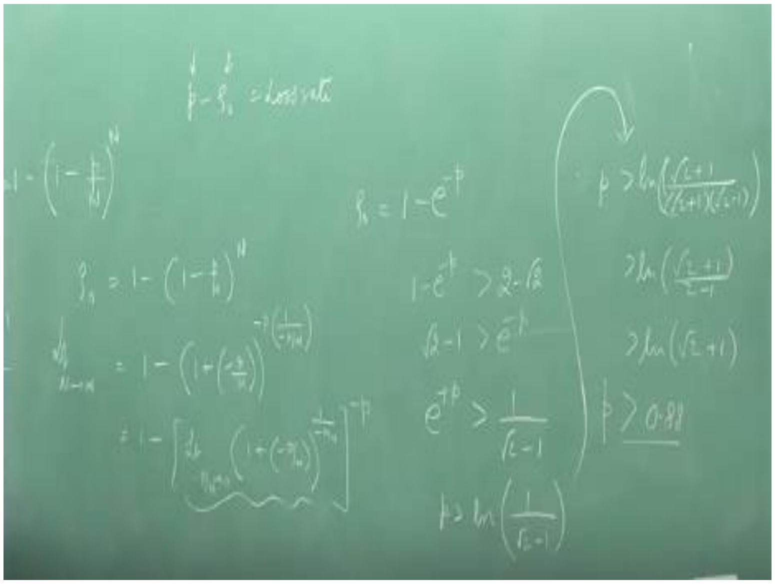

1-p/n that is a probability that packet will not arrive it does not arriving any one of them that is this probability and 1- of this at least one will be arriving or mi will be greater than 0 so this is the probability okay so put trunk utilitarian should equal to this so which implies that ρ_0 will be 1-p/n this power n and since the earlier result which we have computed this for infinite size switch when n goes to infinity let me also take.

So that I can compare what happens when n goes to infinitive for this case okay so limit when n goes to infinity I can write this thing is 1- so 1+(-p/n) in fact so when n goes to infinity p/n goes to 0 actually so I can write here –p 1/-p/n so this can be written has limit p/n goes to 0 so this is nothing but an expression for explanation so this results 1 e^{-p} so ρ_0 has to be equal to this so when this value when this loading condition for a given p this is an out going through put this is the probability level p.

So when this through put is going to be higher than the maximum achievable $2^{-\sqrt{2}}$ so let us find that thing out so 1-e^{-p} now you have to understand ρ_0 is a through put is there link out relation at the output side p is the input arrival rate probability okay so if I for this condition to be troll if you solve it you will get √2-1>e^{-p} e^{+p} >1/√2-1 which is going to be p>natured log of 1/√2-1 and this will turn out to be that p is >.

So this w3hat will be the value now log of √2+1 if you compute is 0.88 so whenever your p actually crosses 0.88 if you can actually get in to a condition where by your through put this through put is more than the through put given by which for the saturation condition before there is a through put will be smaller so which actually and of course now you have to understand p- now ρ is the outgoing rate p is a incoming rate so whatever the different that must be the loss rate. So these many packets per unit time per port we actually will lost at the input so with this we can actually understand that how those switch will be operated so let me now put it down.

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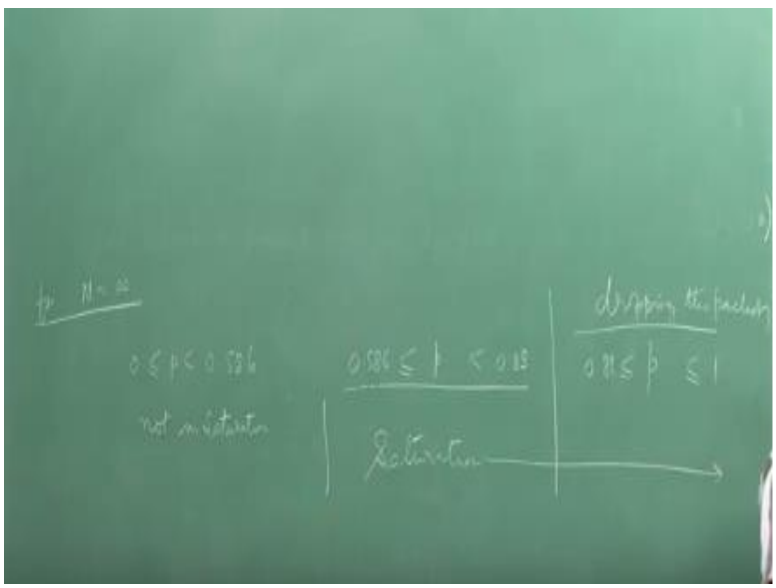

So for n is equal to this only for n is equal to infinitive remember so when your probability is p 0.586 switch is actually not operating in to the saturation it will just res saturation this is operating normally after this if your p is 0.586 but less than 0.88 you r switch is actually operating in saturation in fact after this it is operating is saturation now all throughout but for this doming dropping the ad of the queue is not is does not make sense.

Because we have figured out through put will only improve when you are dropping the packets at the ad of the queue only when p will become greater than 0.88 oaky, and that is now the queue is actually unstable at this in this regime because your p is more than the departure process so you can start dropping the packets at the input so you can have find at q is not find out because when p is more than this the q will build up to infinity.

So the moment actually p crosses 0.88 in this zone then you will start accepting a practice of dropping a packets okay accept one for all other containing packets will be drop form the ad of the queue so instead of the adding of the output of the end of the buffer now you start rubbing from the buffer so every time new setup packets will come you require less amount of the buffer size and you operated hare through put performance of course loss is still going to be happen okay.

So this hare is should actually operate the switch with this now we do understand what is the difference between the input q and the output q remember what we have done is we have started with the cross bar which is not has being use with the packet switch now cross bar actually has limitation and needs a controller, controller has to realize all the headers decide on the operation of the switch in every minute time lot inside the switch this become compare some typically if my speeder factors are high okay.

But if the speeder factor 1 this what you can get at most you start dropping and then your probability and you value your expression will be 1/ e^{-p} okay after 0.88 is actually further improve when will p will equal to 1 so maximum through put will get is 1/1-p that is the maximum which you can get okay this is the maximum value which we can get when p will be equal to 1 so with 0.88 when p is 0.88 you will switch over you will move to this is the maximum through put which is lower than what you will get for the output queuing system.

Now in the next video we have to look in to a scenario because I cannot keep on using cross bars size is a limitation I want to build up I have a through put performance so adder processing time is the bottom link so now we will look in to how to the multi stage inter connection networks actually can be used to build up a packet switching system of course now one of the important thing we normally would like to have we do not want a single procedure doing the job so we will start to build up a distribute system okay so next video will look in to this, thank you.

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