Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 26

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So in this video now we will move had from where we left in the last one so we were suppose to do now the computing, computation of waiting time.

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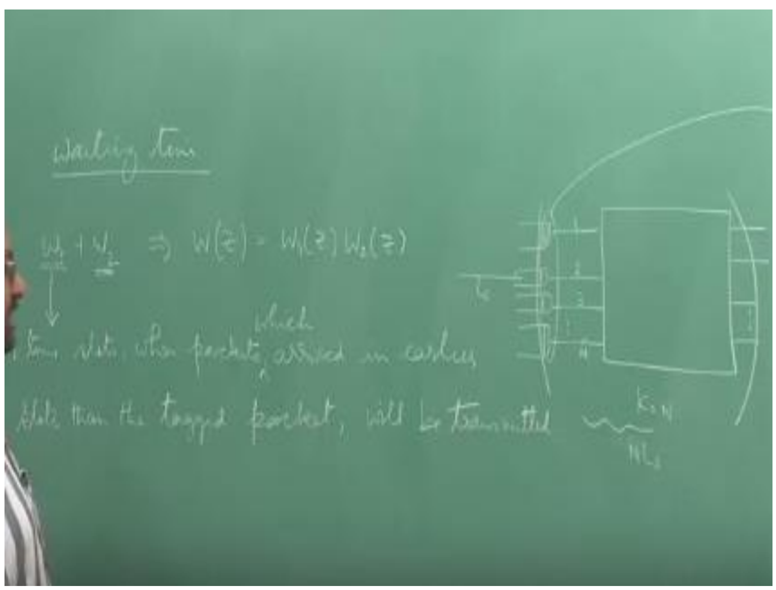

Okay and of course for completing the context we were actually talking about and an output Queued system so the packets are coming here but there can be only one packet at any point of time and my speed of vector I have taken to be n so in this it will be four because a 4n = 4 in this case in general if their large numbers then this will be n so even if all packets which are they are at that front they want to got to the same port they can be moved out in one single slot because internally switch.

Is operating at a speed which is n times the speed which is they are on the line speed actually so the speed which is operating it is n times the line is speed so all packets can be move to the outgoing thing so no packet buffering will be required at the inputs at the output we will have all the packets all n packets will come up in one go so we actually had estimated what will be the q length earlier and that was given by q bar is = n – 1/n $p^2$21 – p okay and of course I told that this expression is for.

MD1 ∞q okay and when I am goes to ∞ this q length will behave as if it is MD_1∞q so where the arrival is Marcoviean remember so packets k had been assumed to be arrived at any particular movement but in reality packet only arrives in time slots they arrive only periodically and we define we do not have arrival rate we define by set by p so p technically works as good as an arrival rate here okay so now let us look at the same thing this is the q context and I have to find out what is a waiting time.

If I take packet I actually mark here I put a marker on this and I observe this packet so what is going to be by expected delay so that is the question so what can happen to this packet this there will be w1 time slots so when this packet third packet is going to be moved so for example to this particular port there must be already packets available which would have been transmit in the earlier time slots okay they were already would had been there then when this third packet will be done it will be also then maybe other packets.

Which may also we pushed along with it so delay will kind be consist of two components so 1 is w_1 we call it so in fact total delay waiting time will be $w_1 + w_2$ so w_1 represents is technically w1 time slots when packets arriving in earlier slots then the third packet will be transmitted so w1 is that part now what is w_2 now since there is a batch there is a certain number which will be going there so they will be added so they will be added now this was the earlier once so this constitute w1 now there is this batch.

And say there is going to be I packets which are arriving your third packet can be any where it can be in the front it can be anywhere and the w_2 component is because of this okay so we need to understand what is going to be the value of for w_2 so w1 is very much clear is basically I have to find out what is a average q length and remember w1 and w_2 both are independent and w1 will depend on the q which we had computed here this q bar essentially will give this so I know the w1 the probability.

Generating this actually has come from probability generating function qz which was $1 - p_1 - z$ $A_z - z$ so from here actually we got this q bar so this will ultimately will also be the probability generating function of W_1Z will be given by this now you have to understand that these are two independent random variables this actually implies the w (z pg) f (w) will be proportion will be w_1(z) x w_2 of z okay so whenever the random variables are added you actually multiply their probability generating functions to get the final probability generating function okay and W_Z is nothing but z transform of W_1 $_{W2}$Z receive transform of w_2 okay so now let us try to find out what will happen to W_2 how to find that thing out okay.

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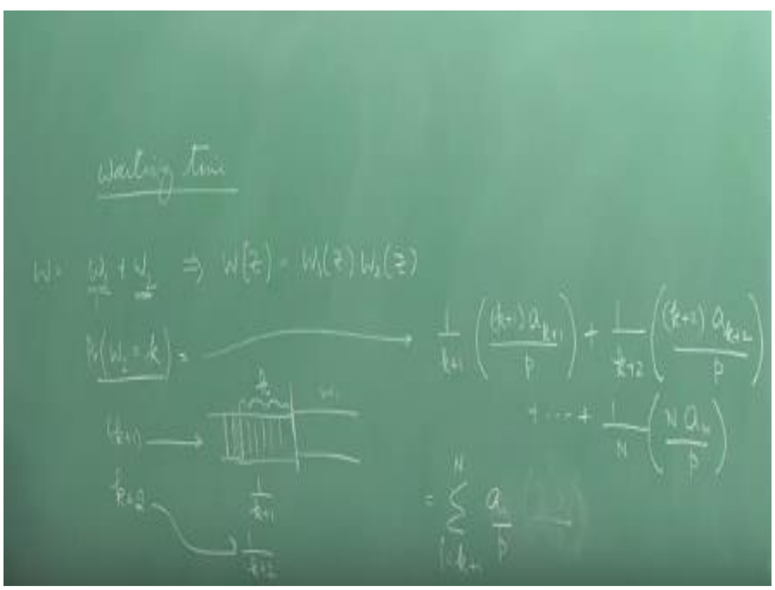

So what we are doing is I am actually assuming that we need to estimate the probability of third packet is in batch of I packets what does it mean is their I packets which are they are in the had of the q they are been pushed out to the outgoing port okay so how this will be find out so third packet will be in the batch of I packet if the remaining so there is one packet which third packet which is already there it is a conditional probability on the arrival of the third packet remember so reaming n – 1.

Out inputs should actually have i – 1 packets directed to the same output port so that is the pro conditional probability that you will have third packet in the batch of I packets so which implies that I have to have n- 1 C_{i-1} and since arrival of a packet is p it is directed to an outgoing put p/n so a packet out of these n- 1 is being direct to this port that probability is p/n so there i – 1 such packets which are being directed so that is this probability and remaining are not been directed to this that is (n-1) – (I – 1) do that will be this probability.

So of course now I can start solving it so this will be n-1 so this turns out to be -1 will cancel out so if I take this bracket –i+1 this one will cancel so I will end up in getting only n-I so this -1 I can take care by taking n/p out, okay. So now I have to do is n/p so I can take this |(n-1)|! (I-1)! n-1 so I can take this n x ?(n-1|)! So let me write it down, so this I can actually write an i on top and this will become i!. So this will be nothing but i/p a1, this is the same ai which we have discussed earlier in earlier video I ai was nC_i p/n so I ended up in having the same thing, so it has become iai/p probability that w2 will be is equal to k, so I have already got the probability that you will be coming in batch of I packets so when you want a delay to be k.

You should be at least in a batch of k+1 or air number, so if you are in k+1 bath if you are in k+1 in that case you can actually have k packets which are coming in front of you, so I am actually discarding these packets which corresponds to w1 I am not considering but here you will have k packets in front of you and you are here k packet is this one, so you will be having delay of k, w2 will be k after which you will be serve.

W1 will be because of their earlier arrived packet, it is possible that I may actually also have k+2 in that case with equally likely probability this 10^{th} packet can be any way remember, okay. So when it is being when you have only k, k+1 your 10^{th} packet will be present here will be given this still be happening with equally likely probability it will be 1/k+1, okay. So when it is k+2 this probability will become.

Because one packet will be here you can be anywhere equally likely and your 10^{th} packet is present here 1/k+2 and so on, so in this case I can write probability of this now can be written as 1/k+1 the batches of k+1 ai(k+1)/p + 1/(k+2) hence so on, so on till whatever the maximum best size which can be there which is going to be and my where k will can take a maximum value of n-1.

So I can write down so last expression will be 1/n (nAn/p) so I can write this as summation I = k+1 to n 1/I, okay. This is what will be the probability that a blue two will be equal to k, so I can this I will actually cancel out so I will have ai/p so I can use this so infact now I can write it closely here and it move it from here, so one I know the probability at the blue two will be equal to k. I can use it to find out what is going to be w2z and use it for finding out what is going to be wz and hence forth what will be the delay, okay. So let us see what will be the wz.

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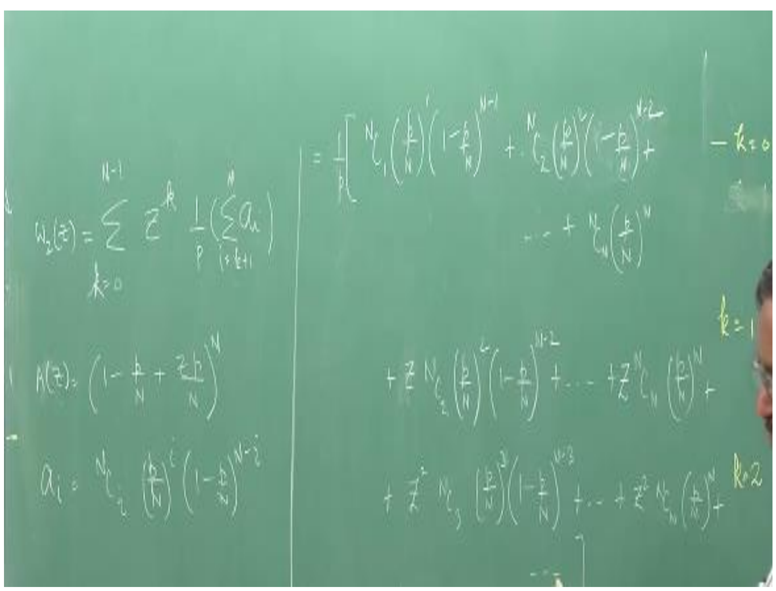

So you will have PGF for waiting time will be Σz^k 1/p(Σi) I goes from k+1 to n, k goes from 0 to n-1, okay. So this has to be remember has to be n – 1 because this is n the last term is k will be n -1 for the last term, okay. So this will be w2z so we have to estimate this, now ai is already known to me ai will be given as $(1 – p/n + zp/n)^n$ so this so this is az you already given to me ai was nCi $(p/n)^i$ 1- p/n.

So now we have actually these three and based on this I have to find out what is going to be wz, so let us see if I can how to do it, this actually forms a last series but we have it still solve it without any issue, so 1/p actually comes out where remember it is a double summation now, so I will take first of all k = 0 so I will have nC1 0+1 is 1, 1/p has been taken out so I can now write down ai is starting from k = 1 so this will be p/n. The last value will be $(p/n)^n$ so this is when k will be 0, now let me write down what will be k = 1.

So this summation so I will write it here, so when k will be equal to 1, so I will actually add up with the second term here integrate at the second term also, that will be great. So the second term will now be, when k will be equal to 1 so I will start with NC2 now this term will not be there, I will be starting from here, so every time I increase my k, my number of terms in this summation will start reducing, okay so I will end up to now z will be multiplied z^1 I will have N_{C2} P/N^2 N-2. Similarly when I will do for k=1, term will start with N_C 3N that will be Z^2 in that case, okay so they rest to be z here, so next row will be Z^2 and so on. So the last in fact I can write down here what will be the terms for Z^2 and so on.

For k is equal till N-1, so I have to just now do the sum of this, so I can write this thing very well as.

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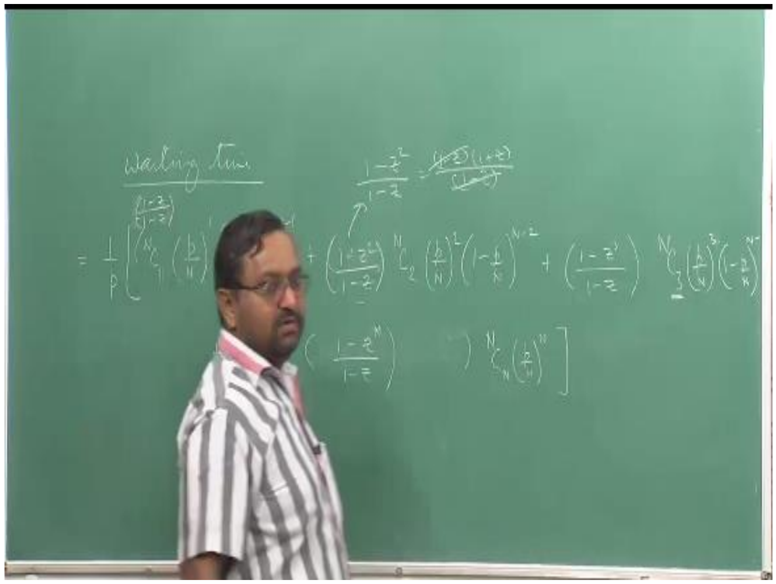

So this term will the only term with i=1, next one will actually have NC2 will have only two terms 1+Z will be there because of that, when you will have the third one you will have $1+Z+Z^2$, the least tern will be $1+Z+Z^2+$, so this will be the term so I can actually now write down the summation of these terms. So in general if k is here, k is for a value of k the series will be now $1+Z+Z^2$ Z^{k-1}. If I take the k because that k 0,1,2,3 here I have changed k=1, okay let me take k is equal to this so I will take it is k. so this summation this is a geometric propagation and for this so rule is the first term the ration which is Z this our total number of term which is in this case in fact let me put k-1 here itself so that I can collaborate with this particular number or let us use it l.

l-1 so Z^l-1/Z-1 so I can very well replace most of this terms by, so 1-Z will come here $1-Z^3$ 1-Z this one will be replaced by $1-Z^N$/ 1-Z, this one actually is nothing but $1-Z^2$/1-Z which is 1-Z and this 1-1+Z is coming so I will also write this thing as $1-Z^2$/1-Z, okay. So let us start solving it what will happen, so 1-Z or Z-1 I can do it as it either we around so, for being consistent with my expression maybe I can do it this only not an issue. So I can multiply this by 1-Z/1-Z, okay so once you do this 1-Z in the denominators can be taken out which is common so I can write.

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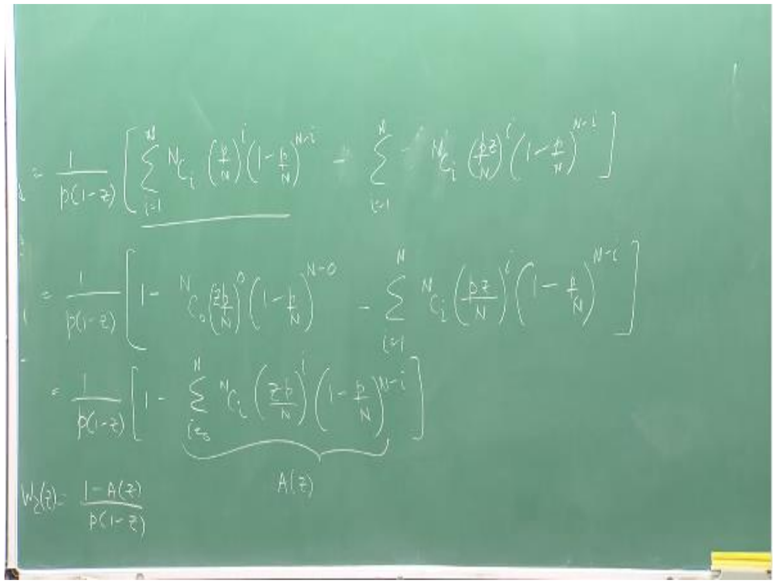

1/p(1-Z) and then of course, all the terms with 1, 1 multiplied by this, 1 multiplied by this, 1 multiplied by this I can add and those will become ∑ N_{Ci} i goes from 1 to N, now there is also terms with –Z into this, $-Z^2$ into this, $-Z^3$ into this so those terms will be also going from 1 to N, Z^i, so I can very well multiply this Z^i I can actually keep inside, okay this what will be the expression. So I can this one is nothing but $1-N_{C0}$ here I made a mistake there should be a minus here so -∑I = 1to n now since it is 0 I can put a z also it does not matter z^0 is also 1 so this will turn out to be nothing but 1(p1-z) 1-∑I goes from 0n so I am actually including this term also inside the summation and this term is nothing but a_z so we can write this thing as 1- a_z /p1-z so that will be your w to z actually.

So once you have this I can now solve it so this will turn out to b first one is q_z so w1' (z)you take the derivative put z goes to 1 that will be your w/ this we had discuss in the earlier video so this will turn out to be nothing but q bar + 1(2p) a^2 –a bar okay this is the second moment around means so that is double derivative divided by 2!so you can always get it you can so this one is actually come from when we have z and p_i so this is z^T actually.

So when you take first derivative you get when you get double derivative the second moment and this will be so I can write it here and when you take limit z goes to 1 so you have i^2 a_i – this,

this actually second moment this is the first moment okay, and that is what it is coming up here when you take the first derivative that is how it is happens okay I have not solved it I am directly writing the relation.

So this is what will be the answer so this is the more or less in the standard result in any Cuming curie book, so a^2 a bar will be equal to p a^2/ will be p^2 + 1 / p/n okay so you w bar will be n-1/m q bar md1 infinitive system plus 1/2p and of course a bar is p so it is –p this p will cancel with this one so again I am using this standard result from here directly putting it here I have also written it in the earlier video.

So this one is again the standard which we can actually get it by using a_z and this value will be p/2 n-1/n so you can actually solve it m-1/n can taken out this happens because of the finest size n when this n goes to infinitive only that this part can be assume to be 1 so this will give p^2 2_1 –p so it is ½ it will be p-p^2 so I can just add them up p^2 canceled and interestingly this one is also nothing but for the same Marcovian arrival deterministic departure single server in finest size q this one is actually the delay for that particular component.

Only impact which will be coming because of the batch arrival process is again n-1/m so when nl it is goes to infinitive this also give the same result otherwise this is the small thing which will happen because of the desecrate Haitian because I am talking about the packet switcher. So that is how the output coming actually behaves they essentially gets converted to md1! for larger size switches.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL, IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production