Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 14

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Okay let us move forward with the clos network so clos network as we all known so far that it is a three stage network.

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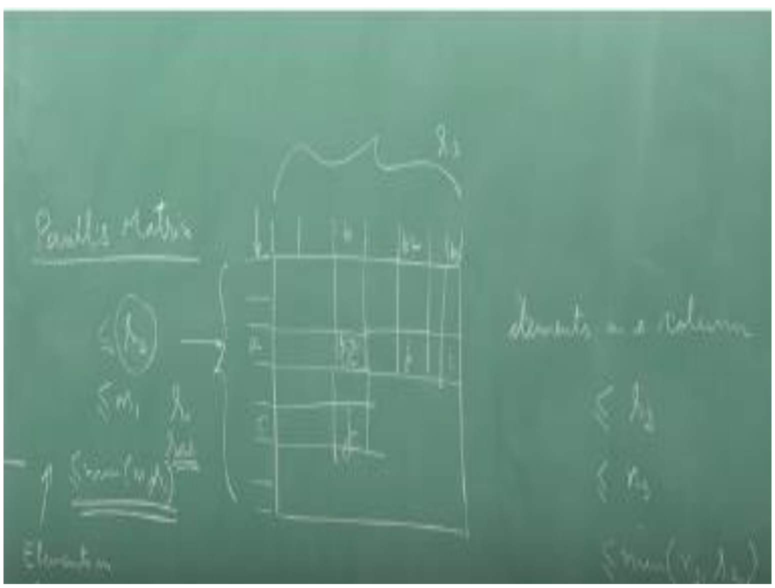

So it will consists of the switches in the first stage and we defined them as m_1/n_1 and they will be r_1 such switches and in the middle stage we will have r_1/r_1 switches each one of them will be n_1/n_1 in fact we had written in the beginning that it will be m_2/n_2 n_3/n_3 so we can actually do that m_2/n_2 and they are total r_2 number of switches we have n3 has output and n_3 input in the last state and m_3 as the input in the last stage and I can connect these so in total there will be r_3 switches here when it is a symmetric situation r_1 should be equal to r_3 m_1 should be equal to n_3 and n1 should be equal to m_3.

So of course we had also written down by looking at the picture or the drawing we can actually establish that n1 should be equal to r_2 should be equal to be m_2 okay there is exactly 1 line which is coming in so clos network as this characteristic of one line and similarly on this side your $_{r3}$ should be equal to n_2 your r_1 should be equal to m_2 okay so this typically is the clos structure.

Now the question is when I actually exampled the condition for strictly non blocking nature of a switch I said there is 1 free port here which can actually connect to any one of this middle say switch for on word making the connections there is one free port somewhere here which need to be which can be connected to the middle stage switches basically if except this one which a free port except this one so remaining once which are going to be m_1-1 can be connected to middle stage switches so that they can be they are connected to some other places on this there will be n_3-1 switch ports outgoing ports which are connected to the again middle stage switches.

So the set to which these busy once m_1-1 ports are connected and set m_3-1 are connected they are different, they are different sets so if have one additional switch that can be used to make set up a call from this free port to this free port and then we came up with a condition that my r_2 number of middle stage switches as to be greater than equal to m1-1 + n_3 -1 + 1 so switches as to be greater than equal to this number we actually then arrived at a condition that r_2 has to be greater than equal to m_1 + n_3-1.

Okay now question is can I represent this whole thing in a different fashion so I did it through an argument which is fine but everybody there should be a better method of visualizing it so one of the best methods which actually can be done is known as Paull's matrix it is pretty powerful abstraction or the mechanic way by way which a cross network actually can be represented all connections okay now how this actually will be done so let me built up clos so what will be the dimensions of the clos network that is the first thing.

So I have actually rows in this so rows will represent what and there columns so columns as to also represent something and in a corresponding cell what is going to be entered so these three things as to be identified. So the way it happens is the Paull's matrix typically will be having for example we will say we will actually call it as a switch I can draw it here so this is switch number A it can use lot of middle state switches okay I can call them as and there is switch here I call it B.

Okay and then these one are F there is possibility of G there is possibility of H so if FGH are connecting in this fashion then I can have other once IJ kind of thing so this is connected to some other one say b_2 this is connected to b_3 say it does not matter to which input port in the A switch this line is connected because this is a strictly non blocking switch so these ports does not matter so I have to worry about from this switch A which particular path as been used to connected to which particular switch in the output stage so these ports actually does not matter here these ports are in material.

So what I will do is I will use this switch ID's to represent my rows so there will be a row A corresponding to the switch A so how many rows will be there in this Paull's matrix so how many possible first three switches which can be there these are r_1 so you can have r_1 rows so I will write down the I will write down when the conditions for a Paull's matrix to be legitimate or legal or the correct matrix to represent a clos network so one thing which is clear from here there has to be r1 rows and what we do is the columns will be representing the switches in the third stage or the output stage.

So how many switches are there in the output stage these are r_3 so will have r_3 columns and I am again not worried about the outgoing ports I will be worried about the switch so a column will be represented by P for example now from A to B and now I have created set up a call wire F wire H so I can actually put F, G, H, A okay so the important thing within a cell A B so row corresponding to row A and column B need to have there is restriction that how many entries can be put there can be more than one entries which can be put here.

Okay and that is what the Paull's matrix is all about And of course similarly of I have connected A to B wire J so I have to find out where the b_2 is b_2 somewhere here may be and I can make an entry J I can find out where the I can make a entry this was I, so I can make an entry I so if you look at it, how many possible entries I can make in row A let us look at that, so that will be govern by number of switches which are there in the middle stage, for every connection now remember what I am considering here is a Uni-cast connection.

There is only one input there is only one output which is there, okay. So when I am connecting one input and output gets consumed one connection goes through so if this connection F has been consumed F cannot repeat F cannot be used to connect to B2 or B3 all three simultaneously I will come to a case when that will also be feasible, so this actually implies the number of entries in a row will be restricted to number of middle stage switches.

So there will be hard two possible entries which can be there that is one argument but how many connections I can set up from the switch A? So those will be basically decided by number of input ports so number of input ports which are coming in is M1, of course number of middle stage which is a number of incoming ports are two different things, so I can increase these I can reduce this.

So my number of elements here cannot exceed RR2 so they have to be less than R2 they also have to be less than or equal to M1, so if M1 is smaller it cannot be greater than M1 actually it has to be lower than M1 number of elements in the row, so which actually implies the number of elements has to be minimum of these two whichever is there M1 and R2, so they should this number of elements in row will satisfy this condition,

Similarly for the columns also if you want to estimate I cannot have because there is exactly one path coming here so utmost all these number of middle stage switches it has to be lesser than that, okay. Now there is a possibility when number of middle stage switches are less than the number of input ports or number of outgoing ports then the minimum of these two will be R2, okay. So similar conditions exist.

This is four number of elements in a row, so similarly there will be number of elements in a column, so they have to be less than R1 or they have to be less than N3, so they will be essentially because they cannot be greater than these two values so I can write down they have to be less than minimum of N3 and R2 sorry this have to be R2 is a very similar condition which also happens on a elements in the column, okay.

If for example this B is also connected to J and then it has been connected here so this is some element say C so I can define a row C and the B is connected where J, okay. So this is how typically the Paull’s matrix will be filled in, now if you look at a Paull’s matrix can you identify whether it is correct or not correct, so one thing which is showed if my number of elements here are more than this then there is certainly a problem.

It is not correct or if number of elements in a column are more than this then there is a problem, okay. So let us look at write down the legitimacy conditions for the Paull’s matrix, so we will use this Paull’s matrix in know all of my theorems to figure out the conditions for a simply known blocking nature and re-arrange non blocking nature.

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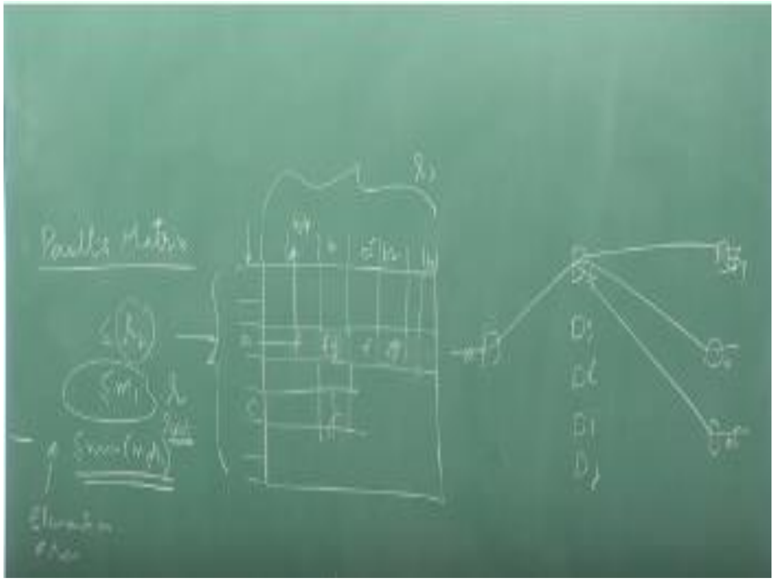

So the first condition as we have done it through discussion so I can very well write it that each row can have utmost M1 symbols, so because maximum number of paths from a switch to the middle stage switches can be equal to number of incoming ports is because of that, second is each column can have utmost m3 symbols so the first one was corresponding to this the second one is corresponding to this statement, okay.

Again this is because those mean number of output ports are only available so those many middles stage switches can only be occupied or can be only connected, okay. The third statement which goes, now this is important, symbols in each row must be distinct, okay. So there after two distinct symbols that is why we can have utmost maximum R2 and why these need to be distinct?

Because if they are not distinct than what is going to happen let us see. So I will have a switch, now a symbol actually has been repeated that is what I am telling that in this case if this is A this is B and symbol has been repeated I repeat a symbol say F here, F here G here, G here what is it mean? I am connecting where F to some switch which can be B4 here I am connecting F, B and F also being used for some B5.

So which actually means I am connecting here and then F is been connecting here, F is been connected to some other ones so this is B4 this is B5 what is it mean? So some input single information is coming is been routed where this cross bar to the middle stage 1 and the middle stage is connecting to all 3 simultaneously splitting the signal, is making three copies and transmitting to these switches.

Which then of course can further split if they wish or they can have only one out going port a splitting here does not matter it because is restricted on blocking switch, okay. So this scenario is a multi casting scenario. So number of elements actually can be more than m1 and R2 but they will and they may not be distinct actually but this number of distinct symbols so if the this limit has to be still satisfy okay so this multicasting is happening and I was not considering multicasting in the beginning.

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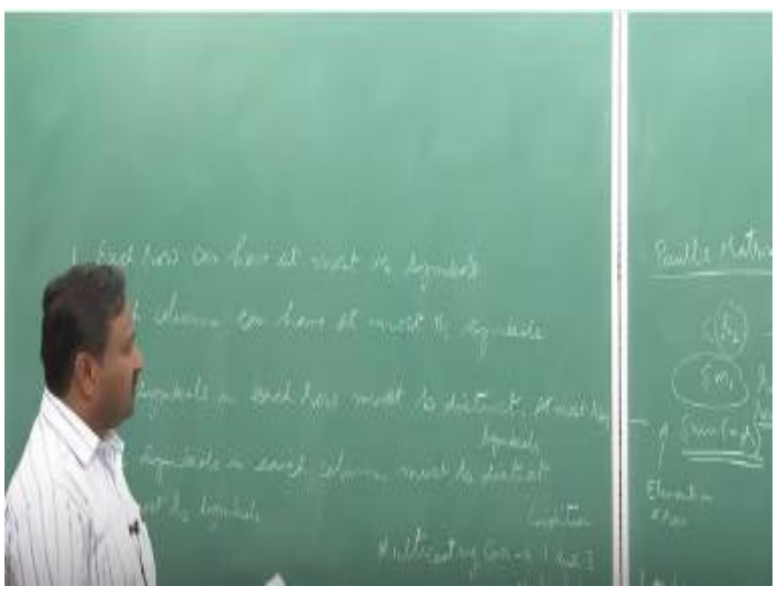

So similarly we will have fourth so all elements which are being used their all distinct so there is no repetition that is what the condition three for a uni-casting case one to one the fourth condition will be that the symbols in each column must be distinct now we have to understand if these are not the sting what is going to happen and that will give us a reason why they should be distinct so let see if the symbol in each column or not distinct they are not different I am repeating a symbol okay.

So I might repeat so I have got fgh I may repeat that from c also I am going to an f okay so yes been used to connect to this so this connection is not required only it is connected to be okay and this is f so f there is a g through that also h through also is been connected but interestingly I am also connecting to b wire b and c also wire f so there is some see which is setting here and I am connecting it to f and connecting it b so there is only one line here and their two lines here it means that two circuits.

Two paths which are coming in their merging here it means two signals to separates inputs signals which are coming from the input side are being put on a same line they will interfere so this is actually illegal this is not permitted it is immaterial whether it is a multicasting or a uni casting even for a multicasting or uni casting both cases this is illegal so column the is elements have to distinct always okay and this actually means the number of symbols are going to be and utmost R2 symbols can be existing.

And in this case utmost are two symbols in case if we do allow multi casting then I have to do away with this condition third condition that the symbols have to be distinct in the row they can be then repeated so their need not be utmost are two symbols okay and of course each row can have utmost mn symbols this condition will also not be true I can actually repeat so f can repeat multiple times in his fourth I can have more than m1 symbols that is possible so that binding condition goes away and they need not be distinct.

So condition one and three need not be satisfied for multicasting so whenever you have multicasting case condition one and three need not be satisfied okay so that is what the Paul's metrics is all about and that is how we can represent a three stage clos network in a Paull's matrix okay now once we have understand what the we have to not look at the Clos theorem so essentially the strict one blocking property for a unicaste case or one to one connection the same

I am going to now formally put up as a theorem so this theorem is because of Carle's Clos and this is known as Clos theorem.

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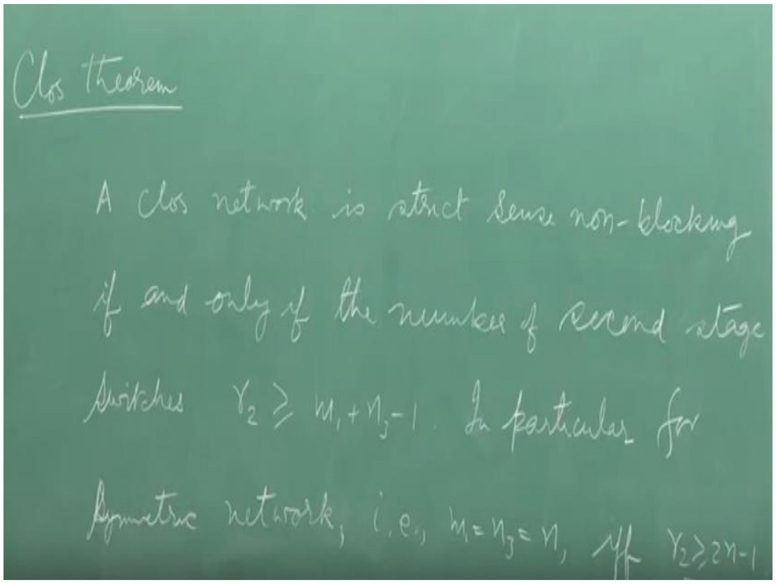

Okay so theorem is states that a Clos network is a strict sense non blocking remembering in strict sense means that existing connections will not be disturb so without disturbing the existing connections we will set up a new we can setup a new connection if the input and the output ports both are free is a strict non blocking here if an so if implies it is in a sensory condition only if implies it is also sufficient conditioning you do not have to satisfy anything else this is good enough.

The number of second state which is r_2 > or equal to m1 + n3 – 1 so when I at explain the same thing to you earlier I have not given you the condition for sufficiency actually I never proved that I only say this is an S3 condition okay and of course in particular for symmetric network that is m1 = n3= n if n only if r_2 is > than or equal to 2n – 1 okay so that is the Clos theorem so let us start with the proof.

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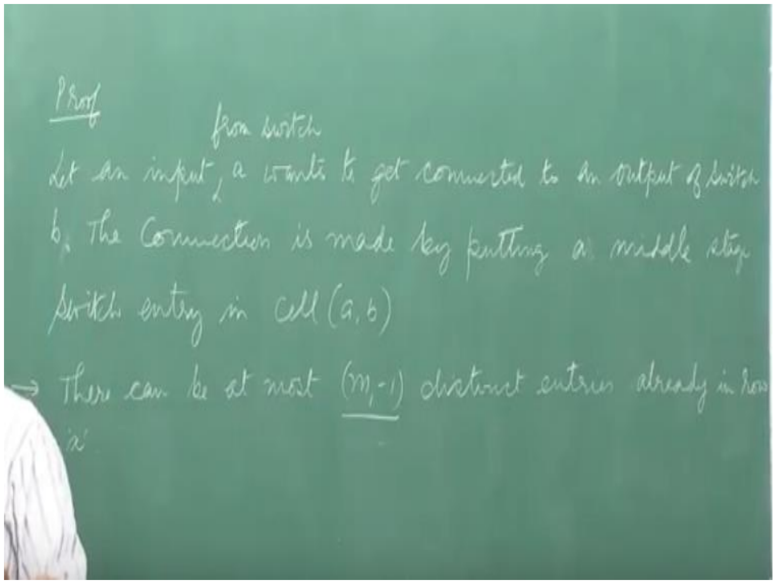

So we will say let an input from switch wants get connected to an output of switch B the connection is made by putting a middle switch entering in cell A B of the false matrix okay, so we have to ensure that we should be able to find out an element without violating the legitimacy condition of a false matrix, so the logic actually goes very simple the argument where already given now for formally writing it okay.

There can be at most m1-1 distinct entries, already in row A and I am talking about distinct entries so there is no multicasting here okay there can be only m1-1 elements which can be there okay, why m1-1 because there is one free port which is there in A which I want to connect to a free port in B okay.

So there is a one free port so there maximum number of incoming ports is M1 so number of occupied ports can only be m1-1, so there can be at most m1-1 one distinct entries already existing in the row A, I can actually take the same logic and this is the first case scenario they can be less also is not that when an input port has to be connected these many distinct entry has to be there, this is the first case scenario when all other neighboring inputs also connected to the switch A or already connected to somebody is a first case scenario I am taking.

In a better cases there can be less number so I am looking at even in the first case scenario I should be able to set up a connection so the next one is using the same logic column B can have n3-1 distinct entries so same logic holds through here so because there is one output port which is free at switch B only N3-1 other output ports in worst case are already connected so correspond to each one those it's a point to point connection there will be n3-1 distinct entries in the column itself .

Okay now this actually means if I draw the matrix so there is a row A which have already got entries which has mn-1 there is a corresponding column which is for B ,I have entries which are n3-1 i need to find out an element which can be put here ,in again worst case scenario these m1-1 an n3-1 the number of entries which are in column which are in row and which are in column they don't have any overlap so the entry is this m3-1 entry is let me call it the entries for B actually so this particular set and the corresponding entries here is are AE entries.

And in worst case scenario AE∩BE is null okay which actually means AE∪BE the cardinality of this particular set should be m1-1 +n3-1 ,I need one extra if I have one extra switch i can put that switch here middle state switch which actually means whatever is the cardinality of this union my middle state switch is have to should be greater than or equal to cardinality of AU,AE BE +1,if that is true then I can actually put that element here I don't have to disturb any existing connection and I can set up the path okay which implies that r2 ≥m1+n3-1 and that's the condition for the clos network this actually means that I this is necessary and sufficient condition for setting up for the path without disturbing any existing connection. Okay so that's what the clos theorem all about.

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