Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

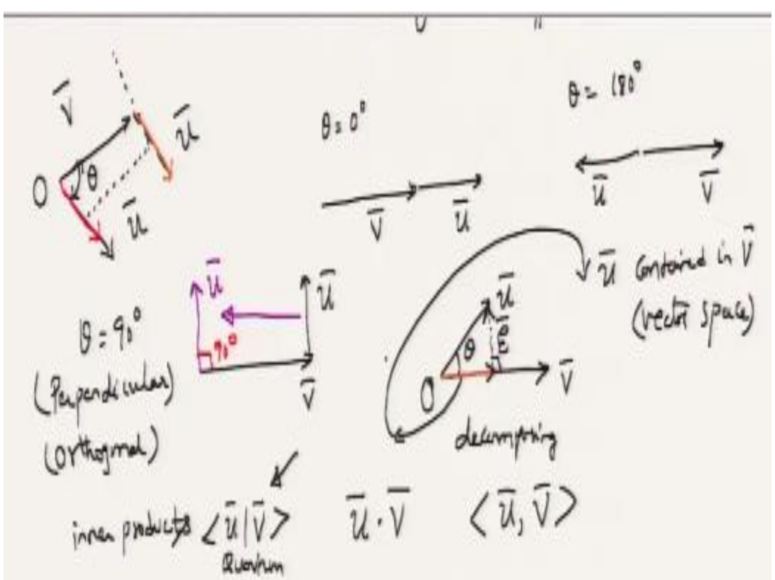
Course Title
Optical Communications

Week – II
Module – II
Review of Signals and Representations – II

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome in the short module we will review some concepts of signals and representation that is necessary for us further study digital communications or rather digital optical communications to begin with.

(Refer Slide Time: 00:30)



Let me start by calming that we consider signals which you know when I say signal you think of a wave form which would be depending on time and it would be changing with time such as s(t) for example we consider this signals as vectors to back up what I am saying because the notion of a signal as a vector is very important in communication systems because it allows us to think of signals which are some continuous functions of time which we assume them to be some continuous functions of time.

In terms of vectors which are you know having certain geometrical property so it is a way of generically understanding what signals are that allows us to further analyze digital communication systems and therefore a briefly review of those ideas with which we associate are we identify signals with vectors is something that we are going to do now to start with let us first briefly recapitulate a few factors or few facts that go with vectors rights so when we say a vector the immediate thing that would come to our mind is physical quantities such as velocity.

And this velocity is quantity that has both magnitude right so we say that the car is moving along a particular direction at 60 km/ h so that would be the magnitude part of it and the direction whether it is moving from Kanyakumari to Delhi in which case they can think of the movement from south to north or you can think of any other directional movement that direction part is the direction property of the vectors.

So we say that a quantity is called a vector when it has both magnitude and direction quantity is that can only be specified or that can be specified with only magnitude such as temperature at a particular point is called a scalar so this a vector if you just have a magnitude for example speed of the car it would be 60km/h so when you specify speed you do not normally specify the direction however when you specify velocity you specify both it is magnitude as well as it is direction.

Now this is something that is quite elementary in our thinking of vectors but we also have other representations of this vector right so one geometric way of representing a vector is to draw an arrow there is a head of the arrow and the tail of the arrow and we are though that the length of this arrow which would be know this particular distance between head and tail is called the magnitude of the vector or the length of the vector.

So we can have a graphical or visual representation of a vector in which we specify the head of the vector in the tail of the vector and the distance between these two the two points or the length of the vector would be called as the length of the vector, now we can you know give a name for our vector for examples since we have talked about velocity as one of the vectors we call this vector as v to distinguish between a scalar and a vector you put a bar or an arrow sometimes.

In print you would write v in terms of it is bold phase you know print okay so to just to distinguish that form the scalar so I will be using this bar the over head on a quantity which then would be mean a vector, okay so this vector which I have graphed here as a particular direction and it also has it is length the vector is given by v with a bar over that.

That would be the way in which I will name this vector then I can specify the length of the vector or I will use a special symbol to specify the length of the vector by enclosing the vector between these two lines okay in more technical language this particular quantity which we have written is called as the norm of the vector and you will see soon that sometimes incident of norm it is a norm squared that becomes important so you can think of norm as essentially length unless we really want to go very technical into all these accepts which we are not going.

So this is a vector v with a bar there and the length of the vector is represented by this particular quantity okay that is what is we call as the norm now when you have two vectors let us say this the vector we and then I have a vector u right you can see that there is some sort of a graphical relationship between the two right so you can see that there seems to be way in which these two vectors v and u are written there is a specific angle between the two although you would Not be able to find that out so easily.

In order to find the angle between the two vectors and in order to even find the intuition behind that one you have to understand that mathematically a vector u or vector v can be slid parallely okay and in the particular direction as long as I am not changing the length of the vector so as long as the length of the vector is unchanged you can slide it parallely and then up and down so when you do that you are not changing well you are actually changing the physical vector if for example this represented a car going in this direction then moving u parallely.

So as to form let us say let me use a different one so this would be the vector u that has been moved right that would be the vector that has been moved these two vector would be completely different because this would specify a car on this road where as the vector that is represented by this u moved would be a car in the same direction but it would be on the parallel road or it would be on the other road physically they might be different but mathematically.

They are identical therefore we consider the vector u moved as equal to the vector u itself okay so this is something that you have to remember you can also slide this vector up and down okay so you could have actually done that one right over here you could have for example take this vector and just slide it just under here so that this would have become the new vector.

Why have I doing all this because it simplifies my analysis or it simplifies my understanding if we consider both vectors v and u to have a common origin o, okay. So if I bring two vectors which are spatially separated but I can bring them such that their heads are intersecting at a particular point which is the common origin point then I can talk about the angle between the two vectors, okay. So I can measure the angle either from the vector v or from the vector u.

And I denote this angle by θ if $\theta = 0^{\circ}$ right then we know that the vector v and the vector u must be in the same direction only then the angle is 0, if $\theta = 180^{\circ}$ then I know that the vector v must be opposite in direction to the vector u, correct? What happens when $\theta = 90^{\circ}$? Then we have what is called as the perpendicular case or the vectors are set to be perpendicular to each other, another name for this is orthogonal to each other.

Or in another words vector v will be orthogonal to vector u when the angle between the two is equal to 0, of course in the way I have written I have put the head of the vector u onto the tail of vector v I could equivalently move this vector u right parallely and then write it in this way, so these two vectors as I said earlier they are equal to each other so the angle between these two which defines the perpendicular thing is 90^{O}.

So the angle between these two lines of these two vectors is essentially 90^{O}, associated with this there is one other concept that is very important we ask this following question, suppose I have a vector v and in general I have a vector u, now I have a common origin to both of these vectors which I am denoting by o, if I ask what is the amount of u contained in v this seems to be slightly strange question to ask.

I have to how can one vector be contained in another vector for a full answer you have to wait for a theory known as vector space but the short answer is that if I were to consider dropping a perpendicular from u to v then I see that this particular length would represent the amount of u contained along v, okay. In earlier class you might be more familiar with this word called decomposing a vector.

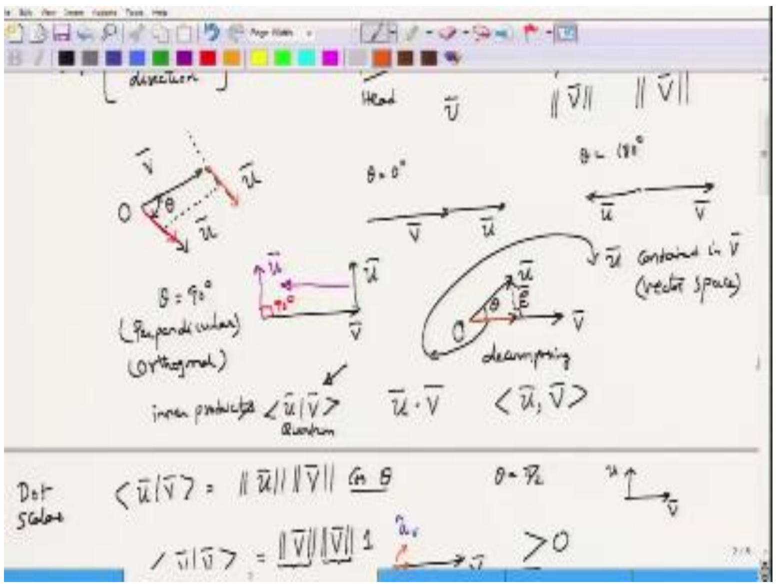
When you decompose a vector what you would have done is, you would have taken the vector u and decomposed it along this particular horizontal vector and the vertical vector and in this case the horizontal vector is the vector v and this length that you have obtained which is shown in this orange color would be the component of u along v, okay. How do I obtain that length?

Suppose the angle between the two is θ then you define what is called as the dot product off the two vectors this is one notation for the dot product you might be more familiar with this notation which is U.V this notation is also sometimes used, okay. With a comma instead of a length l, these are typically also these are actually also called as inner products, okay. And this is the notation that is typically used in quantum mechanics.

And this is the notation that is sometimes used by mathematicians and this is the notation that we normally use this notation so you can pick any of the notations let me pick arbitrarily this particular notation, okay. So I will pick this notation because I am kind of use to this, this will denote my inner product the physical meaning of inner product is that, it gives you the length of the vector u contained in v and to obtain that length you have to take this vector u you drop a perpendicular.

This is perpendicular because if I were to denote this vector by say e then I know that the angle between e and v equal to 90^{O} so if I drop this perpendicular e and then I have find this distance which is given by this orange color then I will obtain the inner product, okay.

(Refer Slide Time: 11:13)



Mathematically the inner product is given as the length of the vector u multiplied by the length of vector v there is a simple multiplication between the two and then there is also multiplication this is also multiplied by a factor Cos θ in fact this Cos θ = 0 then it implies that θ = Π/2 in which case the vector u will be perpendicular to vector v, so if you have to look at the component of the vector u onto the vector v you would find that component is actually 0, right.

So this is the notion of inner product or called as dot product or sometimes called as the scalar product of two vectors, interestingly what would be the scalar product of the vector v with itself, well I have the vector v here if I project the vector v onto itself what I get would be this length square, why do I get a length square? Because mathematically expanding this v on v itself is to obtain then length v you know I have to write it twice.

Because there is u and v here it is v and v and the angle θ will be 0 therefore Cos θ = 1 and we know that this portion is length of v this is length of v so what I get is the length of the vector square and this quantity would always be greater than or equal to zero when will this be equal to 0? When there is a special vector known as the 0 vector or the null vector is involved, otherwise this magnitude would always be greater than 0.

It can have any magnitude but among any magnitude one particular vector is important, this vector is called as the unit vector, the unit vector has a magnitude of 1, okay. And it has a direction somewhere this direction of the unit vector is denoted by a subscript to along which it is supposed to be directed, the cap is to denote a unit vector and the subscript v which we have written will tell us that the vector is directed along v, okay.

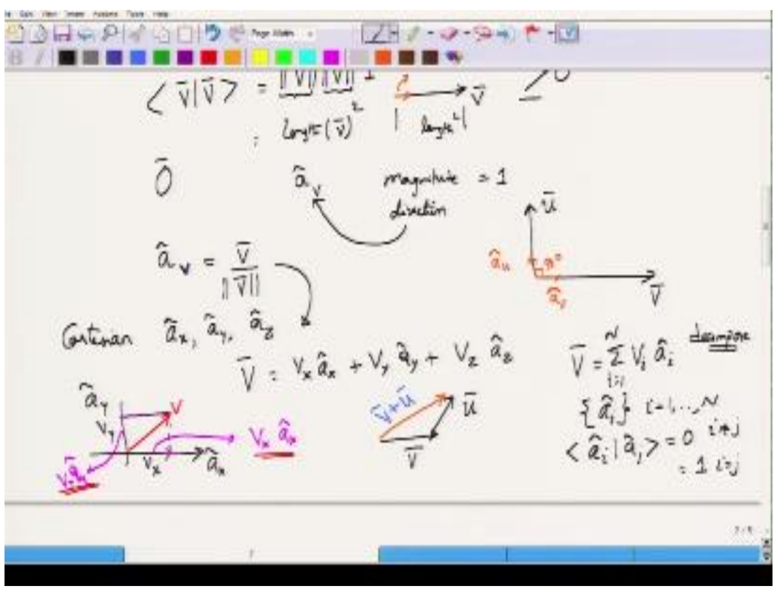
What do we mean by that, when consider this vector v here the unit vector along this direction would be a vector which has a magnitude of 1 and it is directed along the vector v, so this particular vector which we have written in the orange is the vector at av so this would be the unit vector along v, so how do I obtain the unit vector all I have to do this, take the original vector v and then divide by that vectors length, right.

So if I divide the vector by its length then I would have normalized the vector v with its length then I would obtain the unit vector again a kind of seem to be missing that a^ park here so this would actually at along vector V when the two vectors v and u or perpendicular the corresponding unit vectors which is a_v right and this vector au they are also perpendicular to each other so the unit vectors along the directions of the original two vectors which are perpendicular will also themselves be perpendicular notice that I have not actually talked anything about the coordinate system these vectors could be vectors in Cartesian coordinate system.

Which is described by XYZ axis and the unit vectors x^- y^- and z^- or they could be cylindrical coordinate system vector cylindrical coordinate system in which case the vectors would be having unit vectors are hat five and z^ they could be spherical coordinate system you get the point right they could be vectors in any coordinate system what we talked about are the geometrical properties of these vectors which must hold in every coordinate system why do we normally introduce a coordinate system.

Because it will make our life simple to understand and to manipulate this vectors we cannot always go back and keep drawing the geometry so what you do is you introduce a little bit of algebra so geometry is the original vector the original vector directions how they are geometrically related whether they are perpendicular whether they are apart 45 degrees these are the quantities which are geometrical and if you start giving them numbers and manipulate those numbers then you have come down to of particular coordinate system okay in a particular coordinate I can express any vector for example.

(Refer Slide Time: 15:57)



I take the Cartesian system this is the so called rectangular Cartesian coordinate system which is described by three unit vectors in the so called three dimensional space it is described by three unit vector s which are labeled as a_x ^ ay^ and a_z^ and any vector v can be written as some component of the vector along x + some component of the vector along y + some component of the vector along z now if I assume that this vector is only a along the x direction then I know that by simply flipping this equation I know that what I have written implies that the vector v will have a direction or will have a magnitude of vx.

And it would be directed along ax if I assume that vx is 0 and v_z is 0 which means that my vector is actually directed along the y axis then vy will give you the length of the vector v okay in general if I have a vector in the two unit vectors in the two dimensional space which is described by ax^ and ay^ any vector here can be broken down into two vectors which are themselves perpendicular to each other this component is v_x and this component is vy so the corresponding vectors along this directions is given by vx ax^ and this vector is given by v_y, y^ a_y^.

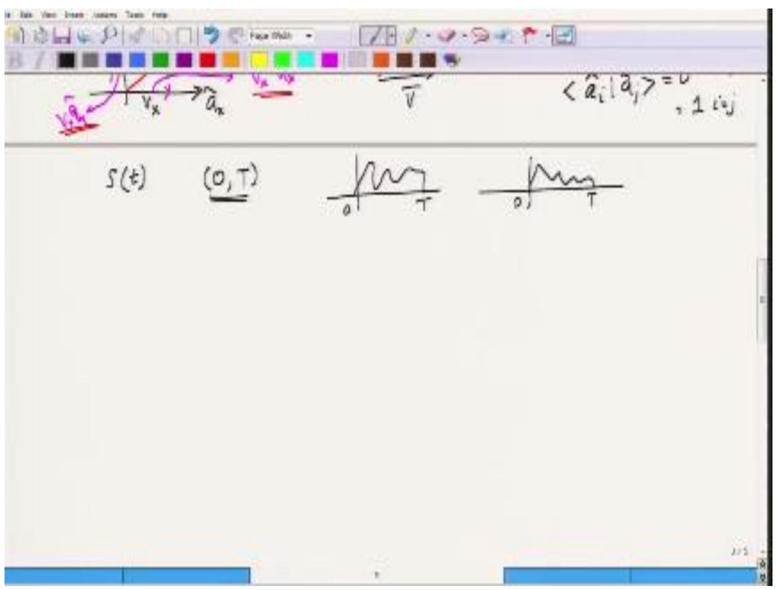
Okay what we have also done is to introduce without telling you that if I want to add two vectors geometrically so called this as vector v and the vector u if I want to add these two vectors geometrically what I would do is to put one vector say u on the tail of the other vector okay so if I put like this and then the resulting vector which would be the some of these two is given by the third vector which is directed from the head of one vector to the tail of the other vector so this vector which I have written will give me the sum of these two vectors.

So if you go back to this particular vector which we have written we here you can immediately see that this would be equal to the vector along x which is given by v_x, x^ this vector and the vector along y which is given by v_y a_y^ if I add these two vectors I get the original vector v of course I can continue to do this one for the case of n dimensional all though it is very, very hard to imagine those m dimensional spaces mathematically I can represent any vector v by giving the components of v along that particular unit vector.

So let say if I have n unit vectors which are a it is labeled as a1 a2 a3 and so on then finding out the components of the vector v along each of those in it vectors I can some those corresponding vectors not the components from the corresponding vectors in order to reconstruct my original vector v okay I can think of this vector v in the n dimensional space as being composed of n vectors which are all perpendicular to each other because we assume that this set a_i^ of the unit vectors i = 12n are themselves perpendicular or mutually perpendicular what we mean is that if I take two vectors a_i and a_j.

They can be two different I must not be equal to j then this fellow will be equal to 0 because they are perpendicular to each other if they are equal then they would be equal to the value one that is this would be when i is not equal to j this would be the case when i = j okay and writing a vector in terms of it is components, component vectors is called decomposing a vector decompose a vector into it is components okay so I hope that these concepts are very clear this was a just a brief review now what we will do is we want to think of a similar things for the signal.

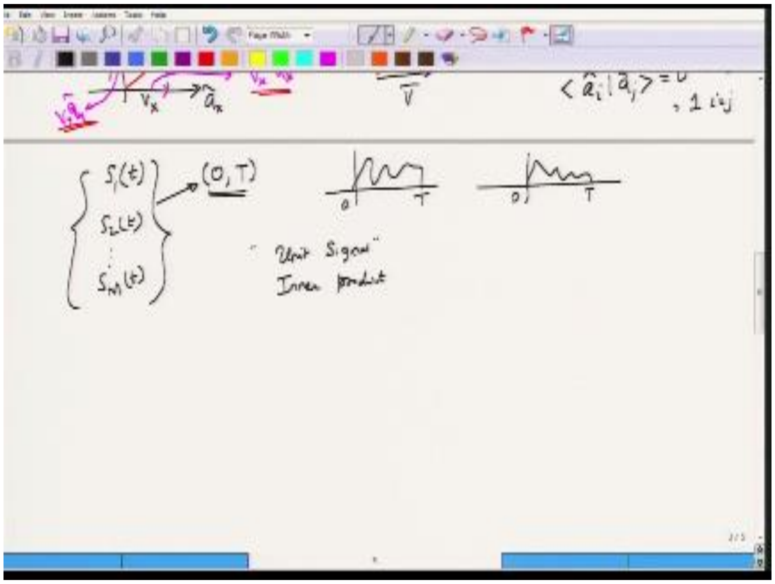
(Refer Slide Time: 20:17)



So let us start by looking at it suppose I have a signal which is a way form a continuous function of time s of t okay so this way for which I have let me assume that this large are this is defined over a time interval 0 to t okay I have, considering this one to be a t second signal the since that over the particular time t this signal s of t is described okay it can have any shape okay it could be for example this one between 0 to t a or you can have a vector which is say this case all of these are assume to the continuous and bounded.

This additional constraint, are just so that the mathematical operations remain finite and they do not run of to infinity in which case it would be difficult o work with those, so I am introducing these additional constrains for that I am introducing the constraint of the t seconds because in digital communication systems every second the source would send out a new wave form, okay that would presumably contain some amount of information, so it contains information and at last t seconds, okay.

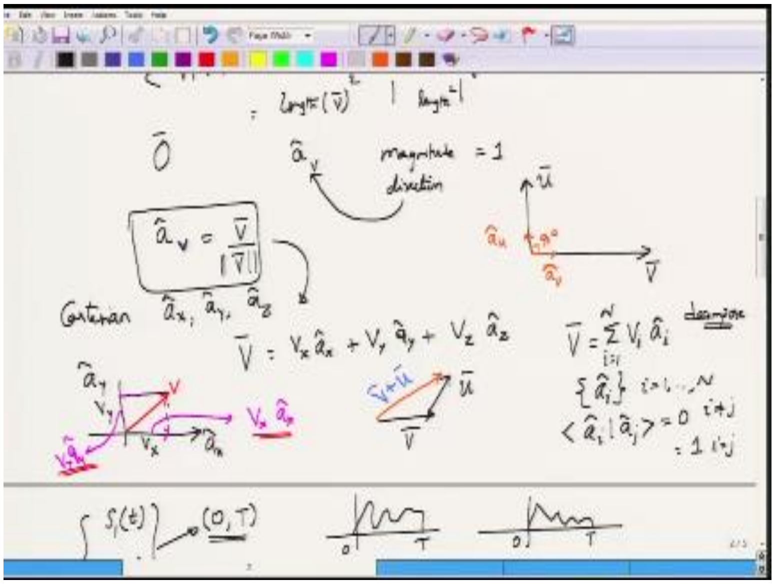
(Refer Slide Time: 21:31)



So for that reason I consider a set of such wave forms so let me label the set of wave forms as $S_1(t)$, $S_2(t)$ and so on upto say $S_m(t)$, okay. So is my set of vectors which all can be different but they have been defined over the same time interval 0 to t. Now these vectors are you know they are just some wave forms actually I am sorry, these are not vectors at this point they are not vectors, these are some wave forms which are functions of time, now how do I claim that they can be vectors, if I have to claim them as vectors or if I have to claim that signals can be treated as vectors then I have to find couple of analogs points.

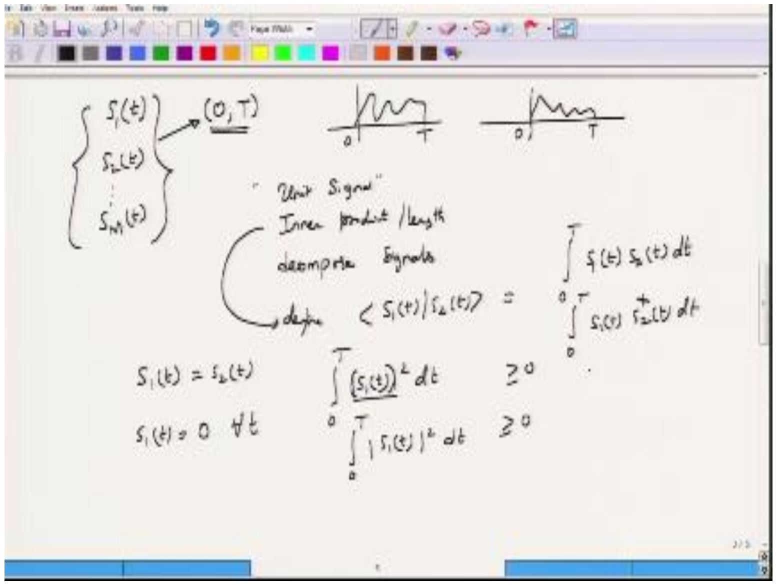
First I have to find a unit signal, okay I have to first of all even before finding unit vector or along with finding vector I have to define inner product of two wave forms although this terminology is not used, so let me put them in course I idea is that I replace whatever that was there with vectors in terms of this signals. So I have to find the unit signal which would capture this particular equation.

(Refer Slide Time: 22:37)



It would capture this equation, right I can take the original vector v divide by its length. So I clearly have defined length as well and I already know that length is related to the inner product of the two vectors.

(Refer Slide Time: 22:50)

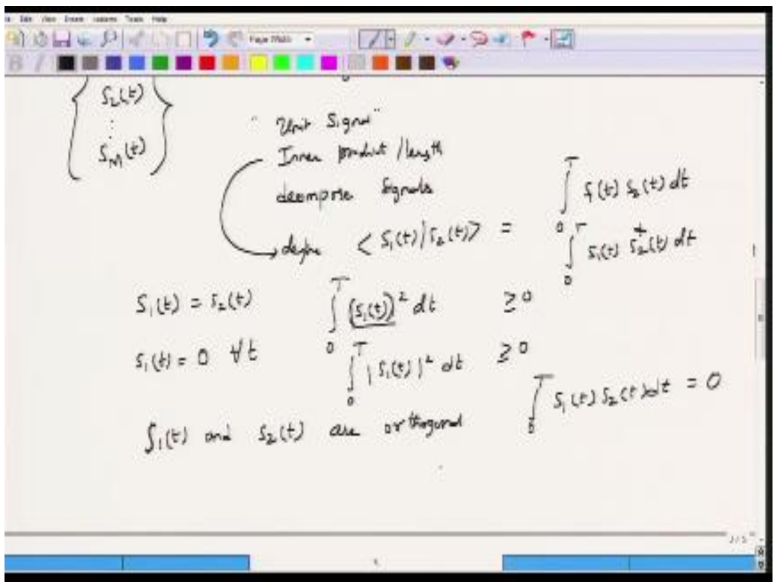


So in fact inner product length can be defined together so I need something that would correspond to the length as well and I need a ability to decompose signals into its component wave forms, so I decompose this wave forms into its component wave forms. So let us tackle all these issues with signals and then see that if we define the inner product I am not relying on anything because there is no geometric intuition to the signals S_1,S_2 and so on up to S_m at this point, so I have to come up with a geometric intuition I do that by first defining the inner product of the two wave forms $S_1(t)$ and $S_2(t)$ assuming that these two are real signals it is very easy to just consider what could happen in, when they are complex signal but let me just give it for both, okay.

I will do it 1 the others can be easily obtained, okay so the inner product of two vectors is given by if assuming that they both are real they are given by this particular integral, okay. If S_1 and S_2 are complex then the inner product is given by S_1, S_2 complex conjugate dt integrated over the same time interval 0 to t, would this inner product makes sense, yes. When you have $S_1(t)$=$S_2(t)$ then this quantity 0 to t $S_1(t)\text{mod}^2$ or rather square dt is $S_1(t)$ is real or it would be $S_1(t)$ magnitude square when $S_1(t)$ is complex these two quantities would be greater than 0, why because the integrate here is greater than 0 so therefore the integral will also be greater than 0, so here I have the notion that inner product of a vector with itself must always be positive. When will this be equal to 0, this will be equal to 0 only when $S_1(t)$ is defined as 0 for all time t.

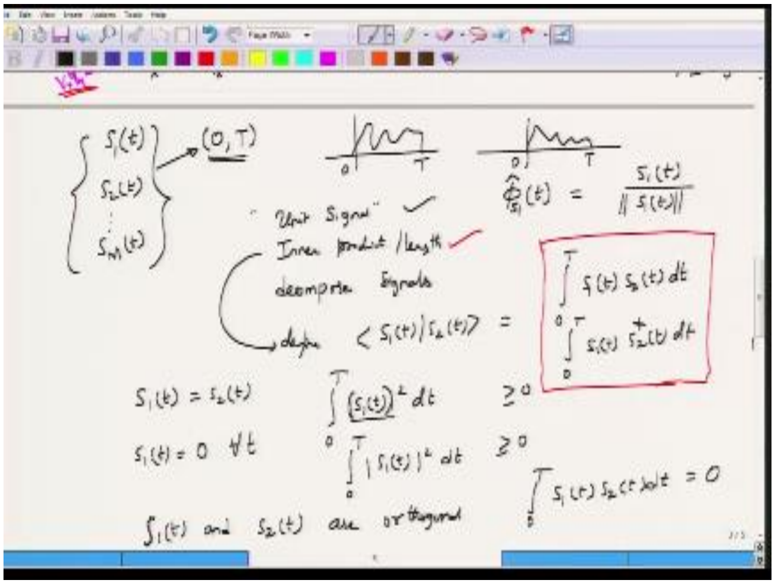
So this is a small mathematical symbol to indicate that this is for all, so for all time t if I define this $S_1(t)=0$ or in general any signal to be equal to 0, then the inner product of that 0 signal will always be equal to 0, right.

(Refer Slide Time: 25:13)



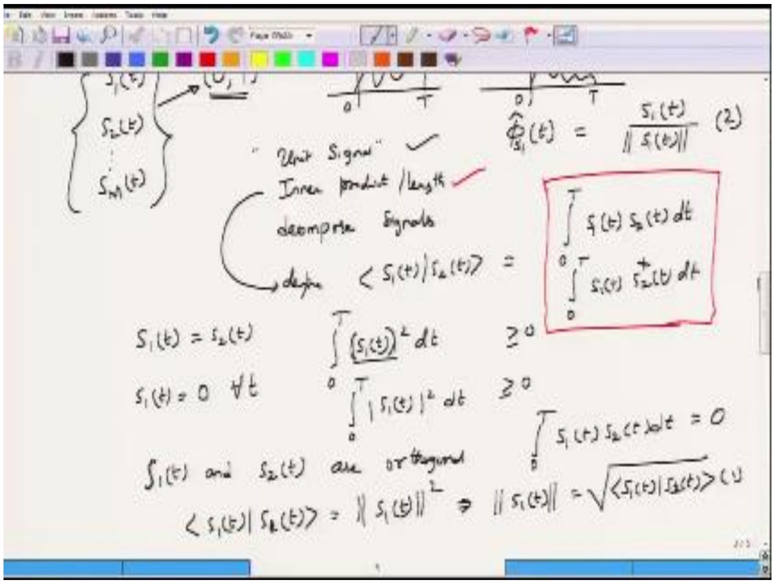
We also define that two wave forms $S_1(t)$ and $S_2(t)$ are orthogonal to each other these are orthogonal to each other when their inner product which is this particular integrant sorry integral vanishes, right so when this integral results in 0 then we call them as orthogonal to each other.

(Refer Slide Time: 25:36)



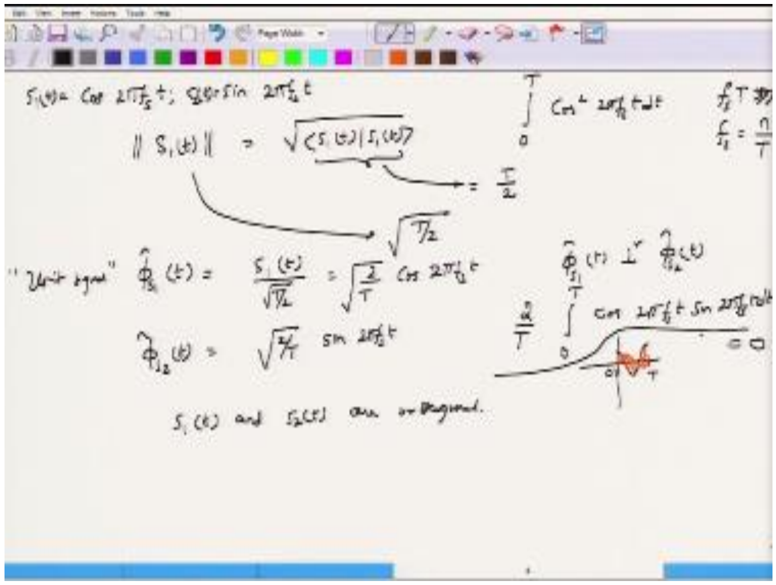
So what we have seen is that this particular definition of inner product, right has allowed us to capture some of the geometrical aspects like inner product and length perpendicularity or orthogonality it has also allowed now we asked to define the unit signal why, if I have want to find the unit signal along S_1 so let me denote that one as ϕS_1 and this of course has to be a function of time t and because this is a unit signal let me put down a hat over this, so if I want to find this I want to take this signal $S_1(t)$ and divide that signal by its length, right. What would be that signal $S_1(t)$ well.

(Refer Slide Time: 26:24)



If I know that the inner product of these two vectors that if I know that the inner product of the wave form S_1 with itself actually gives me the length square, right this is a norm square therefore inverting this relationship I obtain the length of the signal $S_1(t)$ as $\surd S_1(t)$ with $S_1(t)$ itself, so $\surd$ $S_1(t)$ with $S_1(t)$ itself will give me the length. So I can put this length here, okay into this expression so length from equation 1 I can put down into equation 2 in the denominator and obtain this unit signal ϕS_1, okay.

(Refer Slide Time: 27:08)



As an example, consider this signal which is cos $2\pi f_s t$ and another signal sin $2\pi f_s t$, call this one as $S_1(t)$ call this as $S_2(t)$, okay. Let us find out what would be the length of the signal $S_1(t)$ which is obtained as the √ $S_1(t)$ inner product with itself, right. So I obviously have to evaluate this you know argument inside the square root and that argument is basically 0 to t and this would be $\cos^2 2\pi f_s tdt$ I know that $\cos^2 \theta$ can be written as $1+\cos 2\theta/2$ and then that cos will integrate away to 0 because I am assuming that f_s T is much, much, much larger than 1, or if you are not happy with that I can consider f_s to be some integer multiple of 1/t.

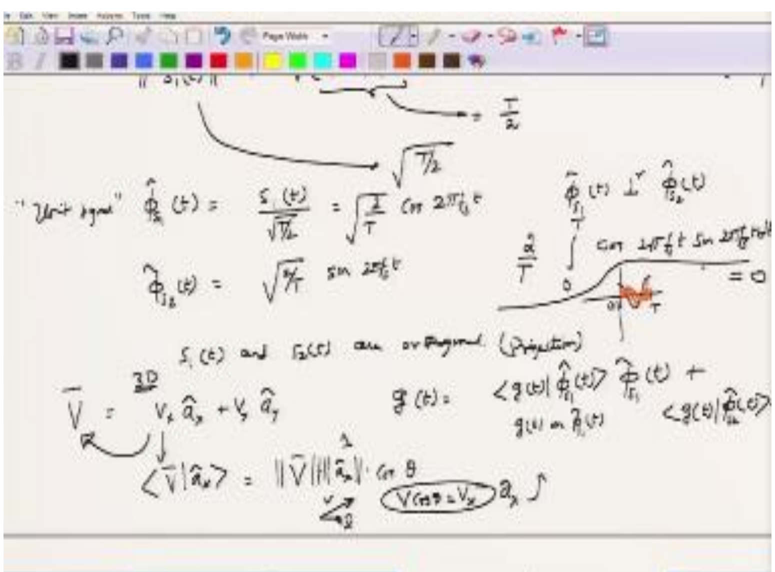
Okay so it must be some integer multiple or it must be very, very large compare to one so that all these high frequency isolations are all gone and what I get with this one is basically t/2. So what I have obtain this just the inner product so what I obtains is the inner product but the length would be if I want to find he length I have to take the √of the inner product.

So I get √t/2 therefore the signal 5 s1 (t) which is the units signal okay this is a unit signal okay is given by the original signal s1(t)/ √t/2 which I can rearrange and write √2/t cos $2\pi f_s t$ similarly you can show that 5s2 of t can also be written as √2/t sin$2\pi f_s t$, now if you want to find out whether this 5s1(t) is really perpendicular this is a short hand notation to 5s1(t)perpendicular orthogonal.

So if you want to find out whether these two are you know orthogonal to each other you simply have to consider the inner product of these tow itself right when I consider the inner product this √2/t would be common to both terms it would come out so it becomes 2/t 0 to t then I have cos $2\pi f_s t$ sin $2\pi f_s t$ clearly cos signal would be over the time duration 0 to t, it would be function of this form whereas the sin function would be so I hope have not written it in properly but if you look at the sin function.

What this implies is to basically take the area under the product of these two and that area actually vanish assort because one of them is our function the other one is even function so this fellow will be equal to 0 indicating that originally s1(t) ands2(t) are orthogonal okay and one other concept is to look at the decomposing the signal in to it is components because the corresponding relation is that if we have a vector v.

(Refer Slide Time: 30:25)



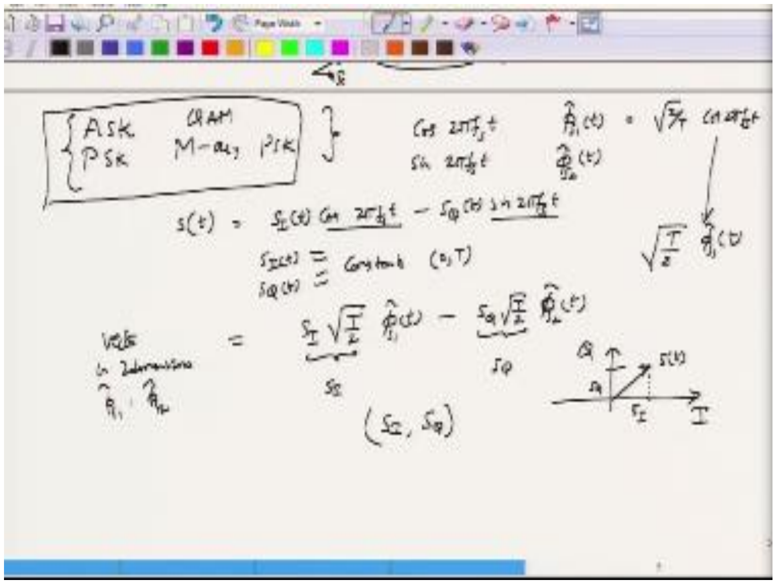
And if I have for example of 3D space then it is possible for me to define 3 vectors which are mutually perpendicular to each other. And in terms of those vectors I can define in term of those unit vectors say $\hat{a}_x$ I can define this right so I let me considered only two such particular cases so $V_x a_x + V_y \hat{a}_y$ I can define this vector V similarly if I want to write down the vector in general let us say g(t) then what I have to do is I have to find out the inner product of this vector g(t) on to the unit vector.

Or the unit signal 5s1(t) this would give me the component of g(t) on 5s1(t) and then this has to be multiplied along the unit vector in the direction of x right so if you back to this one this V_x is nothing but the inner product of v with $\hat{a}_x$ why is that so well this is because if I now look at the length of this vector go back to the definition the length of the vector V ax magnitude or the length of this one and cos of θ were θ would be the angle between this V and $\hat{a}_x$ right this length is one this length is V.

So what I get is v cos θ and this is actually the length of the vector v along x direction but this is just the length if I want to find out on the vector along x I have to multiply this one by the unit vector ax right so looking at the same logic here take this g(t) and then find the inner product of the g(t) on to 5s1(t) we call this as the projection so we say that project g(t) on to the 5s1(t) this will give you the length of g on 5s1 multiply this one with 5s1 of t and similarly you can multiply this you find the inner product of g(t) with 5s2 of t right.

And then multiply this resulting component by 5s2 of t this way any signal s(t) which satisfy certain conditions can be written in terms of these tow vectors which are the unit vectors or the unit signals. So in a sense we think of a signal as a vector much more important discussion would be possible when we discuss the optimum receivers at this point it is perhaps enough for me to discuss this relation or the notion or the signal as a vector.

(Refer Slide Time: 32:54)



Because that is all I need I will be considering next the amplitude shift keying phase shift keying in general a M airy phase shift keying or I will be considering QAM all these modulation methods can be geometrically understood by talking about two unit signals okay our two orthogonal signals these orthogonal signals are cos $2\pi f_s t$ and $\sin 2\pi f_s t$ the corresponding signals which are the unit signals along these two can be label as 5s1 and 5s2 of (t).

This is enough for me because we saw in the last module that any signal s(t) can be described by its in phase component si of t cos $2\pi f_s t$ –sq (t) $\sin 2\pi f_s t$ as long as si of t and sq of t are nearly Constance okay over the time duration 0 to t then we know that cos $2\pi f_s t$ and sin $2\pi f_s t$ are essentially perpendicular to each other right so this fellow is actually given by I know that 5s1 of t is given by √2/t cos $2\pi f_s t$ so I can in fact write cos $2\pi f_s t$ as t/2 under √5s1 (t) okay.

So when I have si of t and sq of t the in phase and quadrature components are call tens write over the duration 0 to their approximately constant then this particular signal s(t) is actually a vector in two dimensions, in this case the dimensions are coming from 5s1 and 5s2 which are related to cosign $2\pi f_s t$ and sin $2\pi f_s t$ right so this would be Si component multiplied by some t/2 5s1 (t), so if you call this entire thing as some Si you know and you have similarly sq √t/2 5s2 (t) then this would be sq.

And you can then define s(t) or capture everything that is there about s(t) by giving the two numbers si and sq which would then allow me to constrict two dimension plan which I will call as I and q okay, and in this I and Q plan I can represent this is and sq as a vector okay this would be my coordinate system the coordinate system has this components of is and sq, so this would be my geometric meaning of s(t).

We will stop her and then we will start with the modulation methods and how do we perform these different optical modulation methods digital optical methods and how do we represent them graphically in the next module so we will stop here and then we will continue in the next module. Thank you.

<u>Acknowledgement</u>

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production