Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – II
Module – I
Review of Signals and Representations – I

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome in this module we will first review some concepts from signals and representation of digital signals or rather signals as vectors and we will talk about concepts such as signal space this is important because we are going to study digital communications and digital communications would involve us to learn about constellations and this constitutions are coming from these signals as signals as being though of us vectors.

This what is sometimes called as the geometric approached signals understanding signals and that is very valuable and widely used in analysis of digital communication systems and that is important for us in the optical communication systems because much of optical communication today in fact 99% of the optical communications today happens in the digital communication format in the sense that we only almost have digital optical communication systems.

So to understand those concepts we will have to review some fundamentals let us began by very briefly recalling what a signal is and then talking about it is Fourier transform okay so a signal we have been talking about time varying wave forms and a signal is essentially a time varying wave from okay suppose S is a.

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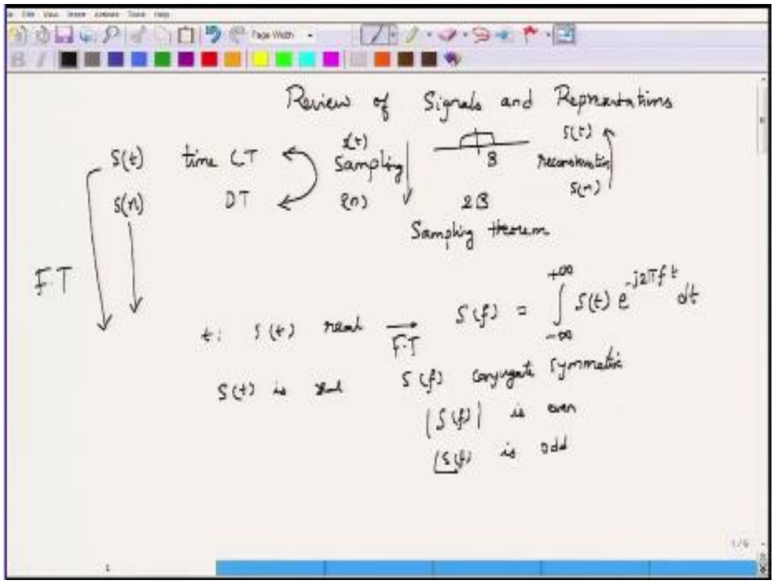

Mapping that I take so you know I have a variable called time so at each time I might have some amplitude or I might have some value it could be power as well so all this values can be captured by writing thus as s(t) so s(t) would represent two notations one it would give the value of this particular function which would be defined by s(t) at the time t it would also give me how the function itself is changing so it is kind of slightly two related notations which we are compressing by saying that s(t) is a signal.

This signal is continuous signal because time t can vary continues signal because time t can vary continuously there is a different signal called as a discrete time signal in which case time can take on only certain samples okay this is coming from the time domain but in fact you can also have discrete signals in a various other context for example you know you are playing cricket the number of the numbers that you score against each ball right would be a example of a discrete time signal.

Right so you can have discrete time signals coming not only as a time dependent thing a discrete signal coming not only on terms of the time but it can also be a function of anything else for example you take a picture that would correspond to a 2 dimensional discrete signal with each variable being the pixel point and the corresponding intensity of the picture would be the discrete signal there it would be a 2D signal.

We will motley will be dealing with only 1 dimensional signal in the sense that we will have only one independent variable and the corresponding output we denote a continuous time signal by s(t) writing and assuming that t is a continuous time quantity or t is a continuous quantity a discrete time signal is denoted by writing it has s(n) where this s(n) stands for the sample at time n as well as the sequence of such samples.

Okay so this is a continuous signal this is a discrete signal we will assume that this n is actually coming from time now there is a relationship between these two signals in the sense that if s(t) satisfies certain conditions then it is possible for us to represent this s(t) not by measuring the value of s at all points t but measuring the value of s at a certain time instant okay in other words I can go from the s(t) representation to it is discrete time version by a process known as sampling okay .

I can go from s(t) to s(n) provided that s(t) satisfies certain conditions namely that it is bandwidth is limited to a certain range of frequencies then it is possible for me to sample fast enough so that s(n) would be a accurate representation of s(t) I can go from s(n) to s(t) so this was the sampling process I can go from s(n) to s(t) in you know what is called as the reconstruction of the signal okay.

So this relationship between sampling rate and the bandwidth if s(t) band limited to B Hz then I need to sample this one by at least twice the rate of B is called as sampling theorem so sampling theorem tell us how to go from continuous domain to discrete domain although in this case we are dealing with only time signals as I said it is possible that the variable could be any other continuous quantity and the corresponding quantity will be discredited okay.

So you can go from continuous to discrete signals now these representations of s(t) or s(n) or what is called as a time domain representations assuming that t and n represent the time then these are called as the time domain representation there is in fact a equivalent and much more widely used representation called as frequency domain representation okay this is very useful because s(t) understanding s(t) in time domain gives you one kind of a picture but many engineers would prefer to understand these signals s(t) and s(n) by going into the frequency domain.

Where they can also think of translation in frequency they can think of filtering a certain band all these operations require us to visualize or to understand what happens to the signals in the frequency domain their representation of s(t) in it is frequency domain the representation of s(t) in it is frequency domain can be optioned by carrying out what is called Fourier transform okay is s(t) is real valued signal that is at every point t corresponding s(t) is real then we call this s (t) as a real valued signal.

If such a real valued signal for such a real valued signal we define the Fourier transform and we denote the Fourier transform by s(f) this s is capital this is s is small sometimes we will denote this by writing this as Š(f) in this case I am just going to use S(F0 itself. So this is s is a small case letter capital S stands for the Fourier transform so for such a real valued signal of course Fourier transform can also be defined for complex valued signals but we will not be interested in the complex valued signals at least for some time.

So s(t) for a real valued signal will be s(f) = $\int_{-\infty}^{+\infty}$ this is the integration over time by multiplying this s(t) by a factor which is s(t) $e^{-j2\pi ft}$ okay this is your Fourier transform representation when s(t) is real the Fourier transform terms have to conjugate symmetric that is if s(t) is real then Fourier transform is conjugate symmetric okay what do we mean by conjugate symmetric or what do we mean conjugate symmetry it simply means that the magnitude of s(f) is an even function and the phase of s(f) remember s(f) is a complex number right.

So for every frequency f you are going to get a complex number which is s(f) because you have a complex signal $e^{-j2\pi ft}$ getting multiplied to s(t) right so this phase of s(f) is odd symmetric okay so this the meaning of conjugate symmetric.

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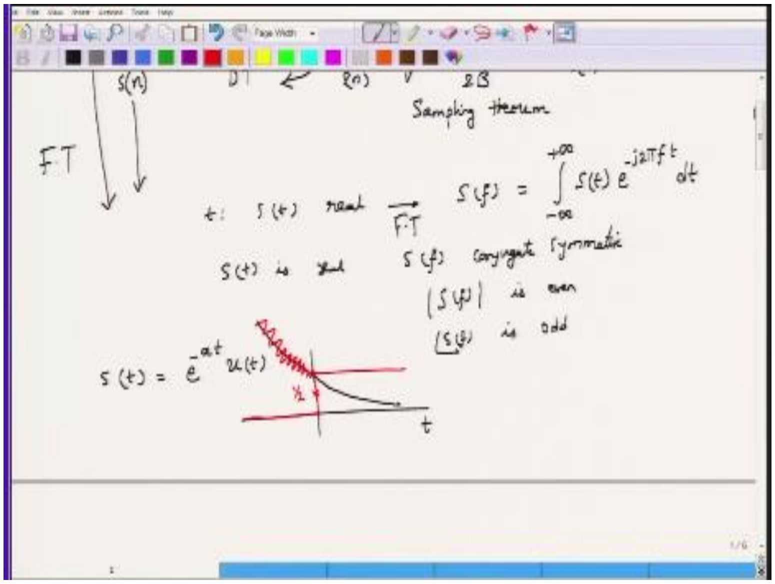

And this is how you can obtain the frequency domain representation of a signal okay as an example consider s(t) to be an expositional function say e^{-at}u(t) e^{-at} is a expositional function if you have simply plot that function what you would see is that at t = 0 this would be 1 and then it would be decaying like this okay however t can also go negative because we did not save anything about t being positive or negative so what it means is that a t= -∞ this fellow would be at ∞.

Right so this how e^{-at} is and it turns out that such a signal cannot really have a Fourier transform okay because the integral will not converge okay in an ordinary sense so what we do is what are we are mostly interested is we want to know what is a Fourier transform of 1 side of the signal right to obtain that one side we introduce this function called as the step function this step function is defined by having it is value = 0 for t less than 0 and it will be equal to1 for t greater than 1.

And at the middle that is a t equal to 0 sometimes this is defined as having a value of ½ which would be the average value to the left and average value to the right okay so this is your u(t) we will when you look at this way you will actually see that because the signal u(t) is 0 for t less than 0 whatever may be the vitiations of e^{-at} B that just gets vanished over this does not come at the output and whatever variations after that would be multiplied by 1 therefore you have a exponential signal that is going like this right.

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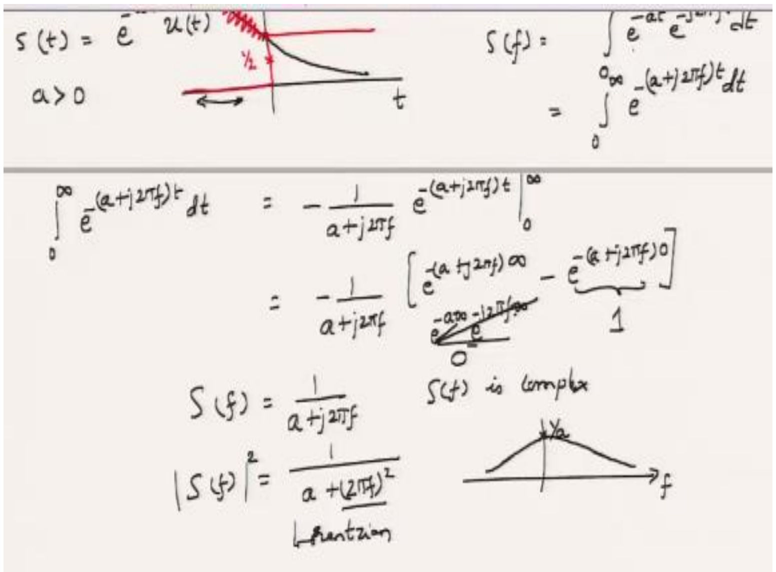

So what you have done essentially is to remove all the values of e^{-at} function so if this 1 function so this function values has been removed when you multiply this one by u(t), okay. What could be the Fourier transform of this? Well to obtain the Fourier transform let us go to the expression s(f) is given by e-^{at} now the integration can go only from 0 to ∞, right because - ∞ to 0 the value of the function is 0.

So there is nothing to integrate so go from 0 to ∞ multiply this one by $e^{-j2\Pi ft}$ because that is a definition then you combine this exponential say no powers right, I mean the argument of the exponential function, okay. Why can I do that because I know that exponential of θ1 and exponential of θ2 is actually exponential of θ1 + θ2, right? So this is the power law of the exponential functions.

I can do that and modify the integrant as $e^{-a + j2\Pi f}$ t is a common variable write this as dt. Carry out the integration I know the integration of 0 to ∞ $e^{-a+j2 \Pi f(t)}$ dt is nothing but 1/a+j2Πf right $e^{-a+j2 \Pi f (t)}$ within 0 to ∞, there is a small correction of course here there has to be a minus sign because integral of e^{ax} will have 1/ a and in this case a is negative you know or maybe you should have used the different one so e^{ex} will have 1/b times e^{bx} here we simply identify that b is negative because you have e^{-} some quantity times t, okay.

Because of this we have a minus sign here and if you now look at what happens to this quantity inside you will see very interesting thing so I have -1+j2Πf and then I have $e^{-a+j2\Pi f}$ times infinity minus $e^{-a+j2\Pi f}$ x 0 this part should not represent as with any difficulty because a x o will be 0, j2Πf x 0 will be 0 so exponential to the power 0 is equal to 1, what about this quantity? Here I have $e^{-a\infty}$ which would be $e^{-\infty}$ itself.

Because a being a positive quantity I was assumed here I am not told you specifically but a must be a positive quantity, so this $e^{-a\infty}$ could be positive, right. Then I also have $e^{j2\Pi f}$ x ∞ right or a – j2Πf x ∞ right this fellow will again became ∞, right. For f positive if you be a positive ∞ for f negative this would be approaching negative ∞ that is when f goes to infinite then this ∞ in the positive sense then this would be $e^{- \infty}$, right.

That is getting multiplied by $e^{-\infty}$ this is a real signal this is a complex signal because there is a j sitting there, however the product of this one will be equal to 0 as f goes towards ∞ right positive ∞ what happens as f goes towards negative ∞? This quantity if actually not growing but this is actually a oscillating signal, okay. However this oscillating signal is getting multiplied by $e^{-a\infty}$ this could not change.

Whether F goes to negative ∞ or positive ∞ this $e^{-\infty}$ will always towards 0, however if this have behaved in very erratic way then we would have had the problem in evaluating this expression luckily $e^{-2\Pi F}$ ∞ will always have its maximum value to be equal to 1, okay. Because of that even as f goes towards ∞ this does not really matter to us because this magnitude will always be equal to 1 and then that is getting multiplied by a 0.

So this quantity is actually equal to 0, so regardless of whether f is approaching + ∞ or f is approaching - ∞ this fellow is equal to 0 this id equal to 1, a minus sign here and a minus sign will cancel with each other and you end up having the Fourier transform as 1/a+j2Πf as we expect s(f) is complex right. So the Fourier transform is complex but now if you look at the magnitude of s(f) we will see that the magnitude is given by $\sqrt{}$ $(1+a^2\ 2\Pi f)^2$ correct?

So this is the magnitude mean write down correctly and what you see whether f is negative or f is positive this s(f) would always be equal to a positive quantity, in fact if you where to graph this magnitude of f(s) as a function of f at f =0 you have a value of 1/a because a^2 and the square root will cancel so you have a value of 1/a and when f is going positive and when f is very large when this quantity $2\Pi f^2$ will become very large compared to a^2.

And the square will cancel and then this could essentially go down to 0, similarly when f is going large in the negative value region that is when f is going towards - ∞ this fellow this $(2\Pi f)^2$ will be much larger compare to a^2 and then you can remove that neglect this a^2 quantity and then the square will go away but because f is getting squared even as f goes negative this fellow will always go towards this would be positive and would be going towards 0.

So you can connect these two and this would essentially be the way in which s(f) would behave such a description or rather when you square s(f) right, so when you square this one then the square root will go away this base or this function is widely used in lasers you know to represent and you can actually show the lasers will have such a characteristic if called as a Lorentzian, okay. So this function is called as a Lorentzian. We will see this Lorentzian when we discuss the line width of a laser later, okay.

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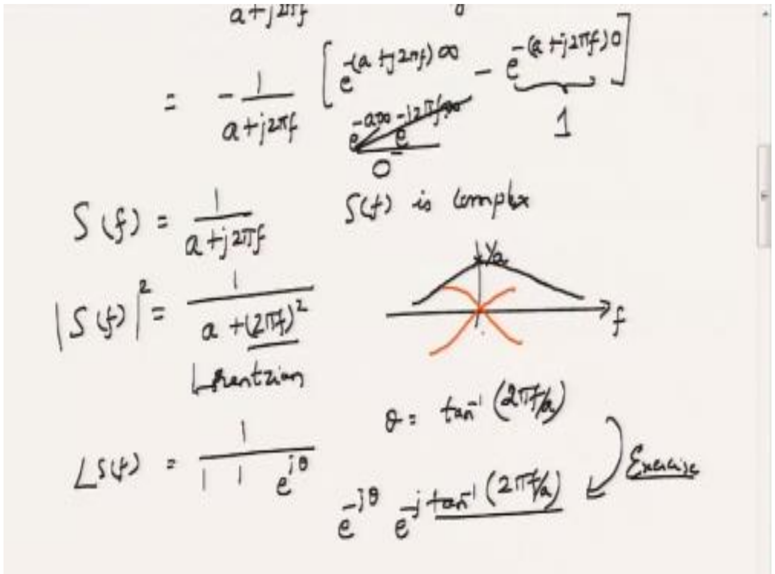

Now what you have to get from this equation is that magnitude of s(f) is an even function as you can very clearly see it is a even function of F but if you want to calculate the angle of s(f) you would be seeing that this would be negative I mean this would be odd symmetric y because I can represent this a+j2Πf in terms of its magnitude and if phase which I can write this has $e^{j\theta}$ where $\theta = \tan^{-}(2\Pi f/a)$ and because this $1/e^{j\theta}$ is there this will go up and become $e^{-j\theta}$.

So the angle will be $\tan^{-}(2\Pi f/a)$ so this fellow will be $e^{-(2\Pi f/a)}$ when you look at this function right and if you start looking at the values of f this would be negative did I get it alright, so I hope that this would be okay, so just you can put a small thing and find out this has an exercise what you should be able to show is that this could be a odd function, so for S(F) positive it would be going in this way.

So for negative F it would be going this way or may be in this particular way I am really not really concerned about which way it goes but essentially to show that this has to be a odd faction of F, okay.

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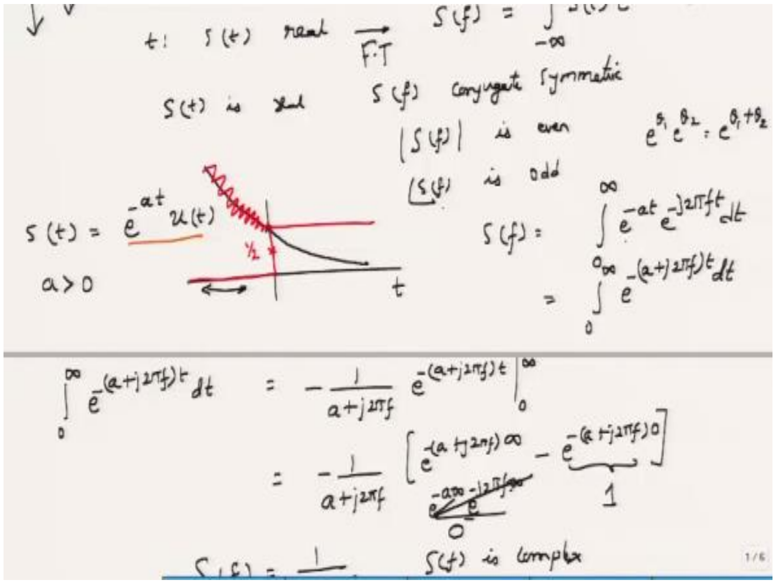

So what I was trying to tell you is that s(f) for a real valued s(t) and certainly this $e^{-at(t)}$ is a real valued signal for this real valued signal s(f) turns out be a complex or you know it turns out to be a conjugate symmetric function because conjugate symmetric implies that s(f) magnitude is even symmetric and s(f) is odd symmetric, okay. So this is what the Fourier transform representation for the signal s(t). Now why is this important for us, well let us look at one example here.

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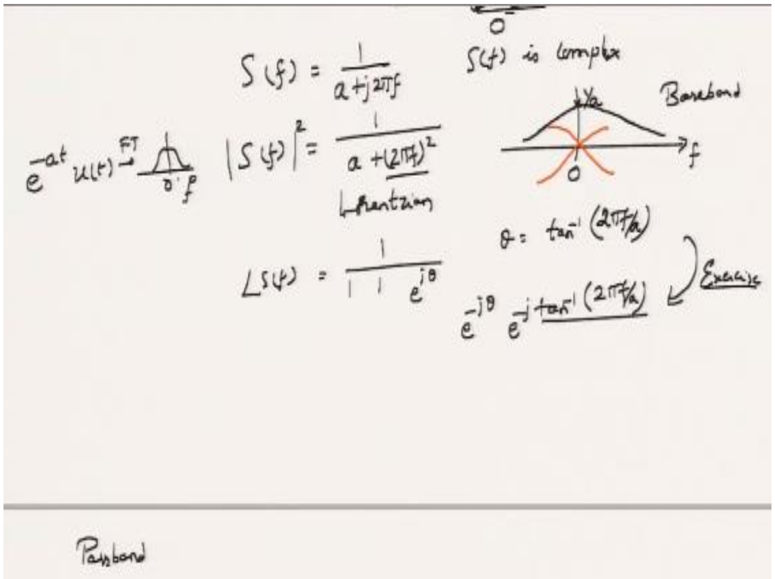

Or let us look at one system here where I start with what is called as a pass band signal, what is pass band signal? Well if you go back to this Fourier transform where is the average frequency of this Fourier transform or where is the Fourier transform centered at, it is centered at o hertz, correct? So such signals are called as baseband signals, so this $e^{-at\ u(t)}$ whose Fourier transform is centered at f = 0 right.

This is the Fourier rid mine so the Fourier transform is centered at 0 is called as a baseband signal because much of its frequency content is located at f= 0 or the DC signals, however there are situations when you consider signals whose fixed centre frequency is located at not 0 frequency but at some very high frequency that is greater than 0, how can I obtain or where do I find such signals well?

Whenever you modulate a signal we have seen that after modulation the spectrum would essentially move towards the higher frequency, consider for example.

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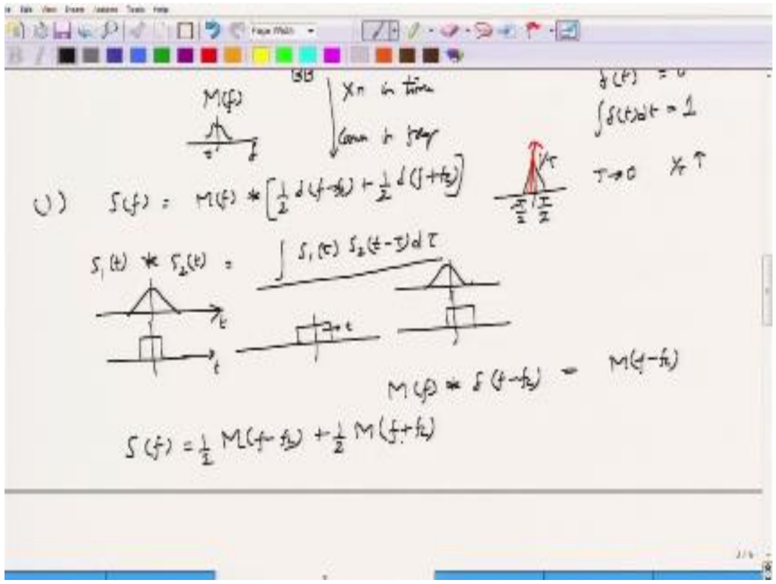

Your double side band suppressed carrier modulation s(t) is given by m(t) x c(t) we know that c(t) is cause 2Лfct and m(t) is whatever the massage signal m(t) that you are sending let us assume that the signal m(t) is base band indicating that m(f) would be centered at 0 frequency okay so /9t) is a base band signal what would be the Fourier representation of cause 2Лfct well we will not going to the details of the Fourier transform there are some issues here but the Fourier transform of this one is given by half δf – fc + 1/2 δf + fc okay this δf is a function which is called as the impulse function which is defined.

In a way which says that if δ(t) is a impulse function then δ(t) = 0 for t not equal to 0 and the area of δ of t will be equal to one we normally consider them to be a normalize to one so what we are saying is that it will be 0 for t0 = 0 and if you integrate this one this would be equal to one and example of an impulse function would be a triangular function okay of I know width t and an amplitude of 1/t but as you start decreasing t you know as you consider sequence of such functions where t goes of to 0 1/t goes no grows and grows and it would essentially start to look more and more like a.

And it would like this eventually it would start to look like a impulse function okay so this an example of an impulse function except that in this case impulse function happens to be in the frequency domain right now when you find out what is the Fourier transform of s(t) you will see that this would s(f) and it is given by m(f) right convolved with ½ δf – fc + ½ δf+ fc you might be wondering what this convolution is convolution operation is defined for you know in this manner so if I have two signals s1(t) and s2(t) the convolution of s1 and s2 of t is denoted by this star and written as or evaluated as s1of tow s2(t)– tow and integrate over this variable t tow is a dummy variable.

All you are doing is take the signal s1 of t and then take the signal s2(t) invert it and then keep delaying it okay so if s1(t) is this write so if this is your s1(t) and this is yours2(t) okay all you are saying is that if you were to invert this s2(t) okay in order to so if you flip this around you will get this signal and you need to delay it okay so you are essentially going to get this signal this needs to be multiplied point twice to this s1(t) so you multiply these two signals and integrate you keep doing this for every value of time shift t and then you will essentially obtain the result of this integration.

Okay more details are of course available for you in the signals courses please referred to that if you are forgotten you can go back and read some signal and system text book you will be able to understand what convolution is or will be able to recall what convolution is okay with that in mind convolution operation can be performed in time domain it can be performed in frequency domain it turns out that this impulse functions have a very special property that if I convolve any signal which is not an impulse.

Write m(s) convolved with δ f- fc right what I get is that it will be m(9f – fc) okay so when I convolve them I will get f of m f- fc ands why this is convolution in the frequency domain is because there is a Fourier transform theorem which tells us that multiplication in time domain is convolution in frequency domain okay so because of that your double side band suppressed carrier signal which is m(t) x c(t) can the Fourier transform of that can be thought of at the convolution of m of the Fourier transform of m of t which is m of f and the Fourier c(t) which is this δ function right.

So if you now look at this convolution I am not proving any of this so if you look at this relationship and then applied to this equation which we will call as one then I can find the Fourier transform s(s) as m(f – fc) there is a half here + ½ m(f + fc) what is this m of f – fc and m of f + fc this is now.

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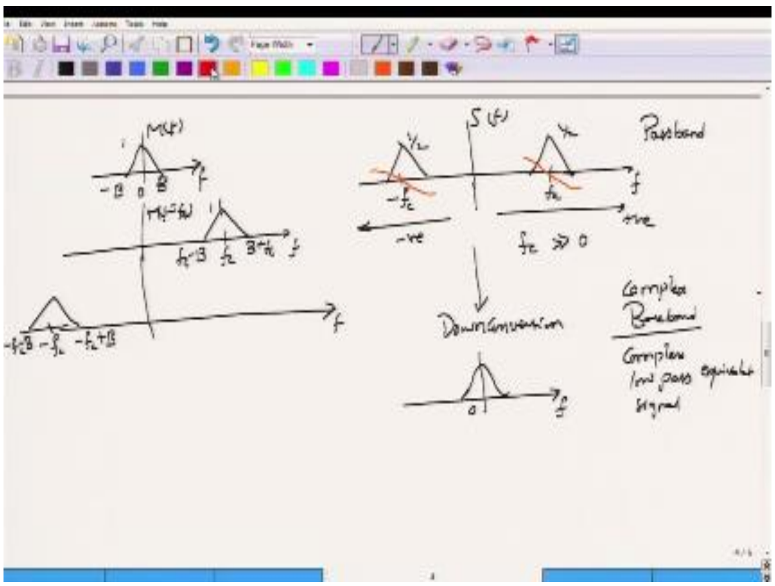

If you start with m of f right so let say this is my m of f I am just plotting this one if you start with this which is center at 0 as it should for a base band signal m of f- fc would correspond to a signal which have been shifted to the right okay so it has been shifted so let say the value here is one so this is one and it is say this value has, has a value of b and this is – b so therefore this becomes fc + b this is fc – b okay similarly so this is m of f – fc okay n and m of f + fc would be a function which would be shifted to the left which will now a centered at – fc and will be at – fc + b.

–fc - b okay when you combine these two what you get is this s(f) when you also change the amplitude from 1 to ½ so you get one at located at fc the other spectral ½ is located at – fc this has a value ½ this has value ½ this is your signal s of f okay now such a signal which is centered not at 0 frequency but centered at a different frequency fc, fc must be much larger than 0 here so are the DC signal is called a pass band signal pass band signals are encounter whenever you modulate your time domain based band signal so when you look at the corresponding frequency domain.

You will see that they are now centered at value fc okay now for one of things that has now happened is if you have taken s of t has an original valued signal as we should obtained when you have when you connect physical modulator the signal that you are obtaining will be a real signal a s of t for such a signal s of t the Fourier transform will show that it has components both in the positive frequency domain as well as in the negative frequency range right.

So you have negative frequency domain so here is all the frequencies are positive here all the frequencies are negative so the corresponding s of f has values non zero values in both but you can already see that this both copies are essentially identical because the magnitudes spectrum of these two are the same and when you look at the phase spectrum you will see that these two were also the identical okay they would also be odd functions so the point here is that it that of seems redundant to you have two copies okay in fact when you take a modulator in the electrical domain what you will find is that if you start with the real valued signal s of t which is a pass band signal.

Okay the first operation that you will do is what is called as the down conversion operation the result of a down conversion operation is a signal that is now centered at 0 frequency we somehow have to make this signal which is centered at non zero frequencies to be centered at the frequency f okay this process of going from bass band to baseband is called as down conversion, now there is a very important fact about down conversion that we have understand the way in which this down conversion we are going to do will result in not as real valued signal.

But in a complex base band signal okay it will result in a complex base band signal or sometimes called as complex low pass signal okay complex low pass equivalent signal these concepts are not only important in communication they are also important in laser theory so if you spend some time understanding this you will be able to understand and appreciate the topics in laser theory also very well so this is called as low pass equivalent signal or co complex low point equivalent representation which will be obtained by down converting from pass band to base band signals.

Now how do we do this down conversion all I can think of is a very simple way I already know one function which takes away all the negative variable values right so if you recall this e- at signal here.

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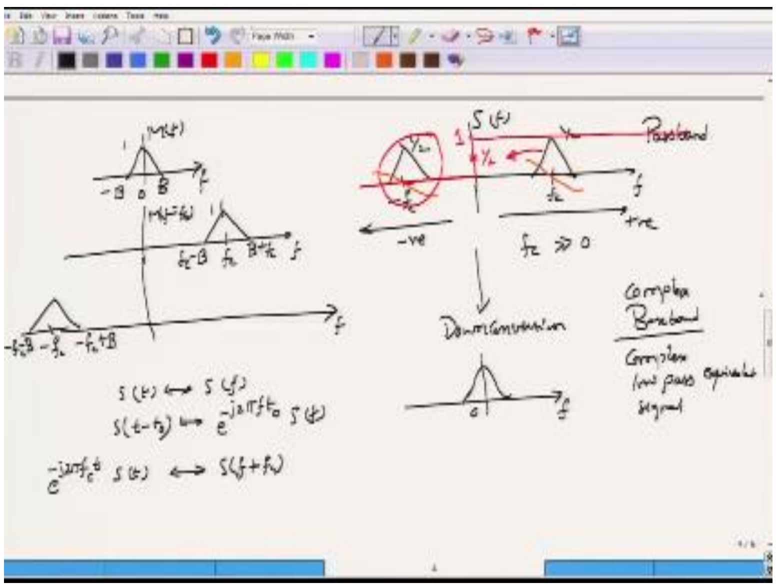

We multiplied this e- at with this function u(t) u(t) what it did it removed all components of e – at which when t was less than 0 now if I can do a unit step function in time I can as well do a unit step function in frequency right so what I have to do in order to eliminate all these components would be to multiply this one by a unit step function okay by a unit function whose value will be equal to 0ne for f greater than 0 and it would be equal to 0 for f less than 0 right at f = 0 sometimes we can have this value are equal to ½ so when you do this all this negative frequency components would vanish after I multiply S(f)/U(f) all this components are gone.

What would be the corresponding spectrum, the spectrum is now having only components in the non zero, I mean in the only in the positive frequency domain, right. Now if you look at that spectrum that would be centered at f_c, now I have to bring this f_c down to 0 what should I do, I have to translate this back in frequency, right.

Here is where your couple of the Fourier transform properties would help, if you take s(t) which have the Fourier transform S(f) and when you shift this one by a factor t_0 to the right or to the left depending on whether t_0 is positive or negative this would be equivalent of multiplying this Fourier transform by this phase factor $e^{-j2\pi ft0}$ ·S(f). Similarly if I start with s(t) which are the Fourier transform of S(f) and then if I consider S(f)-f_c then this would be equivalent of multiplying this by $e^{-j2\pi fc}$ ·s(t).

So multiplication of s(t)/ $e^{-j2\pi fct}$ is equivalent of shifting the spectrum to the right, but I do not want to shift the spectrum to the right, I want to shift the spectrum to the left, so to do that I simply consider f_c to be negative, right so if I consider f_c to be negative then I get s(f+f_c) and this will be a minus signal here.

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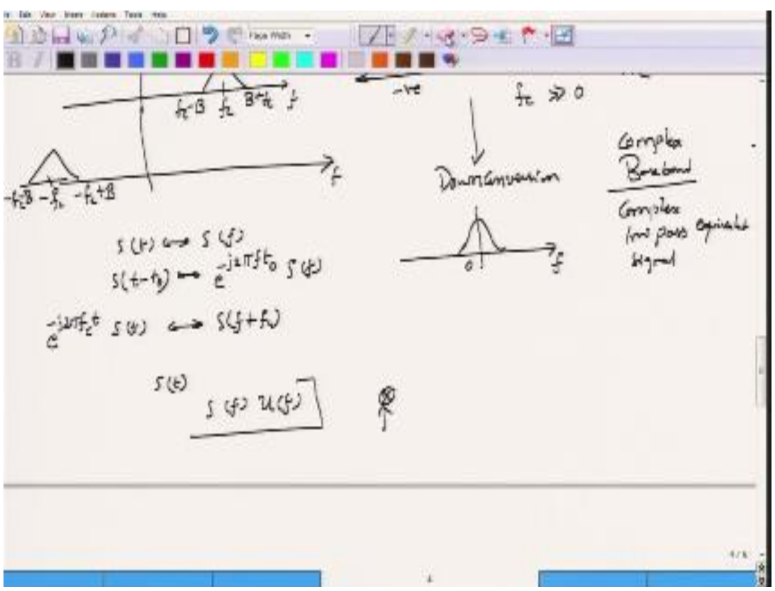

So I have identified two operations that I need to do, start with the real value s(t) which you would anywhere receive to obtain the complex low pass equivalent signal or the complex base band signal from this you multiply the Fourier transform of s(t) which is S(f) by the function U(f), right and then in time domain you also multiply this one, so this is let us call this as let me just rewrite it in the block diagram way, so that we can understand.

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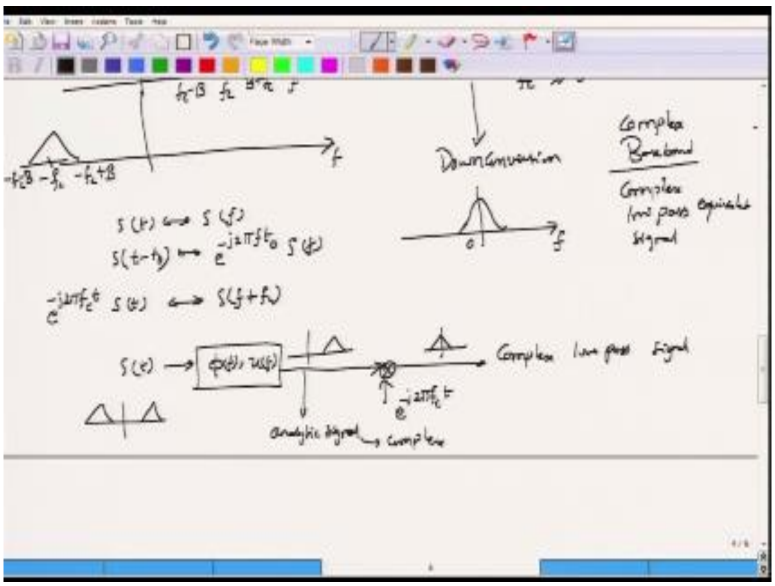

So S(t) is coming in this way that goes into filter who is transfer function $\phi(f)=U(f)$ and the output of this filter will be multiplied by $e^{-j2\pi fct}$, okay what comes out will be the complex base band or the complex low pass signal. In fact there is a name to this particular output, okay name to this output it is called as analytic signal, analytic signal will have its Fourier transform only in the positive frequency range, okay. So you have S(f) here, right and this after multiplying by U(f) you will have an analytic signal which will have components only at the positive frequency region and once you multiply this one by $e^{-j2\pi fct}$ what you get is a complex low pass signal, okay.

Why should I get a complex low pass signal, well this signal by it is or this spectrum by itself will result in a complex valued signal, because the Fourier transform is now not symmetric. If the Fourier transform is not symmetric it is not congruent symmetric then the corresponding time domain signal will be a complex signal, so the analytic signal is necessarily a complex signal, this complex signal is now being multiplied by one more complex signal it turns out that the overall signal that you are going get will be a complex low pass signal, okay.

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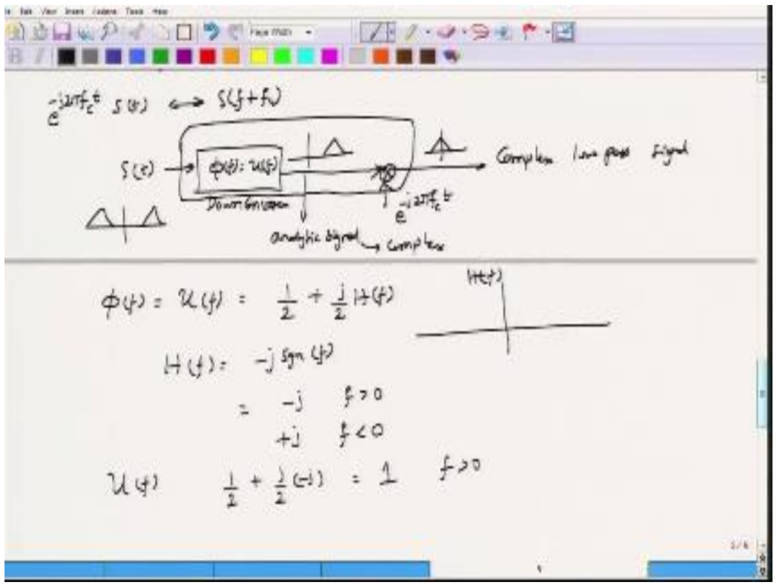

So I hope this is understood once you have understood this low pass signal then we can also understand you know in the time domain expression, if we do not want to always think of the frequency domain in this case, it is necessary to also look at the time domain representations of this signals to do that one let us split this ϕ(f) which was the filter here, which was used for down conversion process or rather this entire block is for down conversion process, okay so this entire block is for down conversion process this filter was removing all the negative frequency components, right this was the U(f) I can split this signal U(f) into two parts, I can write this as 1/2+j/2H(f), okay because I want to discuss the filter properties of this position.

So if I say H(f)=-j signum function of f, okay. What is this signum or the sign function, the sign function basically is positive when f is positive its argument is positive, it would be negative when it is argument is negative, so if you plot H(f) itself you will see that H(f) will be equal to –j when f is greater than 0, it would be equal to +j when f is less than 0, right. So this U(f) when f is greater than 0 will have 1/2+j/2.-j but I know that –j and +j is 1, so I get is this one equal to 1 when f is greater than 0.

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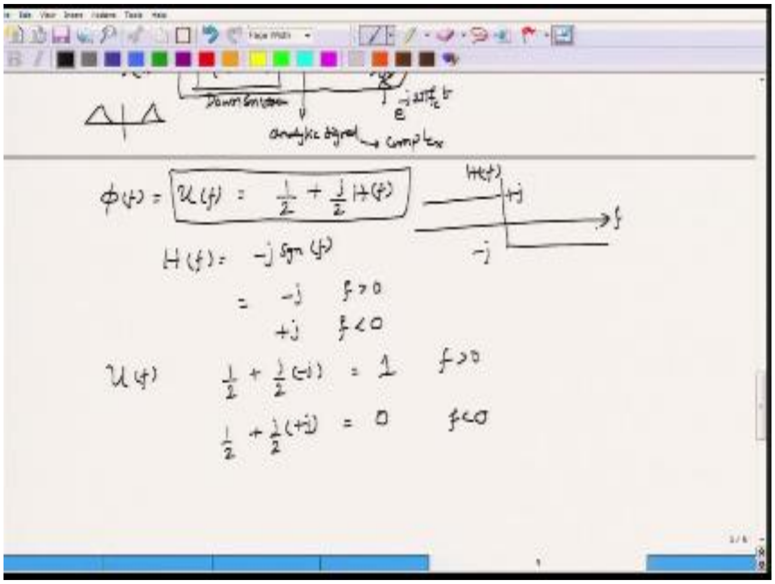

Now when f is negative for f negative this would be 1/2 +j/2 but this would also be equal to +j here, so this would be j and j is -1 so one 1/2- 1/2 will be equal to 0, so I am alright so I can write U(f) as 1/2+j/2.H(f) so that when f is greater than 0 I get 1, when is f is less than 0 I get 0, and if you define further that signum function must be equal to 0 at f=0 that would be the average of these two values then U(f) will be equal to 1/2 at f=0 and we recover everything about U(f) correctly. So I want to write down j(f) this would be –j and this would be +j as a function of f, okay.

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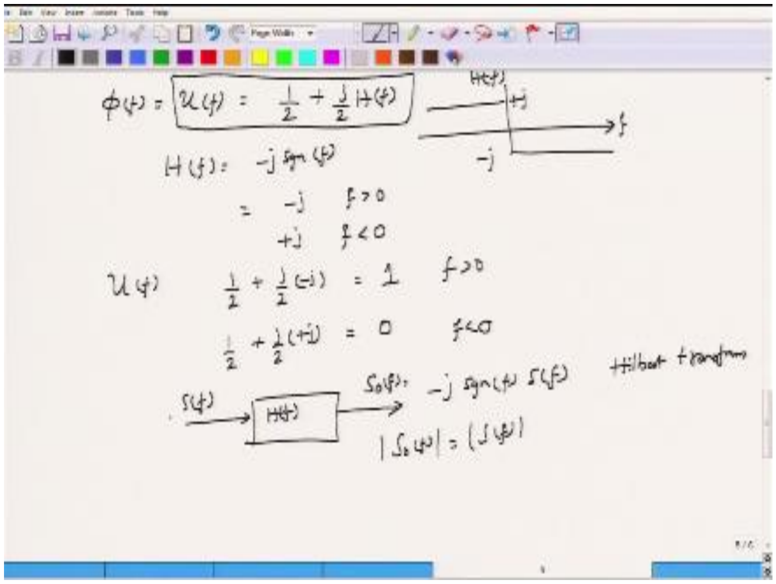

Incidentally if I take any signal and then pass it you know through this H(f) so if I have now a filter with simply realizes this H(f) and then pass this signal S(f) through this H(f) output of this one will be –j sgn(f).S(f), okay this particular signal this particular filter which produces this spectrum is called as a Hilbert Transform, okay. What is the significant of this Hilbert transform you see here that if I want to take the magnitude of this, so let us call this output as S_0(f) of this particular filter, so if you look at the magnitude of S_0(f) I will see that the magnitude should be equal to magnitude of S(f) itself.

Because there is nothing changing the magnitude of –j will be equal to 1 and sign will anyway be a not magnitude will always be equal to 1, so the magnitude if we put of this filter magnitude is equal to the magnitude of the input.

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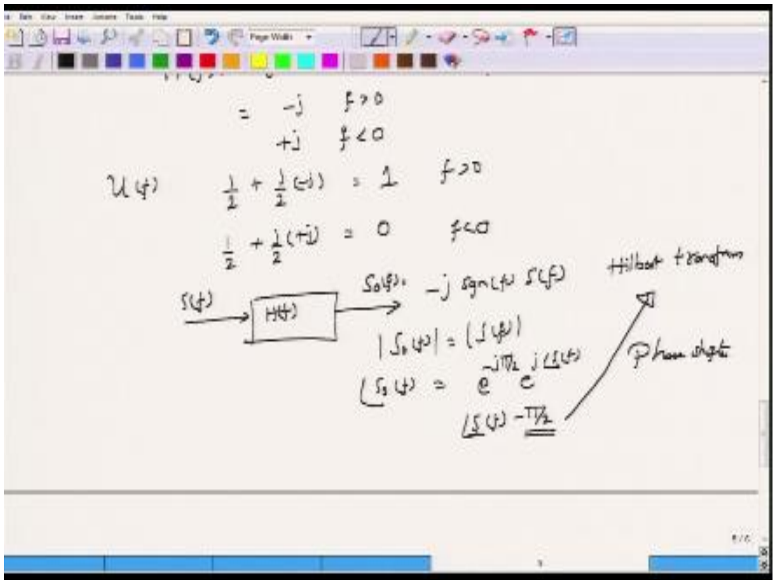

But look at what happens to the phase of $S_0(f)$, the phase will be changed because you have a –j here –j is equivalent of $e^{-j\pi/2}$, right and sgn function will be positive when S(f) is positive and when it would be negative when S(f) is negative, this would be added to the phase of S(f), okay. So the total phase will actually be phase of S(f)-π/2, okay when f is positive you will have to subtract –π/2 and when f is negative again you will have to subtract this –π/2, right. So all the frequency components of S(f) the phase of those frequency components are getting delayed by π/2, okay this is the action of Hilbert transformer sometimes that is why this I called as a phase shifter as well, okay. because its shifts all the frequency component phases by a value of π/2.

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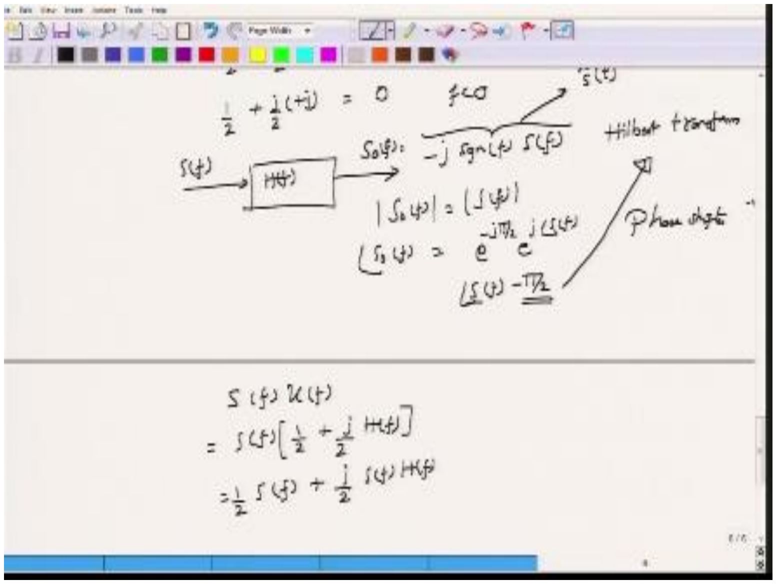

Now what is the significant of that well if you denote the output in the time domain, the output of the filter in the time domain you will see that would be a complex signal which can be written as $\hat{S}(t)$ that is if I take this –signum function of f into S(f) that would correspondingly give me a complex number, so I get S(f) multiplied by U(f) but I know that U(f) is nothing but 1/2+j/2.H(f), okay. So if I expand here I get 1/2S(f)+j/2S(f)H(f) and H(f) itself is given by sign function into f, right.

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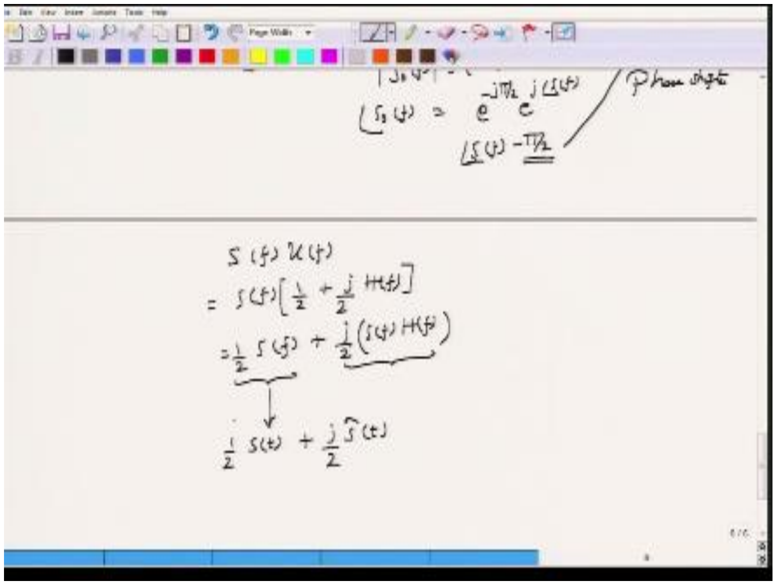

If I now go to the time domain representation this by taking the inverse Fourier transform what I get here is1/2.S(f) which would be S(t) and this flew will be a complex signal, right this will be a complex signal which we will write by writing a cap over S, okay this is called as the Hilbert transform of the signal S(t), okay. This would be the Hilbert transform of the signal S(t) and therefore, the signal that I am obtaining right after this analytic or this is you know after the multiplication by U(f) is called as.

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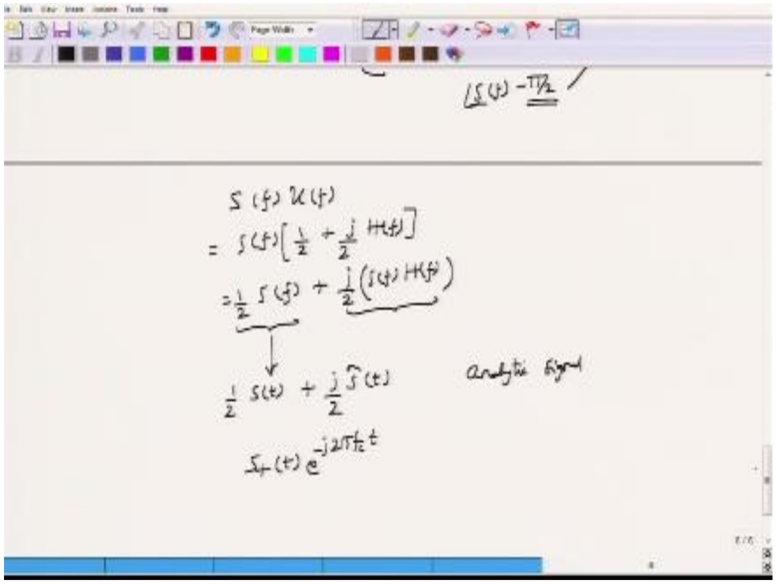

If it is the complex signal this is called as the analytic signal, okay this analytic signal will have its Fourier components only in the positive frequency range, okay. So this is your signal $1/2S(t)+j/2\ \hat{S}(t)$ this we will denote by writing at $S_{+}(t)$, okay this is not the completion to this $S_{+}(t)$ if I multiply by $e^{-j2\pi fct}$.

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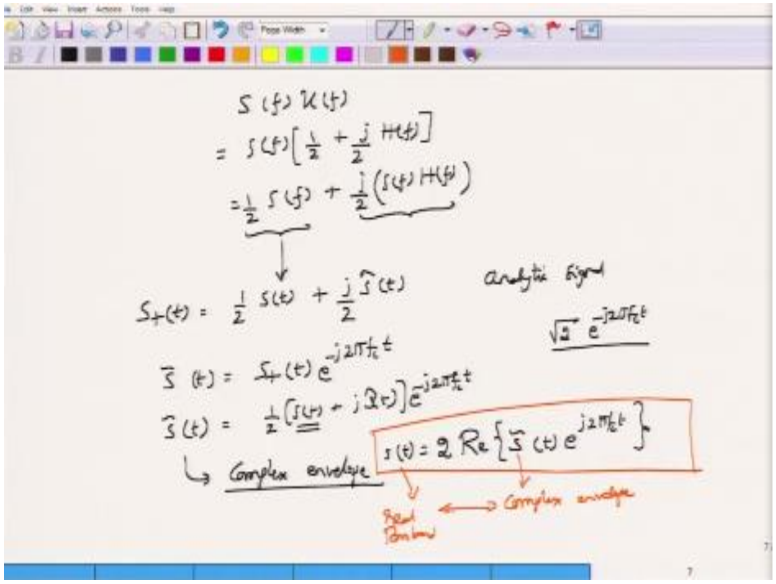

Remember this is the second operation that I am suppose to do to covert the base band signal to a complex low pass equivalent signal, if I multiply this one then what I get is the analectic signal s(t) okay this is given by and if you know recall what s+(t) is ½ s(t)+ jŝ (t) right and then you have it the -ĵ2π f_ct sometimes you know you do not multiplied by e/-j2π the other multiply by a √2 x e^{-j}2π f_ct this is done.

So I have scale up I am not done that one here but in some text some literature you will find in that the multiplication portion will have a √2 in the prefacer of e^{-j}2π f_ct just so that the energies of the complex envelop signal will be equal to the energy of the original pass band signal I have not done that one okay now this is you ŝ(t) called as the complex base band signal or sometimes called as the complex envelop okay.

What is the significance of this complex envelop well the significance of complex envelop is that you have s(t) so let us write it down here itself what would be the mathematical way of obtaining s(t) how do I obtain mathematically what would be s(t)in how to obtain that one I need to simply remove this e^{-j}2π f_ct so to remove that one I should be multiplying this s(t) by $e^{+j2\pi}$ f_ct so when I multiply her this e^{-j}2π f_ct and $e^{+j2\pi}$ f_ct will cancel so that is gone.

And I still have this part to recover this part can be recover by writing this as the real part of it so if I take the real part of it I simply obtain s(t) of course I what I obtain is ½ s(t) so therefore I need to multiply this one by a factor of 2 so I can go from this would be equal to s(t) so I can go from s(t) itself , you know I can go from s(t) itself which is a real valued signal so this follow is a real valued pass band signal.

From this real valued pass band signal I can go to the complex envelop okay and then I can recover real pass band signal from the complex envelop band visor versa.

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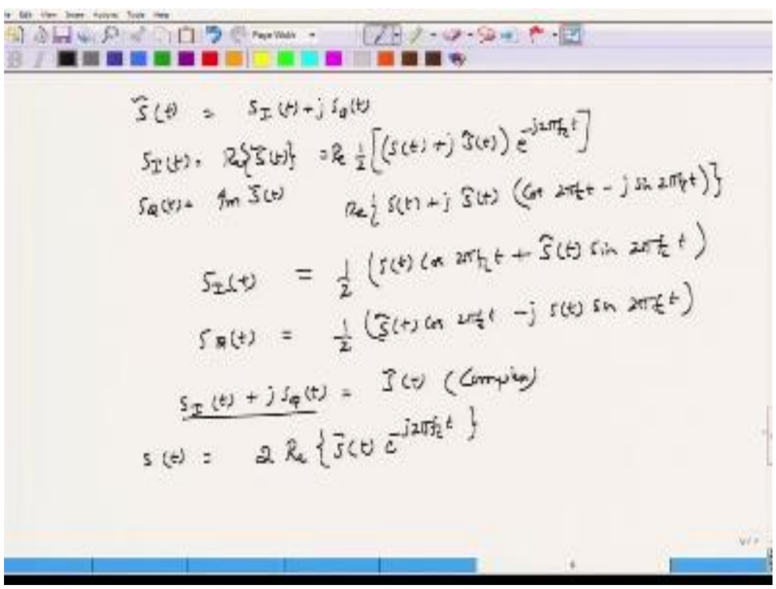

In practice or in you know in vides produce you do not normally write down s (t) as s (t) + jŝ(t) rather than that you write down this ŝsδ (t) which is complex envelop as s_i(t) + s_q(t) and call this s_i(t) as in face component and s_q(t) as the quadrature component, whereas I mean what is that s_i(t)is actually the real part of sδ (t) and s_q(t) is the imaginary part of s_δ(t) to obtain this substitute what is sδ(t) the complex envelop the complex envelop is there is ½ factor s (t) + jŝ(t) time c-$^{j2\pi fct}$.

I am looking for the real part of this which means that I have to expand this c-$^{j2\pi fct}$ and make this as $\cos 2\pi f_c t$-$j\sin 2\delta f_c t$ multiplying s(t) + jŝ(t). Let us forget about this ½ factor for now when I look for the real part of this I see that this would be s(t) $\cos 2\pi f_c t$ that is because s(t) and $\cos 2\pi f_c t$ would be the real number s(t) and -$j\sin 2\pi f_c t$ would be complex of the f4 they will be not be coming in with me ŝ(t) jŝ(t) $\cos 2\pi f_c t$ will also imaginary therefore that will also go away this real operation +j and –j will be +1.

So I get +ŝ(t)sin $2\pi f_c t$ so there is a ½ here this is equal to s_i(t) this is the in phase component what would be the imaginary component I do not have to do many more her I have already obtain the expansion here I just how to pick the imaginary component the imaginary components will be ½ ŝ(t) $\cos 2\pi f_c t$ and then there is a –j_s(t) sin $2\pi f_c t$ okay so this is your in phase component this is the quadrature component you can very well show that if I take fi (t) + j_{sq}(t).

I should be able to obtain this ŝ(t) because that would implied multiplying this sq(t) /j here and that j will be there but j_x-j will be + and then if you add them up clearly you would see that this would be equal to ŝ of sδ(t) which is the complex representation. Well this is sδ(t)but what about s(t) this is the complex envelop but what about my real valued signal of s9t) can I recover it from this s_i + j_{sq} (t) well I can because s(t) is given by two times real part of the complex envelop being multiplied by $e^{-j2\pi fct}$.

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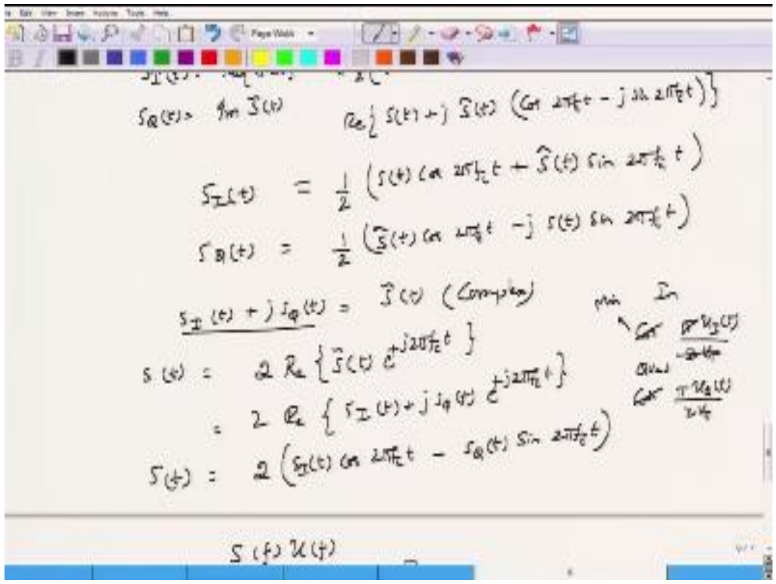

So if I work to just substitute for this s of s for s complex envelop here and write is as si(t) +j_sq(t) and then write down this $e^{-j2\pi fct}$ I will be able to obtain what is s(t). so this would be si(t) cos2πfct and then j_sq(T) cos2πfct will go away but what the real other part that you are going to get is j_sq(T) x –j sin 2πfct –sq(t) this is the factor for 2 sitting here times sin2πfct okay the reason why I should get a – I mean –sin here because this is actually + right I forgot to put a + here cos sδ(t) here the complex envelop will have ae-j2πfct to overcome that one you will have $e^{+j2\pi fct}$.

So therefore this would be + so this would come out and what you get is s(t)now what is the significance of this all thing that we have discussed well you remember this iq modulator that we discussed in the laser module you found that you know you can write down the output has from cos πui (t/2bπ) x cos 2πfct or cos Ω st + cos π uq(t/2bπ) x sin Ω s(t) right so those cos of πui(t/2) and cos of πuq (t/2bπ) or the in phase and quadrature components okay of the iq modulator these are the in phase and quadrature components.

You might question that they do not look exactly like si(t) and sq(t) because there sis cosign function there but remember these are the outputs of the max ender modulator so you need to operate them or bias them at the minimum transmission points and assume that uy (T) is a small number. So you can have this you know you can remove all this cosign and then all this π/2bπ can be observe in to our constant to obtain Uy (t).

Similarly you have to bias fitting such a way that you get a – sin here that can also be done by going to the other operating point and then remove this cos remove this π/2b π this are the constant and you get a –uq(t).

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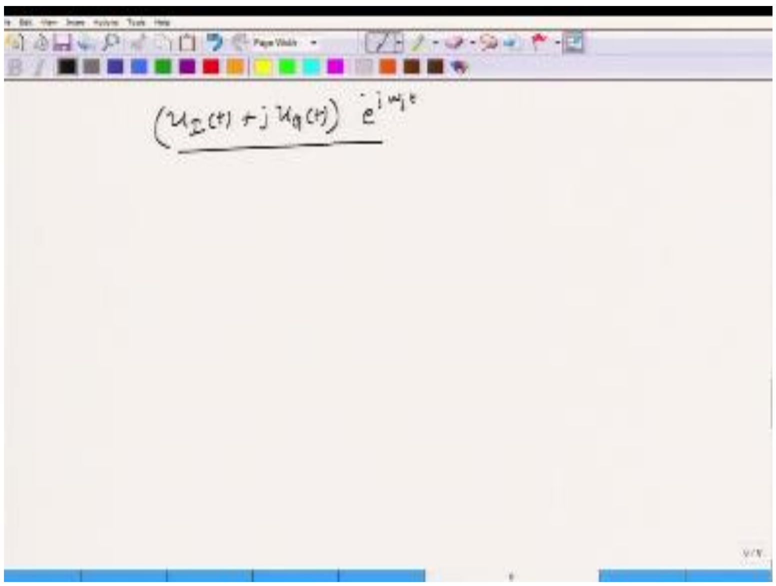

The output of the iq modulator was ui(t) you know approximately +j uq(t) this was multiplied by e^{j} **Ω** s_t so it is possible to go from this you know it is actually what we have seen is that this iq modulator is actually implementing the iq representation of the signal s9t0 so by changing the in phase or the quadrature component or by modulating the in phase and the quadrature component it is possible for us to perform the iq modulation okay.

So this is the significance of complex envelop we will meeting complex envelop we will meeting analectic signal when we discuss lasers and some of the properties of the lasers, so we will close this module write with understanding of complex envelop in the next module we will take up some concepts from signals which is required in order to understand digital communication fundamentals. Thank you.

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Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

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Ram Chandra

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