Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – I
Module – V
Intensity modulation

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In the last 2 modules we were discussing optical transmitters and we looked at three kinds of modulators one was the phase modulator which you can use to modulate the phase of the light wave emitted by the laser and this is used in phase modulated optical systems the second modulator that we talked about was in constructions slightly complicated compare to the phase modulator this was called as the mach zehnder modulator which worked on the principle of interferometer.

This we have seen that the relationship between electric field and electric field input and electric field output of this particular modulator is a non linear function because of the presence of a cosine transfer function within combined phase modulator as far as the mach zehnder modulator in order to form an even more complicated structure which can we used to modulate both amplitude as well as phase and there is actually equivalent to a conventional IQ transmitter that you finding electrical communications systems this modulator was called IQ modulator we will have more to say about IQ modulator later but we let us look at one additional use of mach zehnder modulator today in this module.

The starting with intensity modulation and then covering up how can, we generate optical pluses using mach zehnder modulator okay so we have already seen how mach zehnder modulator can be used.

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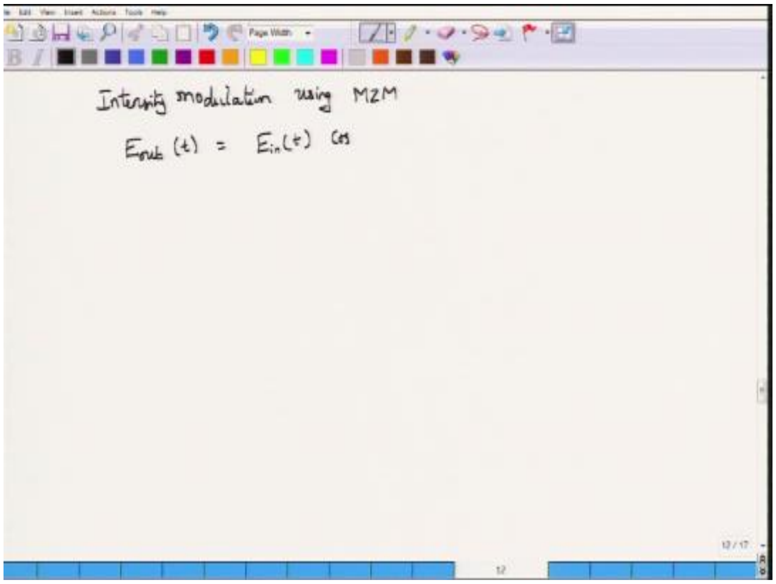

To modulate the amplitude of the incoming light field there we saw that you have to operate the mach zehnder modulator at it is minimum transmission point and at that point the electric field at the output can be approximately literalized as function of the input electric field so that is there is a approximate linear relationship between output and then input when the mach zehnder modulator is operated at it is transmission points assuming that the signals that you are giving the modulating signals that you where providing to the modulator where very small then one can see that output and input where related in a linear fashion.

The drawback of that one was that when you your operating at the minimum transmission point then you have to apply a certain voltage to get a non zero power so all though you obtained a linear transmission linear relationship between output and input which could be used for useful for double side band surest carrier type of modulations what we saw there was that the output power would be very small because the mach zehnder is beast at it is minimum transmission point.

Now in intensity modulation we are typically not concerned with what happens to the amplitude of the signal what we really want is that when I have some information signal then I need to look at the power and this power if it various linearly with the modulation signal that is alight with me the reason why it is okay is because if I put a photo detector in order to detect the signal the signal will detect the optical power.

So if I can modulate the optical power itself then all the variations that I am seeing in the optical power will be directly transferred to the electrical domain in the form of the photo current which can then be used in order to extract the information signal the disadvantage here is that you have to operate the mach zehnder modulator not in it is minimum transmission point or it is maximum transmission point but at the center where it means that when you do not have any variations coming in or you do not massage signal coming in then the modulator will expand same power.

Okay because it would provide because you are operating at the bias in the halfway between the minimum and the maximum transmission points it will produce some non zero power.

However that is quite interesting to look at how one can modulate the optical power in order to do that one let us recall first the relationship between electric field at the output of the mach zehnder modulator which we will denote it as Eout it was function of time this is given by the input electric field coming from the laser times the transfer function please remember that this transfer function is not the same as the transfer function concept that is used in signals courses in there transfer function actually refers to the forum transform of the output variable to the forum transform of the input variable.

However we are using transfer function as slightly general sense what we are saying is this is the relationship between output electrical field and the input electric field this relationship is in the time domain it is kind of how the input is transformed transferred on to the output side that is what we mean when we say transfer function in this context please keep this difference in mind.

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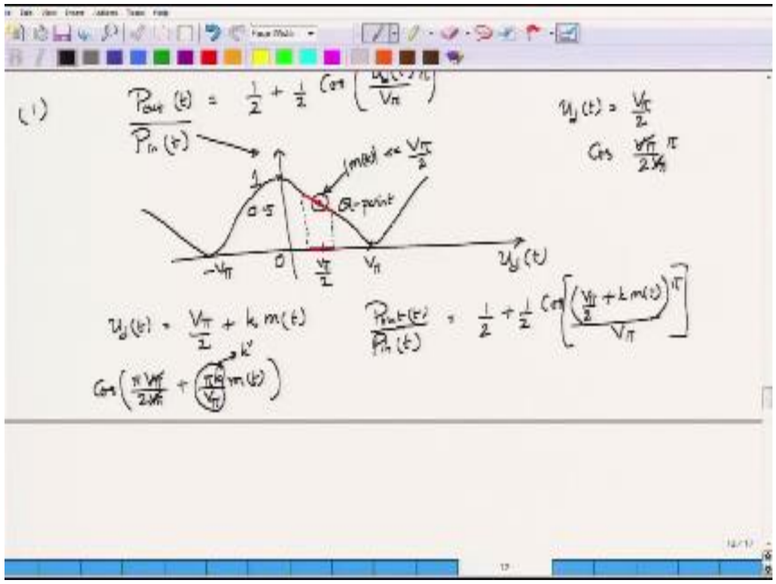

Now coming back to the electric field relationships the output electric field is related to the input electric field in this particular non linear manner as we have seen there is in the argument of course you have cos π ud(t) remember ud(t) is the difference signal between u1 and u2 of a dual drive mach zehnder modulator.

This mach zehnder modulator is being driven by u1 and u2 and ud corresponds to the difference between u1 and u2 so you have cos π ud(t)/ 2v π this is something that we have seen we have also seen how the electric field transfer function looks like when you plot the ratio of Eout by Ein as a function of ud (t) it would be a cosine wave form now as I said I am inserted in the intensity.

That is coming out of the mach zehnder modulator intensity is proportional to optical power in this particular case we will assume that intensity is actually equal to power you know if there is some conversion factor which we will neglect at this point and in order to obtained the optical power as I have total you earlier you simply need to square this particular component if I take the absolute value and then square it so when you square what you get is the output power and I will this as an exert size okay to show that the output power of the mach zehnder modulator will be equal to ½ + ½ cos(ud(t))/ v π x π.

Now you might be wonder where this is coming from sorry this as to be multiplied with the input powers or let us actually divide this one by the input power so that we are looking at the ratio of the output optical power to the input optical power if you are wondering where this ½ + ½ cos ud (t) π by something is coming up you just have to remember that I am taking the square of this right hand side quantity.

So I end up getting something like $\cos^2 \theta$ and I know that there is a trigonometric relationship which say that $2 \cos^2 \theta$ must be equal to 1+ cos 2 θ so if I square it then I know that $\cos^2 \theta$ can be expressed as 1+ cos 2θ/ 2 so that is this ½ coming from this particular relationship and because the argument doubles up the 2 in the denominator goes away and then this is what the equation that you are going to get please verify that you are able to derive this particular equation okay again what do we do about ud (t) ud(t) is again the signal that you are applying let us plot what will happen to this Pout (t)/Pin (t) as a function of this one when you plot it you are going to see that when ud (t) is equal to zero cos (0) will be equal to 1.

So you get ½ + ½ which is maximum so I am actually plotting this particular quantity on the y axis as a function of the applied signal $U_d(t)$, okay. So when you look at this way at cos=0 this would be 1/2+1/2 so it could have a maximum value of 1 at $U_d(t)=0$, now let us make $U_d(t)=v\pi$ when this is equal to vπ, vr in the numerator cancels with the vπ the denominator in the cosine function in the argument of the cosine function, so I end up getting 1/2+1/2 cosπ but I already know that cosπ=-1 so I basically get a 0 as the ratio, right at the output would be essentially be 0 this will happen at vπ.

Interesting thing would be to look at what happens when $U_d(t)=v\pi/2$ so this is the case when this is vπ, so when $U_d(t)=v\pi/2$, okay then you see that the argument of the cosine function becomes cos vπ/2vπ there is also a π, vπ cancels, right vπ cancels here and then you get cos of π/2, when cos of π/2 is there this output power this quantity the ratio that we are plotting will be equal to 1/2, correct so at vπ/2 this power would have dropped or this ratio would have dropped to 0.5 or

1/2 and here you actually have a cosin kind of a relationship which are $cosin^2$ relationship which could go in this particular manner, it could be a periodic function with a fundamental period of –vπ/2 +vπ.

So I hope that this transfer function is alright, now as I said this vπ/2 is something that is getting interesting because you have in this particular case a relationship where if I take the Mack-Zehnder modulator and bios at this point and then in addition to this I apply a certain m(t) the I will be going over this particular transfer function as my m(t) changes here the corresponding changes will be on the transfer function and it would be essentially oscillating are on this vπ/2.

So there is almost a linear relationship which one can have provided of course that this m(t) is very, very small in relation to vπ/2 that is the peak value of m(t) must be absolutely small compared to vπ/2 so that this linear relationship or actually the non linear relationship can be taught of us approximately the linear relationship let us do that mathematically and check what we get, so let us assume that the difference signal $U_d(t)$ which I am applying to the Mack-Zehnder modulator is given by DC value which is vπ/2 plus some constant k, okay do not worry what that constant is times m(t),m(t) is this small signal that I am applying.

If you have studied transistors or diodes you would remember that you have to bios a transitory at a particular operating point, correct this operating point is sometimes called as the Q point and then you have to apply the AC signal which is very small in relation to the DC signal, so if you look at a typical transistors circuit you are operating point will be somewhere in the middle of the operating region of the output characteristic of that transistor.

But the AC signals will be in the order of a few milli volts, so that AC signal is essentially this m(t) which we are applying. So as the voltage changes on the input side $U_d(t)$ starts of at say vπ/2 then increases and until it reaches the peak value of m(t) and then it comes back to the –m(t) value, the corresponding chances that would happen will be happening on this transfer function, so this is the range of input and output that you are operating and over this range you can consider the Mack-Zehnder modulator to be a linear function in the power that is output power is linear related to the input power, okay.

So with that, if I substitute of course we have just said this in graphically way, let us quickly do the math in order to see what we are talking about is reasonably, okay. So you have the different signal $U_d(t)$ which is equal to vπ/2 plus some constant k.m(t), now I substitute this into the expression here let me call this as equation 1, so let me substitute this one into equation 1 and then simplify to find what I get, right so the ratio here will be $P_{out}/P_{in}(t)$, right this is equal to this is the left hand side of equation 1, this would be equal to 1/2+1/2 cos (vπ/2+k.m(t))π/vπ.

I can expand the argument in the cosin 1 let me just take that part here and say cos(πvπ/2vπs) here plus there is a πk/vπ m(t), okay again this k being a small number this πk/vπ itself is very small compared to this πvπ/2π vπ in the numerator cancels with the vπ denominator and this πk/vπ can be considered as another constant, let us call this constant as some k′ there is no significant to k and k′ in the sense that they are just constants which have to be considered in order to, I mean just a denote this number k πk/vπ I am denoting it be k′, okay.

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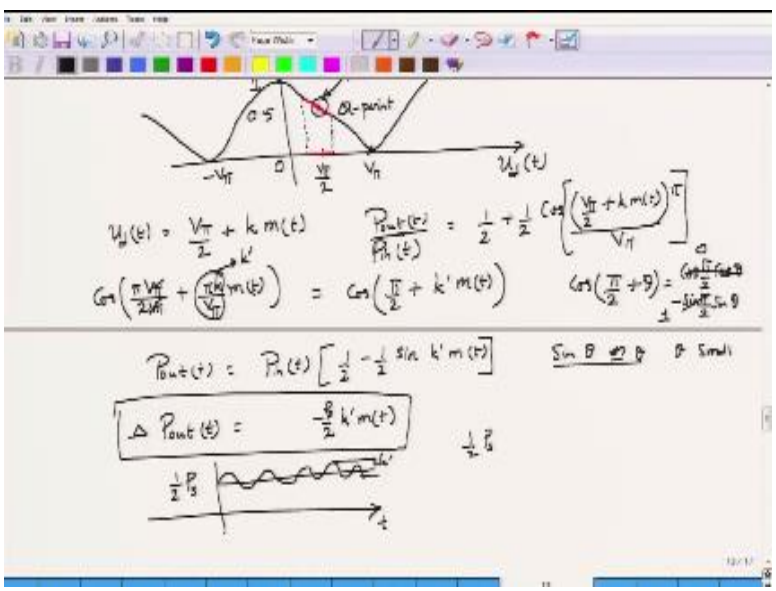

Alright, so with this one I can expand here, right so what I get here is cos π/2 plus sum constant k′ times m(t). Now I know that cos (π/2+θ) can be written as cos π/2 cos θ-sin π/2 sin θ and I know that cos $\pi/2$ is a 0 so that term goes away and I know sin $\pi/2=1$ so this cos $\pi/2+\theta$ is actually equal to $-\sin\theta$, so I can write down this in this way and say the ratio $P_{out}(t)$ or rather $P_{out}(t)$ itself is equal to $P_{in}(t)$, right this is the input power from the laser times 1/2-1/2 sin what is θ, θ is actually k′ m(t), of course we also have another relationship which says that when sinθ is very small sorry, when θ is very small then sinθ can be approximated as sinθ is approximately θ for θ very small, right.

So compare to this 1 if θ is very small then sinθ will be approximately θ, so θ small is probably a better way to write this one so let me just write it like that, for small values of θ sinθ can be approximated a θ. So I can remove this sign from here and I also know that the input power is not varying because I have assumed a ideal laser, ideal laser does not fluctuating its amplitude, okay and it phase is immaterial since we are operating this is in the continuous wave and this laser has a peak power of Ps or the continuous wave power of Ps, so I am going to write this as 1/2 Ps and then I have 1-k′m(t).

As I said do not be alarmed by this minus sign, you have to remember that this k′ is a very small quantity so because this is small we have been able to make this approximations, okay because k′ is a small quantity all you are seeing is that if you able to plot p out of t as a function of time and assume that m(t) = 0 then the c power that your are getting at the output would be half PS so it would be a constant over and above if this m(t) would say for example be a sinusoidal signal the over and above is what your power would be fluctuating so this half PS is the average power assuming that m(t) is something that would average out to 0 so this half PS is the power you can think of this as average power or the power that you would obtained when m(t) = o.

And when you apply m(t) to be as sinusoidal signal of something then the fluctuations will be having the magnitude of k' in the sense that the peak value will be k' and this entire thing would be the peak to peak variation will be 2k' okay so you can see that there is a linear relationship if you forget if are if you are looking at the change in the output power okay rather than looking at the output power itself is or looking at the change in the output power this change in the output power can be obtained by removing this DC component half PS okay and then what you had is – half K' m(t) right.

So I can see here that I have removed the DC part but the change in the optical power can be negative of course this implies that this does not imply that the power itself is going negative but please remember this is only the change in the optical power so what we have obtained here is that we have obtained a oh sorry there is a PS/2 also I am sorry I forgot put that one so this change in the optical power is this follow where k ' is a very small number so this would essentially be the output right so you have now seen that come you know if you are looking at the change in the optical power that can be a linear function of the applied m(t) right.

So there is a linear relationship this can be used for intensity modulation analog signal so if m(t) then analog signal then you can actually use that to use the maximum modulator to obtain the output power which is varying assume accordance with the input message signal m(t0 having seen this intensity modulation that is move on to a different application of m(z) m okay.

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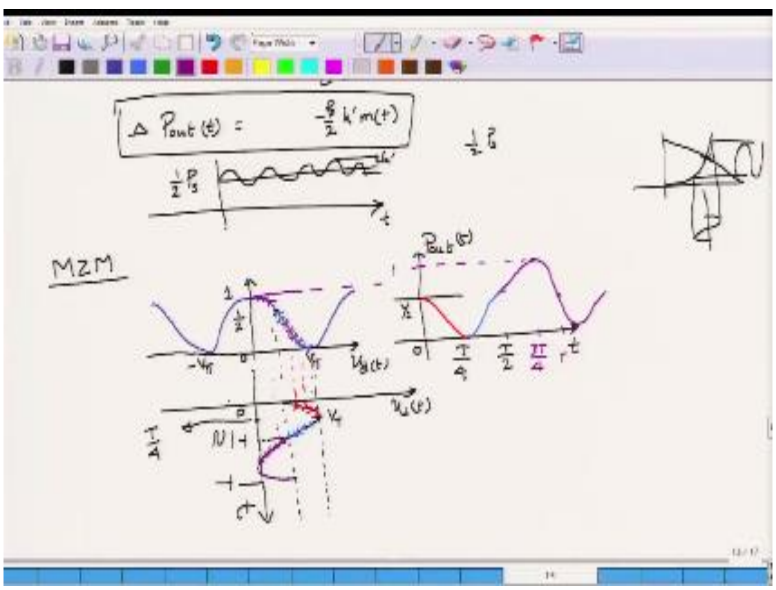

In optical communication systems it is typical of this systems that we transmit information when we modulate the information on to the light signals we also want the light signal to be pulsed so the send out optical pulses carrying information this optical pulses can carry information both in their amplitude phase or in general the quartered amplitude modulation however optical pulse shaping simply implies that I am able to generate optical pulses and then modulate these optical pulses using any of the modulator is that we have discussed now having optical pulses is an advantage because it will let us know.

Where the particular symbol boundary is ending so if I have to have a clock and I am not able to send a clock from the transmitted to the receiver on a different channel or something then I have extract clock from the optical pulses extract clock from whatever I am getting so when I have optical signals in the pulsed form or I am getting optical pulses then it becomes very easy for me to extract the clock in order to in both transmitter and receiver so for this reason we use optical pulses we also change the shape of the optical pulses in order to compact the fiber induced impairments.

As you will see later fiber introduces impairments such as chromatic dispersion and non linear effects so in order to overcome these we also pulse shape not just in any arbitrary pulse shape we have choose specifically that pulse shape which can combat dispersion as well as non linear effects so we do that it is therefore necessary that if we have a continuous wave laser as you would getting most lasers when you purchase then in the market then you need to be able to generate pulses out of this continuous wave laser right, so and already do that one, one can either have pulse laser by itself.

But that would be a different approach but if I have a continuous wave laser then I can use the Mach zehnder modulator to generate optical pulses okay we order to see that one let us go back to the power transport function of the Mach zehnder modulator so I have as a function of the different signal u(d) of t okay the power transfer function is of course something that we have seen here so this power transfer function so let me operate it like this it is periodic as I said these values are VЛ here this is 0 this is – VЛ.

So far let us just use this one that is this region we will use because that is more than n of for us now this is my ud(t) suppose I take ud(t0 and then by as it at half way point in the sense that I take ud(t0 and then have it is average value given by VЛ/2 and then I also have it has a sinusoidal signal okay sorry this should have come and met the horizontal access so have this as a sinusoidal signal who speak value is also VЛ/2 so that the peak value of ud(t) that I am getting will be equal to VЛ and then it is minimum value will be equal to 0 so if this is something that I am looking at this is a function of time.

This is the ud(t) so if this ud(t) is presented to the Mach zehnder modulator what would be the output of the modulator well if you remember again your you know B_{jt} or diodes you would have come across a concept called as a load line so you have a diode right which you would be biasing at a certain operating point and then you vary the input okay whatever it could current or the voltage and the corresponding variations can be obtained by this particular load line concept so you just have to look at what would happen to the voltage here.

And then find the corresponding output voltage and then plot it so this would be the variation of course I am not really drawing them nicely but this is essentially what you are going to do right this is what you do have done in your earlier circuits course as so a same idea we will use what we will do is we will plot this ud(t) as a function of time okay so these are two slightly different graphs so these are two different graph please keep that in mind however I am aligning these graphs so that I am able to capture what I was telling to you about this load line concept.

So let because the ud(t) = VЛ/2 the average value is VЛ/2 at this particular time let us draw first ud(t) that I can draw it in this manner and as a set the maximum value will be v π the minimum value will be 0 so my signal will be between these values right I am drawing only one period of the way that is enough for me to illustrate the concept you can extant this to any periods that you want okay.

So this time t =0 and let us say this has a total period of T okay so this part where it goes to 0 is about t/2 okay so this is about t/ 2 okay this $u_d(t)$ of course at time t =0 the value of $u_d(t)$ = v π/2 what would be corresponding output the output value will be at v π/2 in this case it would be ½ we have just seen that right so this would be ½ the scale is may not match correctly so please keep that in mind the scales are probably not matching.

But I hope that you get the idea of how to draw this outputs that is what I am more interesting, so let me also erase this potion erase this slightly okay her what I do is I put out the output way forms so let us call this as p out and I am plotting p out as a function of time over this particular graph okay so I have align them so that I can better capture whatever that is happening at the input to the output I can actually obtain this kind of a nice plot.

So at t = 0 u_d (t)= v π/2 so the output would be at ½ so this value will be ½ okay then what happens as u_d (t) continuous to raise here and reaches a maximum value of v π at t/4 so this follow is t/4 the corresponding change that would happen on the max ender modulator transfer is that go down here as this increases from v π/2bπ the corresponding output actually goes down here correct that is what you know you can actually do that by paring of each point like this.

So you know pair of these points and essentially when you pair to the point at v π which is the modulator is going through a t = t/4 right the output of the modulator would have gone down to 0 so I have to start from this value of ½ and then come down to 0 at t/4 alright so I hope that this potion is okay. Now look at what happens as the modulating signal u_d(t) or the drive signal u_d(T) is going down right it start as v π and then keeps going down.

But this movement or this change of signal right will be reflected by the transfer function going or raising from 0 to the corresponding ½ correct? So I have to now go back and then get to this potion which any way I was there a t= 0 but now I am repeating that one at t/2 because t/2 the modulating or the drive signal as come back to v π/2. Further as this signal goes down right so here it is going down to 0 the corresponding change that happens on the max ender modulator will be that corresponding to this one.

This is the output so it would actually start to go up and then eventually reach to 1 right so it will go up and eventually reach to 1 so let me just write down the this one so at this point so it is not as you can see the scales are not really looking correctly you know for a half I should have gone up so let me try to do that one.

So the scale correspondingly should have been chosen so let say this is 1 so it would have gone to this portion and then once this comes back from 0 right once it again raises back to v π/2 at time t so this was happening at 3t/4 and then finally when you come down to t the output voltage would have come back here okay, this is how you can use the max ender modulator to plot the output okay.

This does not really resemble a pulse because after it has come here then it would go down and then it would essentially be a you know signal that would be raising again, so it is not really looking like a pulse so in order to obtain the pulse this was easier for me to discuss with you for therefore I used this particular curve but to obtain pulses we will have to operate this max ender modulator data different operating point.

So let us choose first this point which is the maximum transmission point and then have a excursion all the way to v π and - v π so let us apply that signal.

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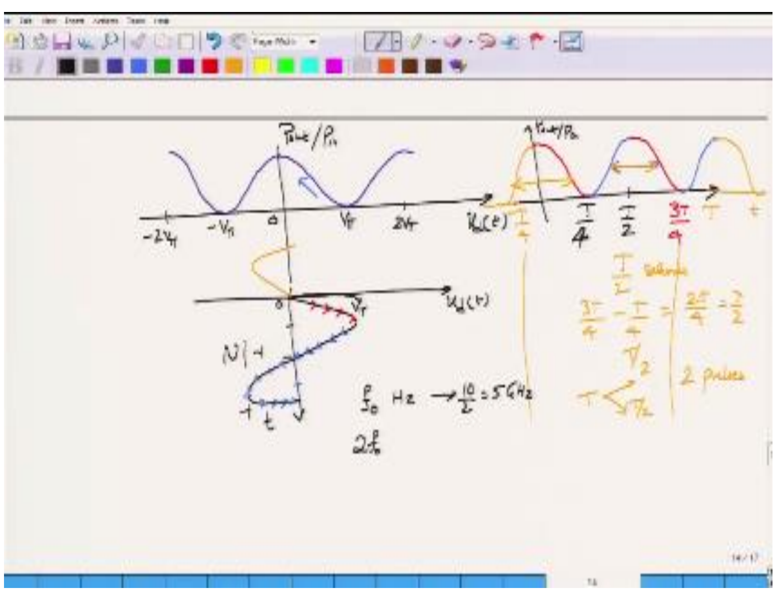

So now that we have understood how to plot I will be little quick here so I am plotting $u_d(t)$ here this should be a transfer function which I am plotting right so this is the transfer function so this is at 0 to this is v π this is a 2 v π this is at - v π this is at -2 v π so this is the transfer function of the max enter modulator this is the ratio p out to p in now I have to put down one other graph which will capture $u_d(t)$ for me right.

As a function of time then I have to also put down one more graph which will capture this ratio p out to p in as a function of time right so I will have to capture this particular thing also, so let us do that in this case I will assume that $u_d(t)$ is 0 initially and then goes all the way to v π so it will go to v π and the go to - v π and then come back oaky so it has peak voltage of v π average value of this fellow is at 0.

So at t/2 it would be so this is at t/2 and this would be at t where it would have made a complete one time period okay now let us plot the corresponding output initially corresponding to this 0 voltage at t=0 the output of the max ender modulator is that is maximum as the voltage continues to raise and reaches v π at t/2 the output of the max enter modulator starts to go down and reaches here to 0 at t/4 so this is at t/4 and then from t/4 the input ud(t) starts to drop.

So it starts to drop and goes down to 0 at t=t/2. So similarly w3hat will happen over here, this will now begin to change from here, as you come down here, is equivalent of going along, you know in this particular direction, on the transfer function of the modulator, will actually go up and reach you peak at t/2, t/2 now reached a peak.

Then what happens the input starts to go negative right, it would continue to go negative and reach –vπ and t=3t/4, however t5he corresponding change on the max sender modulator would be, that you stop dropping in the power from the maximum here, all the way to 0 when you reach –vπ, so at 3t/4 you would have gone down here again, so this would be 3t/4 and finally we will complete this output and will be back to where will we started at t=t.

So t=t this voltage will be 0, you power would go back to peak, now what you can see here is that you manage to obtain once pulse here, you manage to obtain one pulse here, which lasts about t/2 seconds. Why this would be t/2 seconds value you tell, just do 3t/4-t/4, what you get is, 2t/4 which is equal to t/2. So you have now obtained one pulse of duration t/2, but if you look at this way and realize that the signal is not just one period, actually a periodic signal, you would immediately realize that there would be one extension this way and there would be an extension along this side as well.

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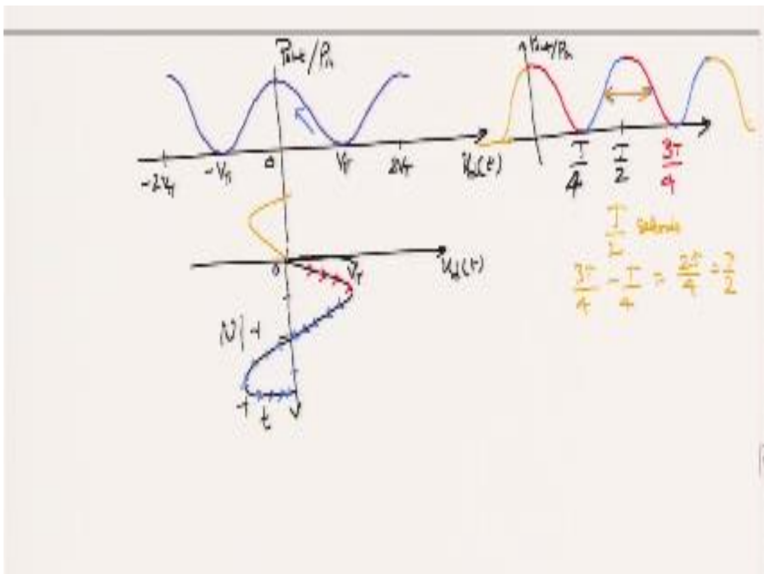

So this is your tπ, so this is your t actually, the total time period t and in that time period t, you actually manage to obtain two pulses, you obtained two pulses, one pulse from t/4 to 3t/4 and one more pulse, if you have to remember that the signal ud(t) is actually a function of time, so you will have obtained one more pulse, so if this would be –t/4, so in the total time period of –t/4 to 3t/4 which will correspond to 1 on time period t, you have obtained two pulses right, so each lasting t/2 seconds you have obtained 2 output pulses.

In other words if my frequency of the signal ud (t) is f0 hertz, if its f0 hertz, the pulses that I obtain will be at the rate of 2f0, right? So I obtain pulses at double the data rate, so for example if I want to generate pulses for my use, for 10 Gbps modulation, 10 Gbps digital modulation and I aqm having one pulse carrying one bit of information, then I need to generate pulses at one pulse I require at, the pulse rate is 10 Gpbs, but the signal that I need to apply will only be half of that.

So 10/2 which is eq1ual to 5 giga hertz, because each half portion of the sin waveform ud(t) is giving me within one period, if this corresponds to 1 giga hertz, there would be two pulses, therefore the data rate which I will be obtaining will be double the modulating thing. So the pulse rate will be twice the frequency of the drive signal, analog semi solid signal that you are driving the ma send4er modulator.

Now this is not the end of the pulse shaping business, that one other pulse shaping that is also widely used. In order to obtain that pulse shape, we again start from the transfer function, this time it will be even more quick, this is the transfer function, as ud(t), I am going to put one more here as a function of time to obtain the output by Pin and then now I consider a signal, this is ud of interfacing time3, I now consider by modulating signal in this fashion, so this is going all the way to 1 second, so I consider the modulating signal which will go to a peak of vπ or the peak value of vπ but at 0, average value of this one will be at vπ.

So the average value here is at is 0, I mean the value of ud(t) is that 0, and this would be time t/2, this would be the total time. Now what would be the corresponding output? Well initially when you are at vπ, at t=0, the initial amplitude of the ud(t) signal is z vπ, so I put vπ here the output would be 0 here, as the input increases and reaches to 2 vπ, this 2 vπ, the corresponding output will reach to maximum.

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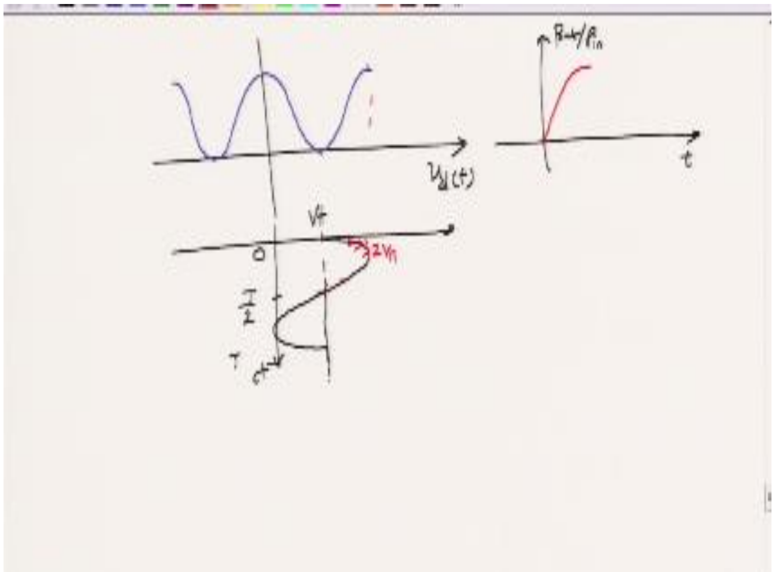

And then once you start dropping, you reach at t=t/2, you would start to drop, and then corresponding output would gone down to zero. So this is t/2, this is 0 and then you would again see that, there is one more pulse, so if you compare these two waveforms, you will c that the area occupied by these two waveform will be different, although I am not shown it very nicely, there actually going to be slightly different and these two are therefore called as 33% and 67% waveforms, which simply indicates the % area that this pulse carry, compare to one period.

SO we will not go further beyond this pulse shaping thing, will talk about pulse shaping in the electrical context later on, but for the optical pulse shaping we will stop with these two pulse shape, because easy to generate and therefore widely used. No I should also point out one additional factor over here that you can not only get these pulse shapes, you can actually get arbitrary pulse shapes, right by changing amount of t or changing ud of t and by choosing different biasing points of the max sender modulator.

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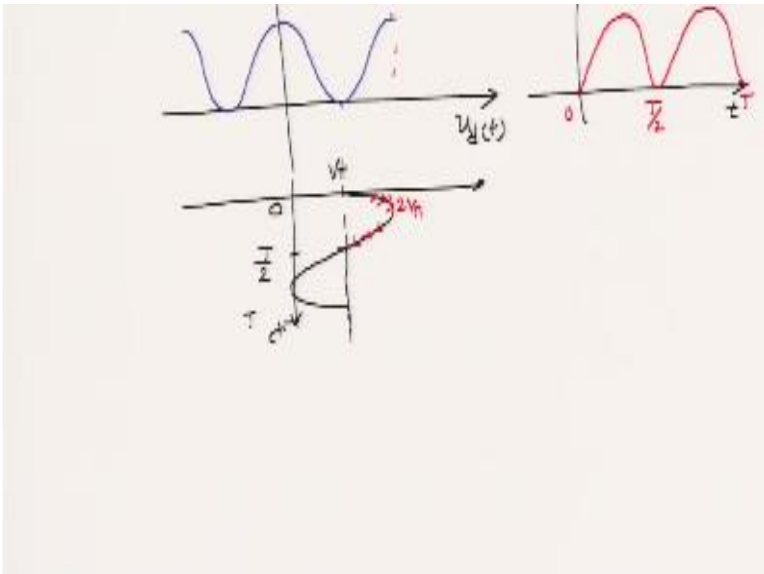

So in this example we are biased max sender modulator at its mini9mum transmission point, it is possible for you to bias it at any value, in fact you can send in any complicated signal, and then find what could be the output of the max sender modulator, now you can do all that in simulation by using matlab and writing the simple program to generate, to give this as one input and write down the transfer function of the max sender modulator, the power transfer of the max sender modulator and then find out what kind of the pulse shape that you are going to obtain.

You can look for such matlab programs in the assignments section and you will be able to generate various pulse shades and confirm for yourself that it is indeed what you are obtaining is something that w have obtained intuitively by drawing here. So you can pick different operating point and therefore get arbitrary waveforms. So we will end this module by optical pulse shape, in the next module we will talk about some fundamental topics that are required in order to later continue the course. So these topic will be from signals, systems and a little bit of communication theory, we will look forward to that in the next module. Thank you

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