Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications
BPSK
Week – XI
Module-IV
Higher modulation techniques (contd.)

by
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Hello and welcome to the mook on optical communications. In the last module we were discussing how to increase the spectral efficiency, we will continue the discussion and introduce an additional way of increasing the spectral efficiency okay. So we have already looked at one method of increasing spectral efficiency that is single carrier plus higher order modulation okay.

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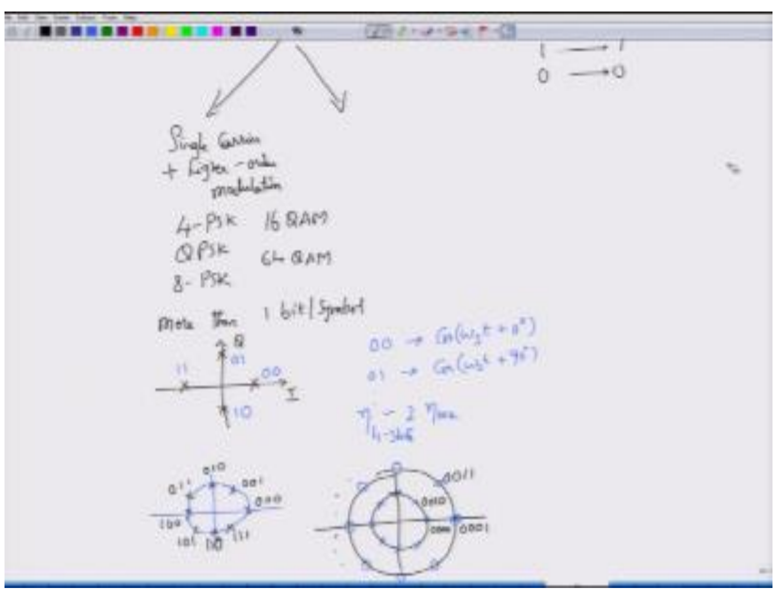

If you have a DWDM channel of 200 bands and each band can then be utilizing the 16 quam or a 64 quam, then the overall spectral efficiency will also increase okay. The catch here is that, so let us look at first the BPSK situation and then come back to the other approach over here.

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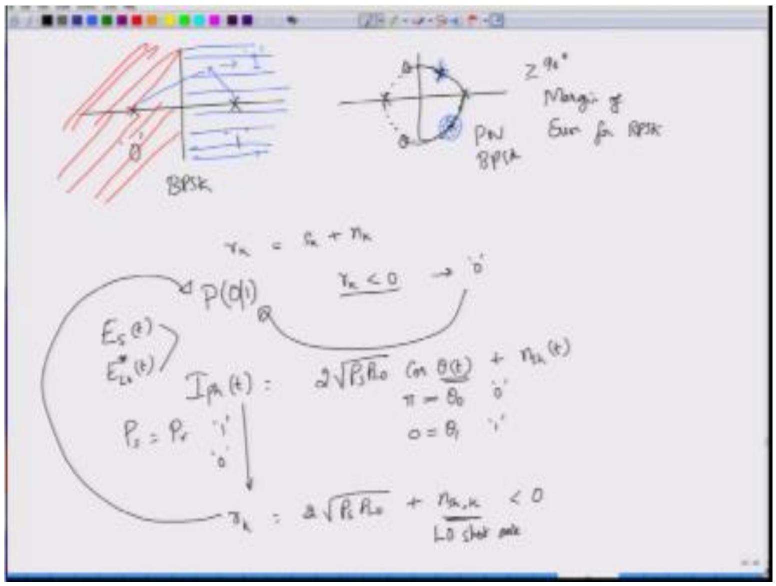

With BPSK what are the symbols that we have, we have one symbol which is symbol 1 and then you have a symbol which is 0 and these two are placed such that their normalized average power or average energy is equal to 1 okay. So they are placed as far as a part is possible, the way to distinguish between 1 one symbol and a 0 symbol for the BPSK is that, if your received signal for some reason happens to be in this region of operation or in this region of a signal space where I have I and Q, you look at the distance between these two points okay.

Clearly this blue dot or the blue received symbol is closer to this 1 rather than to 0 and therefore you can declare this one as bit 1, you can see that this entire region will then be declared as bit 1, because signal falling anywhere within this region will be closer to 1 than it will be equal to 0. And obviously anything that would fall within this region will be allotted bit 0, but interestingly if, let us, you know you have a BPSK transmission.

And then you have some phase noise okay, what would the phase noise do to the BPSK signal is that it kind of starts to rotate the constellation points okay. So the received signal can be lying anywhere on this circle depending on how strong your phase noise is, it can lie anywhere on this circle okay. One can see that as completely cover the circle means that the phase noise has to be large, if the phase noise is not very large then it would be only over one particular section okay.

So let us say over this segment your received signal can lie anywhere, but still notice that you are not making any errors here okay. If you add some AC noise around that, then this would be the situation that you have okay. So you have some AC noise coming from the amplifiers, coming from the interaction of amplifier with everything else that we will look at, your received signals can be anywhere here and of course they can anywhere be rotated by this particular value okay.

So you see that with this received constellation you are still not making much of an error okay. and primarily the reason for this, that is happening is because the phase noise is still manageable, the phase noise is over here it is still manageable, because it is only covering one segment over here okay. If the received signal has to go to the other region okay, wherein they are not supposed to go right, then the phase noise variance or the phase noise RMS value must be greater than 90° greater than or equal to 90°.

So this 90° is the margin of error that we have okay, for BPSK signals. So with BPSK signals the margin of error is 90° and this is a pretty, pretty large margin that you are going to get. Suppose you have received the signal okay, so received signal is the photo current which I am representing this as R_k this received signal will be sum of the signal that you have actually transmitted which is S_k plus you have some noise N_k right.

And if you look at when you are going to get this an error when this r_k so what would be the probability of mistaking the received samples as 0 when you have transmitted a 1 would be when this r_k becomes less than 0 right so if this particular scenario happens then you can see that the received signal r_k when value becomes less than 0 the it would be declared as 0 and this would clearly be the error that you are looking for otherwise you are on the set so as long as this part is okay right you are in this particular area you would not have made an error.

But the movement you go from there to here you would a have made an error and that error would happen when r_k is less than 0 okay if you look at the coherent receiver output right the photo current that you are going to obtain would have the signal component right what would be the signal component there it is two times $\sqrt{P_s\, P_{LO}}$ and in this case Ps will be the same value so whatever the Pr the received signal value for both 1 as we will as 0 because in case of both 1 as well as 0 you are transmitting a certain amplitude sinusoidal signal to the receiver both carrying the same average power of Ps okay or carrying the same power of Ps okay.

So the received signal will be $2 \sqrt{P_s\, P_{LO}}$ I am of course assuming that this responsively r is = to 1 in this case and then you have cos $\theta(t)$ where $\theta(t)$ is the phase difference between the signal that is the signal field which is Es(t) and the local oscillator field which we have assuming to be having 0 phase okay so this phase difference is cos $\theta(t)$ and clearly this $\theta(t)$ must correspond to either θ_0 or θ_1 θ_0 when you have transmitted bit 0 and θ_1 when you have transmitted a bit 1 okay + there will be short noise terms if you want to sample this signal to generate your r_k you see that this would be $\sqrt{2\, P_s\, P_{LO}}$ and assume now that you have transmitted a bit 1.

So when you have transmitted bit θ_1 must be = 0 θ_0 of course is equal to π when you are transmitting bit 0 so this would be $2 \sqrt{P_s\, P_{LO}}$ cos of 0 will be 1 + whatever the short noise sample that you are get okay if this received signal because of the noise that you have which is mostly limited by the Lo short noise that we are assuming that this is Lo which is giving the largest component of the noise then if this combination is less than 0 then there is probability that you would have mistaken the received signal as a bit 0 although you have transmitted 1 you are going to declare the bit as 1 which would be in error right.

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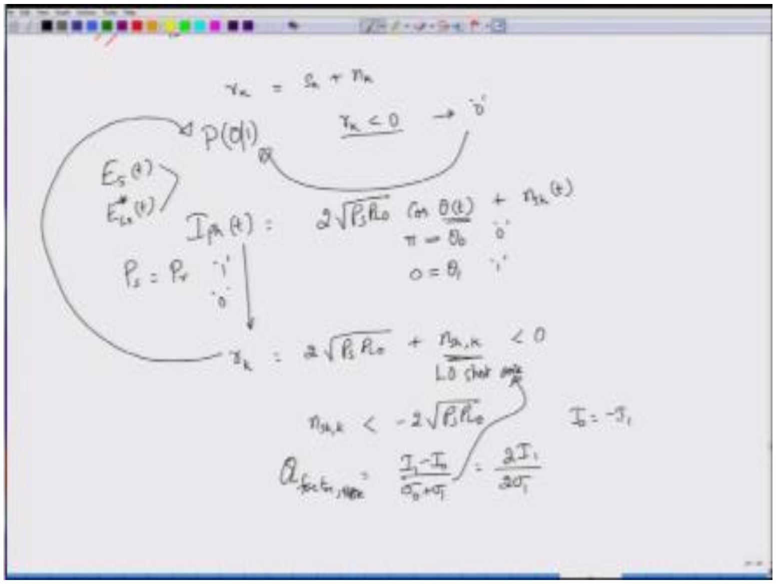

So you can rearrange this equation and then say the condition for this to happen is when the noise actually falls below this minus$\sqrt{}$ 2 P_s P_{LO} and then you can integrate and do the appropriate you know calculation to find out what would be the probability of 0 to obtain to a 1 and express all this in terms of the Q factor okay so the Q function the Q factor for the PSK system for the BSK system can be defined by looking at I_1- I_0 / $\sigma_0 + \sigma_1$ and we know that I_0 will be = to – I_1 right.

So you get two times I_1 / 2 times σ1 because the noise for 0 and noise 1 in the Lo short noise limited case will be approximately equal okay and Io will be = -I_1 that is when you transmit a bit 0 you will get minus of the current trend the amplitude will be negative so you get – I_1 and 2 , 2 cancels over here and the Q factor simply becomes.

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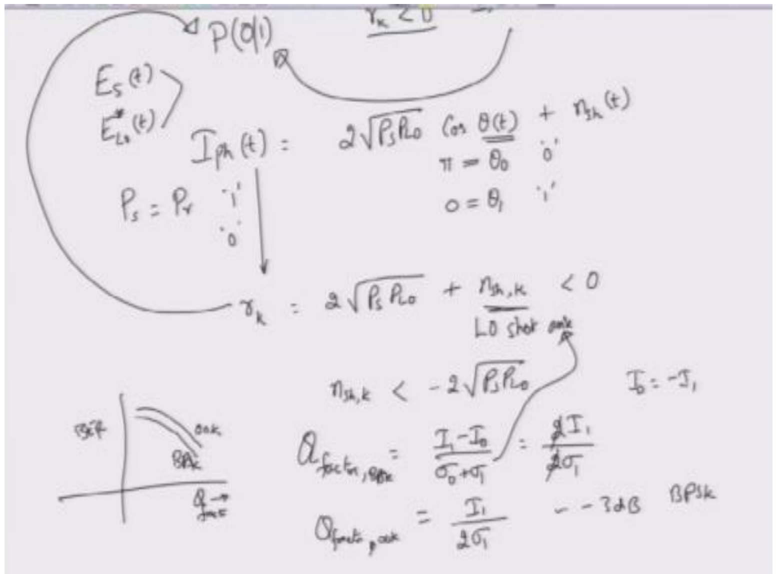

I_1/ σ1contrast this to the Q factor for ON-OFF keying okay with the ON-OFF keying I0 was 0 σ0 was approximately σ1 and your I the Q-factor would have been I_1/2σ1 okay, so you can already see that ON-OFF keying is -3db worth compared to the VPSK system, okay. So if you look at the Q-factor verses you know the bit error rate you will see that as the Q-factor increases the bit error rate drops out.

The first thing would be the highest sensitive would be for BPSK then you get a sensitivity which is 3db degraded for the ON-OFF keying, okay. This is all nice a for a BPSK system okay, I am not going to look at the expressions for this probability of error they are not very important.

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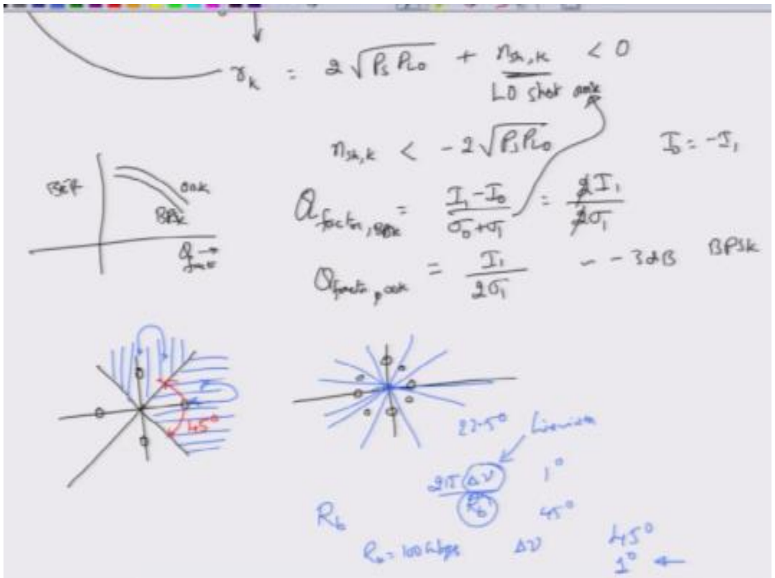

But this graph is very interesting, right. So BPSK seems to be performing better and the margin of the phase error in this case is about 90 degrees which is very interesting, now you go to from BPSK you go to the 4 PSK here the constellation points are located here and then the decision regions will now be located at these points right, so if may received signal happens to lie within this particular region.

Then anywhere if it is lies in this region then I will declare that one as this bit sequence, okay. On the other hand if my received sequence lies anywhere over here then I will declare this one as corresponding to this particular bit sequence, similarly you have additional two regions as well okay. You can calculate the error factors over here but what is interesting to observe is that, if only phase noise is present then the phase noise margin of error would now have been reduced from 90^0 to 45^0 okay this is very important, so you are now cannot go more than 90^0, so 22.5^0 on this side 22.5^0 on this side I think that is what 45/2 is and roughly you are going to get 45^0 up here, okay.

If you know start going to higher order modulation, for example we talked about an 8 PSK modulation, right. So you can see very clearly that the decision regions corresponding to this would then be even more closely spaced okay sop this is one more decision regions so you are going to get that should have been over here, but the point is to note that the noise margin or the phase noise margin starts to decrease again here, right.

So from 45 it should become now 25.5 ^{0} So as you start packing in more and more number of constellation points the lower phase noise margin decrease, now if you say my phase noise margin should be just 1^0 there is actually a relationship between the phase noise for variance and bit rate as well as the line width right, for the same bit rate R_b okay the phase noise variance is given by $2\Pi\Delta\nu/ R_b$, okay.

So if you fix R_b then and say this should be 1^0 versus say 45^0 clearly the constrain that you are going to get will be on the line width of the laser, okay it would then become very difficult to fabricate lasers with line widths as small as what we5 are looking for okay, for a case of $R_b = 100$ Gbps I will leave this as an exercise for you to find out what would be the variance or what would be the line width $\Delta\nu$.

Such that the margin of error is 45^0 and the margin of error is 1^0 and you will be surprised to see that the kind of $\Delta\nu$ that is required for 1^0 phase margin or the phase noise variance would be very tight and to achieve such low line widths is very expensive, so it is actually very expensive thing to realize this 1^0 variance kind of a system, okay. So the drawback for increasing the spectral efficiency for single carrier system is that.,

As you start packing in more and more number of points okay the lower noise margin and the system specification also starts to increase also with one degree variance that we assume for the phase noise okay, or we want for the phase noise the main drawback would I mean apart from of course constructed in the laser the other major drawback will be that how are you going to synchronize the transmitter and the receiver if the margin of error is just one degree out there, right

So that is the kind of trouble that one has to look for when you go to higher order modulation, but this has not stop people from going to higher order modulation and up to 64 quam the only thing is the complexity of the receiver starts to increase dramatically.

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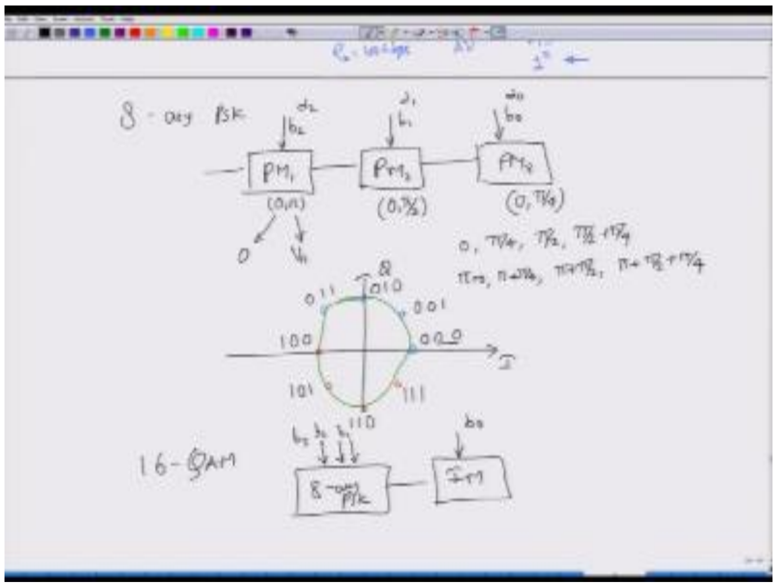

Before we go to the other approach as quick way so we were looking at how to implement an BPSK system so if have already not looked at how to implement an BPSK system all you have to do is to take three phase modulators call them as PM1, PM2 and PM3 connect them in cascade this is one possible realization I am talking about and sending the bits here b2, b1 and b0 this is for a straight binary encoding, if you send in d2, d1 and d0 which stands for the gray encoded symbols then that kind of head ache is up to you, okay.

You configure the first phase modulator to give you phases from 0 to π remember how to do this one this is by sending 0 voltage signal to obtain a π phase shift you have to send in vπ, okay. Similarly you configure this phase modulator to give you 0 and π/2 this one to give you 0 and π/4, okay and you now look at what kind of consolations I am going to get in the IQ plane, okay when this b2=0 so with b2=0 the total possible phases that I am going to get here is 0 when both of them are 00 you get π/4 you get π/2 and you get π/2+π/4 okay.

When bit $b2=1$ to these phases you have to add π, so get $\pi+0$, $\pi+\pi/4$, okay $\pi+\pi/2$ and $\pi+\pi/2+\pi/4$ let us mark these points 0 phase goes here $\pi/4$ phase goes over there, $\pi/2$ goes here they all have the same amplitude, okay. Although it might not look very nice as a circle you have to assume that I am putting everything on a circle $\pi/2+\pi/4$ is this one, okay so what are the bits that you are going to get this is for 00, this is for 01 this is for 10 and this is for 11, okay.

Next what do I have, I have to translate everything by a value of π or rotate everything by a value of π so 0 becomes this symbol this one goes through here, this one goes through here, this one goes to this side, okay.

You can now complete if you are interested you can complete a circle over here and see that they all have to lie on a circle unfortunately this is not a very nice circle but the corresponding bits that you are going to get are so 00, this would be 10, this is three bits right, because you have actually three bits to work with, so you have 000 so these two are b2,b1 and b0 you have 0 which is b2, 01, 010, 011, and then you get 100, 101,110 and 111, okay.

So this is how you generate an eight array system, now to generate a 16 star QAM okay, all you have to do is take a 8-ary PSK system over here, okay and then have an intensity modulator to this 8-ary PSK you supply b3, b2 and b1, okay to this intensity modulator you supply b0, b0 being the least significant bit, okay.

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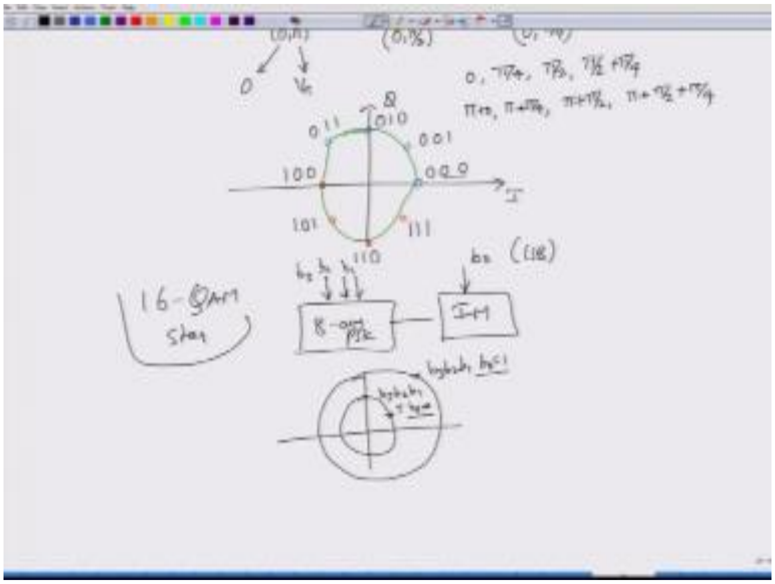

So you supply the least significant bit and what you would essentially generate is one BPSK correspondingly I mean define by b3, b2 and b1 and the same BPSK but this time with the different amplitude okay and that would be so any point here is all this with B0 = 0 and the points over here will be B3 B2 B1 with B0=1 okay so this is with B0=0 so the inner circle with B0=0 outer circle with B0=1 and you will be able to generate the 16*QAM additional types of generation including IQ modulator is all not important for our module here so we are not going to discuss that.

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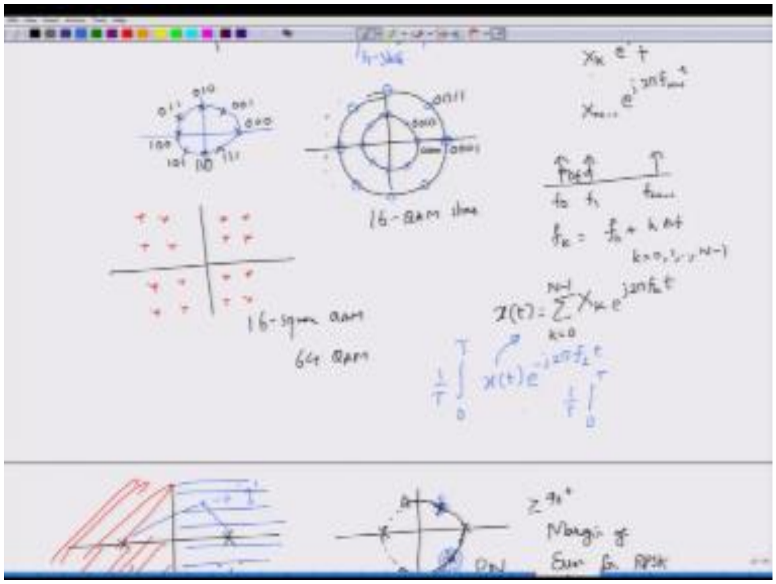

Having looked at one way of improving the system performance okay in terms of the spectrum efficiency we will now look at an additional or a different type of approach this approach is called as multi carrier modulator approach or in earlier situation it is also called as sub carrier multiplexing approach okay this two are not exactly same but our purposes they can be consider to be almost the same.

What we do with multi carrier modulation is that suppose I have some symbols let us call this symbols as X0 X1 X2 all the way up to X n-1 so there are this N symbols these are all complex symbols that you have if you are looking at BPSK modulation then X0 will be real of course because it is + to – 1 but if you are looking at a QPSK the X0 X1 X2 any of these can come from 1 +j we have -1+j you have 1-j when you have -1 –j okay so this can be any of this X0 X1 x2 all way up Xn-1 can be any of these for the QPSK modulation and for addition modulation they can be any of the complex numbers okay.

What we do is over a particular bit period over the symbol period so let us say 0 top T okay let us multiply each of these symbols with a bank of modulator so let us say if I take this X0 x $e^{J2\pi}f_0T$ x X1 with another signal sinusoidal signal complex sinusoidal signal of course with $2e^{j2\pi}f_1T$ and then you have X2 ej2πf2T and so on at the k^{th} time you have $Xke^{j2\pi}f_kT$ okay finally you have the last one as X_n -1 $e^{j2\pi}$fn-1 T okay.

Where this frequency components f0 f1 f2 are all equally spaced okay so this is f0 this f1 the spacing between them is called as the sub carrier spacing this f0 f1 are all called as sub carriers the last one the fn-1 so clearly the kt^h sub carrier frequency is given by f0 +k x δ f k goes from 1 0 all the way up to n-1. Now that you have multiplies all these signals along with their bank of modulators what to do is if some all of them you sum and call the resulting signal as x(t) so the x(t) is what is called as a multi carrier signal and in short we can write this as sum /k=k=n-1 xk $e^{j2\pi}$fkt.

Now you can question how may I going to receive this x (t) so after this you transmitted to the receiver side okay so you can say how do I obtain this xk well you can so one thing take this x(t) and multiply it with e-j2flt where Fl is one of the frequency components that you have okay integrate over one period and average it by making it b1/t okay if you carry out this integral by putting x(t) inside what you end up is the integral okay apart from whatever the summation that you are going to get the ∫ will be $e^{j2\,\pi\, f0t}$ $e^{j\,2\,\pi(k\text{-}l)\,\Delta\, f(t)}$ okay, $\int_t^t$.

So if this case is equal to l, then this would be equal to 1, this $e^{j2\,\pi\, f0t}$ with f0 is normally taken to be equal to 0, then will give you t, t/t=1, and simply what you are going to get her after doing all this integration will be the data symbol X_l, okay. For this to happen, of course you need to have only k=l. And when k≠l, this ∫ should go to 0, and that will happen when you look at the orthogonally of the basis signal which are this $e^{j\,2\,\pi\, fkt}$

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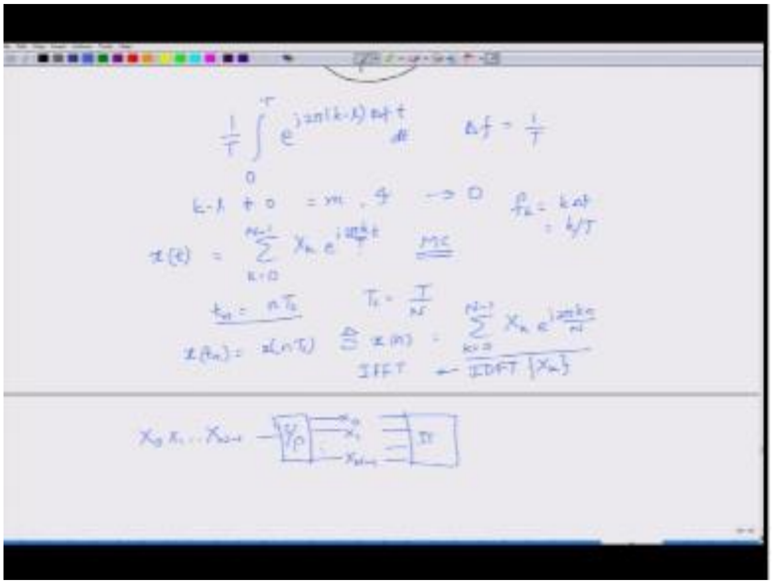

And recall that if you take this $\int e^{j\,2\,\pi\,(kl)\,\Delta\,ft}$, okay and take Δ f, this $\int$dt, so if you take Δ f=1/t, and then consider k-l $\neq$ o, and rather it is equal to m, where m is some integer, then this would be some integration of sinusoidal signal, which has m cycles inside one period, okay.

And because of a sinusoidal signal having m cycles within one period will integrate out to zero, when k $\neq$ l, okay this result will be equal to 0, so clearly you can what x(l) is? However is that not as simple as we talk about, so we go back to x (t), and recall x (t) = k=0 to (n-1) xk $e^{j2\,\pi,}$ I know what is fk, right? fk is now given by k Δ f, but Δ f is nothing but i/t, therefore I can regret this as 2 π k/T, and t, okay.

So this is the multi carrier signal that you have, okay now do one thing instead of taking this continue signal x(t), you sample this okay, you sample this at the n^{th} sampling time which is given by nts, and you take this sampling time t is as, the symbol period T/n, okay.

So you take your symbol time t, when you break it out into n equal parts, and when you break them into n equal parts you are generating Ts spaced samples, okay and you look at what happens x(t) there, so the sample signals, x(t_n), which is given by x(nts) and further compressed in notation writing as x(n)is given by Σ k=0 to n-1, xk, there is no change there.

But in place of t, you have to go to the discrete time which is nts, which is Nt/N, there is a T in the denominator here, there is a T here those will cancel out, what you get is $e^{j2 \pi kn/n}$ and if you know your Dsp, then you know very well what is this expression? This expression is nothing but IDFT of the sequence Xk, okay.

IDFT can be computationally implemented using IFFT, so what you get as the time samples that you are transmitting from transmitted to the receiver, are simply the IFFT of the sequence, okay when this samples go through the fiber, okay the fiber impulse responsible convolve with the samples of the signals of x(n).

And what you get be the outputs sequence y(n), and these y(n), if you now apply the FFT operation and assuming everything has gone okay, with the we will see what those module in the next module, but if everything else goes correctly then after taking the FFT you are able to extract the data sequence x(k), so at the transmitter you start with the symbols and then you know, you start with the symbols x0, x1 all the way up to xn-1, you convert them into a parallel form using the serial to parallel covers ion or serial to parallel converter you get a x0,x1, all the way up to xn-1. So this you can connect or you can take the IFFT of this.

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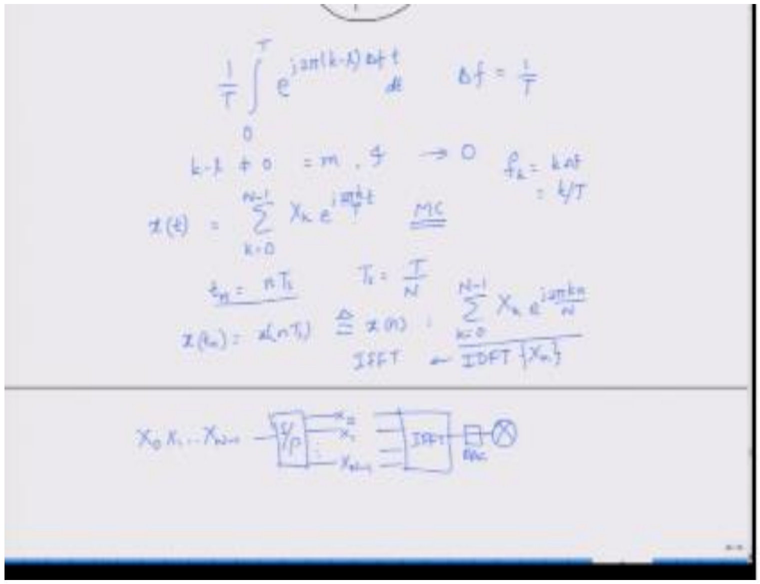

And then take it through a dack ,which is digital to analog converter to obtain the continuous waveform or the approximate continuous waveform, and then multiply it by some $e^{j2\pi f0t}$ to up convert it on the (Refer Slide Time:) domain and further up convert it by using an MZM on to the optical domain. Where in to the MZM we are going to give laser diode and then transmit it through the fiber. So this has to block, called parallel to serial conversion, it can come after dack or before dack depending on how your implementing.

But t the receiver side, what you have is, you know, the signals which re coming in sequence, which would be the samples y(n), y(n) will be x(n) convolved with h(n), h(n) I the impulse response of the fiber, if you take these samples y(n), before you get to this one, you should actually have a photo detector, so from the photo detector, you know, you extract these samples y(n), which would then represent x(n) convolved with the impulse response h(n).

Put them through a serial to a parallel converter again, so extract the parallel component, take the FFT of the sequence, then again convert this FFT into a serial, after FFT you convert this into a serial using a parallel to serial combination and what you get are the data symbols x0, x1, all the way up to xn-1, this of course assume that everything the world has gone right, so that there is no noise, there is no offset, there is no phase error.

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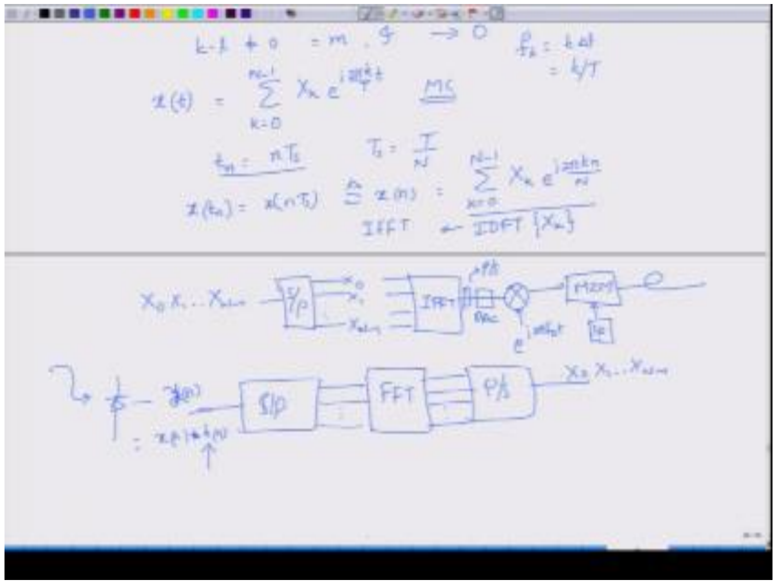

We will see all those effects in the next module, but if everything else goes so well, this system is able to re produce your transmitted symbol at the receiver side, simply by exploiting the orthogonality of the basic sets which re the $e^{j2\pi fkt}$, the cosine complex sinusoids as the basic function and this system is called as the orthogonal frequency division multiplexing or OFDM, OFDM is the special case of multicarrier modulation and for optical communication, the advantage of OFDM is that it can combat the chromatic dispersion in the fiber, so it can easily be or relatively easily one can compensate for the dispersion in the orthogonal frequency by multiplexing.

The drawback is that, the phase noise has to be still critical here and any residual phase error or the frequency offset error will be detrimental, but thanks to lot of work that people done in the wireless and in the side for OFDM, one can utilize many of the algorithm that are developed there to mitigate the problems with phase offsets, phase errors, frequency offsets, frequency errors and any other kind of impairments that OSDM system go through.

An additional dis advantage of OSDM system is that they are subjected to what is called a the high peak to average power ratio and because the optical amplifiers, I know the erbium doped amplifiers, respond differently when you have one of the band saturating power, this could be a little bit of problem and there are techniques to avoid this PAPR or at least minimize this PAPR and will talk about some of the techniques very briefly, I'm going to mention that in the next module.

So the next module we will come back and finish this discussion OFDM, we have not introduced a another very important aspect of OFDM called cyclic prefix, we will introduce that, complete the discussion of OFDM and ten take up the further impairments that a n optical fiber can produce to your signals, nice the signals are propagating. Thank you very much

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