Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – XI
Module-I
Determining BER in OOK system

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Hello and welcome to the mook on optical communications. In this module we will be talking about the bit error rate determination of that in optical on-off keying systems. So this will, after that we will review all the different methods that are available for on-off keying systems to detect the on-off keying systems and then move on to the higher order modulation techniques.

So let us begin by looking at what kind of, you know module that we would like to consider in calculating the bit error rate of on-off keying systems. In one of the earlier modules we have actually discussed BER on PIN receivers, EPD receivers, receivers with coherent, you know or the local oscillators, or the coherent receivers, and receivers with optical pre amplifiers right.

In each of these receivers there are different noise processes which are involved and which dominate over all the other noise processes, they have quantified the operation of the system, the performance of the system by defining what is called as the Q factor to which we have related the bit error rate of the system.

However, we have not told you how to calculate those BER, you know expressions and it is instructive to do that for a simple on-off keying system, because it brings out the difference between a simple wireless communication system which employs an on-off keying type of modulation versus an optical system that employs on-off keying modulation.

We will see that, the kind of assumptions that go into a wireless on-off keying system are slightly different from the assumptions that go into the on-off keying systems in optical domain okay. with that let us look at what is the signal model that we are looking at.

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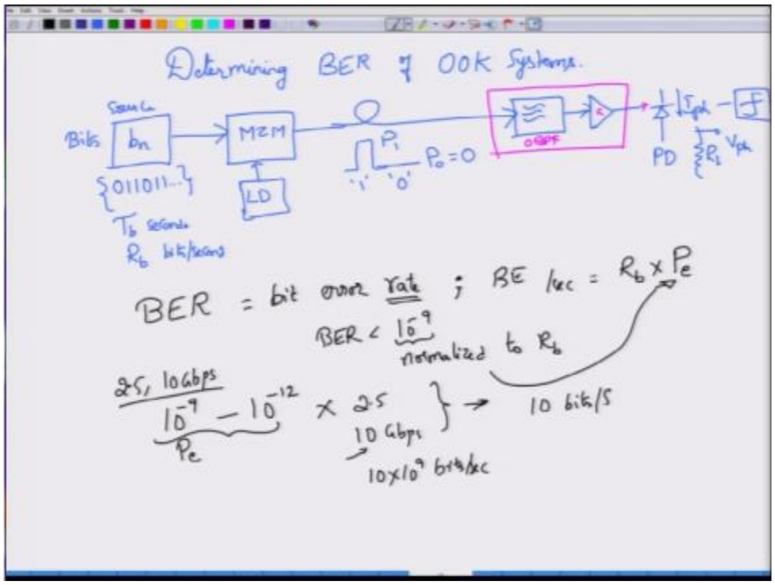

So we have a certain data sequence or in this particular case the data sequence is the same as the bit sequence, this bit sequence will be converted into a certain wave form correct, this wave form conversion happens because you actually take a laser diode and then you take a external modulator or you can directly modulate the laser diode itself, but in whatever way you do, you are going to generate this abstract bit sequences, bit sequence could be 01, 10, 11 and so on right.

These sequences are coming out every T_b seconds which means that this source is putting out these abstract bits right. So these are the bits, these abstract bits are the bit sequence at a rate of R_b bits per second. So this is the rate of the source which is generating the bits. However, these bits as I said cannot be directly transmitted over the optical fiber in order to transmit them, you have to encode these bit information onto the amplitude of the power of the optical carrier.

And if you take the power of the optical carrier and then, you know switch between two levels, you are sending a certain optical power say P1 when you want to transmit a bit one and you are sending an optical power of P0 which is usually taken to be 0, later we will see that this will not be exactly equal to 0 for this case okay, for now we will assume that this is equal to 0.

So you encode bit 1 by transmitting an optical pulse okay, you can have various pulse shape which we have discussed, but for now we are not even bothered with the pulse shape okay, we will assume that the pulse shape has been chosen and we have the receiver bandwidth that is sufficient in order to reproduce this pulse at the output, of course the pulse would not be exactly reproduced they will be noise that comes along depending on what kind of receiver you are going to consider right.

So with bit 1 represented as an optical power P1 and bit 0 represented as an optical power of P0 which is very, very close to 0 that we actually take it to be 0 this is then transmitted over the fiber after transmission we will assume that there is only one fiber link between the transmitter and the receiver okay it comes to the receiver front end at the receiver front end you might actually have a band pass filter an amplifier or you can inter change the amplifier in the band pass filter because the unit to first amplify the signal and then filter out the ASE components from that one and after that eventually you are going to have a photo detector correct.

So this photo detector would produce a photo current this photo current flows through load voltage and then this output across the load voltage would be the voltage that is generated corresponding to bit 1 and bit 0 and looking at this voltage and comparing it against a certain threshold you are able to deicide weather the bit is 1 or the bit is 0 so this essentially the link model that we are considering for now you will assume that this amplifier and the band pass filter combination that we have so this is the amplifier and the band pass filter combination does not really enter into picture right.

In fact it is not required because the detection can be done or the basics of the bit or rate can be established without reference to this effect of having a gain and an optical band pass filter can be added later or can be discussed later so we will look at this model okay and what we are interested is this quantity called bit error rate so if you expand this stands for bit error rate and this is a quantity that is normally found on commercial modems okay.

So you are anything that you are going to purchase or in system design where you are asked to keep the bit error rate to be less than 10^{-9} however the world rate should tell you soothing else right rate is something that with respect to time when a quantity changes with respect to time that is the way it changes with respect to times is captured by this rate however when we code BR we normally say that this is10^{-9} how and this does not seem to represented with respect to time what we mean here is a normalized bit error rate okay normalized whatever the bit rate that we have chosen.

Okay although we do not specify the bit rate this factor is actually normalized to Rb so to obtained the bit error rate in seconds/ second quantity you have to multiply the bit error rate that is normalized which we can call as the probability of the bit error that is if you transmit a bit 1 for or, bit 0 what is the probability that bit will be incorrectly received at the receiver so you take probability of error and multiply it with the bit rate in order to tell you how Many bits you are making or how many bits are there in error so let us remove this E.

So how many bits are there in error/ sec for example in earlier 2.5 and 1o Gbps systems that standard that was used Br value that was used was anywhere between 10^{-9} to 10^{-12} with 10^{-12} more common for a 2.5 Gbps system and 10^{-9} for a 10 Gbps system with this one as the bit probability of bit error okay what we have actually specified is the normalized bit error rate which is actually the probability of the error when you multiply this one either by say 2.5 or 10Gbps the kind of bit errors that your making would turn out to be.

So let us calculate this one for the 10Gbps system so 10 x 10^{-9} bits/ sec is what you are actually transmitting when you multiply this one 1ith 10^{-9} this 10^{-9} will cancel 10^{-9} and what you get is you are making 10 bits or your received signal you have 10 bits in error at each second, okay on an average of course this is not exactly a sign saying that I mean this is not exactly saying that this is 10 bits will be in error it is on an average 10 bits will be in error, okay.

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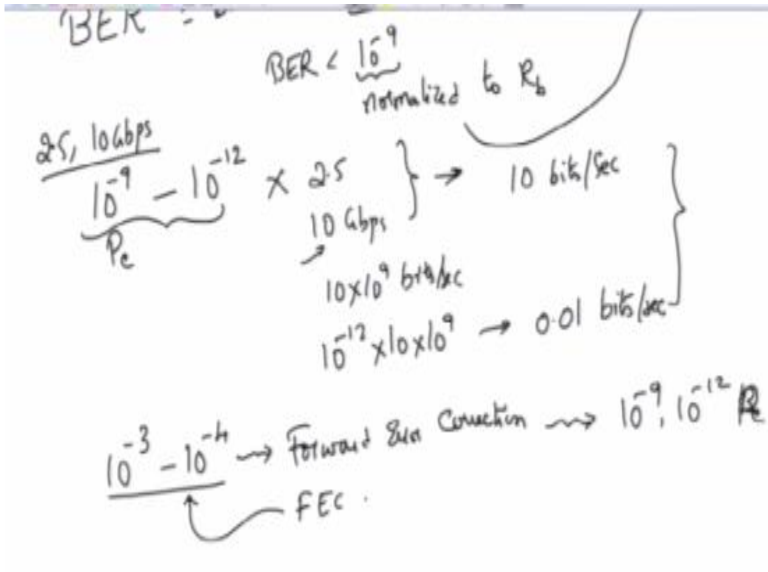

When you have 10^{-12} then the error that you are making per second will be even smaller, right. So this would be 1 x 10^{10} 10 and -12 is there which would be 10^{-2} so which is 0.01 bits per second, correct. So these are the typical values that you would have seen about 15 years ago or 10 years ago when you had a commercial 2.5 Gbps and 10 Gbps systems in operation these values turn out to be very nice, however in modern optical communication systems we normally fix this probability of error or the normalized bit error rate to $10^{-3} – 10^{-4}$.

Now you might question with 10^{-12} and 10^{-9} – you are seeing bit errors of 0.01 bits per second and 10 bits per second would not we have increase a number of bits in error by relaxing the requirements for probability of error you would be right, we have relaxed the requirements of probability of error, however there is another process called as forward error correction which is supposed to take care of okay.

So this is forward error correction which is supposed to take care of the extra gain or extra loss that we are experiencing and applying the forward error correction you can change the systems with an initial 10^{-3} or 10^{-4} probability of bit error into 10^{-9} or 10^{-12} bit error probability or probability of error okay. So this process is actually accomplished by what is called as forward error correction.

And therefore the FEC limit, FEC stands for forward error correction this FEC limit is typically 10^{-3} and 10^{-4} in modern coherent optical communication systems, okay. Having established what BER is let us actually look at how to calculate this probability of error, the bit rate is of course is fixed by the system designer there is no control of that one for us but we would like to see what can be learned more about how to obtain the probability of error and that is what we are going to do now, okay.

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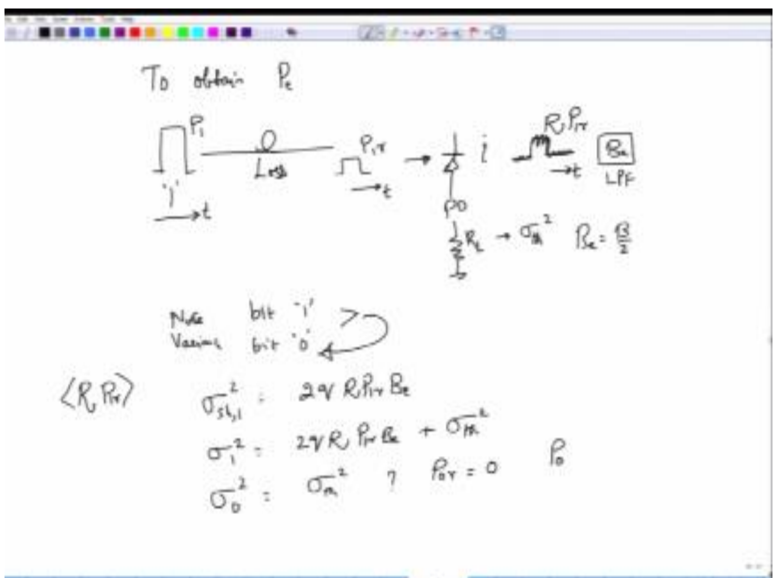

To obtain probability of error we need to first understand what is exactly happening, okay when we will get an error? Suppose you have transmitted a certain optical power P1 okay which stands for a bit 1, once this propagates through the fiber the power actually kind of reduces obviously right, the power at the receiver side would be P_1 r and this power would be less than whatever the power that we have launched at the transmitter side.

Because of the loss in the fiber, of course fiber also has dispersion which means the pulse will spread out from its original time slot and interfere with the next pulse but we will assume that those dispersions has been compensated and therefore we do not really worry about that, okay. Please also note that this is the waveform with respect to time okay, so here you have a pulse which is varying with respect to time.

And let us assume that for all practical purposes is a same pulse which is same pulse variation is maintained except that the amplitude has now decreased, okay. Again this is something that is varying with respect to time, if I put in a photo detector here what is that I am going to generate at the output, I will be generating the current waveform okay which looks very similar to the received optical waveform, okay.

And it get multiplied by this responsivity of the photo detector so you have a photo detector the current waveform that you are generating which will again be a function of time, okay. Will be R times P_1r is this is what has happened it would have been life would have been very easy for us we could have simply amplified this current.

And then did all are deduction you know later on. Unfortunately, we know that this photo diode is going to add some noise, correct so it is going to add some noise, okay and this noise in addition to the noise added by the photo diode short noise because the current is suppose to flow into a certain load resister which we can take as the input impedance of the per-amplifier or the amplifier post amplifier as we call it.

So that post amplifier input impedance is R_L, okay there is a certain thermal noise associated with that R_L as well, okay this thermal noise is of course independent of what optical power you have transmitted it simply depends on what is the temperature at which the resister has been kept and this resister or the equivalent temperature and this resister with the value of the resistance itself, okay. So this is the waveform that you have received and if you transmitted in the next a lot 0 you would also see some amount of noise.

Now compare to a typical radio receiver which you would find in any of your cell phones or in the older radios the noise levels for bit 1 and bit 0 are different in fact the noise that you see the variance the noise variance of bit 1 is greater than the noise variance of bit 0, can you guess why that is so. The answer is bit 1 when you have transmitted you have received a certain power P_1r and the current that you have generated is RP_1r, okay.

The average current that you have generated during this particular position of the pulse would actually determine the short noise and this is given by $2qRP_1rB_e$ where B_e is the bandwidth that we have used, so we are assuming that after this you are going to use a low pass filter whose band width is B_e which we normally take it to be RB/2 where B is the bit rate, okay so B is the bit rate and we normally take that one to be equal to the band width of the electrical filter to be equal to B/2, okay.

The total noise when you have transmitted 1 actually is given by $2qRP_1rB_e$ this is the short noise component plus there is a thermal noise component. However, when you transmitted bit 0 the only noise that you are going to get is thermal noise, why because we have assumed that the received power when a bit 0 is transmitted is equal to 0 that is because we assumed that the transmit power when you are transmitting bit 0 itself is equal to 0.

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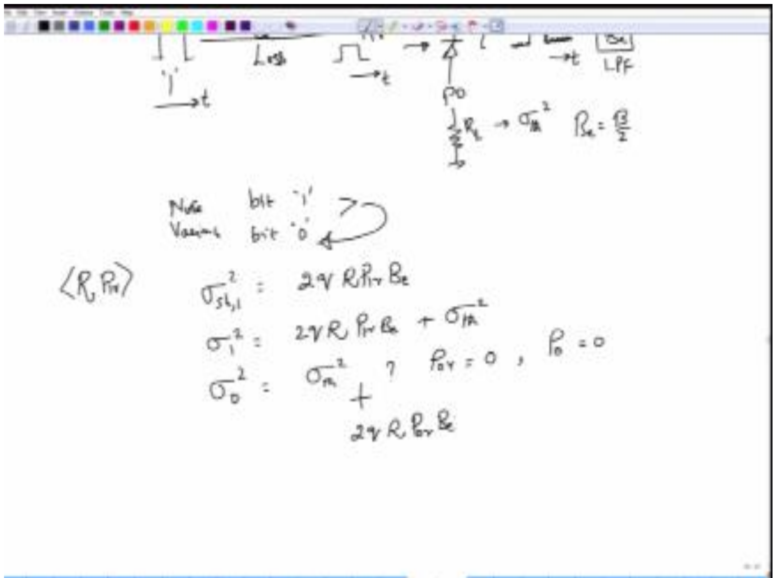

This was the transmit power we had assumed that P0=0. In case your laser is not able to make P0=0 in addition to this there will be a short noise component that corresponds to how much power you have received when you have actually transmitted bit 0 flowing in the same bandwidth, okay flowing the same resister and having the same bandwidth, okay.

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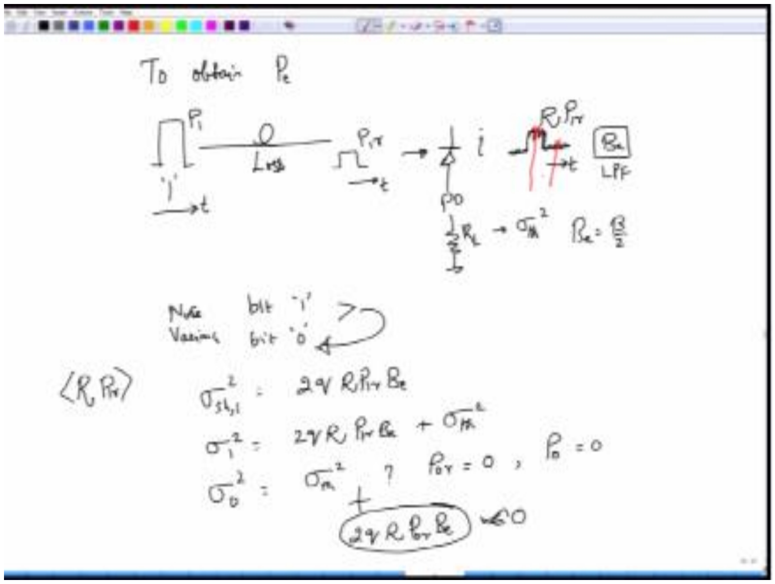

However, in practice this term is very, very small so this is actually kind of very close to 0 okay, if not you can also include this particular thing as an extra noise. But what are these noise that we have been actually talking about, well if you go back to the link here what happens is after you have this current waveform you do not exactly use the waveform to detect, I mean you do not use the entire waveform to detect what you do is you sample the signal at it is best sampling time, okay.

If it is a 1 you normally sample it at the middle of the bit period so that where the eye is quite wide open we will talk about eye diagrams in another module and you again sample at the middle of the bit for in order to obtain the next voltage or the current sample, okay.

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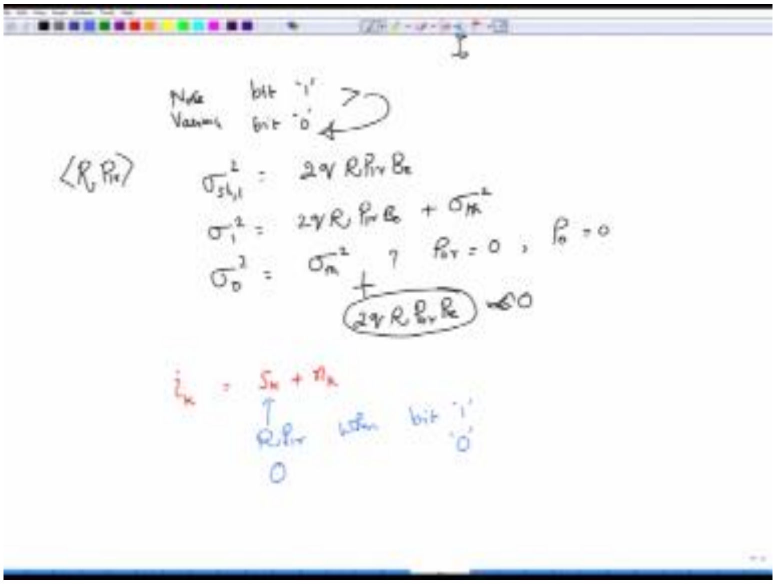

We will assume that your sampling the current and therefore this current which you have sampled at time t_k is given by i_k, okay this is a photo current and the sample time at tk=1 where you have sampled is given by ik+1 okay of course at difference time instances you are actually sampling the current and it is this current that you are interested in what would this current consist of this current would consist of whatever the transmitted signal that you have let us call the transmitted signal at the time k as sk plus there would be noise component nk.

Okay so this is the signal plus noise that you are looking at, now here is an interesting question what would be sk, sk would be RP1R when bit 1 is transmitted and we will continue to assume that p0 r is 0 therefore this would be 0 when bit 0 is transmitted or when 0 is received okay.

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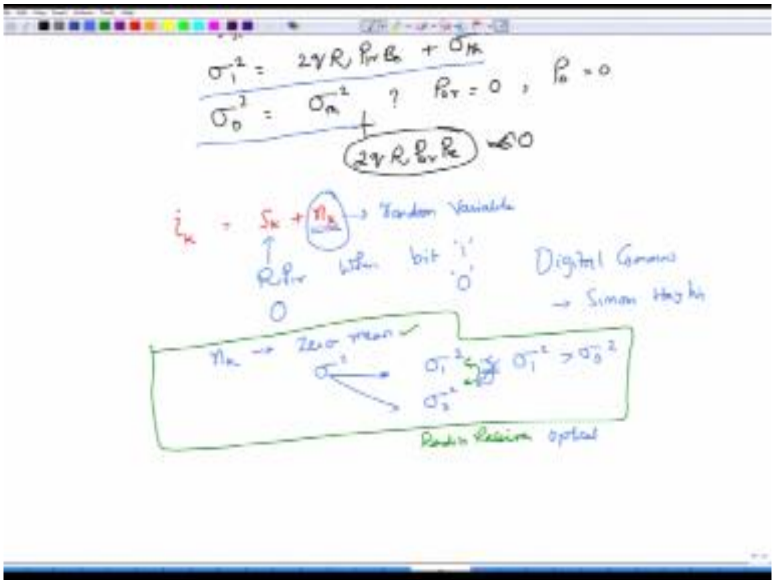

Now this noise unfortunately not something that can be pin down at the beginning of the transmission so noise is actually what we call as a random variable okay noise is a random variable because predicting its exact value is not possible we can only talk about the statistical proper case it is like taking a coin okay and tossing it so if I take coin and toss it right then beforehand I will not be knowing what would I received you know what would the coin be there on my pump whether I would have a head or a tail received on the coin I will not be able to know that okay.

Or if I take this pen and I through right there is no guaranty that I will always catch this pen at the correct stage you know maybe I would catch but maybe I would not catch I can only talk about the probability of cactichi8ng or the probability of the head showing up or the probability that the noise voltage at the point where we have sampled across the load register RL is greater than 3.3 volts or it is less than -1.1 volt okay.

Or the noise current would be less than 2.3 amperes, so I can only talk about the probability of these events and of course I can talk about on an average what happens on an average what is the value of the noise voltage across a load register if it is a white thermal noise a Goshen distributed noise then on in average it would be 0 that is the voltage if your average or the positive and the negative you know value time where you get the positive voltage and negative time voltages in the longer and the average voltage will be 0.

But there will be variants associated with that there is a variation or a fluctuation associated with that so we can determine what is the fluctuation of the noise process okay so you can learn more about all these random variables in any of the other courses or you can refer to this chapter on probability and random process in this book called digital communications by Simon Hykin, okay so if you look at this books it is gives you nice idea of what probability is what random variable are and how this random variables are distributed.

For us noise is a random variable and as I said this noise variable nk is usually modeled as a 0 mean which is on in average the voltage of the noise is 0 but it has a certain variants σ okay,
This variants for when transmit bit 1 will be $\sigma1^2$ when you transmit bit 0 this would be $\sigma0^{2,}$ usually $\sigma1^2$ is large than $\sigma0^{2,}$ now this kind of a behavior is not at all you know seen for a radio receiver.

In the radio receiver the noise will be 0 mean which is perfectly fine but whether you transmit bit 0 or bit 1 the variants will always be the same, so the variants will be equal to each other for a radio receiver okay. However for an optical receiver these values are not equal right so this values are not equal for an optical system, that is because the noise $\sigma1^2$ the noise variants is different from $\sigma0^2$ is the noise variants when you have transmitted bit 0- and that noise just includes the thermal noise plus σ 1 includes both thermal as well as the short noise component.

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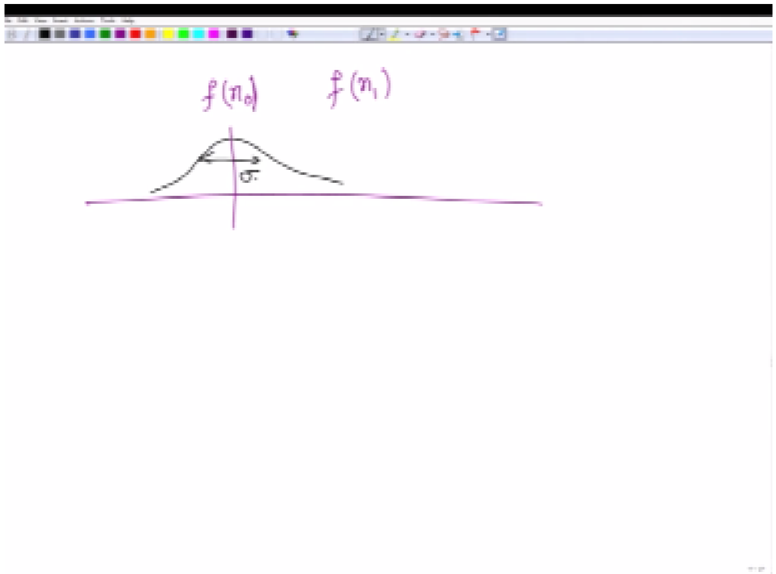

So that variants itself depends on the input that you have transmitted and if you sketch the probability density function of this noise process so let us say there are this know this noise process corresponding to bit 0 and the noise process corresponding to bit 1 okay. F(n0) is basically the noise process when I have transmitted bit 0 and then f(n1) is the noise process when have transmitted bit 1 and you look at these process or the probability density functions you will that the corresponding probability density function we are assuming it to be Goshen so if you look at a Goshen function this is the noise that you are obtaining or the noise probability density function when you have transmitted bit 0.

It would be situated at 0, and it will have a certain variance of $\sigma\ 0^2$, where as the probability density function when you have transmitted bit 1, will be situated at a mean or it would have a mean value of RP_{1r}, why is this so? Go back to S_k, $S_k=P_{1r}$, RP_{1r} when you have transmitted bit 1, so the received current I_k, will be given by RP_{1R}, plus the noise component.

And what is the average value of this I_k, which is obtained by operating this expectation on this one, since this is a deterministic quantity so this would be just R1 and P_{1R}, okay, plus the expectation or the mean value of the noise voltage, but this is exactly equal to 0, however the variance of this noise process I_k, when you have transmitted bit 1, okay is given by $\sigma\ 1^2$.

Therefore the noise process corresponding to this one will be different and it actually has a variance that goes something like this, okay so this have actually different heights simply because $\sigma\ 0^2$ is less, therefore its variance should smaller, but the area under this curve, $f(N_0)$, or $f(N_1)$ should be equal to 1, okay.

Now coming back to, what is this P_e, $P_{e,}$ you know? Is the thing that happens when you receive a bit and decide the bit in the favour of 1, but when the actual bit that was transmitted was 0, right. So you commit an error whenever you onerously decode the bit, or determine the bit, so this P_e, can be written as, what is the probability that I am going to decide 1? When actually the bit 0 was transmitted and what is the probability that 0 itself is transmitted? Plus, what is the probability that the receiver or I will decide the bit as 0.

But when we have actually transmitted bit 1 times, what is the probability that you have actually transmitted 1 itself, okay so this is these are known as conditional probabilities, and these are the conditional probability errors, and multiplying them by p(0) and p(1), removes this pre conditioning.

If you are not catching this particular point, you can refer back to the AC noise derivation, there we had the photo current given certain optical power was received and then we removed that pre conditioning thing right, we removed that conditioning thing that is what you get over here. Normally you assume your transmitting 0's and 1's, equally that is on the longer then this probabilities are equal to ½, so you are left only to calculate what is probability of 1? Given 0, was transmitted and probability of 0, when you are actually transmitted 1. Okay.

So let us actually consider first this case, probability that the receiver things of this as 1, while the transmitter has sent a 0, okay when you have send a 0, right what would be the received current after sampling? So the current corresponding to bit 0 right if you have transmitted bit 0, you should at the K^{th} sampling time, the current that you are going to see I_k0 will be equal to $0+N_{0,}$ where S_0 is actually equal to 0.

Because S_0 is nothing but R P_{0r} but we have not transmitted any power, so this would be equal to 0, so when you sample the current waveform, when bit 0 is transmitted at that particular time you are only sampling the noise voltage, and that is voltage we have call it has N_0, this would be the 0 mean right and irt has a certain probability density function, which we have assume to be Gaussian.

Which means that the probability density function of the noise current of N_0, is given by $2\pi\ \sigma^2$ or $1/\sqrt{2\pi}\ \sigma\ 0^2\ e^{-N02}/2\ \sigma\ 0^2$ this is what we have plotted in this graph, so this $f(n_0)$ is the probability density function that we have plotted. When will I make an error? Suppose I have received this 1 okay, when will I make n error? Well a I said I'm going to put up a threshold, correct? So I will put up threshold and I will subject this particular voltage that I have received when I transmitted bit 0 to the threshold.

In case this $i_{k,0}$ happens to be greater than the threshold, the current threshold, with this happens I would have declared this a bit 1 and that happens when this current sample becomes greater than the threshold current that I have sent. However $i_{k,0}$ is nothing but n_0 because s_0 is actually equal to 0. Therefore this probability can be refreshed in terms of noise, whenever you sample the voltage, if for sample the current, if the noise in the current when the bit 0 is transmitted becomes higher than the threshold current, right? If the noise is so strong that it actually moves up the sample value beyond the threshold, there will be an error.

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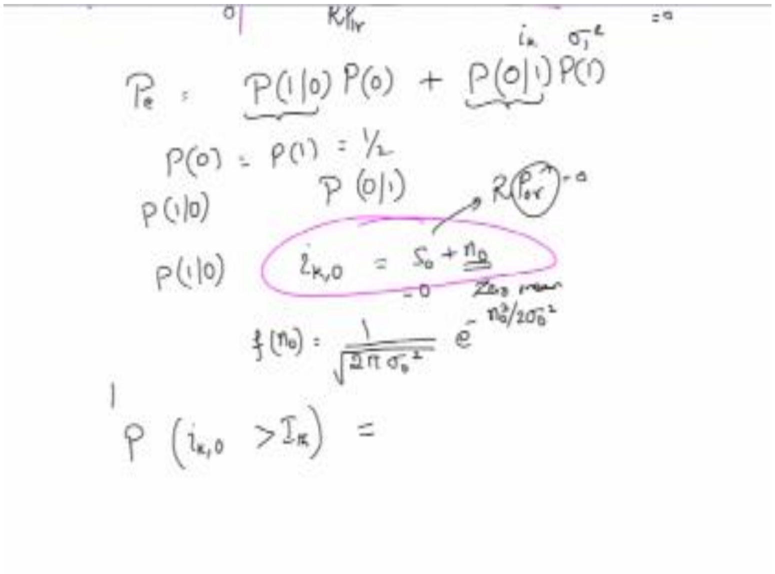

So the probability that you are actually declaring a bit a 1, when you have transmitted 0, just the probability that the noise would be greater than the threshold, okay and this can be easily obtained, why? Because I know, what is this probability, if I take probability density function that look like this, okay. As function of independent variable x, o this would be f(x), probability that an event lies between x1 and x2 is given by integrating the probability density function from x1 to x2, okay of this f(x)dx.

Similarly if I want to know, what is the probability that noise is greater than I_{th}? I'm looking at the probability that you know of this entire area. So the probability within this entire area is simply the probability that, the value x is greater than x1, okay. The probability that your output I greater than x1 is obtained by integrating this x1 and moving this x2 to 2 infinity, so you can simply write down what you get here, this would be the probability that noise is greater than the threshold current.

So this is actually given by I_{th} to infinity, that I the noise current will be greater than the threshold current, there is $1/2\pi\sigma_0^2$ over here which I forgot to put and then $e^{-no2}/2\sigma_0^2$ dn_0, okay, n_0 of course recognized as the dummy variable, I could have written x here and nothing would have actually changed over here, I can relate this probability, okay or this expression, what is called a the q function. The q function is defined as Q(x) is given by 1/√2л integral from x to infinity $e^{-t1/2}$ some double variable, this fellow okay.

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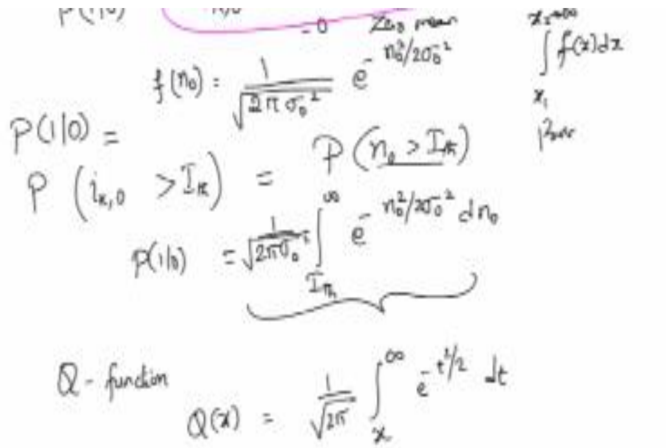

So this particular Q function is simply telling you the probability that 0 mean unit variance random variable, this is to be 0 means unit variable random variance, what is the probability that random variable will take on a variable will take on a variable is greater than or equal to x and that is what the Q function will tell you. So you can rearrange this one, we are going to talk but that in the next module and complete the expression for the probability of error. Thank you

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