Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

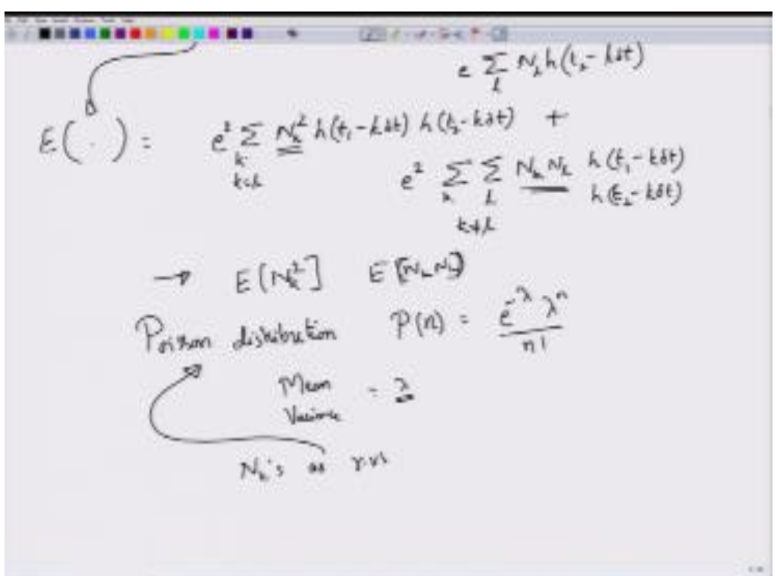
Course Title
Optical Communications

Week – X
Module-IV
Noise in optical amplifiers (contd.)

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome let us continue the discussion on optical amplifier and noises in the optical amplifier. Last time where we left we were actually trying to determine the auto-correlation of the current I(t) the photo current under the assumption that we have actually given some optical power to, and of course, if there is no optical power there would not be any photo current. And we ended up with an equation that looked like this.

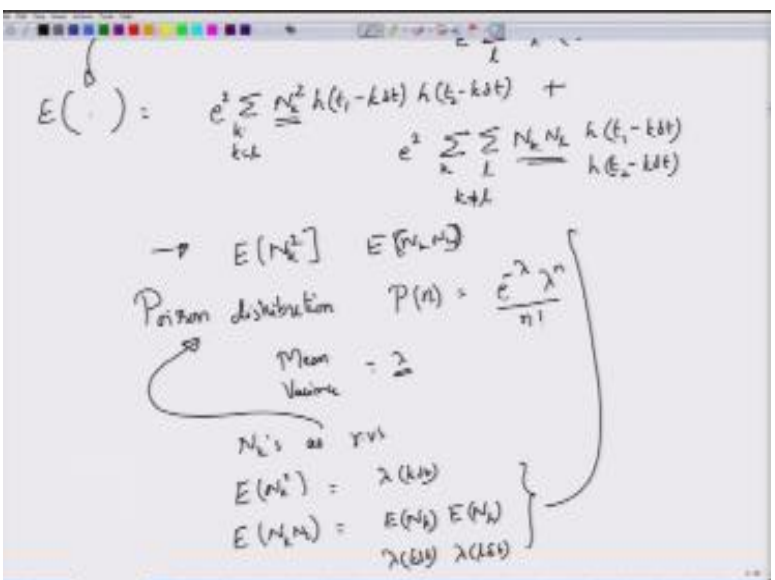
(Refer Slide Time: 00:36)



We have two terms one term, you know had nk^2 because k was equal to L and the other term corresponding to all the non-overlapping intervals. Now here is where we will use one of the Poison distribution formulas okay. The distribution is Poison and not a Gaussian distribution, because Poison distribution is well suited for photon counting applications and this is something that actually ties up to that one.

The idea of a Poison distribution is that in a given time interval the probability that you actually receive n photons or, you know generate an electrons any n type, n such things is given by some $e^{-\lambda} \lambda^n/n!$ where λ is the average value okay. It turns out that for Poison distribution both mean as well as variance, both are equal to the average value λ or the value λ okay. So going back to that, if we consider all these n case as random variables which are Poison distributed okay. So these are Poison distributed random variables.

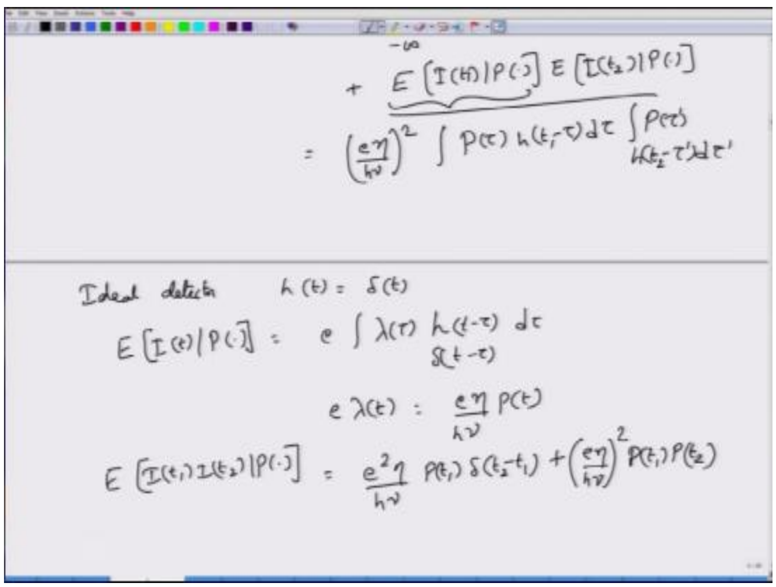
(Refer Slide Time: 01:46)



Then $E[N_k^2]$ which is the variance of this particular random variable N_k is given by $\lambda(k\Delta t)$ okay. So λ evaluated at $k\Delta t$ it is a non-uniform Poison distribution which means that it simply depends on time okay. And what about $E[N_k]$ and N_l well these are actually the ones which are independent Poison random variables. Therefore, their expectation would be slightly different.

So the expectation here would be that $E[N_k]$ times $E[N_l]$ so it would be independent variables, independent random variables, therefore they would be product of the individual variables so this is $\lambda(k\Delta t)$ and this is $\lambda(l\Delta t)$. So substituting these three into the equations that we have right, so you remember the two term equations that we have.

(Refer Slide Time: 02:40)



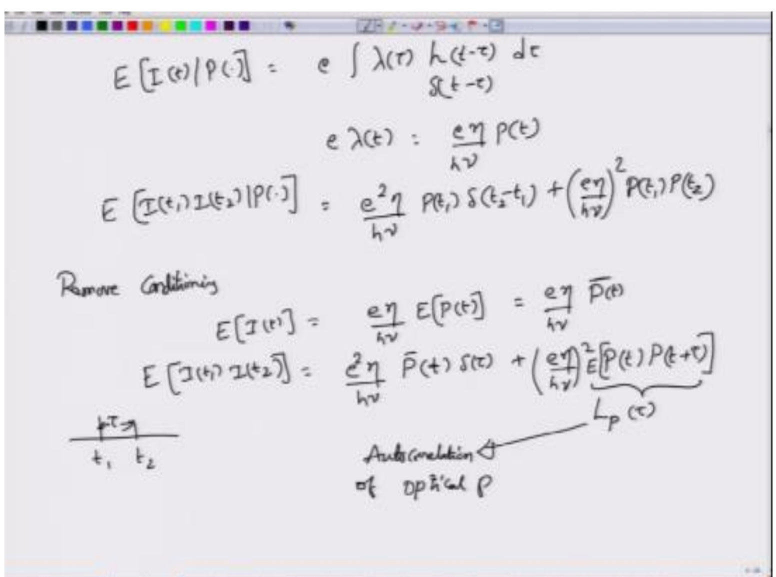
And simplifying what you get here is that E I(t1)I(t2) given the fact that you have some optical power is given by e^2 integral $\lambda(\tau)h(t1- \tau)h(t2- \tau)d\ \tau$, where τ goes from minus infinity to plus infinity plus 2, because of this $\lambda(k\Delta t)$ $\lambda(l\Delta t)$ the result will be two integrals which when you integrate is going to be E I(t1) given that you have send some power, times E I(t2) given that you have send some power okay.

Nut we already know what these are, remember these are nothing but $E\lambda$ and integral of that will, so there is going to be two times E of that one. So we can rewrite this second term as $(e\eta/h\mu)^2$ 1 $e\eta/h\mu$ coming from one expression the other $e\eta/h\mu$ is coming from the second one okay. And then you have two integrals, you have t(τ), h(t1-tou)dτ and P(τ)’, h(t2-τ)’dτ’. Now let us consider an ideal impulse detector.

In this ideal detector scenario I have the pulse to be just an impulse function okay. So the pulse shape in that a current or an electron generates is given by the impulse function δ(t) for which the integral scan now be simplified the mean value of the photo current that you are going to get provided that you have some optical input is given by e∫λτ h(t) – τdτ but I know that h is nothing δ function so δ (t) - τ as this property that it would shift the value of **λ**τ out it would skimpily pick the value of λτ because that is where it will be informally that would be non zero right this is actually e times **λ(**t) that is it okay.

But **λ(t)** is nothing but p(η)p/hν so I can rewrite this e η/ νp(t) okay similarly if I look at the auto correlation it would be I(t_1, t_2) conditioned up on the fact that we have received some optical power and we can easily show that this is given by e^2 η / hν p(t_1) and a δ function that depends on the time difference between t_1 and t_2 that is δ(t- t_1) + you have the second terms which is eη / $h\nu^2$ p(t_1) p(t_2) okay once you have.

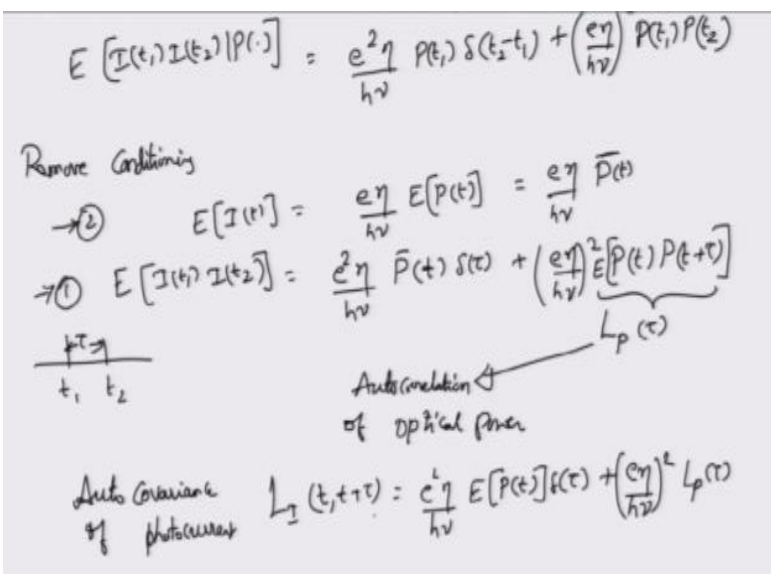
(Refer Slide Time: 05:23)



Now we can remove the conditioning on the fact that we have received an optical power so we can remove this preconditioning that we have done okay and when we remove the condition you see the mean photo current I(t) that you obtained it is given by $e\eta / h\nu$ the mean optical power that you have received okay.

We can rewrite as $e\eta$ / h ν time p bar where p bar (t) is the mean optical power that is incident similarly e I (t_1) $I(t_2)$ which gives you the auto correlation is given by $e^2\eta/h\nu$ and p bar (t) because the expectation goes on to that and $\delta(\tau)$ where τ is defined as $t_2 - t_1$ so τ is defined as t_2 – t1 is the lag that we are considering okay + so we are considering 2 units time t1 and t_2 and you can see that this you know is a function of the lag τ so you can write this as p^t which is the mean optical power times $\delta(\tau)$ the second term is e η $/h\nu^2$ p(t) and $p(t+\tau)$ because $t_2 = t+\tau$ and there is an expectation sitting on to this one this exception let us call this Lp (τ) where Lp (τ) is the auto correlation of the optical power auto correlation of optical power over here okay.

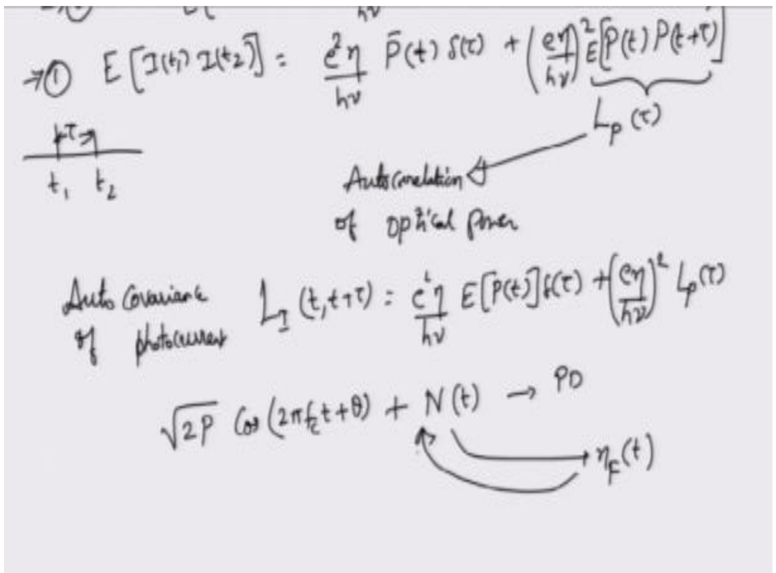
(Refer Slide Time: 07:07)



Now this is mean we have auto correlation we have we can find what is called auto covariance of the photo current so auto covariance of the photo current I(t) is obtained by subtracting this equation okay which let us call this as one and subtracting two from 1 okay so if you subtracted

2 from 1 what you get here is the auto covariance of the photo current and that turns out to be the auto covariance of this one turn which we can write it has $L_I(t, t+\tau)$ okay which is t_1 and t_2 terms it is $e^2 \eta / h \nu$ expectation of $p(t)\ \delta(\tau) + (e\eta / h\nu)^2 \quad Lp(\tau)$ okay so this of course would be the function which is just a function of the lag τ okay, now here is where we need to go back to what is the power that has been incident.

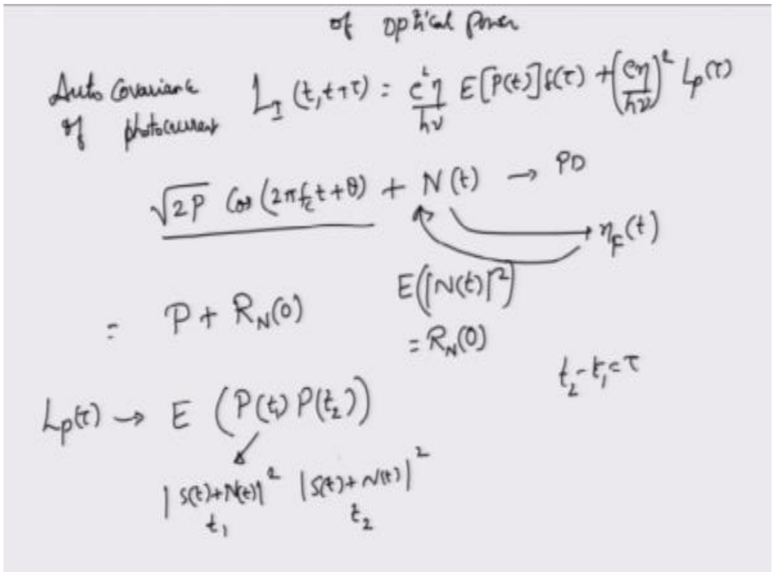
(Refer Slide Time: 08:05)



What power we have incident, remember the input signal power to the amplifier was S(t) which is given by $\sqrt{2P}$ Cos $2\Pi(f_c\, t + \theta)$ okay where θ was a random variable and then the power of course will be proportional to this, however to this output of the amplifier will have an addition to this one it will also have the noise component N(t) so this would actually be the total optical field that is coming out of the amplifier.

So you will have this as the signal field and then you have the optical filed over here, so if you look at what is the power that is present you know like this is the one that is going into the photo detector correct this has been the filter so in the previous module N(t) we had written it has $n_F(t)$ we have simply renamed that one as N(t) because that is that filtered noise is assumed over here, okay.

(Refer Slide Time: 08:57)

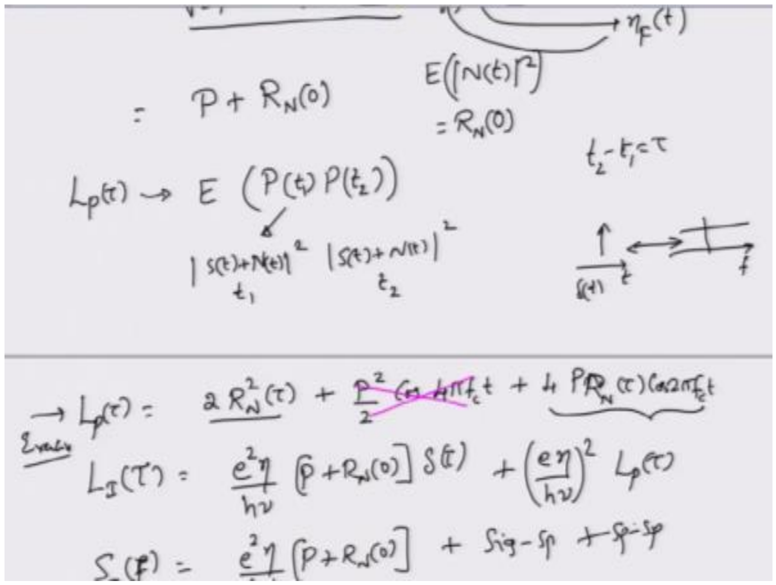


The power that you are going to get here can be evaluated by looking at the magnitude square of this term and when you do that you see that this is a $(\sqrt{2P})^2$ and there is a Cos ^{2} $(2\Pi f_c t + \theta)$ so when you break that on into 1+Cos 2θ you are going to get a ½ that ½ factor will cancel with two and what you get is the power P in this term and here you get interestingly $E(|N(t)|)^2$ where E is the expectation operator.

It turns out that this is nothing but the auto correlation function of the noise process evaluated at a lag of 0 so this is $R_N(0)$ which would be the component or the average power that is coming because of the noise that is filtered noise okay, this is what you are going to get there are of course the other two components that is coming because of the signal noise and the noise-noise beating. So what would those terms be, to evaluate those terms let us look at this $L_p(\tau)$ which comes from taking the expectation of the power optical power that you have so $P(t_1)P(t_2)$ where you remember that $t_2 – t_1 = \tau$ okay, when you write down this equation the total optical power that is coming in from this side will be the optical power, right. So that would be $|S(t) + N(t)|^2$ where evaluated at t_1.

So it would be S(t) + N(t) you also have $|s(t) + N(t)|^2$ evaluated at t_2 so this is the optical power $P(t_1)$ this is the optical power $P(t_2)$.

(Refer Slide Time: 10:38)



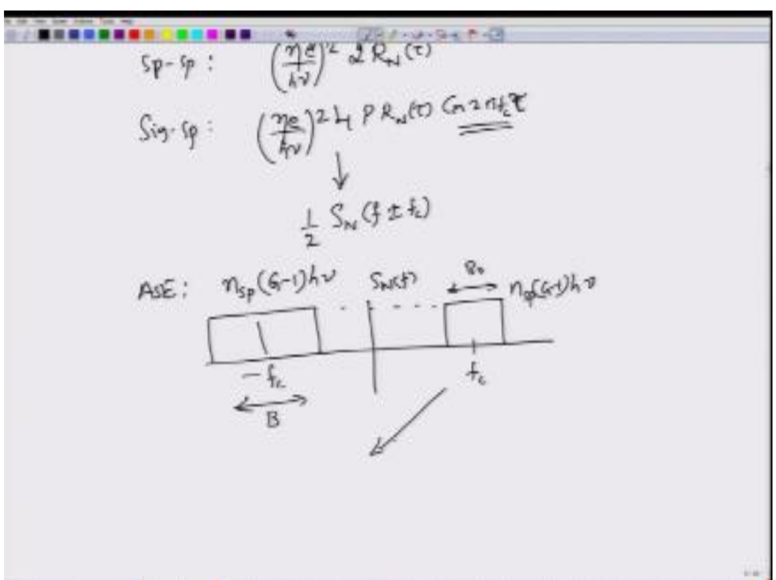
When you do this product and I leave this as an exercise to you, you can it is a little bit of a tedious thing but I will put this as a assignment problem so you can solve for this one and see for yourself that this can be written as $2R_N^2(\tau)$ where $R_N(\tau)$ is the auto correlation of the noise process plus you are going to get a term which is actually fluctuating at twice the frequency F_c okay twice the signal frequency.

Plus you are going to get the beating term $4P\, R_N(\tau)\, Cos\, (2\Pi f_c t)$ which would be the band pass process sitting at F_c this is the one that is centered at F_c, $R_N(\tau)$ is the auto correlation of the noise process okay, substituting this $L_P(\tau)$ into the equation for the auto covariance of the photo current you will see that this can be written as $e^2\, \eta/\, h\, \nu\, [P + R_N(0)]$ remember this was the one that actually came out from the short noise component.

And this is $P + R_N(0)$ where is this shortness component located, this actually has a $\delta(\tau)$ function, right because it was $\delta(t_2 - t_1)$ then the next term that you get is $(e\eta/h\, \nu)^2$ times $Lp(\tau)$ where we have looked at $Lp(\tau)$ which have lifted as an exercise to you to show that here, okay. Because this term is varying a twice the frequency component $2f_c$ so twice the signal frequency component this can be safely eliminated, okay. Now that we have eliminated that you can actually find out what would be the overall like the power spectral densities, so if you take the power spectral density of this process you are going to get $S_I(F)$ where F is the frequency so it would be $e^2\eta/h\nu P+R_N(0)$ this term is independent of frequency.

Remember, the Fourier transform is such a way that if you have $\delta(t)$ as the time domain function, time frequency domain or the Fourier transform is basically a constant, right so for all the frequencies it would be a constant. So $\delta(\tau)$ Fourier transform is 1 so this is the term plus there are two terms one term is $R_N^2(\tau)$ and other term is the signal and noise beating, right the signal is $P\cos 2\pi f_c t$ the noise is $R_N(\tau)$ so this fellow will be the signal noise beating or the signal spontaneous noise and then you have the spontaneous, spontaneous noise term that is beating, okay.

(Refer Slide Time: 13:14)



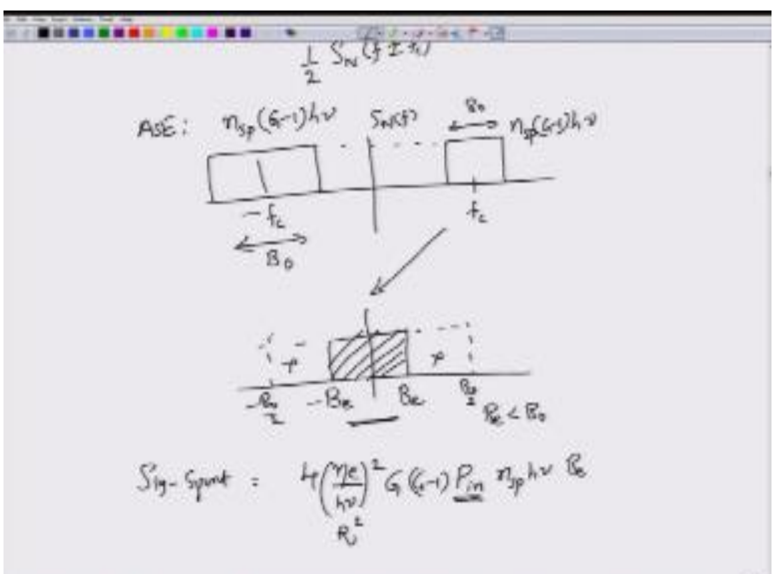
And the spontaneous, spontaneous emission noise process is given by $(\eta e/h\nu)^2$ which is coming from this one times $2R_N^2(\tau)$, okay and the signal and the spontaneous emission term is given by

$(\eta e/h\nu)^2$ and there is this $4PR_N(\tau)\cos 2\pi f_c\tau$ the Fourier transform of this one should be the Fourier transform of the noise $S_N(f)$ but that will be shifted by $\pm f_c$ right, so you are going to get two terms one will be at f_c, one will be at $-f_c$ so if you assume now the ASE noise of the optical amplifier has a power spectral density that is given by $n_{sp}(G-1)h\nu$.

Remember, this is for an amplifier having a gain G and a center frequency f_c which is equal to ν having a spontaneous emission factor n_{sp} this is the power spectral density this power spectral density will be two sided what we mean here is that over the bandwidth B_0 which is centered at f_c so this band width here is B_0 the power spectral density here is given by $n_{sp}(G-1)h\nu$, okay. There will of course be one more term at $-f_c$ so you are going to get over the same band width B this would still be the same value of $n_{sp}(G-1)h\nu$.

But when you shift them this is the power spectral density of the noise, but when you shift them to $+f_c$ and $-f_c$, right.

(Refer Slide Time: 14:47)

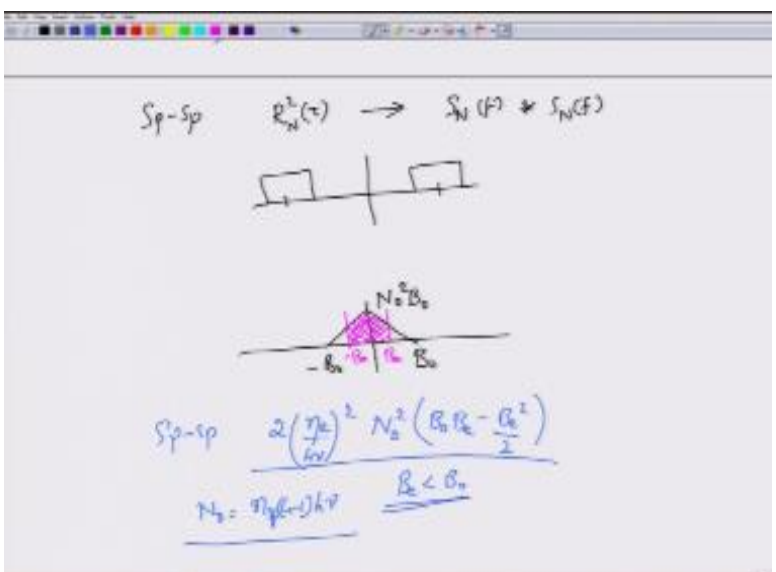


By shift them up by those values the one that is at $-f_c$ will be shifted to the center, so if you now shift this one to $S_N(f)+f_c$ this term will be shifted down to the base band level this term will also be shifted to the base band level and one term will be thrown out into $2f_c$ and $-2f_c$. So effectively, what happens is that this spectral density gets pushed into the lower frequency 0 frequency at which point you are going to put a electrical low pass filter having a band width B_e, B_e is considered to, I mean B_e is taking to less than the optical band width B_0 sorry, this band width is B_0, right.

So when you shifted down and then evaluate the total power by obtaining the area under the curve for this one you are going to obtain the total signal to ASE noise beating term. I am not going to show this one but you can again I will put this as an exercise for you, you can see that once you have transmitted it down then all that is required is to multiply this by a factor of 2 times B_e, okay.

So when you do that the signal spontaneous variance you are going to get will be $4(\eta e/h\nu)^2$ please remember that this is nothing but responsivity R^2 $G(G-1)P_{in}$ where P_{in} is the input power okay n_sp h η times B_E the optical band width will be sitting here right so this would be the optical band width and this area is not accounted for so this is at –B0/2 and b0/2 but is b is lower than b0 and therefore this would be out, so this is the signal spontaneous.

(Refer Slide Time: 16:26)



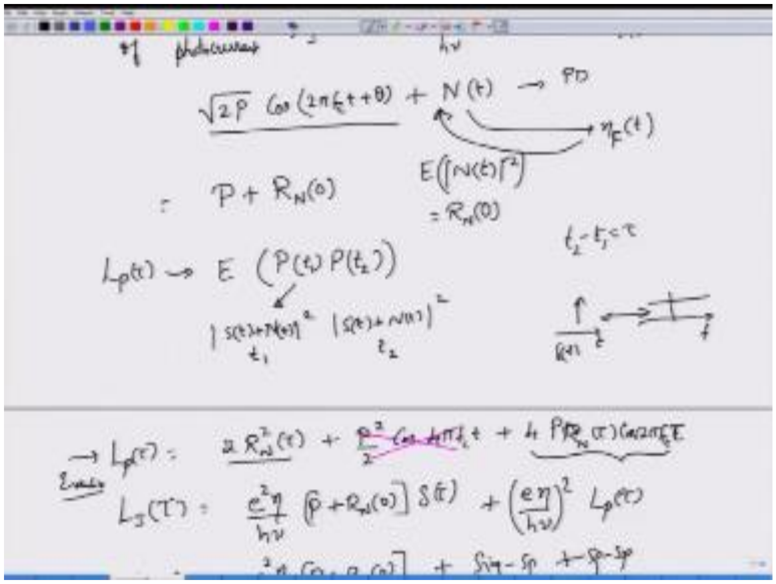
The spontaneous noise process is actually slightly interesting and slightly more tweediest evaluate you see that this will have a R_n^2 (τ), if you look at the power spectral density this turns out to be the convolution of s_n with itself okay and you actually have rectangular shapes for the power spectral density when you convolve this with itself what you are going to get is a triangular shape okay.

So you are going to get triangular shape you can do this for tour self the width gets extended from $B_0/2$ to B_0 to $–B_0$ and the Amplitude here is $N_0^2B_0$ okay and now you are looking again at the power that is contained within this band width within this –BE TO + be to find out this area under the curve you have to do some integration which I will leave as I said as an exercise to give you the spontaneous, spontaneous noise term which is given by $2(\eta_e/h_n)^2$ $n_o^2B_0$ B_E- $B_E^2/2$ okay.

Please remember that we have derive this under the condition that $B_E < B_0$ okay so for this condition you see that this is given by this and n_0^2 is the noise power that is actually sitting inside the optical short noise itself okay, so this is the way in which and n0 is basically given by n_sp g-1 x h_n okay, so this is all that we wanted to talk about for this difference noise terms so it is important to look at what is happen to over the last two modules please remember that we have looked at the auto co relation or the auto co variance of the noise of the photo current.

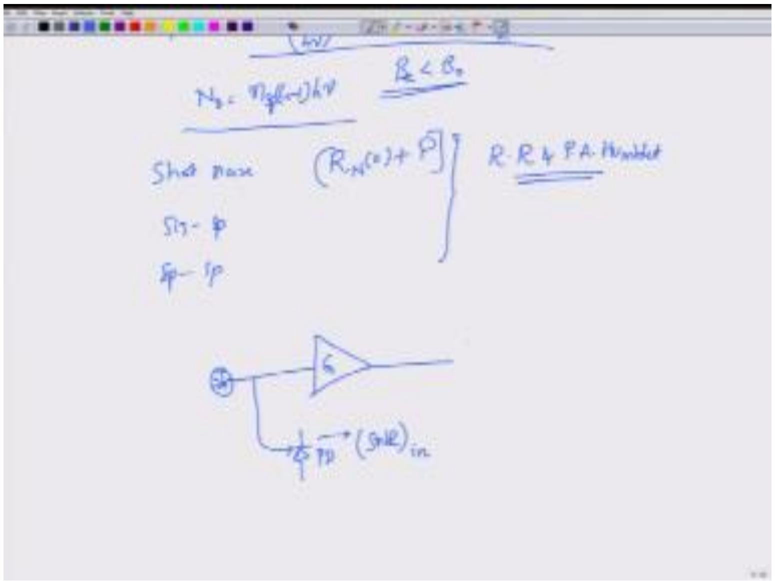
Photo current has a certain amount of noise associated with that and from there we were able to look at two terms one is the signal to spontaneous noise variance or you know that comes from this one and then you had another term that is coming from the spontaneous- spontaneous emission term okay, that is this fellow.

(Refer Slide Time: 18:31)



And finally you had a signal term okay, so signal terms was fairly simple and that came from the short noise component so when you actually look at what is the output of your amplifier noise you will see that you will have signal term you will have signal spontaneous as well as spontaneous noise beating okay.

(Refer Slide Time: 18:49)



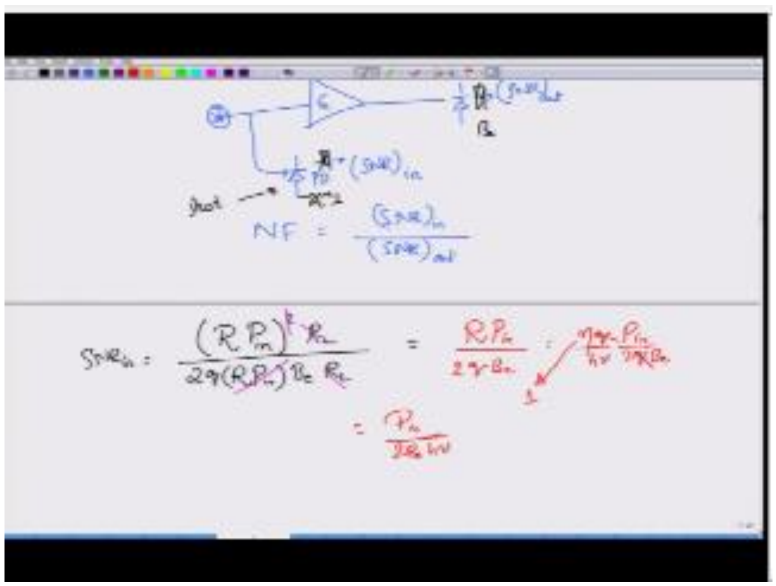
So the total noise variants that you are going to get from this one will depend upon the short noise component okay the short noise component was given by R_n (0) +P okay and then you have the signal spontaneous term and then spontaneous – spontaneous term okay. Anything that I have missed over here I would again refer you back to the paper that we talked about R. Rama Swamy and P.A. Hamlet, so this paper in the appendix actually derives all this power spectral densities.

Once we have this power spectral density we might be interested in finding out what is the effect of putting an optical amplifier on the signal noise ratio okay, to evaluate that one let us look at this situation okay we have some optical input to this okay and then there is an amplifier with the Gangy this optical input might if you give it to a photo detector will produce a certain photo current and therefore there will be certain signal to noise ratio here if you assume an ideal photo detector then the ideal photo detector will not introducing any noise but it will not introduce any this one but it will introduce a little bit of a noise but that noise can be characterized by looking at the input signal to noise ratio.

If you now putting one more photo diode, okay, this photodiode would also produce a signal, okay. The photo current which it is now because of the amplifier could be higher, but there will be a signal to noise ratio at the output.

We define the noise figure, as the ratio signal to noise ratio input to the signal to the noise ratio at the output, let us quickly evaluate these two terms, okay.

(Refer Slide Time: 20:28)

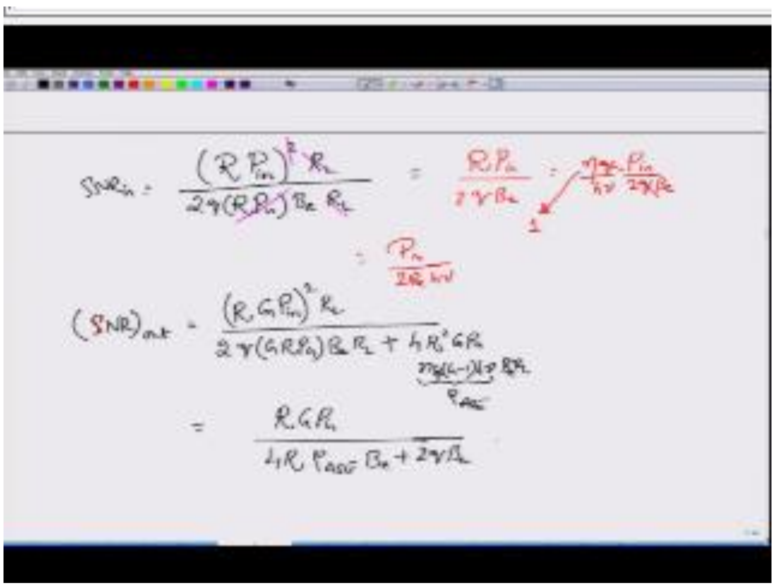


If you just have a photo detector of course followed by some load resistor that you have and assume that the input signal is fairly large enough, so that this is short noise limited, okay. So if it is short noise limited, the signal power is given by R, which is the resposivity, times whatever the power with which you have coming with tight, so the input optical power is let say RP_{in} , this is the photo current, photo current time square R_l give you the signal power, divided by noise spectral density or the noise power is given by 2q RP_{in}, which is the photo current , times B_e is the electrical band pass filter, okay.

We are implicitly assuming that you have an ideal low pass filter, after the photo detector having the bandwidth of B_e times, R_l, because this noise also gets dissipated in the load resistor itself, if you look at this would be the signal to noise ratio at the input which we are quantifying, clearly R_l will cancel with each other, and then one of the RP_N cancel with one of the RP_N, and then you get signal to noise ratio as at the input as RP_{in}, which is the photo current divided by, $2qB_e$, that okay?

R itself is given by ηq/h ν, times $P_n/2qB_e$ $_{you}$ can clearly see that this Q basically cancels with each of that, and then if you assume that this η =1, you get $p_n/2B_E$ h ν, okay, so you can assume that this is given by 2 B_e h ν, okay,

(Refer Slide Time: 22:10)



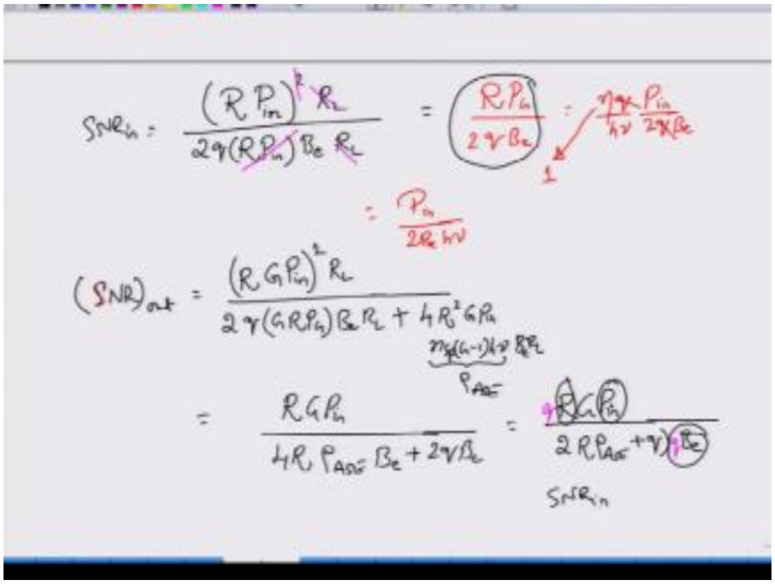
Now how about the signal to noise ratio at the output? Well signal to noise ratio at the output is given by, so at the output is given by the signal power at the output. Now the signal power as increased from P_{in}, the signal power as gone up so the optical signal that you are going to get at the output of the amplifier is G times P_{in}, G is gain of the optical amplifier, times R_l so this entire thing square times R_l , is the signal power at the output of the amplifier, divided by the total noise that is contributed by the, short noise as well as the amplifier noise.

In the amplifier noise you have, signal spontaneous and spontaneous, spontaneous, if you neglect the spontaneous, spontaneous as being too small, then the dominant component is only the signal power, correct, and of course in addition to this there would be the short noise process, with the short noise process, is fairly simple to see, this is given by 2q GR P_{IN}, that is the signal power as increase now, B_eR_L is the short noise component, and signals spontaneous noise component we have seen is given by $4R^2GP_{IN}$, ns P G-1 h ν, this comes from the previous one.

Times B_e, times R_L so this is your signal spontaneous noise, term that has come in, and you can now cancels some of these factors and realize that this can be written as RGP_{IN} R_l cancels any way on both sides, divided by 4R, let us call this nsp G-1 h ν, as the power spectral density of the AC noise process, so this would be 4R ϱ_{ASE} within a bandwidth of B_E+2q B_E, you can further simplify this equation, by looking at what is signal to noise ratio input is given by, $RP_{in}/2qB_e$ okay, so you might actually rewrite this one.

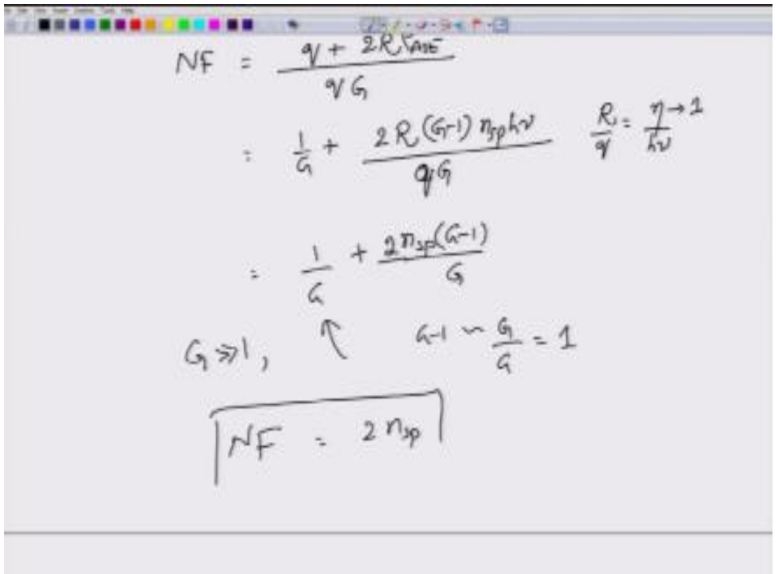
So B_e on common both sides, you can write this a as $RGP_{in}/(2R\rho_{ase}+q)$ B_e can come out and then multiply and divide by q here, so if you multiply and divide by q , you are going to get RP_{in}/q B_e, there is a 2 factor of course, this is 2q and 4R, two factor is there, so these terms are $P_{in}2$ q B_e is nothing but signal to noise ratio at the input.

(Refer Slide Time: 24:52)



So now taking the ratio of signal to noise ratio, the input to the signal noise ratio at the output, the noise factor or the noise figure can be simplified and written as $q+2R\rho_{ase}$, just this term in the denominator divided by q times G, which can again be split into two terms into is 1+G+2R and $R\rho_{ase}$ can be written as $G\text{-}1n_{sp}h\nu/qG$, R can further be re written as R/q is nothing but $\eta/h\nu$ and then you can consider an ideal photo detector with $\eta=1$ and the $h\nu$ here cancel out with $h\nu$ in this numerator and you get even more simplified expression of $1/G+n_{sp}$ cannot be removed (G-1)/G.

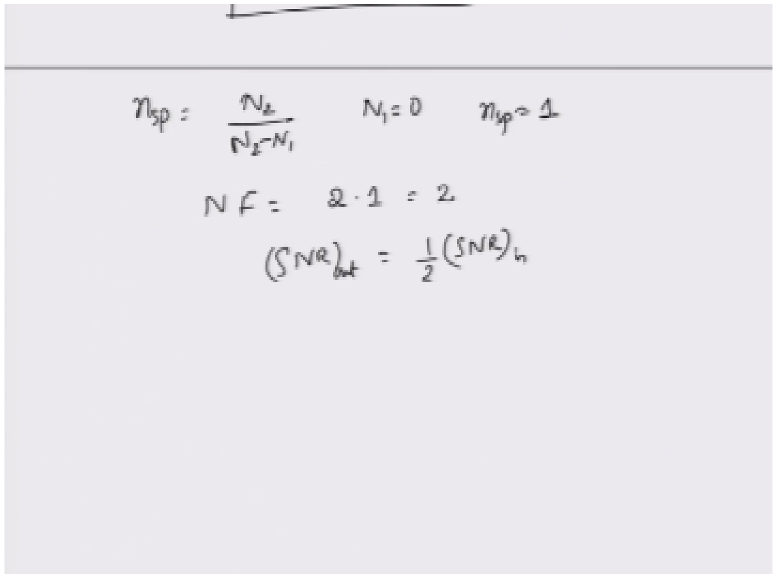
(Refer Slide Time: 26:11)



If you assume G to be much larger than 1, which is what happens in practice, first term can be neglected and in this term you have G-1 which can approximately written as G and then you have a G in the denominator, there is an 2 in the numerator, o this will be again be equal to 1, then what you get is 2 n_{sp}, so this noise factor turns out to be two times n_{sp}, n_{sp} is the filling factor which is defined in the previous factor, given by N2/N2-N1 okay. In an ideal scenario N1=0, so that n_{sp}=1, okay.

And what you see here in the noise figure is that, this would be then equal to 2 times right, n_{sp}=1, so 2*1 =2, so the signal to noise ratio at the output of an optical amplifier followed by a photo detector I actually only half the times of signal to noise ratio at the input. O you can see here that just because, we are putting an amplifier, it does not mean that only signal is getting amplified, it actually has an effect of having both signal as well as noise which worsens the signal to noise ratio.

(Refer Slide Time: 27:00)



And this worsening I by a factor of half in the best possible scenarios and half if you take the Db scale which turns out to be 3d, so even in a ideal kind of situation, your noise figure is still wore by a factor of 3Db. In practice you get a noise figure anywhere from 5 to 6 DB okay, and this complete are discussion of noise figure of the amplifier, In the next module we will putting up all the terms that we have look at, all the components that we have looked at and analyze what happens to the receiver side okay.

We analyze the performance of the systems with or without amplifier, what could be the limitations of putting many, many amplifiers in the link, how should they be placed and then we will look at the statistics of the receiver signal. Thank you very much.

<u>Acknowledgement</u>

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production