Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – X
Module-III
Noise in optical amplifiers

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Hello and welcome to the moke on optical communications. In this module we will discuss some aspects of noise associated with optical amplifiers. We will talk about the signaled and the amplifiers spontaneous emission noise beating and we will also talk about the noise because of the amplifier spontaneous emission beating with itself okay. We will evaluate the noise figure of an optical amplifier and show that for an ideal optical amplifier, the noise figure turns out to be 3 or rather 3dp okay.

So let us look at before looking at the noise in the optical amplifier let us put down the setting that we are interested in.

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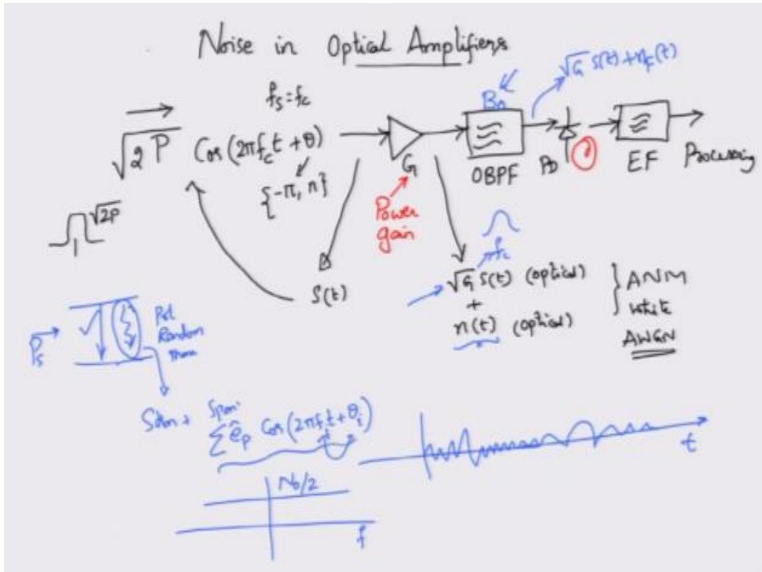

So we know that from the transmitter side you are actually getting an input signal okay. This could be modulated wave form or it could be just a constant. Now we will simply assume that input signal that is coming in has a certain form which is given by some power okay, and this power so we are going to use a slightly different notation that we have used. So here we will assume that the amplitude of the signal is under root of 2xp, this of course if centered at the optical carrier which is F_C so cas2ЛF_Ct the carrier has a certain random phase θ okay.

This expression could very well tell you the pulse that you might have received when you are transmitting using a pulse on-off keying system okay. When you transmit a pulse okay, then the amplitude of the pulse would be square root 2P and then it of course, has a certain center in terms of the frequency it will be centered at the frequency F_C.

In the previous classes we might have use F_S=F_C, so we might have just used a different notation, here we are using this notation of F_C the information is actually residing in the power and not really in the phase of the information, the phase is completely random, it could be anywhere between –Л2+Л okay. So this is my optical signal which is arriving at the receiver, what we do here is that we take this one and then send it through a amplifier okay.

The amplifier is assumed to have a gain G and there are certain noise processes that are associated with the amplifier which we are anyway going to talk about it slightly later. After you have amplified it, you then filter it out okay, so this filter is actually implemented as a band pass filter in the optical domain. So please note that the signal that you have received is optical after amplification the signal still remains in the optical domain, that is because you are actually using a optical amplifier.

Optical amplifiers, amplifier signals in the optical domain and once you have amplified the signal in the optical domain then you are filtering again in the optical domain okay. After you have filtered that in the optical domain, you are going to put this one through a photo detector okay. So that you may convert the optical signals into electrical signals, you might also want to put up some electrical filters here in order to limit the out of band noise or the electrical filter could be part of the trans impedance amplifier design that goes with the photo detector itself.

And what you get at the output would obviously be the signal or the photo current that you are interested in okay, the filtered photo current is what you are going to get and whatever the processing that you want to do, you can do the processing onto this photo current okay. So this is our model, now let us first write down what would be the signals that you are going to see at different points on this chain. And then look at the statistics of the noise processes that go along with this okay.

Now to see that one, let us call this signal that we have received which is square root of $2P\cos 2\pi F_C t + \theta$ as the signal and after that it would obviously be amplified by a certain factor please remember that this factor G is actually defined as the power gain which means that amplitudes actually go as $\sqrt{G}$ why do we the amplitudes has to go as $\sqrt{G}$ because remember that amplitudes square is giving you the power not the amplitude itself so if you want the power gain to be G the amplitude better go as $\sqrt{G}$ okay so this signal at this point is just whatever the signal that has come in and this $S(t) = \sqrt{2P} \cos 2\pi f_c t + \theta$.

But what you get here would be an amplified signal okay this would be an amplified signal but in addition to the amplification you are also going to get a certain gain okay so this would be $\sqrt{G}$ s(t) which is the amplitude of s(t) that has been scaled up by a factor $\sqrt{G}$ but in addition to that you are also going to get a noise process we will assume that this noise which of course will have a much larger band width compare to the signal is adding to the optical signal please note that this is also optical noise right noise in the optical domain this is also optical signal that you have except that the optical amplitude as now scaled up by $\sqrt{G}$.

The two optical fields are getting added so this model is sometimes called as additive noise model okay in addition if the noise process happens to have a flat power spectral density then the noise is set to be white in addition to that if this flat or wide power spectral density the noise is also Gaussian distributed then you talk about additive white Gaussian noise and this is the model that we are going to use so our optical amplifiers are described by additive white Gaussian noise process or additive Gaussian noise models.

The noise adds to the optical signal the noise as a very flat power spectral density much larger than ideally of course it will be infinite flat over the infinite frequencies but for our purposes if it is flatter over the entire band pass filter band width then one can consider it to be white noise and in addition to that the noise is Gaussian distributed okay so this is a AWGN model what you have to take away is that after you have amplified the signal you do not just get the signal right it would have been wonderful if we just had this as the output unfortunately you have to also cope with the noise that the amplifier itself adds to it tight.

And we have seen the reasons why the amplifier would add noise the amplifier as spontaneous emission remember the optical amplifiers are more or less like lasers except that the feedback as been eliminated or the feedback as been made very small there are still two process 1 is the stimulated emission and the other one is a spontaneous emission taking place expect that the stimulated emission dominates over here because you are sending in the external input signal however the spontaneous emission still keeps taking on okay.

This spontaneously emitted photons will have a different polarization they will have the radium phase that is associated with them with the result that the total field can be written as the stimulated + the spontaneous one and the spontaneous is actually a summation of certain random optical fields which have random polarization okay and then so you can call the polarization as some Ep so they have a random polarization of course in this case it would be polarization as either x or y polarization but that is self is quite random.

So it does not you do not know beforehand weather the spontaneously emitted photon would be x polarized or would be y polarized it would also have fluctuate in the frequency because it will have although the center frequency would be f_c because the amplifier itself has a white bandwidth it will have its frequency which is completely kind of fluctuating or changing in addition to that there is a different noise as well, so you might imagine that the overall spontaneously emitted field could be written something like this, okay.

Where Fi stands for different frequencies within the bandwidth and θ_i stands for the random phase that goes with that, the net effect of this spontaneous emission is that if you look at the process spontaneous emission process itself it will correspond to in time domain some noisy waveform, okay. So this is in time domain to correspond to noisy waveform and all that is happening is thanks to this F_i and θ_i.

If you look at it frequency domain spectra the noise actually has a flat power spectral density which is defined as $N_0/2$ please recall our discussion on the power spectral density that we talked about in the earlier modules, okay. So this is in the frequency domain and because it is flat we call this as white noise process, okay. So at the output of the amplifier you have the signal which you want.

Plus the noise which you do not normally want okay, of course you do not really want the noise to go to come out of the amplifier but it is unfortunate that you have to cope with the noise, what happens after the band pass filter, remember that the signal is centered at F_c and is usually having a certain finite bandwidth, right. So if you choose your optical band pas filter having a bandwidth of B_o.

Which is just enough to pass the signal, okay. Remember the signal is coming from the transmitter all the way propagated through the fiber, now with the fiber you might actually see that a fiber has both dispersion as well as non linear it is with the result that the spectrum of the signal may not be exactly the same as the spectrum at the transmitter side the spectrum would have slightly broaden.

Ad here filter optical band pass filter should be sufficiently large to accommodate the extra broadening that the spectrum might have experienced, okay. So the bandwidth of the optical band pass filter is B_0 which is just sufficient in order to handle all the information content within that, so that is why we model this as a ideal band pass filter, after filtering the noise would not be just N(t) it would be a different version.

While the signal is more or less unaffected over here because the signal as we have said is within the bandwidth therefore it passes through the band pass filter with very little distortion we will assume no distortion here but the noise gets filtered, let us call that filtered noise as $N_f(t)$ finally at the output of the photo detector, okay so let me write down the output of the photo detector down here.

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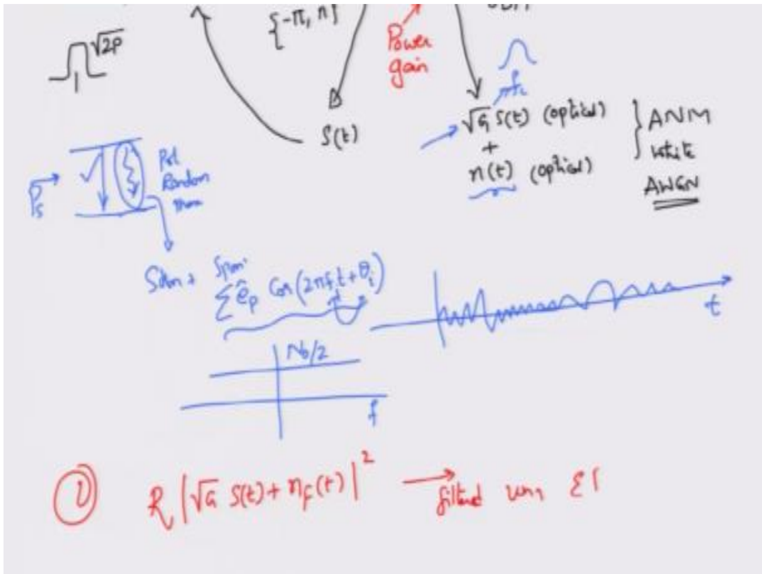

What you would get is that you have √G s(t) which is the input signal or the information baring signal and the filtered noise the photo current that comes out of the photo detector will be proportional to this particular factor, correct? It would be given by responsivity into optical power; optical power in this case comes both in the signal as well as the noise so raising this to the magnitude square will give you the power.

Multiplied by r will give you the photo detector, this can be further filtered okay using the electrical filter E_f.

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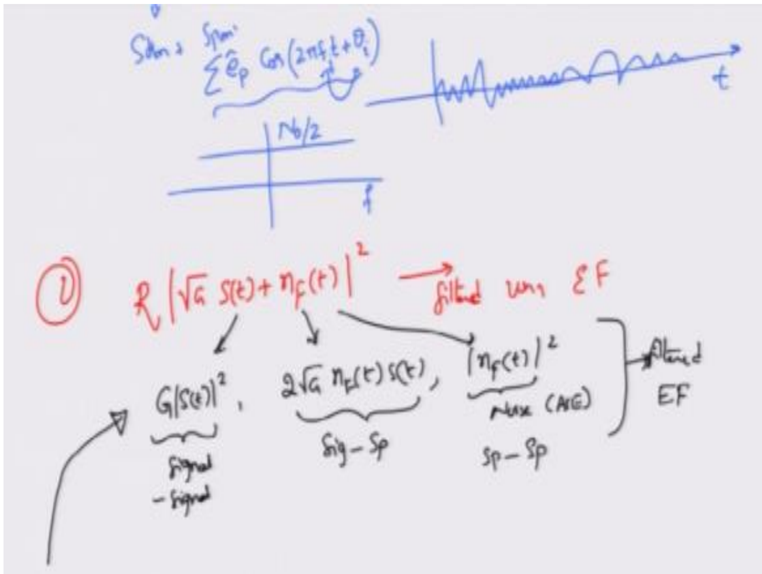

But let us look at what terms are you going to get here, expand this magnitude square and you get one component here which is G $|s(t)|^2$ the other component would be $2\sqrt{G} n_F (t)\, s(t)$ and the third component is basically $|n_F(t)|^2$ clearly this is just the signal component and because this is signal multiplied by a signal or a signal complex conjugate we call this as signal and signal bating.

This is just the noise and the noise source for us is the amplified spontaneous emission source or sometimes denoted by the spontaneous emission noise $S_{p.}$ So this term is actually the Sp that is spontaneous emission noise talking to itself or beating with itself this is called as spontaneous, spontaneous beating term. Finally you have a cross coupling between signal and spontaneous noise which you call it as a signal and spontaneous noise.

Our objective would be to try and look at all these terms individually and then find out the noise power and the signal power so that it allows us to write down what is the signal to noise ratio of this amplifier, okay. So we are going to do that one please remember these are just the terms that are coming out from the photo detector, okay after this they have to be filtered further so we are going to look at what happens when you put them through the electrical filter later on, okay.

So as a start I would taught with slightly different version of the short noise that we have talked about, remember short noise well, we talked about the short noise and we said that.

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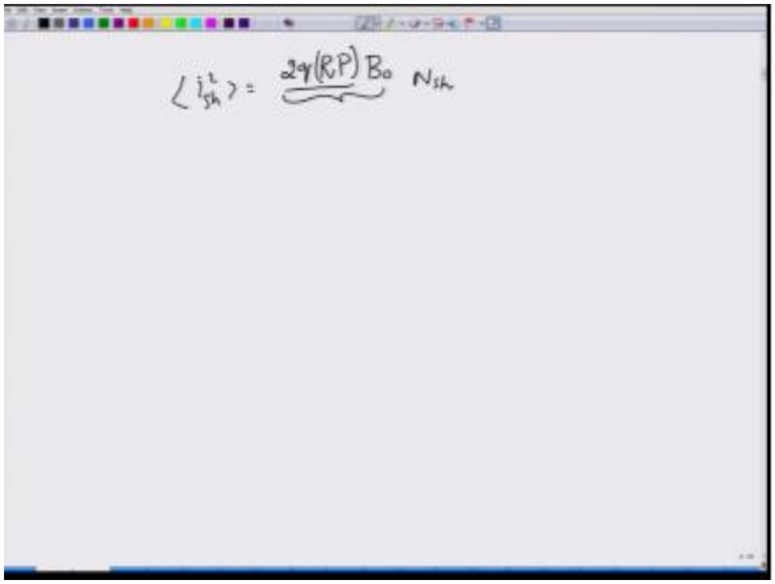

The power spectral density of the short noise I am not sure whether we use the same notation as ρ_{sh} but you know does not really matter, the short noise was given by 2qRP where P stands for the optical power times so this is spectral density, right so it would, it was actually given by 2q(RP), right so this is the power spectral density. When you multiple this one through the band width that you are looking at that would give you the short noise power, okay.

So that is what we had looked at, and the short noise spectral density by itself is given by 2qRP multiply that short noise variance or the short noise power spectral density over the total band width is going to give you the variance of the short noise so I think we had represented this as i_{sh} for the photo current, we had represented this one as the variance of the short noise process and that is the variance of the short noise process, okay. So this is what you are going to get.

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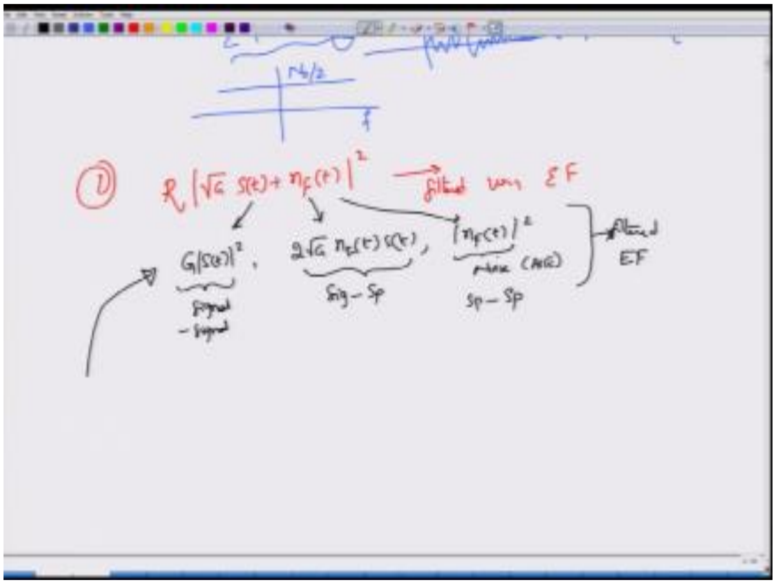

Now our signal or rather our optical field actually has one short noise components sitting right because of the signal itself, right because there is a certain non zero signal power this non zero signal power will in turn introduce a certain short noise component. However, these noise processes do not exactly look like short noise processes because there is a cross multiplication of the filtered noise and the signal optical information varying signal and this one is noise and noise beating. To understand and appreciate what actually goes on with those terms let us completely forget about what we have learned about the short noise, okay.

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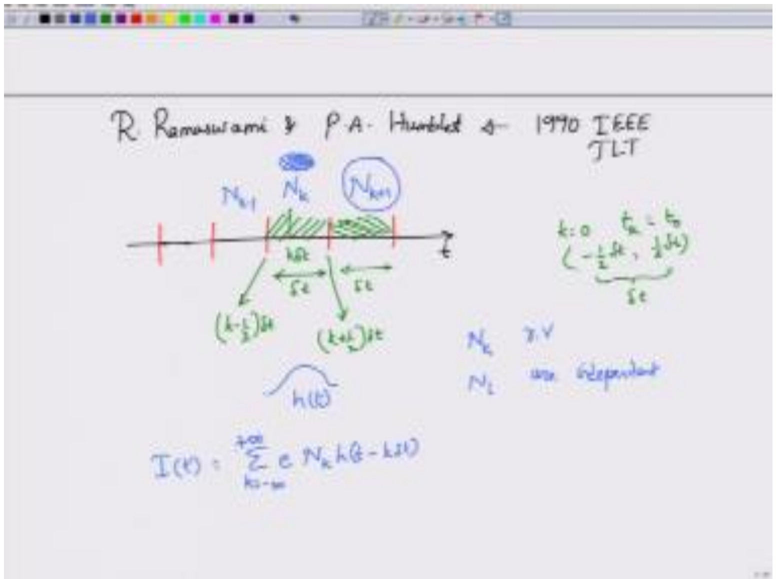

This following presentation is actually taken from a very interesting paper by two people Rajiv Ramaswamy and PA Humbled so I would request in case you have a little bit of a doubt over what we are going to talk about for the next 15 to 20 minutes to have a look at this paper, this was in 1990 I think it was IEEE journal of light wave technology, I think it was cross talk something I do not exactly remember the full detail I will post the links of this paper on the web page, okay.

So whatever we are going to discuss is derived heavily from the paper that was authored by these two, okay. the idea is here very simple you have over time so you have a time access, let us imagine putting up short windows of time, okay so we want windows or intervals of time these time intervals are all located at say this one is located at k times δt by this way you could have easily guess that the width of each interval is δt, so every interval is the same width as δt, okay.

So the for the k^{th} time instant the width will be δt but the n points of this interval will be $(k-1/2)\delta t$. And this fellow is $k+ \frac{1}{2} \delta T$ okay does it make sense well let us look at what happens when k =0 so for k=0 you are looking at a width of $-1/2 \delta T$ or the end point of $- \frac{1}{2} \delta T$ and $+ \frac{1}{2} \delta T$ certainly the width of this one is δT and T0 which is when k=0 which T 0 right happens to be at 0 time axis right. So this is how we are considering this intervals what is important is that one unit of this one does not really overlap with the other one.

So these are called as non overlapping intervals in this non overlapping intervals let us imagine that at each interval okay I am going to get some number of electrons generated. Okay where is this electron generated it is in the photo detector right so let us say the photo detector which of course has to actually look at the number of photons coming in and each photon will generate a carrier based on the quantum efficiency of the photo detector correct so if the quantum efficiency of the photo detector is η then for each photon that comes in η times the photon number would be the number of electrons or number of electron hole pares that are generated.

And these electrons when they are generated they will have to move to the one end of the other end of the photo detector material to constitute the current ands this process can be thought of the actually having a pulse that is emitted okay. So this pulse let us called this as some h(t) okay so this basic pulse that is emitted so at each time you actually have some electrons that are present there will be the number of electrons times this h(t).

So the basic pulse is also emitted along with that at each interval you actually have different number of photons okay so at the k-1 interval you might have had NK-1 we will also assume that the number of electrons that are received or generated here not received generated her in the k^{th} interval is completely independent that the number of electrons that are generated in any other interval in other words nk if you consider this as a random variable then nk and nl where k and l are two different non overlapping time intervals they are consider to be independent okay.

What would be the total current remember this is just the number of photons that are emitted so the current itself is given by it is the number of electrons times E where E is the charge on each electron so this times E will be the number of electrons or the number of charge carrier that are generated and these charge carriers are going to be travelling through the photo detector to constitute the current pulse.

The pulse here itself would be shifted to the time k δ T okay because this is we are considering k^{th} interval and you simply sum this entire thing from – infinity to +infinity.

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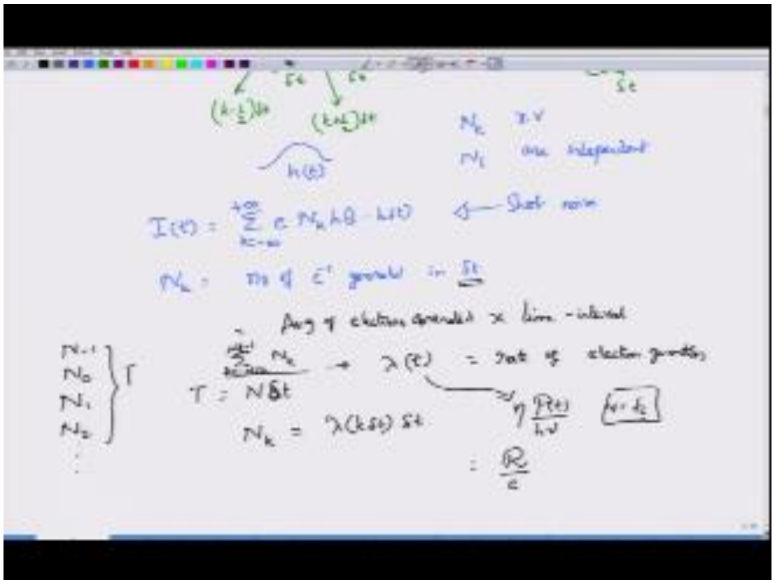

So this is how you can view the current short noise current that is being generated from the photo detector okay but what can we talk about nk, nk is a number of electrons generated in time width of δ T but this can be obtained as average number of electrons okay that are generated multiplied by time interval right how is this coming about well you imagine that I actually take I know you have made measurements so in the first in the 0^{th} interval you got n_0 in the -1 interval I got N-1 N1 N2 and so on okay.

Now if I take the time width, that goes on all of them, then let say i do take a length of N and then N times Δ t, would be the total time width, find out what is the number of electrons that I have been generated, so here k = -N/2 N/2-1 let us say, average or find out the total number of electrons that are generated, divide that one by the total time unit.

What you get will be the average number of electrons that are generated per unit time, or this is called as rate of electron generation, okay this is rate of electron generation and this rate times Δ t, is going to tee me the number of electrons on the average that generated in that particular time interval.

So Nk can be written as the rate which is λ of course this is a non uniform thing that is in some this λ is actually a function of time, it's not a constant generation of the electrons, so this has to be evaluated at the K^{th} interval, what is the value of the rate at the K^{th} interval, multiply this one by Δ t, which is the unit width of the interval of the K^{th} interval, right.

So this would be giving you the, this will give you the number of electron that are generated in the K^{th} time, but what exactly is the rate of electron generation? Well we said that electrons are going to be generated depending on how many number of photons are converted into charge carriers, and that depends on the quantum efficiency η, so η times the number of photons, which would be given by h ν, please note that $\nu = f_c$ in our case, however I am going to use ν because that is kind of giving me the H ν gives the energies so that's something I have used to, okay.

So this would be the rate of electrons that are generated, you might have of course guest that this kind have been easily return as R/e, simply because R is the responsively given as η e/h ν, correct? So η/ h ν , is given by R/e, therefore you are comfortable in the fact that the rate at which electrons are generated, actually depends on the photons that are being emitted, photons are being received by the photo detector, okay. So with this let us go back and substitute for λ in here, and rewrite the current equation.

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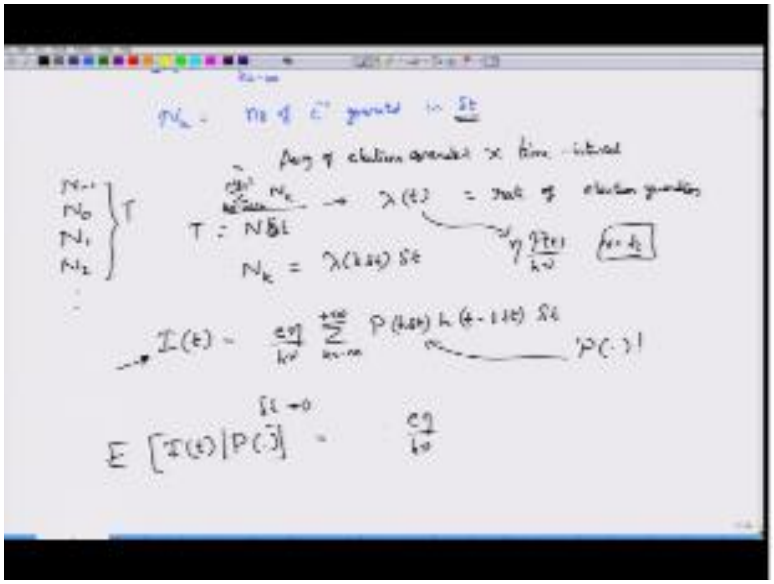

So I have the short noise current which is the photocurrent that is generated, given by e η/h ν,Σ k= $-\infty$ to $+\infty$ p(k Δ t), and then you have h(t)- k Δ t$\times\Delta$ t because this is rate and rate has to be multiplied byΔ t, of course this kind of is a Σ , over an infinite numbers , but you can get to contentious distribution by taking Δ t tending to 0, when you do that, what you get is? The current I(t) p(.) given that certain optical power has been transmitted, okay.

The average value of that is given by this limit of Δ t=0 because please note that this equation would be true only when you have received a certain power, if you have not received any power then you have not generated any short noise, therefore the noise that you have generated when you go to the contentious domain is actually conditioned upon the fact that you have received an optical power, okay.

We will initially look at the current conditioned on the fact that you have received an optical power, or conditioned on the fact that optical power is non 0, and you have received it then we remove this conditioning factors, so we remove this conditioning later but with the conditioning and going from discrete to the contentious domain when you apply domain Δ t tending to 0 limit, this can be rewritten this Σ can be replaced by the integral and the equation simply becomes e η/ h ν ,we are assuming that these are all constants

Integral of P (τ)h(t- τ) d τ where τ goes from - ∞ to + ∞. Of course there has to be an expectation operator, this is not an exponent this is an expectation operator. You can write down this current here in this particular way, o this of course would equal to taking the expectation of operator inside and recognizing that the only random stuff that is coming here is P(τ), E P(τ) would be the average power that you have received.

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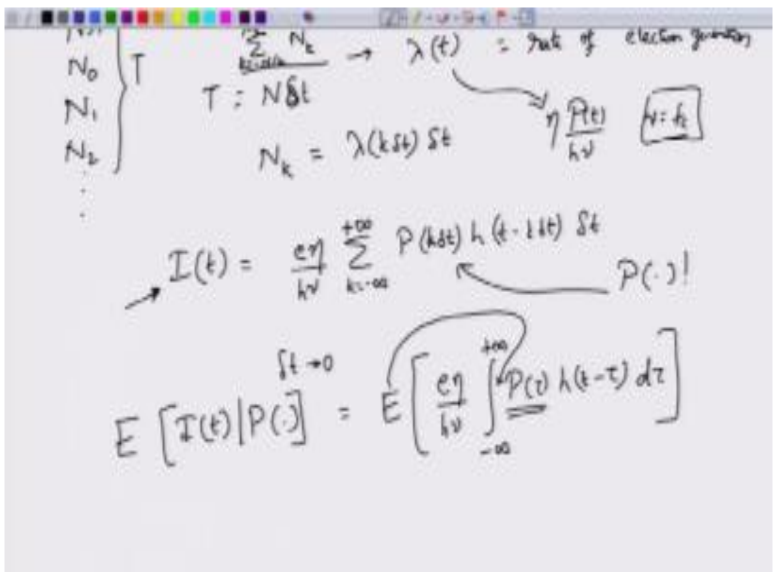

So this would be the average power that you have received, we will do this calculations later, because more important that the mean I'm interested in the auto correlation. The auto correlation is defined as I (t1) I (t2) for time intervals t1 and t2 again condition on the fact that we have received a certain optical power. If we have not received an optical power, we would of course not be able to evaluate this expression. You can substitute, you know from the previous this one, so let us just look at what is I (t1) given t2.

We can put in the expectation operation anytime we want, so you go back to I(t) here and this is I(t), here you substitute I(t1), I(t2) and then multiply and that is going to be 1 Δt over here and this $\eta/h\nu$ $P(\Delta t)$ is nothing but λ and you can write down the equations in this following way, so you have e before you go to the case of λ, let us go to the first notation, so I have to look at this short noise expression, evaluate at time t1 and t2.

Which are considered to be two different times, there is no relationship between t1 and t2. Later we will see that there is a relationship that we want to impose, so here you get e(k) N_k h(t1-k Δt) and you have one more summation coming from, so let us also not put the limit Δt now itself, so will put the limit later. So you have e l and N_L, because when you multiply to the units write two things, you have to also take care of the cross products that can come.

Therefore you have to use the another dummy variable l, so you have N_L h(t2-l Δt), this double summation, this is actually a product, so this double summation can be written in two terms, one term when k=l and the other one when k is not equal to l, when k=l you have e^2, summation is still there but k and remember k =l and you get N_k^2 because k=l here, h(t1- k Δt) h(t2- k Δt)) this is first term plus you have one more term e^2 because you have that double summation continues, k into l, you have N_k N_l h(t1- k Δt) h(t2- k Δt)).

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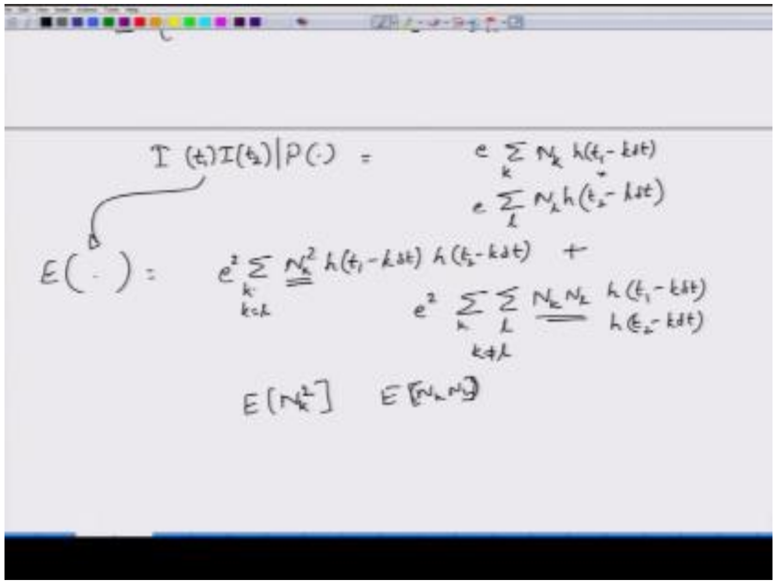

Clearly k must not be equal to l in the second expression, now we are ready to put in the expectation operator, so if you put in the expectation operator onto this one, you see that the random variable are coming from N_k, so if you put two variable $E(N_k^2)$ and $N_k(N_l)$ and we are going to discuss this one, so the discussion continues in the next module, don't go anywhere, we are going to look at the variance of each of them and then talk about what happens to the auto correlation of the current. Thank you very much.

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