

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

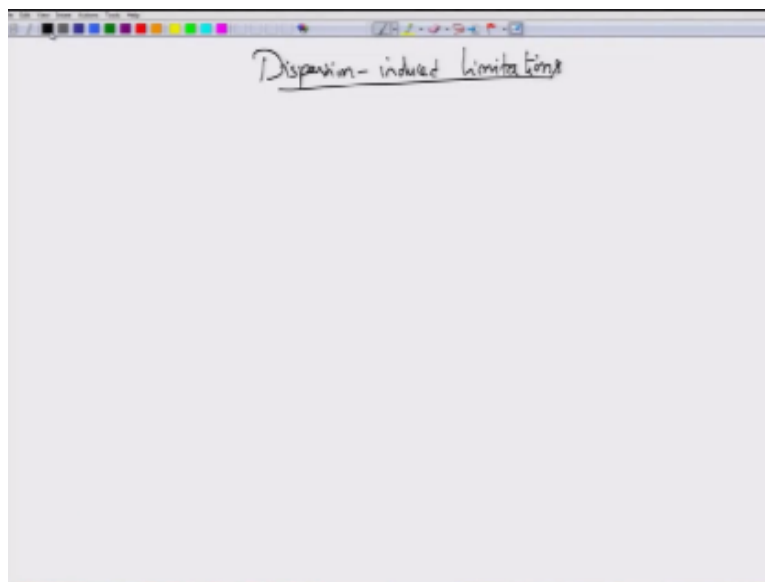
**Course Title  
Optical Communications**

**Week – IX  
Module-V  
Dispersion induced limitations**

**by  
Prof. Pradeep Kumar K  
Dept. of Electrical Engineering  
IIT Kanpur**

Hello and welcome to the module on optical communications. In this short module we are going to discuss dispersion induced limitations.

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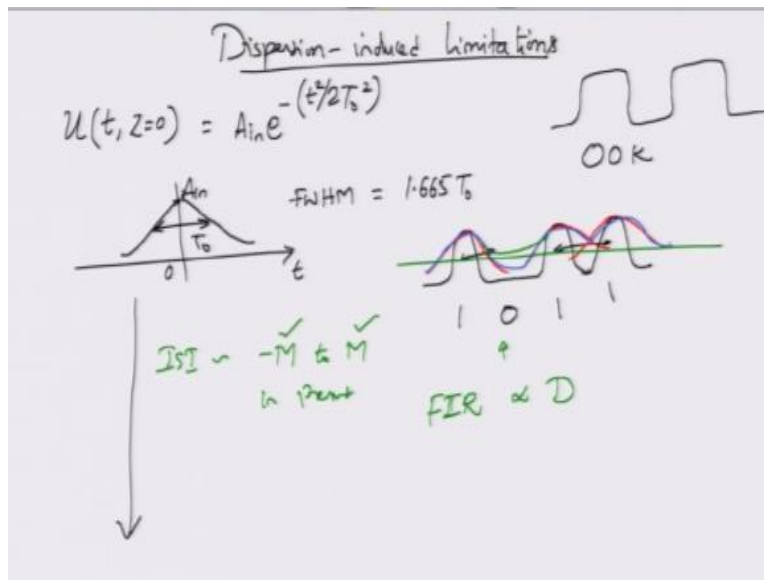
We have already looked at dispersion induced limitation in the optical communications systems earlier, because when you have a fiber which has a certain amount of dispersion its immediate effect is to cause pulse spreading. So you might transmit a pulse in a certain time slot, after that

pulse you want to transmit another pulse carrying the information. However, when you transmit these pulses through the optical fiber because of dispersion the pulses will spread and go beyond their initial time slots and start to talk to each other okay.

So there is this cross talk which we denote by this phenomenon called inter symbol interference. One symbol tries to interfere the other symbol, because the pulse actually spreads beyond its initial time slot okay. So this dispersion in a standard single mode fiber occurs because of the fiber dispersion which we have already seen to be coming from material dispersion as well as the wave grade dispersion, both these dispersions are functions of wavelength.

And because when you modulate a certain carrier and transmit it through the fiber the modulated data occupies a certain amount of spectrum. And this spectrum basically influences the overall dispersion, because that spectrum will not be nonzero if you want to transmit any modulation.

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In time domain we can start to think of the transmitted electric field okay, whose amplitude if we denote it by U and I would say at  $Z=0$ , can be described as having a Gaussian amplitude okay.

Although in practice the pulses are not really Gaussian, but more or less this type of a pulse shaping that we normally use in the on-off keying systems it is still okay to consider Gaussian because they are quite easy to work with to get some analytical ideas or analytical formulas to help us understand how dispersion affects the system so assuming that this is a Gaussian pulse shape we can write this as exponential minus  $t^2 / 2 T_0^2$  we will now assume that there is no loss in the optical fiber okay.

So earlier we looked at the case where loss was the major factor now we are going to assume that there are no loss in the fiber and whatever that we obtain limitation is simply because of the chromatic dispersion okay of course we are considering single mode fibers therefore that is the only dispersion we are interested in okay we are not interested in the multimode dispersion induced in the multimode fibers there is some amplitude to this electric field which you can write this A in this is inconsequential because we are assuming no loss of amplitude in this system but more importantly and this one that is important you actually have the initial pulse width given by  $T_0$

So if you sketch this you know as a function of time you will see that at a time  $T=0$  you have an amplitude of A in and then you basically have this sort of distribution right the width of this in terms of the FWHM that is the first width at half maximum when the amplitude as actually decays to half is given by around roughly 1.665 times  $T_0$  so this is the optical pulse that is actually launched of course you actually what you launch is a sequence of optical pulses here the bit is 1 bit is 0 but is 1 sometime right after the first bit you are going to get a 1 here indicating that.

So indicating that these two time slots which are very close and characterized by having the same bit or likely to suffer from inter symbol interference then these two bits because this can be allowed to expand a little bit what dispersion does is, to expand the pulses around here but when you expand these pulses you will see that the two start merge together.

And the overall pulse shape that you expect out of this bit sequence looks something like this, okay. So clearly these two are much more strongly influenced this is also getting influenced because if you for example have a threshold somewhere here then you can see that because of

the pulse spreading from the previous slot and the pulse spreading from the next slot and probably the other slot is now adding up to increase the overall amplitude when the bit is 0.

So sometimes if this pulse envelope reaches above the threshold then even this bit will be erro9r, in fact you normally define the inter symbol interference or model the inter symbol interference by a certain memory length indicating that  $-m$  of the pervious slope that is previous  $m$  slots to the next  $m$  slots times slots are the once which are interfering in the present slot, okay. You can think of this entire thing as a finite impulse response filter.

With appropriate set of co-efficient in order to model this and this FIR filet model of a dispersion depends on what is a dispersion co-efficient the larger the value of  $D$  the largest will be the value of the time slots which are actually talking to each other, okay. Anyway so we have launched this optical power or this optical pulses into the fiber and what happens as they propagate through the fiber.

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Handwritten notes on a slide:

- ISI  $\sim -M$  to  $M$  in present
- FIR  $\propto \frac{D}{\beta_2}$
- $z=L, T_L^2 = \frac{T_0^4 + \beta_2^2 L^2}{T_0^2}$
- FWHM  $1.665 T_L$
- $\frac{z=L}{z=0} \sim T_L/T_0 \quad T_L > T_0$
- $T_0$  small  $T_L^2 \propto \frac{1}{T_0^2} \uparrow$
- $T_0$  large  $T_0^4 \Rightarrow \beta_2^2 L^2 \sim T_0^2$  limit data rate

Is something that we have already looked at we see that the amplitude decreases okay because of the dispersion but more importantly your pulse time or the pulse duration actually spreads

because after propagating a distance  $z=L$  the pulse width will now be dependent on the  $\beta$  of the, or the dispersion coefficient of the fiber  $D$  is related to  $\beta_2$  as we have discussed in the previous module, so you have  $\beta_2^2 L^2$ , okay.

So in some sense  $\beta_2 L$  is the effective or is the additional spreading that you are going to get, I mean divide this one by  $T_0^2$  would be the actual  $T_L^2$ , the FWHM here after  $Z=L$  is of course given by  $1.665 T_L$  so if you look at the FWHM at  $Z=L$  over FWHM at  $Z=0$  this would be equal to  $T_L/T_0$ , right.

And because  $T_L$  is greater than  $T_0$  the FWHM at  $Z=L$  will be greater than FWHM at  $Z=0$ , okay. Now you cannot of course start to choose  $T_0$  to be very small, because if you choose  $T_0$  to be small then  $T_L^2$  being inversely proportional to  $T_0^2$  will increase leading to a problem, you cannot choose  $T_L$  too large as well, because you might simply get a situation from the numerator the  $T_0^4$  is larger than  $\beta_2^2$  into  $L^2$  okay, and if you put in this conditions you will see that  $T_0$  too large will lead to  $T_L^2$  which is approximately  $T_0^2$  which any where will limit the data rate, okay so you cannot choose  $T_0$  too small or  $T_0$  too large.

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Handwritten notes on a whiteboard:

$$\frac{T_L}{T_0} = \frac{T_L}{T_0} \quad T_L > T_0$$

at  $z=0$

$T_0$  small  $T_L^2 \propto \frac{1}{T_0^2} \uparrow$

$T_0$  large  $T_0^4 \gg \beta_2^2 L^2 \sim T_0^2 \rightarrow$  data rate

$$\frac{dT_L}{dT_0} = 0 \Rightarrow T_{0,opt} = \sqrt{\frac{\beta_2 L}{\alpha}}$$

RTS with a Gaussian pulse =  $\frac{T_0}{\sqrt{2}}$

( $z=L$ ) =  $\frac{T_0}{\sqrt{2}}$

What you do in practice or you know at least in this problem is to find out what is the optimum length at for the given value, okay so you can do this optimum length by differentiating  $T_L$  with respect to  $T_0$  and sitting the result to 0, if you do this you find that the optimum value for the initial choice of the pulse is given by the fiber and the length of propagation now this is very crucial you actually have a length of propagation which is determining the optimum pulse width, okay.

So this optimum pulse width if you want to choose then the root means square width of the Gaussian pulse which is the way in which we normally talk about the width of the optical pulses is actually given by  $T_0/\sqrt{2}$ , okay. So this can be related to the width of the Gaussian pulse at  $Z=0$ , okay and the same RMS width at  $Z=L$  will be given by  $T_L/\sqrt{2}$ , okay.

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Handwritten notes on a whiteboard:

$$z=L, \quad T_L^2 = \frac{T_0^4 + B^2 L^2}{T_0^2}$$

FWM 1.665  $T_L$

$$\frac{z=L}{z=0} = \frac{T_L}{T_0} \quad T_L > T_0$$

$T_0$  small  $T_L \propto \frac{1}{T_0} \uparrow$

$T_0$  large  $T_0^4 \Rightarrow B^2 L^2 \sim T_0^2$  limit down rate

$$\frac{dT_L}{dT_0} = 0 \Rightarrow T_{0,opt} = \sqrt{\frac{BL}{c}}$$

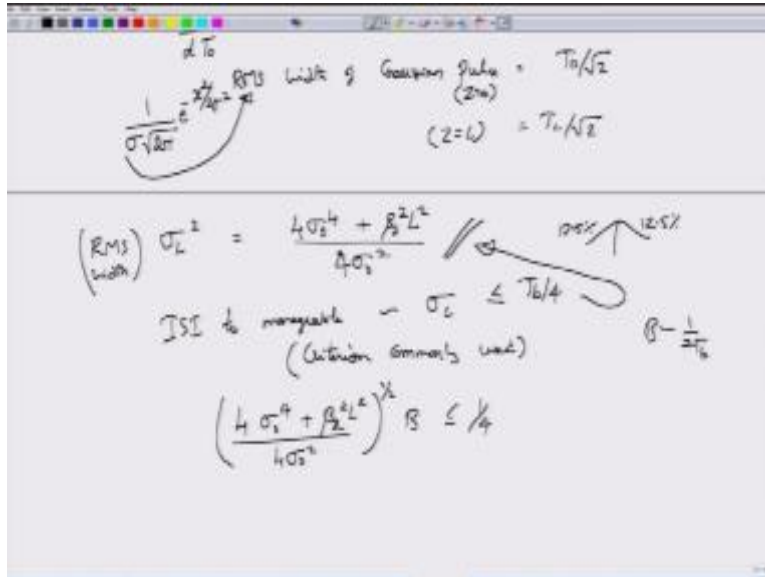
$\frac{1}{\sigma^2} = \frac{4}{T_0^2}$  RMS width of Gaussian pulse (2 $\sigma$ )

$(z=0) = T_0/\sqrt{2}$

It is a slightly different way of calculating this one is rather easier because as general Gaussian random variable is given by  $1/\sigma\sqrt{2\pi} e^{-x^2/2\sigma^2}$  and the  $\sigma$  is essentially the root means square width that we are looking at, and therefore this is kind of easier to deal with in that particular case, okay. So what you get is the width the RMS width this is not the short noise variance that we were looking for, this is the RMS width after propagating a distance  $z=l$  can be written as  $4 \sigma^4$

$+ \beta^2 L^2 / 4\sigma^2$  what we have done is to simply replace  $t_1$  and  $t_0$  in this expression by the corresponding variables for  $\sigma L$ .

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For ISI to be manageable okay of course nothing is grate that ISI should actually be equal 0 bit in case you cannot hope to get a 0 to manage the ISI the necessary requirement that is commonly used is that the pulse spreading okay.

After propagating a certain distance  $L$  has to be less than the bit period by 4 that is you are allowed about a 12.5 % spreading on to the right hand side and 12.5 % spreading on to the left hand side that is you pulse can spread 12.5% to the next lot and 12.5% to the previous lot so this sort of a 25% spreading is what we are allowed to have this is you know a criteria that is commonly used this is the criteria that is commonly used in practice okay.

So you have a 25% pulse spreading allowed and if you substitute that condition on to this one and rearrange the equation what you get and remember also that  $t_b$  is inversely proportional to  $b$  as  $1/2t_b$  therefore  $t_b$  is given by  $1/2b$  okay so because of this relationship you can then take a look at what will happen to  $\sigma L$  and  $\sigma$  you know in terms of  $\sigma L$  and  $b$  so you get  $4\sigma^4 + \beta^2 L^2 / 4$

$\sigma^2$  which as we know is  $\sigma L^2$  which we have just written in on top. So this to the power  $\frac{1}{2}$  because you are looking at  $\sigma \times b$  so this one should be times by should be less than or equal to  $\frac{1}{4}$  okay.

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Handwritten notes on a whiteboard:

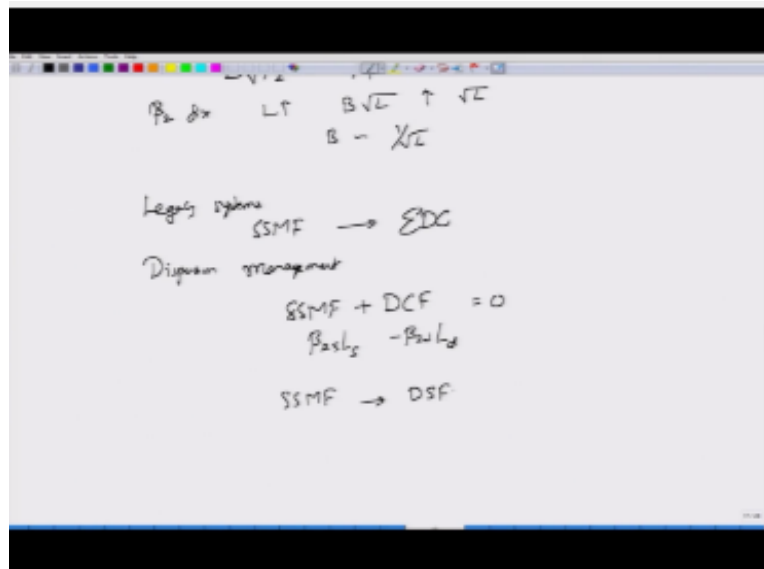
- ISI is negligible -  $\sigma L \leq T_b/4$  (Criterion commonly used).  $\beta \sim \frac{1}{2T_b}$
- $\left( \frac{4\sigma^4 + \beta^2 L^4}{4\sigma^2} \right)^{1/2} \beta \leq 1/4$
- $\sigma_{opt} = \sqrt{\beta L^2/2}$   $\left\{ \begin{array}{l} T_{0,opt} = \sqrt{\beta L} \\ \sigma = T_b/4 \end{array} \right.$
- $B\sqrt{\beta L} \leq 1/4$
- $\beta_2 \propto L^{-1}$   $B \propto \sqrt{L}$   $\uparrow \sqrt{L}$
- $B \propto \sqrt{L}$

So if you assume that this would be the condition and substitute for the optimum value of  $\sigma$  which will be given by  $\sqrt{\beta L^2/2}$  remember the optimum  $t_0$  value was given by  $\sqrt{\beta L}$  and  $\sigma$  is related to  $t_0/\sqrt{2}$  so with that you can see that the optimum  $\sigma$  the initial RMS pulse width must be of this nature and when this is the case you can simplify the relationship slightly here and write this as  $\beta \times \beta L$  under  $\sqrt{\quad}$  should be  $\leq$  or equal to  $1/4$  okay if you fix  $\beta$  right then what will happen if you fix  $\beta$  then the bit as the transmission distance  $L$  increases the product  $b \times \sqrt{L}$  will increase only as  $\sqrt{L}$  okay.

So that transmission distance or if you work to now fix  $b$  then the transmission actually decreases by  $1/\sqrt{L}$ . So as the length increases linearly the transmission perform, so the transmission maximum transmission distance only increase or decreases by a factor of  $1/\sqrt{L}$ , what can we do in order to combat this ISI practice there are lot of techniques are used,



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For most legacy systems okay where in you don't have any dispersion compensation then so the transmitter and receiver are just link by a single mode standard single mode fiber, you simply apply electronic dispersion compensation, at the output end that is the receiver and in order to compensate dispersion.

Many dispersions systems are actually managed by dispersion managed systems, so in order to do dispersion, management you have two options okay, what you do is you take the single mode fiber and at after once pan you know after where you have to reshape the signals you put a dispersion compensating fiber, the dispersion compensating fiber will gave a  $-\beta_2$  which will be in magnitude different from, the  $+\beta_2$  to that you have for the standard single mode fiber.

But if you choose the length of the DCF appropriately and the length of the standard single mode fiber appropriately then you can completely cancel dispersion at the end of the span, okay, the other way would be to actually let single mode fiber be completely replaced by a dispersion shifted fiber, okay.

Where in and then operate everything in at the 0 dispersion wavelength, okay so if you operate transmission system at 0 dispersion wavelength then the impact of dispersion can be minimized, okay these are some simple ways in which have talked about the actual application will involve, properly detailed way in which the dispersion changes over the fiber that is installed.

And the nature of the DCF that you have and how do you actually you know? Connect them together okay. So that is something that slightly beyond the scope of the course here so we will stop at this point and note what we have done, we have looked at in the earlier module what happens when the losses the major factor that limits your performance, and then you come to the dispersion case which again is related to the length of the fiber.

Although the effect of fiber dispersion will scale as  $1/\sqrt{l}$  the effect of length, will show up in terms of degraded BER performance of the systems thank you very much.

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