Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – IX
Module-IV
Advantages of coherent receiver

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Hello and welcome to the moke on optical communications. In this module we will look at the improvement that a coherent receiver provides over the simple direct detection receivers in the on-off keying systems that we considered. If you remember in the on-off keying systems that we considered in the last module, there you saw that the receiver just consisted of a photo diode followed by an amplifier in the RF case in, RF amplifier if you want or it would just be given to the decision circuit right.

And there we found that if you do not have any other impairments except for the noise of the receivers which can be short noised when the bit one is transmitted as well as thermal noise. However, only thermal noise when bit 0 is transmitted, in addition to that noise the only limitation that the fiber of versus in terms of the attenuation. We found that as the length increases the transmission distance between the transmitter and the receiver increases then the performance, the as measured by the bit error rate degrades for these systems.

Let us see if coherent receiver is able to bring out any kind of an improvement over the direct detection receiver.

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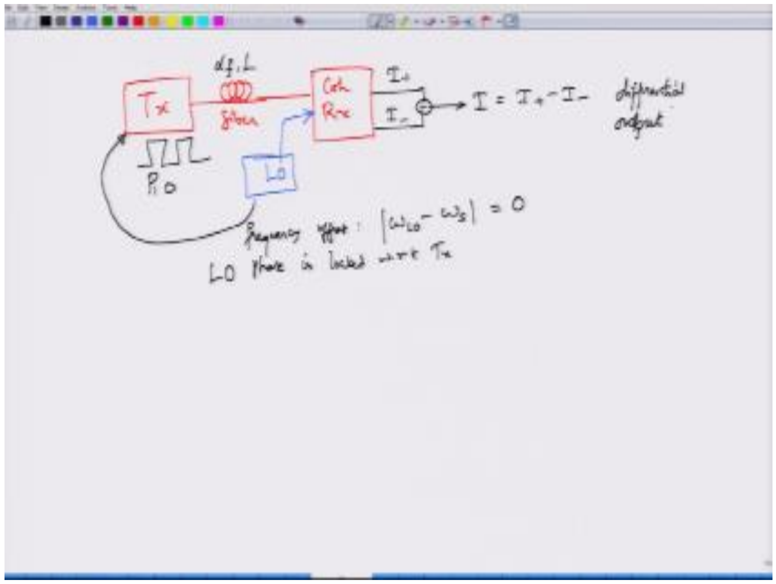

In order to do that let us look at this picture which shows you the block diagram sense of what is happening with the coherent receiver. So you start with a transmitter okay, it produces a certain power, so the transmitter is producing an optical power of P1 and 0 when you transmit a bit 0 P1 when you transmit a bit 1, this passes through the fiber, the fiber is still characterized by only the fiber loss coefficient which is αf and the distance between the transmitter and receiver which is given by the length l.

At the receiver what we are assuming at this point is that there is a local oscillator which is a very strong optical signal coming from a local oscillator whose frequency is exactly matched to that of the transmitter. Moreover, both not just the frequency is matched, but also the phase is locked to that of the transmitter. So the frequency offset which one might define as the difference in the frequencies between the local oscillator and the signal frequency of the transmitter laser is assumed to be 0.

So we assume a frequency offset of 0 and we assume that phase is locked with respect to transmitter. So which phase are we talking about, we are talking about the local oscillator phase which is locked with respect to the transmitter phase. What do we mean by locked phase, what we mean here is that, if this is how the phase of the transmitter is varying okay, then apart from that fixed offset the phase of the local oscillator also tracks the phase variation of the transmitter.

Of course, it will never be sinusoidal like this, but this was just used to illustrate the concept that local oscillator phase is locked with respect to the transmitter phase okay. Under these ideal conditions and the only limitations that you are going to get is the loss in the fiber as well as the noises that will be appearing in the receiver circuit. This part which you see I+ and I- correspond to the output of the photodiodes and two photodiodes are used in the balanced coherent receiver as we described in one of the earlier modules okay.

The coherent receiver takes in as two inputs, one is the local oscillator and the transmitted signal which is now at the receiver, and produces two photo currents I+ and I-. The actual or the final photo current that you are going to get will be the difference between the two. Because you are taking the difference between the two, any DC value over which the signal is riding from the I+ and I- will be cancelled out.

So this one you can think of as differential output, so this differential output will only give you the signal that is encoded in the, or the signal that is riding the AC part of the signal, the DC offset values in I+ and I- which are very nearly equal to each other will be cancelled out. Of course, this, if this was the case then our life would have been very simple, unfortunately with this I which is the photo current that you are looking at, you also get noise okay.

So both I+ and I- because they are the outputs of a photodiode will be subjected to short noise and the overall current I will be subjected to short noise plus thermal noise okay. What we will do is that we will examine the output of the coherent receiver and then see whether some interesting cases can be, you know realized which will allow us to make some concrete predictions about how the coherent receiver would perform.

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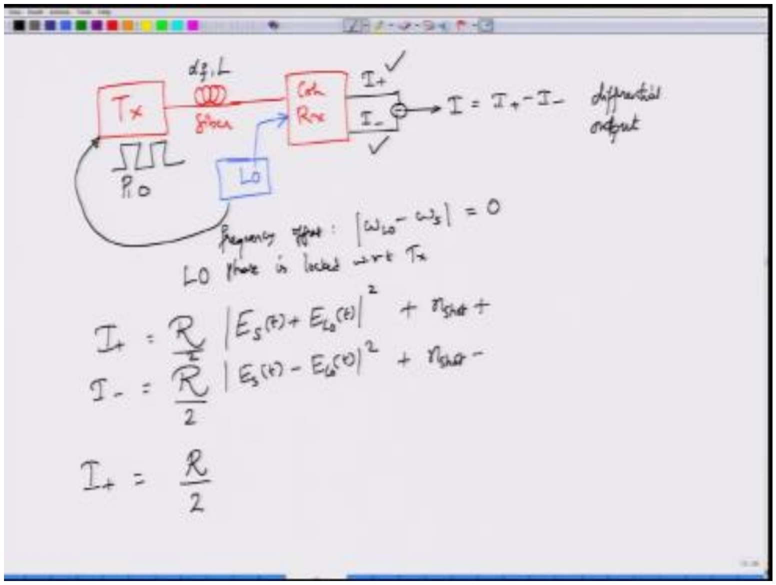

So in order to do that first we need to recall the expressions for I + and I – and let us do that by remembering that the I+ is the output of the photo detector therefore there is a responsively factor R multiplied by whatever the power that is input on that and power is input on that if you remember the coherent receiver that we talked about you have two electric fields which are talking to each other you have the signal electric field E_s (t) and the local oscillator signal which is you know E_{LO} (t).

And the magnitudes square of the sum of these two will be the optical power so optical power multiplied by R will give you the current I+ if you want to find out the current I- thus current will be equal to R and expect for a + sign the rest of the terms are equal right in fact E_s= t / E_{LO} (t) and E_s (t) –E_{LO}(t) are coming from the output of the splitter and each of the splitter outputs are in turn feed into the photo detectors because the output power is coming through the splitter there is a factor by2 here for each of them.

So you have I+ and I- given by R/2 this factor of course as I said this just the optical signal part okay but there is also short noise coming because of the two photo diodes right so there is a short noise + and a short noise – these short noise are coming from the photo detectors that are connected to the balanced photo detector remember the coherent receiver actually has two photo diodes and each of these photo diodes will in turn look at the mean optical power and they will generate a certain short noise okay and that short noise is what is that is you are getting from both I+ as well as I- to proceed further let us just evaluate this the magnitudes square terms so consider just I+ for now I will leave the corresponding expression for I- as an exercise to you so I+ is given by R/2.

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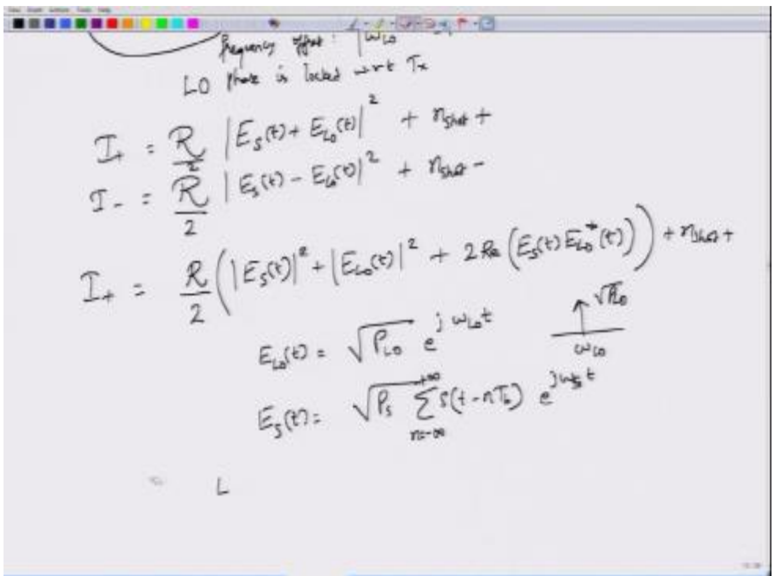

Open up this one you are going to get $E_s(t)$ magnitudes square + $E_{LO}(t)$ magnitude square these are the two powers that the average powers that you are going to see with the signal and the local oscillator + there is a two times real part of $E_s(t)$ E_{LO} complex conjugate of t okay and therefore of course the short noise term that we are going to any have what is this $E_s(t)$ E_{LO} conjugate of t we have E_{LO} (t) which is the local oscillator output having a power of $\sqrt{}$ PL okay.

And then you have $e^{j\omega_{LO} t}$ we are going to assume that this local oscillator is you know having a constant phase and remember that constant phase is the difference between the transmitter and the receiver phase which we are going to assume to constant okay so as for us the receiver is concerned local oscillator is simply a carrier signal okay or sorry a local oscillator signal which is a sinusoidal carrier which as a frequency of ω_{LO} and a amplitude of square root E_{LO} similarly you have Es(t) given as $\sqrt{P_s}$ which is the amplitude that you are going to get and there is also s(t) – nTb summed over N from - ∞ to +∞ clearly this s(t) is the pulse shape that we have used and we are essentially transmitting one pulse after the other okay all these pulse are centered or modulated at $e^{j\omega_s t}$ ω_s is the transmit laser frequency.

So this is $E_s(t)$ and if you ,look at the case when you transmit bit 1 then you know over 1 time period let us actually look at this over one time period for which we can remove all this summation and everything okay so this in the particular time period that you are interested in this simply becomes s(t) and s(t) is the optical pulse okay this optical pulse will be none zero when bit is 12 okay and it would be equal to 0 when bit is 0.

So essentially you get a sequence of optical pulses this would be normalized to 1 we have moralized this one to one without saying it so but this is how the optical pulse shape is going to look like when you are bit 1 want when you transmit bit 0 there won’t be any pulse so that the total optical power during the interval when bit is 0 is actually equal to 0.

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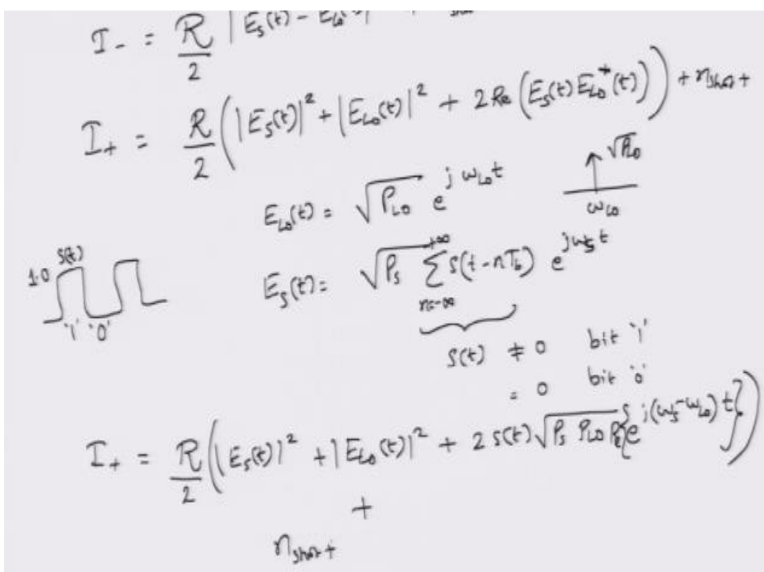

So going back to what is happening to I+ you can write I + as R/2 $|E_s(t)|^2 + |E_{LO}(t)|^2 + 2s(t) \sqrt{P_s P_{lo}}$ I am assuming that these sample woods are being to be constants therefore the signal power is Ps and the local oscillator power is P_{LO} so that essentially enters into the equation, okay. Then there is of course the case of $e^{j\,\omega st}$ right so if you look at Es and E_{LO} * you have $e^{j\,\omega lo(t)}$ which is getting conjugated so you get a minus sign here you have $e^{j\,\omega st}$ that would be there as it is.

So you can combine these two to write this as $e^{j\,\omega s-\,\omega Lo(t)}$ of course you are taking the real part of it so let us basically take the real part of it, okay. This entire thing plus whatever the short noise that you are going to get this would be I^+, okay.

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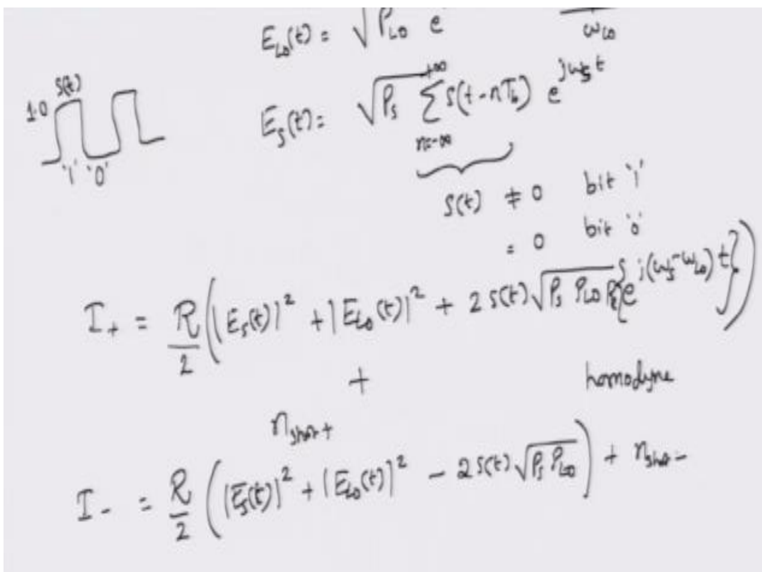

You can write down I^- in a very similar fashion so you have R/t you have $|E_s(t)|^2$ plus the local oscillator signal in place of a plus you get a minus sign here you have s(t) $\sqrt{P_s P_{lo}}$ and here recognizing that $\omega s = \omega Lo$ remember we have assume that frequency is locked to the transmitter frequency and the frequency of the local oscillator is exactly equal to the transmitter frequency.

So now frequency oscillate this goes to 0 such a scheme is in fact called as homodyne, okay. It is a lightly older terminology homodynes means that the transmitter frequency is exactly equal to the local oscillator frequency if it was not equal then such a scheme would have been called as a heterodyne frequency scheme or heterodyne deduction scheme, so you have $-2s(t)\sqrt{P_s P_{lo}}$ coming for I^- then the exponential is equal to 0.

So the exponential of J0 is 1 and real part of one is just one so I am just going to close the bracket over here, of course I cannot forget the short noise that exists and we can write this as short noise minus.

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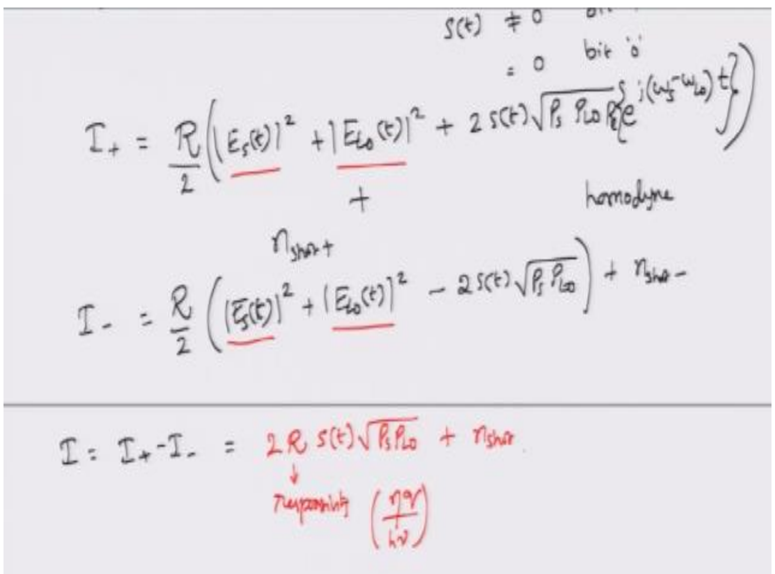

The total photo current that you are looking at will be the difference between I^+ and I^- and you can easily see that when you take a difference like this these terms are going to cancel with each other this $E_{LO}(T)^2$ is going to cancel with this $E_{lo}(t^2)$ so what you get in turn is 2-(-2) becomes a +2 therefore this comes 4 times s(t) there is an amplitude factor out there, there is an R/2 here therefore R/2 x 2 that 2 and 2 will cancel and eventually what you get is 2 times R please remember this R is responsivity, okay.

This is responsivity which is related to quantum efficiency in this manner ηq/hν as we discussed in the photo receiver circuits, so you have 2Rs(t) I do not really have to say s(t) as summation signal. Because I am considering only one period and in that period bit period s(t) can be either +1 or 0 depending on whether we are transmitting bit 1 or bit 0, okay. Times $\sqrt{} P_s P_{lo}$ and then you have the total noise component which can be written as n-short, n –short will de defined as n-short + (-n-short (-)).

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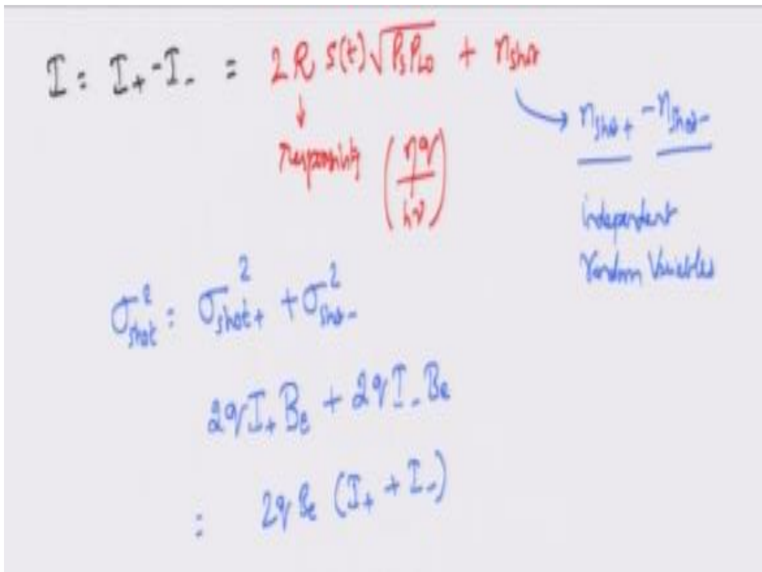

That is to say this fellow is actually n-short + (- n-short)- okay so this is the overall noise variance that you are going to get and this variable the difference between the two noise is, is denoted by n-short, okay. So this is what you have in for the photo diode and what about the short noise variance, the overall short noise variance is going to be $\sigma_{short}^{2} + \sigma_{short}^{-2}$ if you are wondering where this kind of relation is coming from you have to remember that $\sigma_{short} + \sigma_{short}^{-}$ represents the short noises in the photo diodes.

Which are used at the two branches of the coherent receiver these are independent random variables and the way and these variables have both 0 mean and they are distributed according to certain shortness distribution that we have seen but for because they are independent random variables. The total variance of these two will simply add up, okay. What is the variance of short noise plus this is given by 2qI+ which is the photo current.

That you are looking at distributed over the bandwidth of B_e where B_e stands for the electrical filter that you are going to put after the photo diode circuit so you are actually looking at the total noise into that bandwidth of Be, similarly short noise minus will also be equal to 2qI-, I minus being the photo current that you are going to see in the other branch of the coherent receiver.

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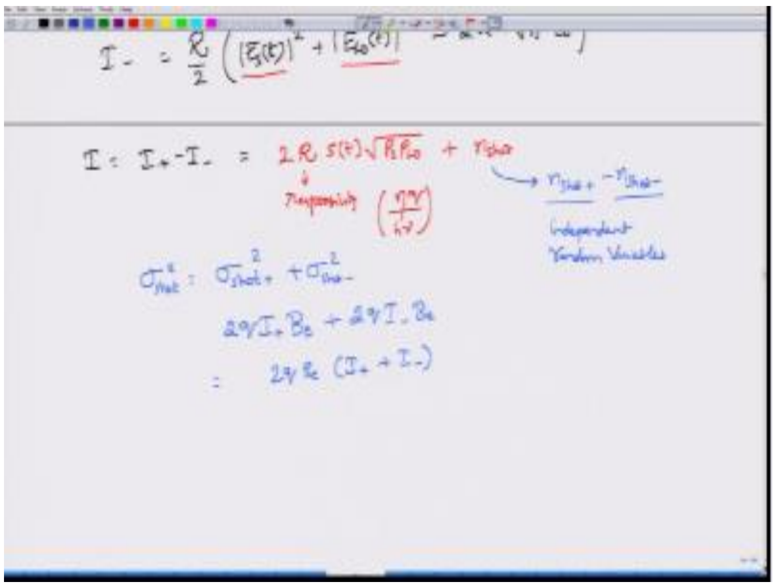

So you can write that one down, you get 2qI-Be you can factorize all this components out and you get I+ + I-. Now look at this, here you have I+ and.

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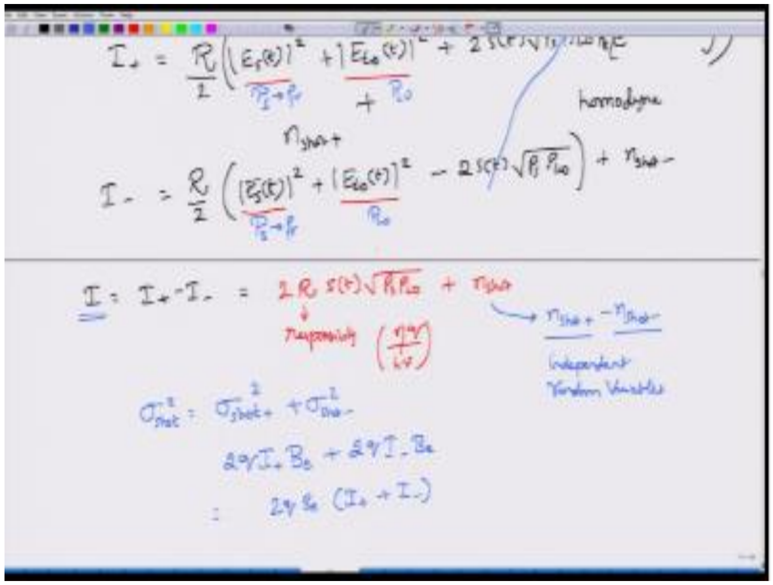

To that you are adding I-, right in order to obtain the overall photo current you have to subtract the 2 but when you get so the noise variances I+ adds on to I-, right so you have an I+ adding on to this when you add them these terms will vanish, right because these terms are and opposite signs so they will vanish, this one is the power that you are seeing in the received signal this is the power in the received signal actually this should be rewritten as P received, okay and we also know that this power that we are going to receive will be non zero only when you have transmitted a bit 1.

Therefore, I can write this as P_1r and P_1r, whereas this is a constant power that you are going to get which is P_{Lo} on both side, so when you now add these two there is R/2 times P_1r+ R/2 times P_1r so this going to be.

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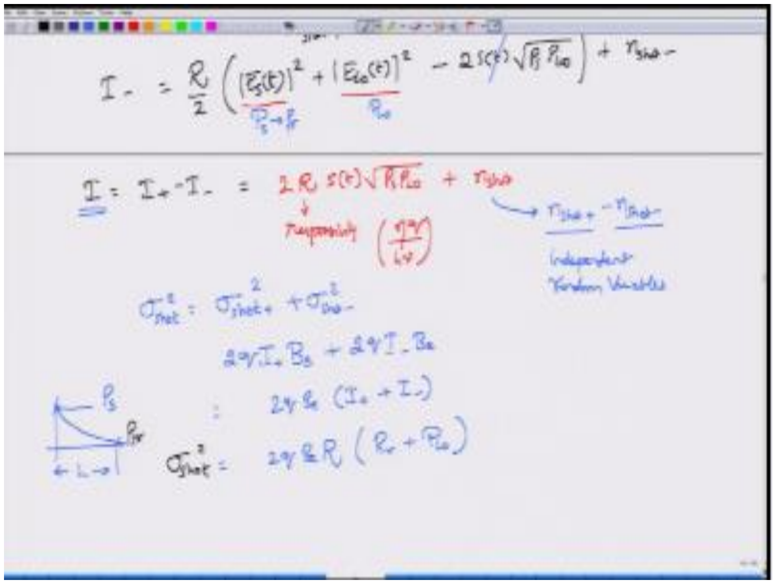

You can write this as 2qBe I+ is R(P_1r+ for the local oscillator that would be RP_{Lo}, why did I write P_1r because remember you might actually start the transmission at Ps but that is not what you receive, what you receive here will be depending on the length of the propagation or the length of the fiber what you receive will be the power P_1r which is going to be, so this power is Ps which is where you start however the power that you receive will be this P_1r which is the actual power that you receive after a propagation length of L. so this is 2qBeR P_1r+P_{Lo} being the total short noise variance and please remember this particular equation, okay.

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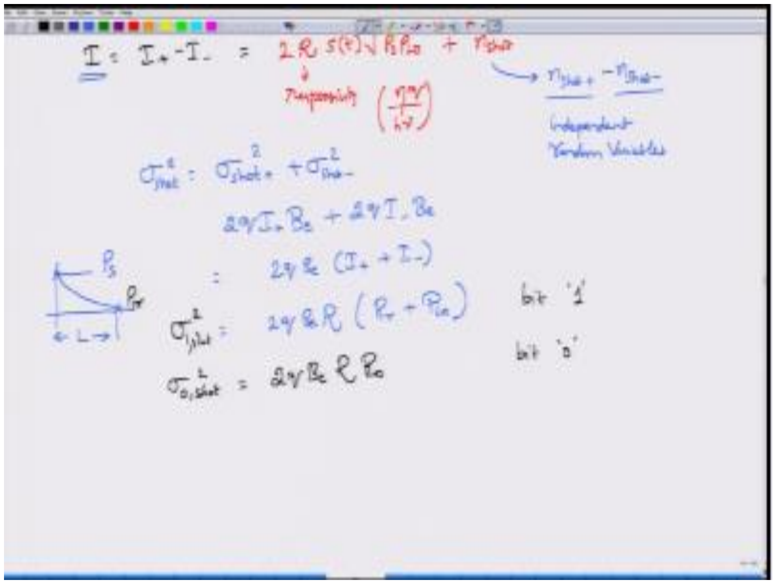

And this equation is also true when you have actually transmitted bit1, what about the case when you have bit 0, so let us also designate this one as short noise corresponding to bit 1, so that we can write this as σ_1^2 okay, and we just have the short noise. The thermal noise of course does not depend on the local oscillator power nor it depends on the received signal therefore, the thermal noise for both bit 1 and bit 0 will be the same, so you can you do not really have to put them down here, okay.

What about the short noise when you have the bit 0, so for bit 0 of course there is no photo current because your received signal P_1r is going to be equal to 0, while the received signal is equal to 0 the short noise unfortunately is still going to be the same except that P_1r will be equal to 0, so the short noise here will be $2qBeRP_{Lo}$, right this is kind of the primary difference between what you are going to get for bit 0 and bit 1. The short noise for bit 1 will include P_1r+P_{Lo} that would be the total mean optical power that you receive when you transmit bit 1.

Whereas when you transmit bit 0 there is no power to receive however the local oscillator which is sitting at the receiver is going to contribute to the total short noise and corresponding to these short noises or in addition to the short noises they are also going to be thermal noises.

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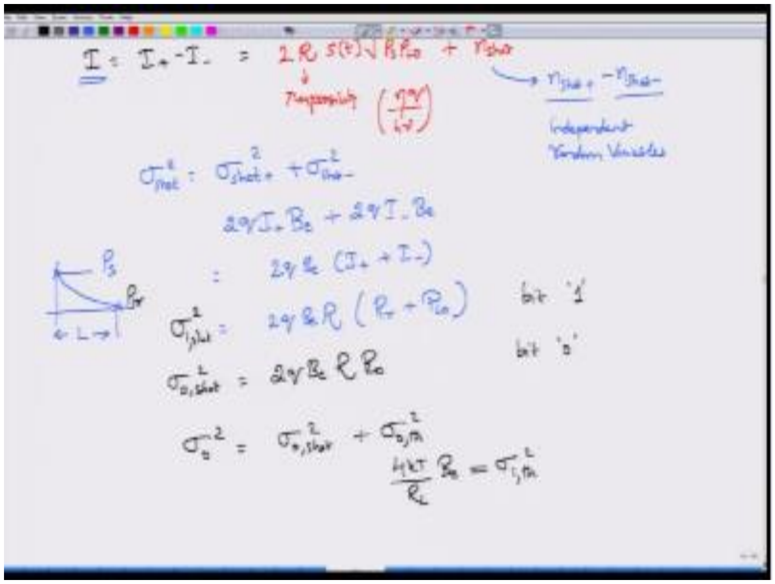

The thermal noises are independent of the optical power, but if you are looking at the total noise σ_0^2 this would be the short noise that we have just written down plus there is a thermal noise, right. So you can write the thermal noise here and then you have a thermal noise spectral density of $4kT/R_L$ multiplied by the total band width that you are looking at which is equal to Be, so this would be the thermal noise which is also the same as the thermal noise for bit 1.

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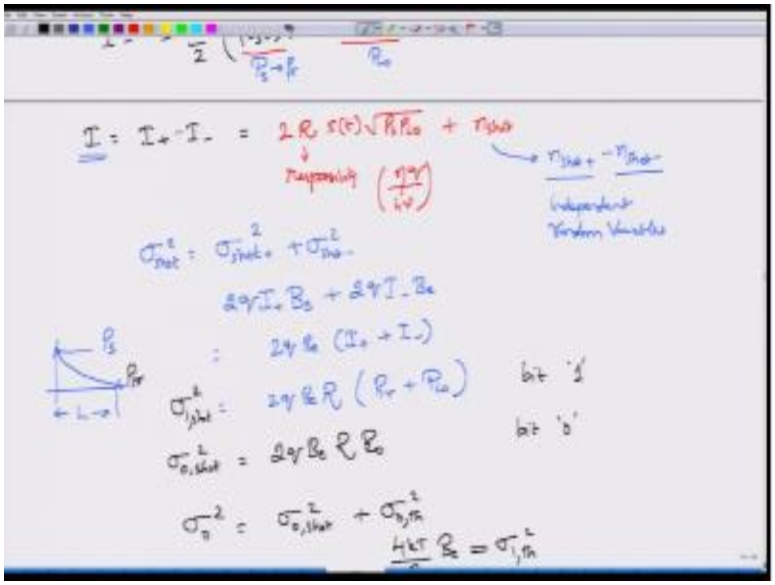

So we have looked at the total short noise and the total noises here, and you can actually this is all that is required for you to come ahead and calculate the bit error rate, you know by calculating the q factor and then applying the relationship between the q factor and bit error rate that is what you can do. However, it would be very, very interesting to see what actually turns out in practice, in practice what we do is we want to operate the signals at what is called as the short noise limit. Why do we want to do that, suppose we operated them not at the short noise limit.

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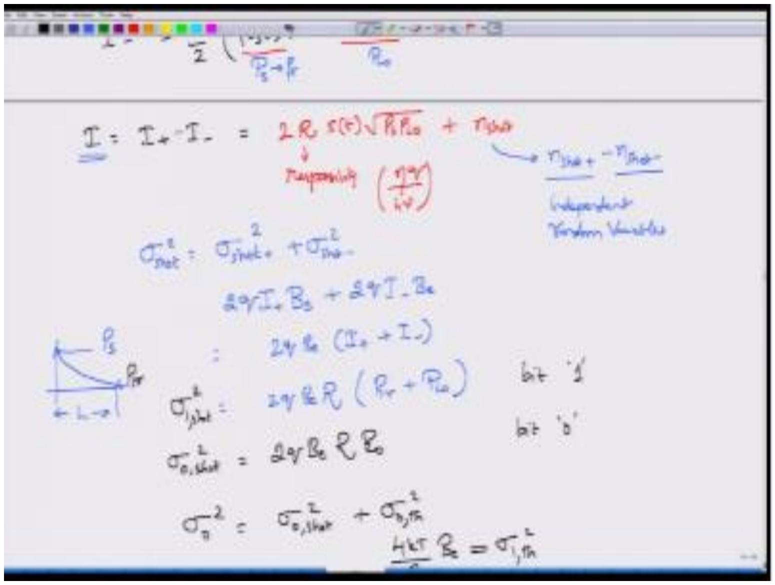

But the thermal noise then bit 0 because there is no optical power input to it in terms of P_1r the variance of that will be slightly lower compared to the variance of bit 1, and this by how much this difference would vary because σ1 short noise square plus thermal noise that is there σ0short noise square plus thermal noise so thermal noise contributions are equal for both okay and you do not really want to operate with the thermal noise limit because that limits the speed of operation so you want to operate in the short noise limit.

But while you are operation in the short noise limit σ1 is slightly larger than σ0 because there is p1 r sitting in σ1 right so you can see that.

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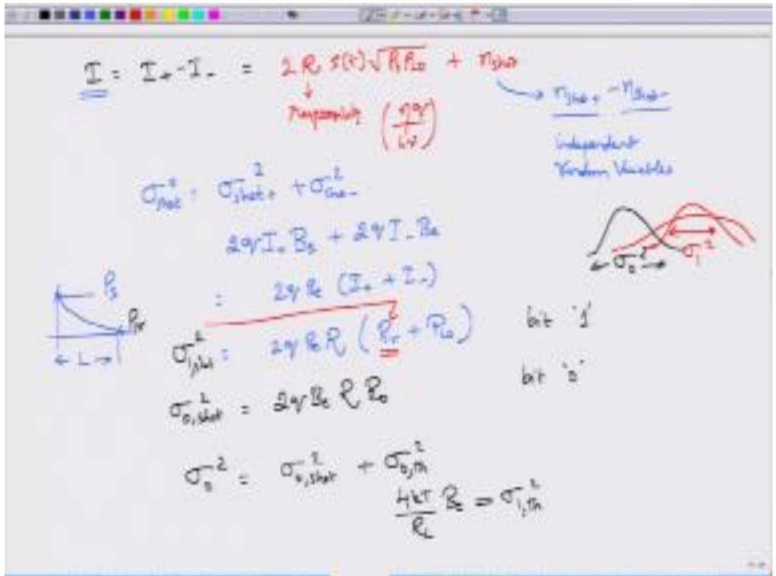

$\sigma1$ short noise square is given by p1r+ plo because of this p1r+ plo this would always be greater than $\sigma0$ short noise square however this of course sets your problem with putting the limits so in one incidents you have a Goshen random variable that is what we have assume this is say $\sigma02$ this is going to be phase this not going to be change right with the fiber length because this is all determine mainly by the or purely by the local oscillator power.

However the power corresponding to or the short noise variants corresponding to $\sigma1$ will change depending on the variants of that depends on the received power okay. When the length of the fiber is very large then p1 r will be small so you can kind of say that it would have the same short noise variants as the 0 bit short noise variants, however at lower powers it is not really so, right so you actually have a variants which is changing with respect to so you have actually have a variants which is changing depending on the length of propagation right.

So this variants of $\sigma12$ depends on the length of propagation simply because p1r depends on the length of propagation so setting the threshold will now be very difficult you cannot have a simple you know coherent receiver circuit and say okay this is the coherent receiver circuit go ahead and use it because then someone also has to tell you what is the length and you have to do a manual adjustment of the thersholding. Thersholding that separates bit 0 and bit 1 in order to avoid this problem what we normally do is.

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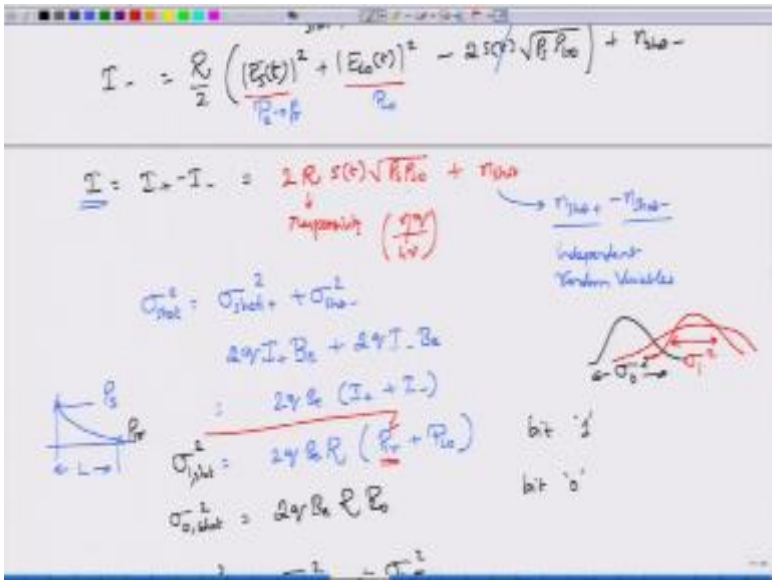

We have a plo the local oscillator completely dominating the received power even under the worst case scenarios you have plo which is much larger than p1r the advantage of that in terms of the signal is that you have plo which is much larger that this received signal po1r of course p1r = ps when you transmit bit 1, so you are essentially getting some sort of a gain without actually using an optical amplifier you are getting some sort of a gain because you are using this plo okay.

So the total photo current increases of course a noise also increases but consider what is happening when plo is much larger than p1r the variants of 0 bit is exactly or very, very close to the variant so bit 1 and because these bits are the same you can simply set the threshold right as the half wave point for example if my p1r =0 dbm which is 1mw and p0 = 0 you can set the threshold right at half mw voltage or half mw current depending on what you are looking at.

So this sort of a equalization of the variants is very important and in order to achieve that you want to operate.

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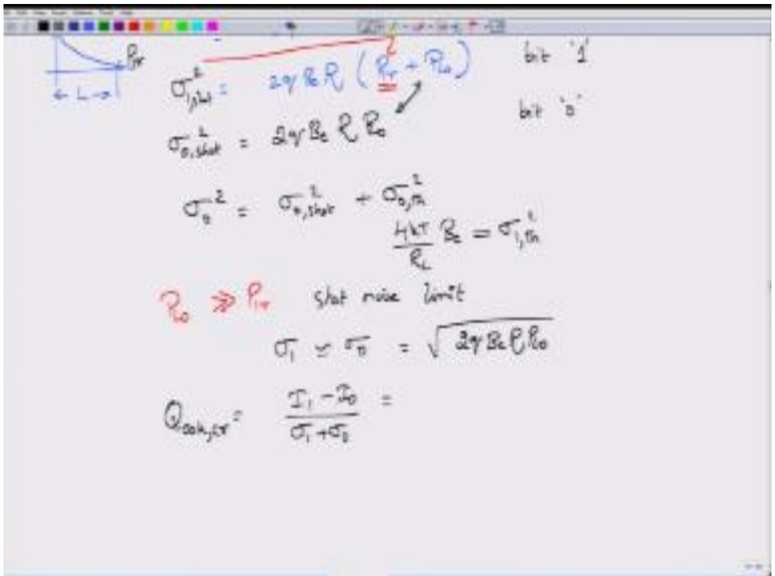

This in under the condition that plo is much larger than p1r. When you do that σ1 is approximately equal to σ0 of course the short noises are approximately equal to each other the thermal noise was anyway equal to each other but we are also assuming that plo is such that we are operating under the short noise limit so we can limit or we can negate the thermal noise okay. So σ1 will be approximately equal to σ 0 is give by √2qBe RPlo.

Now if you recall the q factor that we describe for the on off keying systems with coherent receiver so I am going to write this as coherent receiver on off keying thing this we had defined it has i1 –i0 i0 being the photo current when you transmit bit 0/ σ 1 + σ 0 here σ1 is = σ0 and i0 = 0 what about i1.

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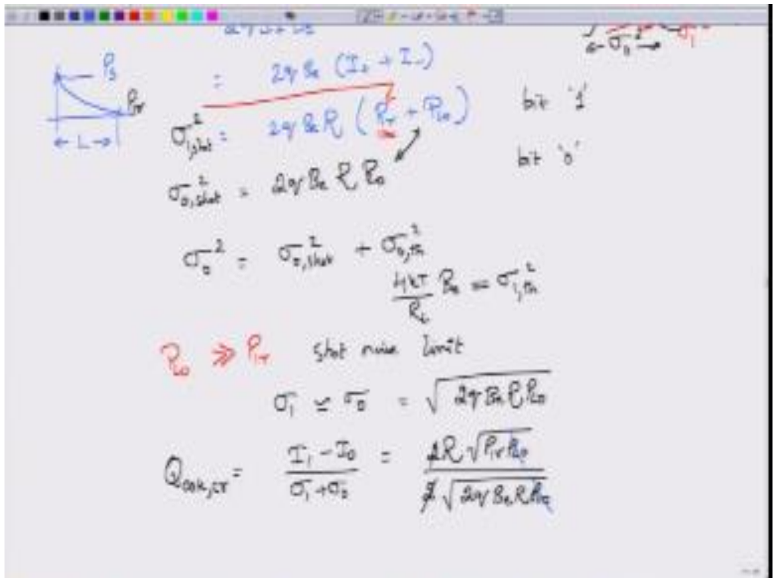

I1 is given by 2 times responsively and s(t) =1 when you transmit bit 1 and ps becomes p1r so substituting this values what you get here si 2 times r √ p1r plo / σ1+σ0 is the thing but two times σ of either of the two right so it would be √ 2qbe r plo 2 cancels on both sides you can clearly see that there is a plo√ here in the denominator which can be canceled with a plo in the numerator all though plo was use to establish the short noise.

It finally turns out that in the Q factor P_{lo} does not make an entrance, okay so your results are independent of P_{lo} as long as P_{lo} is much larger than the received signal, so that both the photo diodes and the entire coherent receiver is operating in the short noise limit, okay. Further simplification of these equations can be made.

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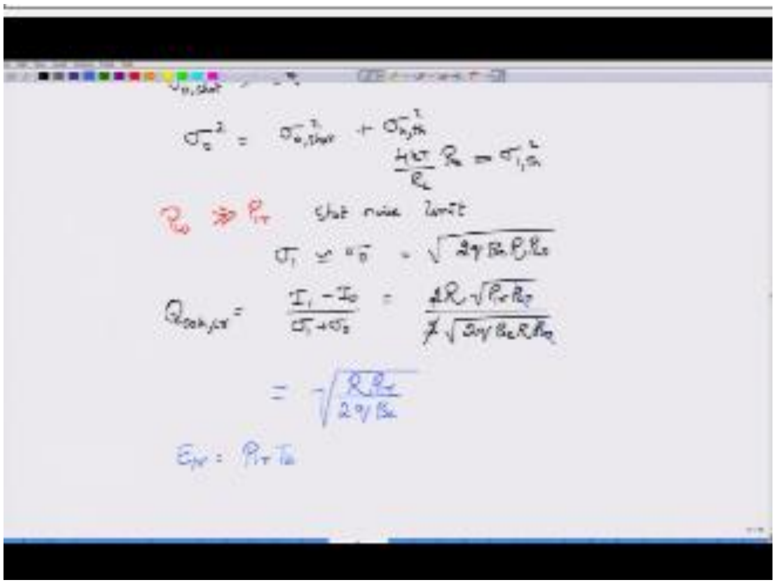

And you can show that because there is a $\sqrt{r}$ here, there will be $\sqrt{r}$ in the numerator, there is a P1r here, and you can show that this is going to be $\sqrt{R\, P1_r / 2q\, B_E}$, now the energy $P1_r$ can re written as the $E_r = P_{1R}\, T_b$ of course energy being power in to time, that is where so I had confused a little bit earlier.

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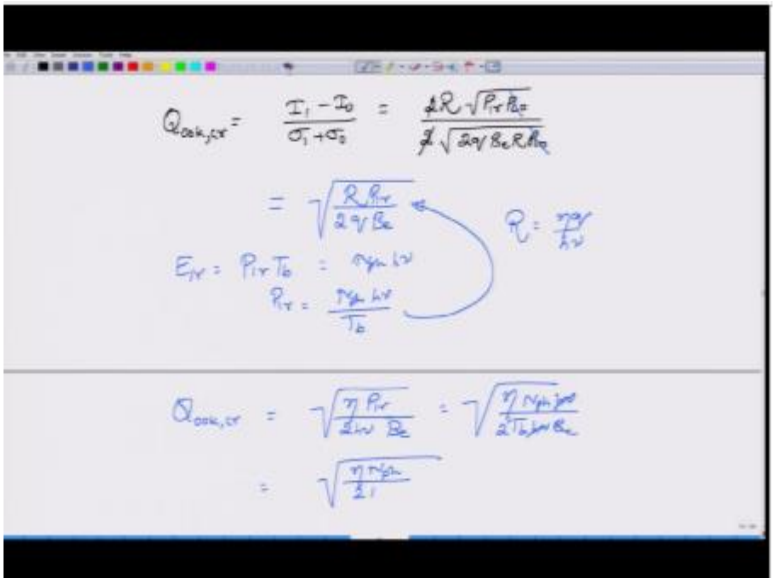

So you can write down the energy as the number of photons that you are going to receive, times H ν, so this allows you to write $P1_r$ as number of photons that you received times H ν /Tb, you can substitute that equation into this one also substitute r= η q/h ν, and simplify the relationships what you get for the on-off keying coherent receiver is an expression which is very interesting you get η P1r/2h ν Be, okay.

Of course you can substitute again as I said for P1r which could be equal to √η Nph h ν/2Tb h ν Be, h ν cancels on both sides, now you get √η Nph/2Be Tb, if you are using inquest pulse shaping then to transmit signals at a rate of,

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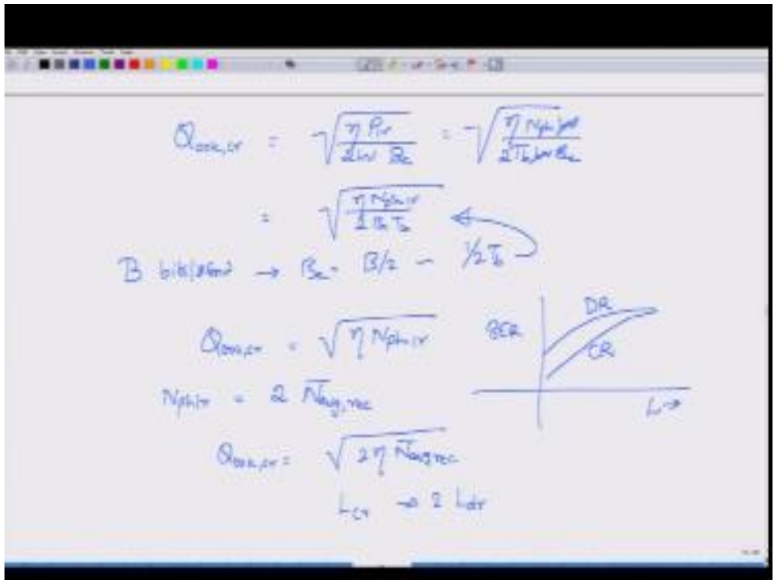

B bits per/sec, okay you need a bandwidth of about B/2 or 1/ $2T_b$ assuming the equiseta pulse shaping, this is what the bandwidth that you require, so you can substitute Be 1/2Tb this expression and see that the denominator simply cancels out, right or becomes equal to 1, so the ON OFF keying for the coherent receiver is given by $\sqrt{\eta}$ N_{ph}, the number of photons that you are receiving when you transmitted a bit.

So let me just modify the notation and then say this is Nph1r, where Nph the number of photon that you have received with 1r, and this is what you are going to get and you can recognize the number of photons that you receive when you transmit a bit1 will be two times the average number of photons that you are going to receive okay.

And therefore the Q! For the coherent receiver is given by$\sqrt{2}$ η, the average number of photons that you receive okay, and this is the quickly evaluate this okay, you will see that the "Be" are of course increases but there is a 3Db improvement with the direct detection receiver and the coherent receiver which is operating under the short noise limit, okay.

As the fiber length increases you will also see that the approximately the length with a coherent receiver will be twice of length with the direct detection receiver, and all this is happening because you are using this extra degree of this one because of the coherent receiver, okay. So we will stop it this point and consider the limitations due to dispersion in the next module thank you very much.

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