Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

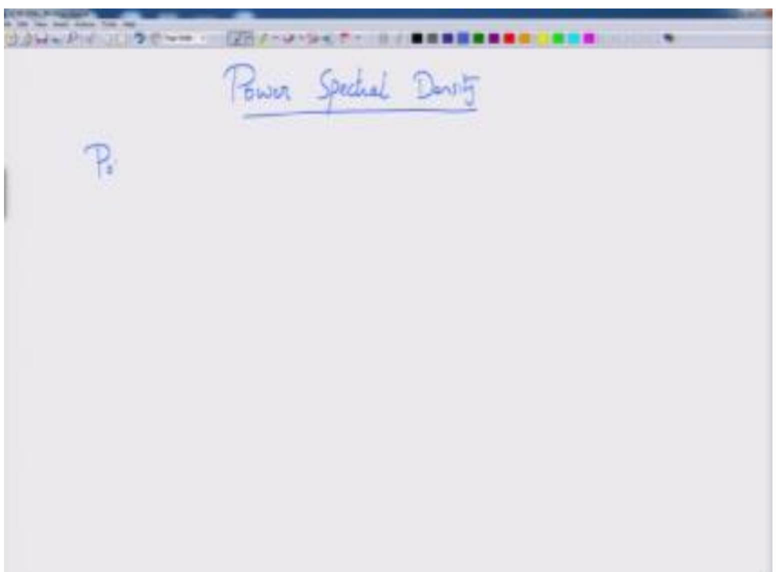
Course Title
Optical Communications

Week – IX
Module-II
Power spectral density

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome to the moke on optical communications.

(Refer Slide Time: 00:22)



In this module we will discuss power spectral density of the pulse shapes that is a continuation of the previous module, where we had discussed the type of pulses that are normally used especially when you talk about baseband communications of course, we are not going to really do baseband communication here with optics, fiber optics, but it is necessary to know what would be the pulse shape that would have, because it would have an impact on the bandwidth and hence the performance of the system.

So we have already seen that we have different pulse formats available or the line codes that are available, the popular ones are the polar format right and then you have the unipolar format and we have also discussed bipolar format okay. If you are concerned with pulse shaping at the optics level, then you are pretty much going to use a unipolar format, because if you are concerned with the pulses in the optical domain, then you are pretty much going to use the unipolar format, because in the polar format or in the bipolar format the optical power or the amplitude signal, amplitude could be positive or negative while that is still true in the optical domain if you are looking at the intensity modulation as one of the modulation that you are looking at then you cannot really have an optical power which is negative.

So you end up almost using a unipolar format and if you were to try and use different pulse shapes that we had talked about in one of the earlier modules right. So we had talked about in the earlier modules about 67% duty cycle pulse, the 33% pulse shape or a 50% pulse shape these pulse shapes in the optical domain will have a certain bandwidth in the optical domain again.

And the available bandwidth versus the bandwidth occupied by the pulse has to be adjusted appropriately. So if the bandwidth available is fixed, then you have to choose between different pulse shapes in order to be within the bandwidth that is allocated to us. However, the electrical signal that comes and modulates the optical carrier, so you have an optical carrier such as emitted by a laser which then goes into an external modulator right.

This external modulator has another input in the form of the RF or the voltage signals RF voltage signals. These RF voltage signals will also have to be encoded in an appropriate way, because what happens is this RF voltage which is carrying the information is pulse shaped okay goes changes the optical carrier, the optical carrier which is now modulated move through the fiber, at the receiver you receive the optical field and once you have received the optical field you put a photo detector, you put the photo detector and get the electrical wave form back.

So if this optical carrier is simply carrying the electrical data to us, if you look from the electrical data point of view you take the electrical data pulse format it, put it into the optical carrier and recover the electrical wave form back here. So from the electrical data point of view, if you are looking at it you can think of this as an equivalent electrical system okay. The optical part can be dissociated with this.

And here again, in order to perform a proper electrical reception of the signal, you know you have the voltage wave form that you have received which is carrying data typically what to do is you put a match filter. So if you want to put a match filter you have to know what is a pulse shape. Again the pulse shape, the electrical domain will have an impact on the bandwidth will have an impact on the performance of the system.

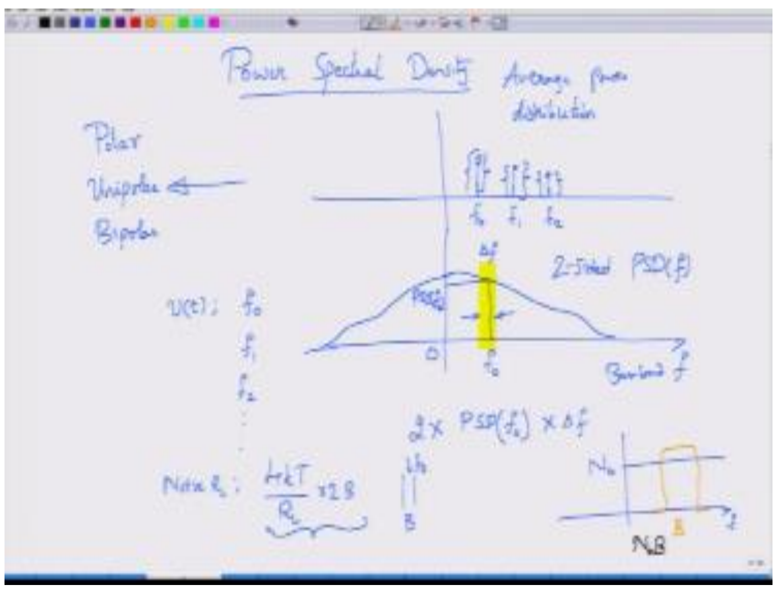
So it is necessary to pulse shape both in the electrical domain and in the optical domain, sometimes you do not do pulse shaping in the optical domain that is something that you do when you are modulating the phase of the optical carrier, there you try to keep the envelope of the optical carrier constant. However, the earlier generation optical communication systems utilized intensity modulation or on-off keying digital modulation wherein the intensity of the optical carrier was changed.

So in the digital communication case there was, we were sending a sequence of optical pulses if the pulse was present it would represent a bit one and if there was no pulse it would represent a bit zero right. So these are different reasons why we pulse shape and that is necessary that why we are studying this pulse shape. So with that in mind let us look at the power spectral density of the pulse shapes.

What is this power spectral density, we know what is power right, if you consider a simple sinusoidal signal and imagine that simple sinusoidal signal is actually being generated by an ideal voltage source. You give that across a resistor then there is an average power that is associated with that sinusoidal signal correct if you the instantaneous of course will be V^2/R and if you c consider for mathematical simplicity $R=1\Omega$ kind normalization then this normalized power that you're looking at will be the normalized instantaneous power will be V^2 right if V is the voltage of sinusoidal wave form and if you look at the average power because that is what you are interested in the resister actually dissipates this average power.

So if you are looking from the average power then you have to average the result but associated with the pure sinusoidal signal there is an non zero average power now if I change the frequency over here right and the amplitude of the sinusoidal signal the average power will also change okay but because the amplitude as changed if you keep the same amplitude it would be the same power however this power is now associated with this frequency right. So what power spectral density will tell you is.

(Refer Slide Time: 06:03)



What is the distribution of the power how is the power or the average to be sure so what is the average power how is it distributed across different frequencies okay you can imagine that you generate a voltage wave form okay V(t) corresponding to a frequency f(0) then corresponding to frequency f_1, f_2 and so on of course there are an infinite number of such frequencies and corresponding to each frequency you are going to plot the power okay if you see the power spectral density going something like this actually tells you the average power associated with the f0 component is so much.

The average power associated with f_1 component is this average power associated with f_2 is this one of course because there an infinite number of frequencies and it is better to work in the continues domain in fact power spectral density is usually a continuous quantity which tells you how the power is distributed so for example this is called as two sided power spectral density PSD is the short form for that and of course power spectral density is the function of frequency f okay so this is the 0 frequency.

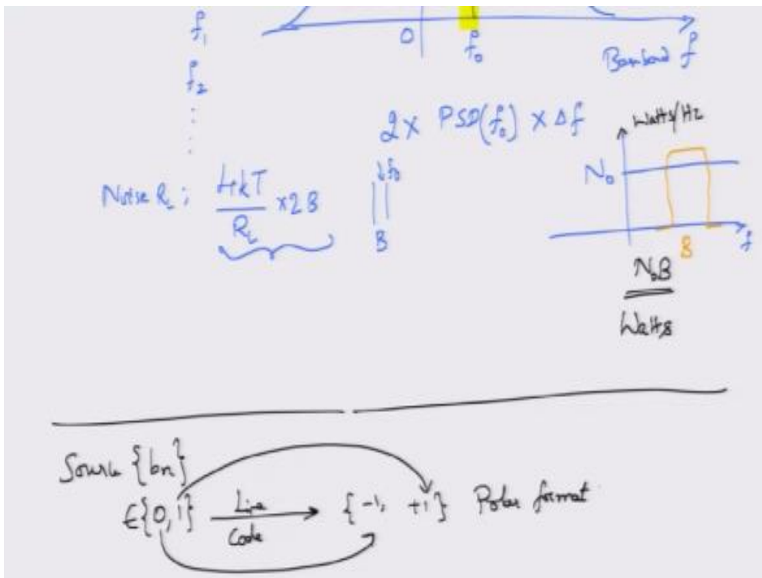
And you can see here that the power spectral density is more or less uniform which is a characteristic of a real autocorrelation we will not get into that detail here so if you want to look at how much power is actually being associated with the band of frequencies which is cantered at f0 and let us this is the band that I have okay so this is the band that I am, looking at so in that band the power that is associated will be obtained by looking at the band width which is Δf you know around f_0 and the value of this power spectral density here may be PSD(f0).

So the power that is associated with the band of frequency is Δf is obtained by looking at the power spectral density at the center frequency multiplied by Δf because this a two sided power spectral density you also have to consider an equivalent power or in equal amount of power in the negative frequency side so you usually end up having a two times PSD (f0) times Δf in fact when we talked about noise we wrote some spectral density formulas right so we said the noise associated with a load resister RL is given or is has a power spectral density of 4KT/ R_L.

And if you then consider a particular band of frequencies which had a band width of B centered at some f_0 in this case the power spectral density is flat it does not depend on f_0 so this is the same value but you multiply this one by 2B you are going to get the total power inside you are band with okay so this is the useful ness of power spectral density it tells you how the power is distributed across the frequency band okay because you are looking at frequencies around 0 this is the base band signal or the power spectral density of a bas band signal hence it is two sided you can also have a one sided power spectral density.

For example noise wave form having a one sided power spectral density is given by N_0 okay as function of frequency then if you consider a filter okay whose band width is b right then total noise in this band width is given by N_0 x B okay so this is the power spectral density and it is usually measured in Volta square per hertz in the typical noise phenomenon or you can measure this one as watts/ Hz okay because N_0B.

(Refer Slide Time: 09:52)

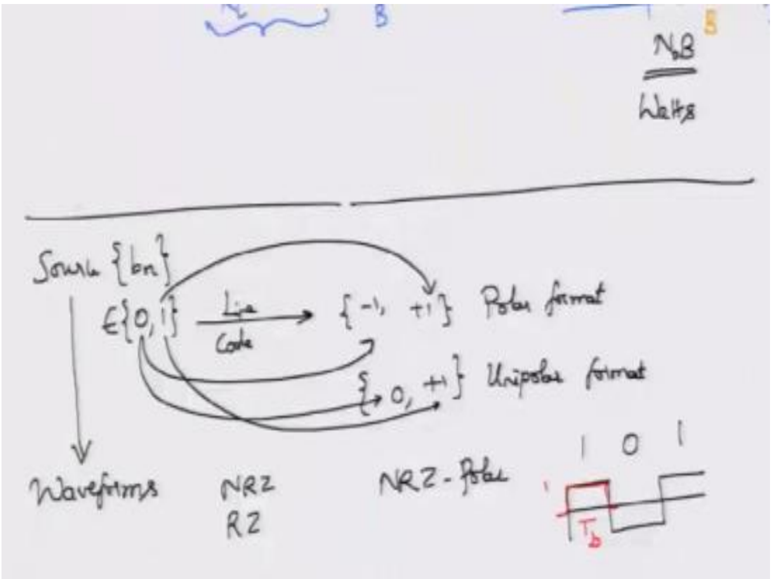


As to turn out to be in watt so you can measure the power spectral density in watts / Hz okay. So this is about the idea of power spectral density now let us jump right into finding the PSD of some of these waveforms that we are looking at we will be particularly interested in polar and uni-polar formats other formats which are important will not be covered in this course but they will appear in the exercises.

So you will get a chance to look at the power spectral density of those formats as well, so let us begin by looking at PSDs of a general waveform okay that is we had not yet defined PSDs we have only given you the intuition behind PSD we are going to define PSD now, we imagine that we have a source remember this source is putting out bits at the n^{th} time it generates a bit BN okay.

Since we are going to use binary line case this BN sequence will be 0 or 1 of course the modulation format or the line coding will change this what is the line coding do, bit 0 will be represented by -1 bit 1 will be represented by +1, what format is this? This is the polar format correct, so in the polar format we replace the symbols by numerical values and this is the mapping that we are going to do.

(Refer Slide Time: 11:22)



If you are looking at uni-polar format a 0 will be represented by number 0 and a 1 will be represented by a number 1 okay, so this is your uni-polar format but this is not just what is happening I given in the previous module we have talked about how this bits B and R generated you know from non data, non electrical data, electrical data, and all that process we have looked at.

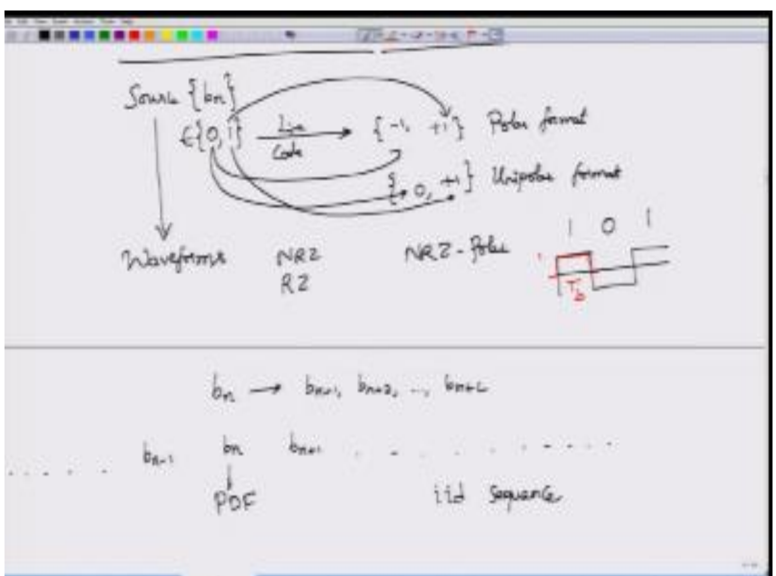
But the sources cannot you know you cannot just communicate numbers I cannot take an optical fiber and say +1 nothing the optical fiber will not do anything, right because it cannot recognize a number, so we have to convert this into waveforms, right and this is where we talked about

NRZ type of waveform RZ type of waveform, right. So if my modulation format happens to be NRZ but polar format.

If I have a sequence 101 then the waveform that would look like the assuming that this is the rectangular waveform that would look something like this correct, so this is my waveform, here you can identify the basic pulse shape the basic pulse shape but I am using is a rectangular pulse shape what is the duration of the rectangular pulse shape, TB seconds okay at each TB second I take the rectangular pulse shape.

Multiplied by the appropriate bit if the bit value is +1 I generate a positive rectangular pulse of may be amplitude 1 and then send it out, okay. If the bit happens to be 0 then I have to invert this pulse because 0 is represented by -1 so that is the net result I take -1 and multiply it to a positive rectangular pulse I get a minus or a negative amplitude pulse and I send it out as it is okay, so that is what I am essentially going to do.

(Refer Slide Time: 13:13)



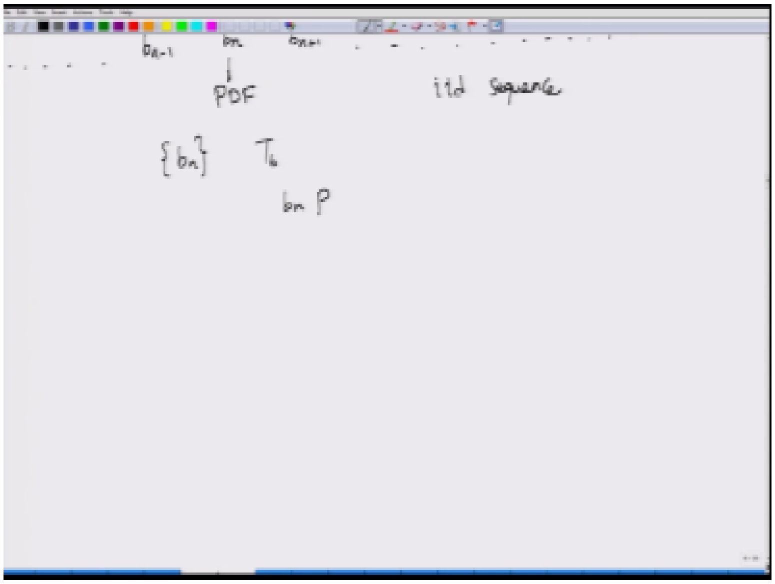
So if I am sending a sequence of such pulses and I do not know what is beforehand which bit is going to appear, right because the bit sequence if I know then there is nothing to communicate so in a sense you want randomness in the system and you get the randomness in the form of not knowing which bit comes next, we have also set something else we have said that it is possible that bit b_n at the nth time might influence bit at n+1 bit at n+2 and let us say bit at n+l.

If I am considering a ward of l letters okay, if I am word of l letters then it is possible that this b_n might tell you or might be influencing all this as I said a proper representation of this is to assume that you are looking at a source with memory source with memory is called as mark of source modeling, okay. A simple modeling on the other hand would tell you that the occurrence of b_n is independent then the occurrence of $b_n + 1$. It is also independent of $b_n – 1$.

And in fact each bit that appears will be completely independent this in turn generates a sequence of random numbers okay or random bits the distribution of each bit will be according to a certain probability distribution function okay, so this sequence which consists of a sequence I mean the entire sequence which is nothing but a sequence of random numbers distributed equally identically distributed.

But occurring independently is called as a IID sequence an IID sequence tells you that it is independent and identically distributed sequence we are going to assume an IId sequence, okay.

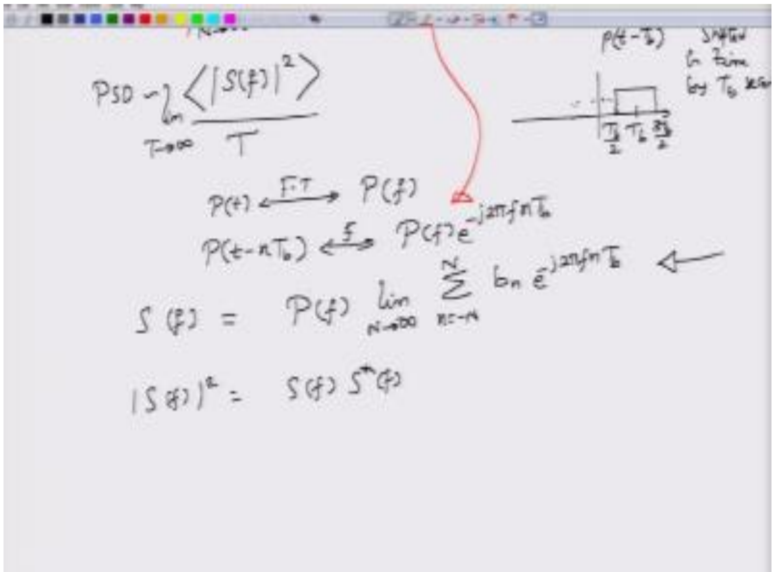
(Refer Slide Time: 15:08)



So we will assume that the sequence b_n keeps you know being generated every T_b seconds and you are going to multiply this b_n with an appropriate pulse is the pulse at 0 time you know some origin of reference that we are going to consider will be lasting from 0 to T_b, right pulse in time duration 1 or the time slot 1 will be from T_b to $2T_b$, then you have $3T_b$ to $4T_b$, $4T_b$ to $5T_b$ and so on, so you just given an appropriate number is a kind of think of this other train having this bogies right so each bogie will be represented by number -1,0,1,2 and so on.

You can even think of the bogie shape as the pulse waveform, right so if I am looking at the n^{th} bogie or the n^{th} time slot I am down here and I am looking $n-1T_b$ to nT_b that is the duration, right. So if it is a third times, third pulse I am sending it should last between $2T_b$ to $3T_b$ that is essentially what I am doing, and mathematically you can represent this train of pulses you know as we call them.

(Refer Slide Time: 16:14)



The train of pulses can be represented by multiplying the pulse that occurs at you know n^{th} time slot which is given by $P(t-nT_b)$ from your signals course you might know that if this is the basic pulse shape so let us call this as time t and let us say this is some $T_b/2$ this is $-T_b/2$ here I am showing you a rectangular pulse. If you are fancy enough you can actually have this pulse, right does not matter but the duration of the pulse has to be within $1T_b$ units, okay then what so if this is P(t) what would be $P(t-T_b)$ this would be shifted in time by T_b seconds right.

So the pulse would actually look like, so in the duration of $-T_b$ to $+T_b$ to there is nothing, but from T_b onwards you actually get this pulse, right so this is T_b from $T_b/2$ to $3T_b/2$, okay. So this corresponds to pulse which has been shifted by one time duration, okay similarly you can think of the n^{th} time and you call this as some S(t) this would be the transmitted signal of course first let us consider a finite duration train, okay train of pulses going from n=-N to +N there is nothing specific about –N to +n you can take at 0 to N does not matter, 0 to N-1 does not matter.

So here is what I am considering of course what I want is not just this finite sequence of pulses because you assume that communication has been happening from time $t=-\infty$ and it will go on up to be $t=+\infty$ to obtain that I take the limit N to ∞, right so I keep adding more and more number of pluses into this, this is your signal that is being transmitted, okay. Now the power spectral density is defined as the Fourier transform of this time limited signal, okay the magnitudes square of the Fourier transform which is then averaged and divided by the total duration of the pulse train, okay.

And you let this total duration go to ∞, okay here if P(t) is the basic pulse shape from Fourier transform theory we know that the pulse P(t) has a Fourier transform of P(f), okay so let us substitute that into this and we also know that the Fourier transform of $P(t-nT_b)$ will be in the Fourier domain that would be P(f) it is a pulse that is shifted in time therefore in frequency it gets multiplied so you have $e^{-j2\pi fnTb}$, okay. So let us substitute these values into the expression and then see what we are going to get.

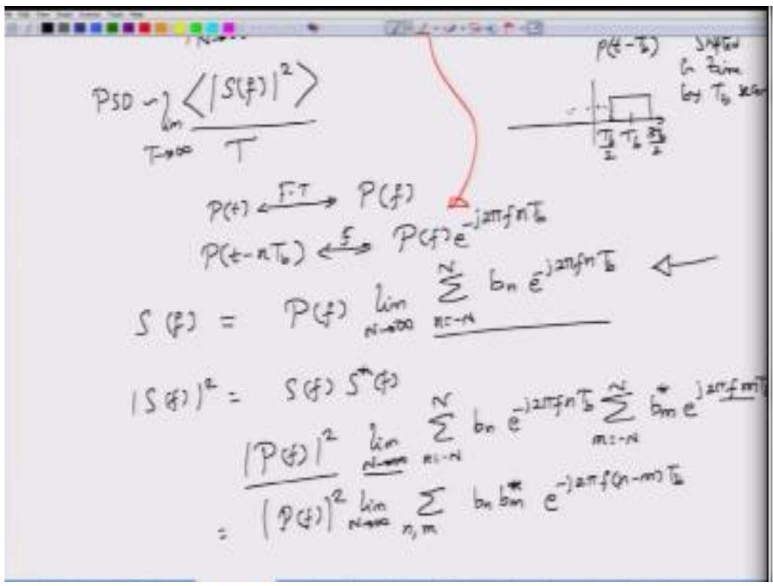
So I have S(t) here, the Fourier transform of S(t) is S(f) and I can obtain the Fourier transform so if I have to do the Fourier transform I take the Fourier operation over here to the entire pulse, then I interchange the limits of Fourier and f, okay and I go to the Fourier transform b_n is a sequence that does not have a Fourier transform whereas the Fourier transform of this $P(t-nT_b)$ is precisely this fellow, so if I take the Fourier transform what I get is S(f)=P(f) and this does not depend on the time slot so I can just pull it out you have the limit N going to ∞ Σn going from – N to +N you have b_n.

And then you have $e^{-j2\pi fnTb}$ this is a Fourier transform but we do not want the Fourier transform what we want is magnitude of furrier transforms square is actually is s(f) x s complex conjugate of f okay so s complex conjugate of s you take this equation and conjugate the complex number s okay p(f) might be a complex number corresponding to a given value of f this is the furrier transform which is complex so p(f) becomes p complex conjugate of f.

The bit sequence bn is not really complex it is a real number that we have chosen but for notational consistency let us put a conjugate there also to remind our self that this is a conjugate that is coming from. And when you take the conjugate of this fellow – becomes + sin only thing is while you have you can take the conjugate here you have variable n okay when you have a summation like this if you want to take the square you have to actually put one more summation right because this is some of like it is like a+ b and you want to take the square of that.

So you can obtain that by writing a+ b then multiplying it by a + b okay so you get a^2 + ab + ba+ 2 sorry b^2 so that is the actual way of writing it right.

(Refer Slide Time: 21:25)

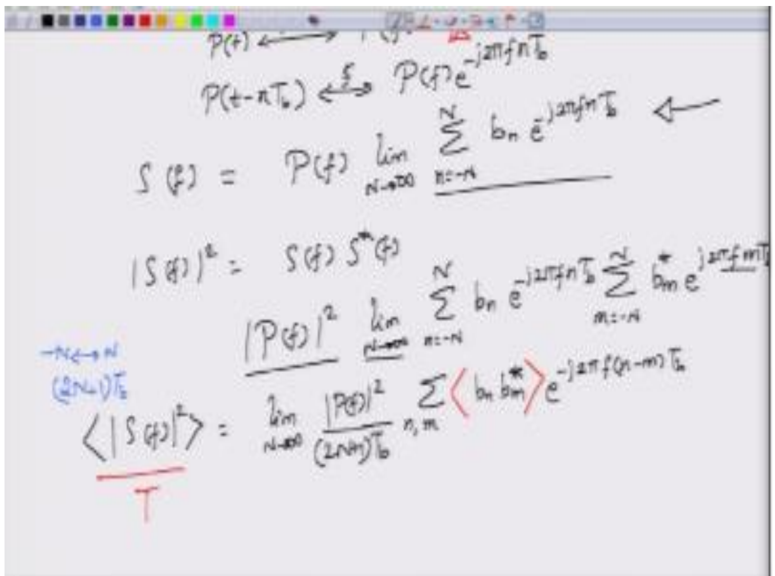


So you can write $(a + b)^2$ as a + b and a + b okay this is square this is not magnitude square therefore you just wrote the same thing twice and this can be written as some ai I = 1 to 2 and this can be written as aj j = 1 to 2 okay so using this same ideas we are going to write down the s(f) mode square and then put the averaging operator. Okay so this $s(f)^2$ = p(f) magnitude square why is p of f magnitude square well you are going to get p complex conjugate of f there right.

So this would be that and then you have limit of n going to infinity you have n = -n to + n right you have bn $e^{-j2\pi\ fnTb}$ you also have some m = -n to + n you have bm right $e^{+j2\pi\ f}$ bm complex conjugate +j2πf m Tb you can then rearrange this summations okay so would be as it is the limit would also remain the summation her can be pull to the left so I am going to use a shorter concise notation I say summation of n and m okay and the limits of –n to + n is kind of understood I am not going to write that one.

Just to simplify the equation how it looks okay so this is what it is and then you have bn bm complex conjugate okay and her you have $-j2\pi$ fnTb her you have a +fmTb but you can write down this by writing this as n-m tb okay, now do not forget to put the averaging operator.

(Refer Slide Time: 23:12)



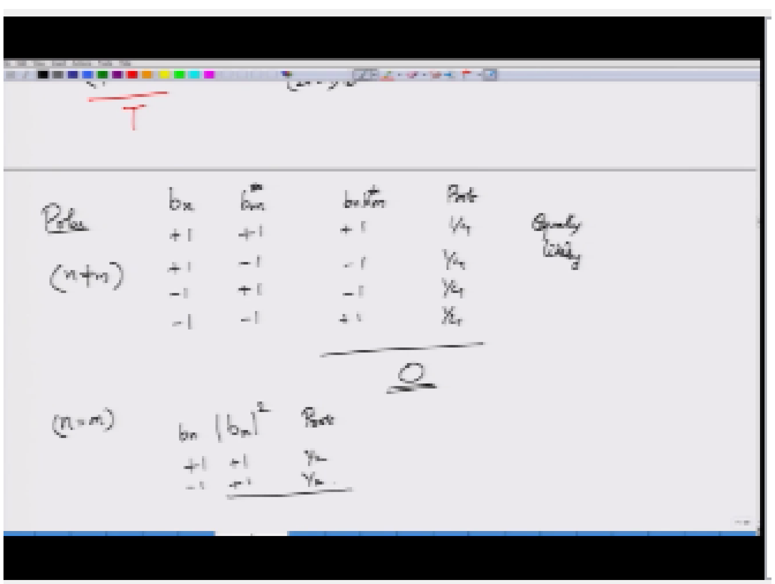
So if you put the average operation you are going to put on the sample average for what you mean by on sample average please refer to any of the probability courses that are being conducted we would not unfortunately do not have the time to spend on that one and you can also show that when you do the averaging here actually suppose to put the average on the random variables these are not random okay.

This exponential $-j2\pi$ fn-Tb is not random so the random is comes only from the bit sequence which is random which we do not know so that is the random or the averaging operation that we have put ion there are some mathematical constrained when and how you can inter change you know the average operation when you can write down the summation before average of operation later but that is of not really much of a concern to us now okay.

After this how to obtain the PST have to divide this one by t which is the total time duration what is the total time duration well I have transmitted how many pulses my index n or m goes from – n to +n right which means there are 2n+ 1 pulses and therefore the duration is 2n+ 1 x tb okay I will remove this or I can just keep it this way but of course n has to go inside so I have to pull this limit operator on to the outside of this one.

So I can just do this one rewrite the whole thing I will write this as limit of n going to infinity you have p (f) magnitude square divide by 2n+1 x tb okay. So please note that the denominator is increasing if the numerator has you know become finite and if the de dominatrix keeps on increasing than the PST might actually equal to 0, let us see weather that would be the case.

(Refer Slide Time: 25:10)

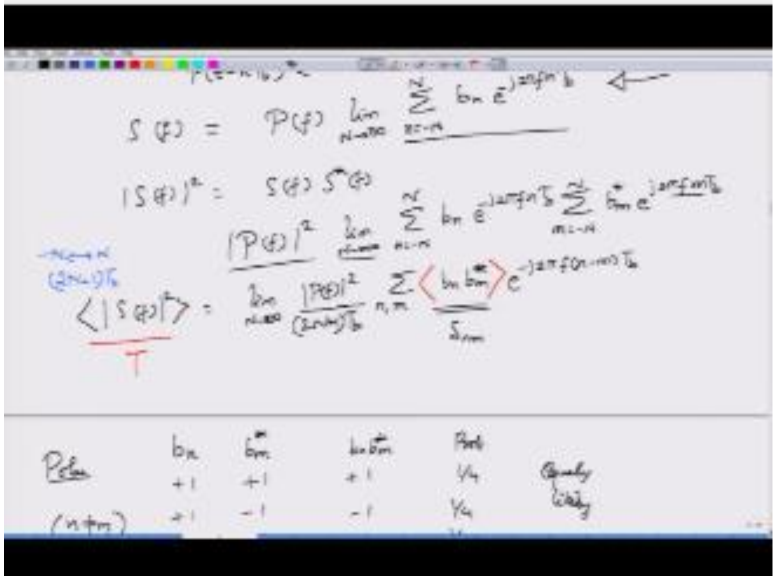


Now everything else seems to be alright only thing is how do we evaluate this averaging process? What is the meaning of that? What it means is that at the n^{th} sequence, you are looking at b_n, at the m_{th} timeslot you are looking at b_m complex conjugate, then you are looking at $b_n \times b_m$, complex conjugate product , and then you are looking at what is the probability of this particular sequence according, okay. In what possible ways b_n can vary, it can be for the polar form which let us consider as the first example, b_n can be +1 the next the sequence at the m^{th} slot can be +12 right, you can have +1 here may be -1, you have -1, you have a +1, and you have a -1 and a -1, okay.

The product will be 1, +1,-1,-1and +1 there are an equal number of +1's and equal number of -1's. If you now assume that all the bits are equally likely, that is to say all bit combinations are equally likely to occur then this probability would be ¼, ¼, ¼ and ¼, thus when $n \neq m$ this happens when $n \neq m$ of course, then if multiple the probability with the value add it next to the probability with the value probability value and sum it you are going to get a big0.

This is the averaging process what happens when n=m that is on the n^{th} time slot itself what can happen? Well then in that m=n, and you are looking at magnitude of bn^2 and then you are trying to average this, if b_n can take on +1, magnitude square will be $+1^2$ which is +1, if b_n takes on -1 the magnitude square will also be +1 because its $-1\ ^2$ right? And if you assume the probability to be equally likely as we have assumed of +1 and 1, this would occur with the probability half, this would occur with the probability half, therefore when you add them to gather way them, or find the average value.

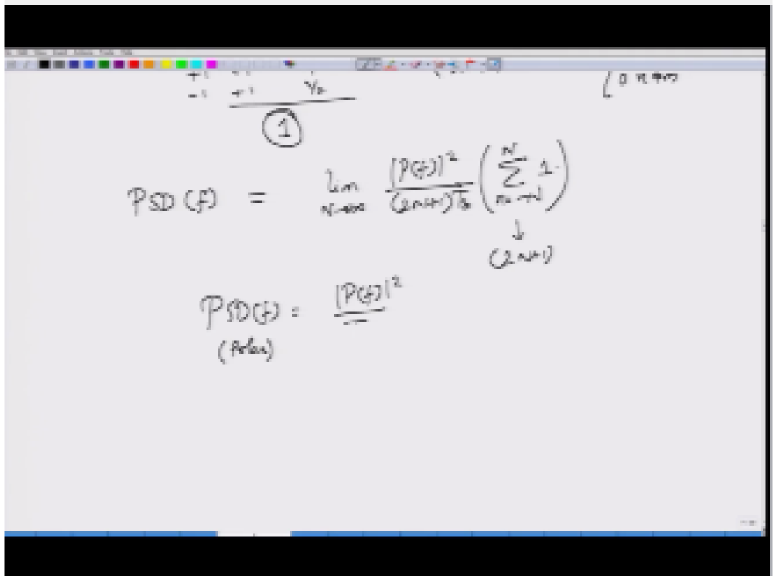
(Refer Slide Time: 27:22)



The average value will be equal to 1, okay these two conditions can be concisely return as $<b_n b_m>= \delta$ nm =1 , where δ nm is the chronically δ function which would be equal to 1, n=m it would be 0, when n≠m, okay so for the polar format this is what we obtain so you can actually go back and see what is the meaning of that.

So here you can replace this b_m complex conjugate by δ nm, so when you replace that what you are saying is, out of the two summations that you have that one on n, and ne on m, you choose only those terms where n=m, through away all the cases where n≠m, so if you take only those cases which where n=m you are going to get the path spectral density as.

(Refer Slide Time: 28:17)



So let us call this as parse spectral density PSD(F) =lim (n -∞) $p\{(f)\}^2$ / 2n+1(Tb), and then you have a summation, right which could be when n=m that could exists only that case so this is the case n=m, but n is going from –n to +n, and there is a 1 here , what happen to the exponential, well the exponential vanished, why did the exponential vanished? Because when n=m this fellow will be equal to 1, its exponential of j(o)=1, and what is the summation of this one? This is simply 2n+1.

Correct this is simply 2n+1, therefore PSD that you are looking for is given by, this is for the polar format, okay so it is given by $\{p\ (f)\}^2$ /Tb, in the next module we are going to look at the meaning of this PSD as well as we are going to look at what happens to polar RZ format and NRZ format, thank you very much.

<u>Acknowledgement</u>

Ministry of Human Resources & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL, IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production