

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title  
Optical Communications**

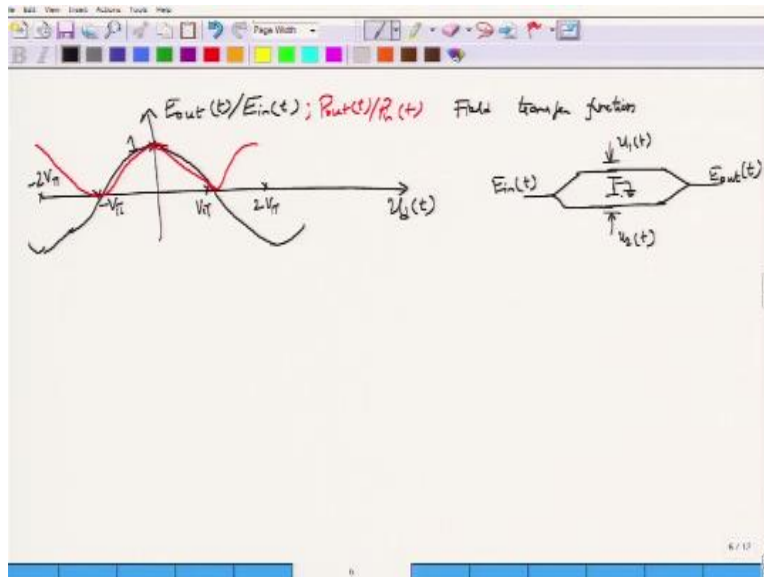
**Week – I  
Module – III  
Optical Transmitter- II**

**by  
Prof. Pradeep Kumar K  
Dept. of Electrical Engineering  
IIT Kanpur**

Hello and welcome in this module we will continue to discuss the optical transmitters particularly I would like to discuss a little bit more on the mach zehnder modulator and then we will see how we can use phase modulator that we discussed in the last module as well as the mach zehnder modulator that have modulator that we have been discussing in the last module as well as this module how they can be used for modulating both analog signals as well as digital signals.

That is to say we can use them for analog modulation as well as for digital modulation if you recall from the last module where we stopped you know discussion on the mach zendher modulator.

(Refer Slide Time: 00:53)



We derived the input and output electric field for this modulator the structure of the modulator at it is very simplest sense we have not looked into the physics yet would consists of the input wave guide through which we couple light from the sources such as a laser and then this light is split into 2 paths or 2 arms each of these arms are themselves phase modulator so you have one phase modulator at the top which is being driven by a voltage  $U_1(t)$  and there is a phase modulator at the bottom which is being driven by a voltage  $U_1(t)$  and then you get and if you combine these two phase modulator outputs.

You will get the electric field at the output of the mach zendher modulator if you now plot the transfer ratio of this in the sense that if you take the electric field of the mach zendher modulator output and then divide that one by what you free ding as the input electric field and see plot that one as a function of the difference voltage  $U_d$  so there is d in the sub script here to indicate that this transfer function is actually a function of whatever the difference between the voltages  $U_1$  and  $U_2$ .

(Refer Slide Time: 02:04)

MZM

$$E_{out}(t) = \frac{E_{in}(t)}{2} e^{j\frac{\pi U_d(t)}{2V_r}} \left[ e^{j\frac{\pi(U_d(t)-U_d(t))}{2V_r}} + 1 \right]$$

$$U_1(t) - U_2(t) = U_d(t)$$

$$E_{out}(t) = E_{in}(t) e^{j\frac{\pi U_d(t)}{2V_r}} e^{j\frac{\pi U_d(t)}{2V_r}} \cos\left(\frac{\pi U_d(t)}{2V_r}\right)$$

global phase

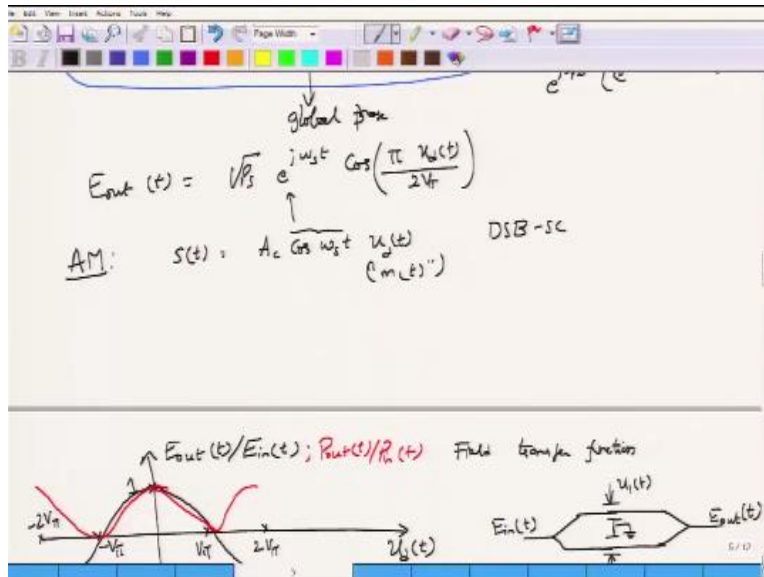
$$E_{out}(t) = \sqrt{P_s} e^{j\omega_s t} \cos\left(\frac{\pi U_d(t)}{2V_r}\right)$$

AM:  $S(t) = A_c \cos \omega_s t u_m(t)$  DSB-SC

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$   
 $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$   
 $e^{jx} + 1 = e^{jx/2} (e^{jx/2} + e^{-jx/2})$

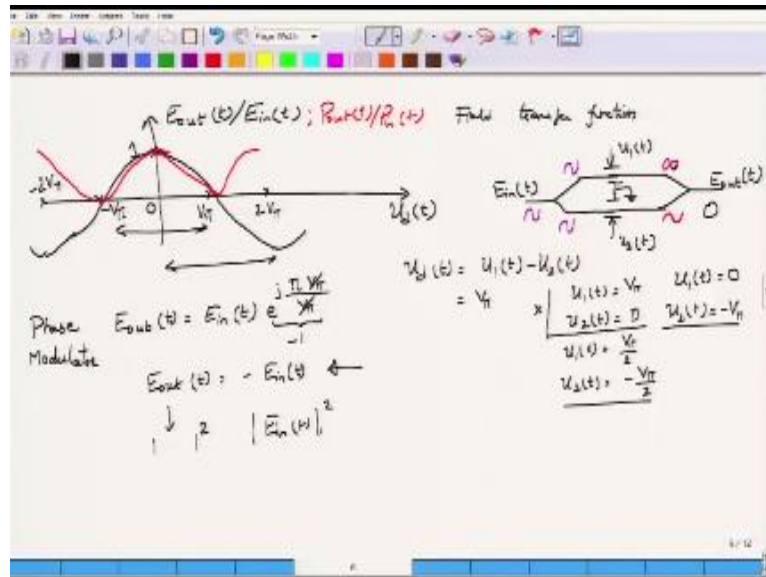
So here we have defined  $U_d$  as the difference between the two inputs  $U_1$  and  $U_2$  so this difference voltage  $u_1 - u_2(t)$  is the differential voltage.

(Refer Slide Time: 02:15)



And we have seen that the electric field at the output is related to the input electric field and that is getting multiplied by a factor which is cosine of  $\pi u_d(t) / 2V_r$  so this cosine which is getting multiplied is making this Mach-Zehnder modulator transfer function between the input and output electric fields into a non linear characteristic.

(Refer Slide Time: 02:39)



This non linear characteristic is very evident over here because this is not the characteristic of a linear system so this is the characteristic of a non linear system so this a non linear relationship between Eout and Ein if you now take the square of Eout or rather the absolute square of Eout what you get is the output power which again is a function of time because the output electric field could also be a function of time because  $u_d(t)$  could be function of time and if you take absolute square of the input electric field  $E_{in}(t)$  you would get the power  $p(n)$

So now when you take the ratio of the output to the input power in this case we will assume that the input power is held constant so whatever changes in the output power that you're seeing at the output of the mach zendher modulator is happening because of the time varying input voltage so as  $u_d(t)$  changes so initially let us say  $u_d(t) = 0$  at which point the output power is maximum the reason why this could be maximum can be seen if you look at the structure here see the light power is coming in okay.

This is some constant power this gets split into 2 equal parts however the fields are the once which are really splitting not the power so the fields split here these two fields are still in phase with each other now as the upper portion of the light passes through so let me try and give you

that graphical interpretation hopefully you would it be better to understand that so this power which is coming in or which is given here would sorry this electric field which is input to the modulator gets split into two parts.

Both are in phases okay in there simple model we consider them to be in phase later we will see that on a wave guide construction there as to be a 90 degree phase shift between these two arms okay however that 90 degree phase shift is offset by another 90 degree phase shift at the coupler therefore the overall phase shift will be 180 then there have to be some additional delay that you have to put in one of the arms so I have to make the total phase shift without any application to be equal to  $2\pi$  or 0 or it is multiple to  $2\pi$ ,  $4\pi$  and so on.

So in this case since the input electric field is split we are assuming we are modeling it as being split into equal parts in the sense that the amplitudes of these two arms are equal and they are in phase right so if they are in phase while this is rising this will also rise well this is falling this will also fall so essentially these two arms the electric fields and these two arms are in phase with each other as they propagate this fellow will undergo some phase shift okay so let us say if this is my reference electric field then after phase modulation let us say this is my phase shifted version.

Okay of course I am considering a time which is very small so that I am able to denote it this by a phase shift actually this phase shift will be a function of time because  $u_1(t)$  will be changing with time a similar thing can be written down for in this case also because  $u_d(t)$  is equal to 0 so at this  $u_d(t) = 0$  indicating that this  $u_1(t)$  is exactly equal and is in phase with  $u_t(t)$  therefore whatever the phase shift that the signal in the upper arm undergoes the signal in the lower arm will also undergo the similar phase shift.

So at the output of the phase modulator as well these two light waves in the 2 arms of the interfere meter are in phase now when you combine them you essentially get the combined output which is in the full strength right so because the phase shift of the two waves which are interfering happen to be zero, when the phase difference happens to be zero then you get the maximize the interference pattern.

This is something that you know from your high school or you know first or second Year College where you might have done some experiments on young stubble slit interferometer. So you know that whenever you have two waves which are interfering with 0 phase shift you get maximum of the power or the intensity will be maximum there, and that is the reason why you're getting maximum in the transfer characteristic over here.

And because I'm looking at the normalization, the output power is divided by the input, the maximum power that you can get here is 1, this max sender modulator is not an amplifier, it is just a modulator. It is an interferometer which is not an amplifier, therefore you will not get any more power than what you're putting in, and we don't have any gain elements in the modulator. By the same way can we understand why the modulator, transfer characteristic must go to zero when, the difference voltage will become equal to  $v\pi$ , we can understand that.

Now one way to understand this characteristic here is that, we recall what  $u_d(t)$ ,  $u_d(t)$  is  $u_1(t) - u_2(t)$ , now if this has to equal to  $v\pi$ , we can have it in many ways, all we are asking is the difference between the two signals must sum up to a constant. Now I can have this in one case by  $u_1(t) = v\pi$  and  $u_2(t) = 0$ , I can also have the same situation by having  $u_1(t) = 0$  and  $u_2(t) = -v\pi$  or I can split this  $v\pi$  into half of each, so I can have  $u_1(t) = \pi/2$  and  $u_2(t) = -v\pi/2$ , so that the difference between these two voltage is sum up to  $v\pi$ .

So you can see there are an infinite number of choices and we will simply assume that these choice to discuss the modulator transfer function as why we are getting a 0, as why is the modulator output going to 0, as when the difference voltage =  $v\pi$ , to do that go back again to the same characteristic, what happens here is that, at the input side everything is alright, so you have the light coming in, this light gets split into two parts, both are in phase at this time, which is perfectly fine.

However now look at what is happening, because  $u_2(t) = 0$ , whatever signal that you're getting at the output of the below phase modulator, that should be equal to the input of the phase modulator that should be equal to the input of that phase modulator because there is no phase shift, so as long as the voltage  $U_2(t)$  was different from 0 you would have obtain some phase shift between the output and input. Now since that  $U_2(t)$  is 0 here whatever the output would be will be the

same that is it would be in phase with the input signal and it would not have change its phase or you can say that the phase would be  $2\pi$  but in again nothing would actually change, okay.

So you have the output of the bottom phase modulator to be exactly the same as the input, so I am writing that by showing both this super imposed on that, okay so both input and output are essentially the same signals, okay. Let us keep this simple model in mind, now when you look at the top phase modulator unfortunately that is not exactly same why? While the input is this one what could be the output of the top phase moderator when  $U_1(t)$  is  $v\pi$  to understand that let us rewrite our or let us recall the relationship of the phase modulator so I have  $E_{out}(t)=E_{in}(t)$  this is the phase modulator equation that I am writing, okay.

So this is phase modulator multiplied by  $e^{j\pi U_1(t)/v\pi}$ , this was the relationship between phase modulator so  $U_1(t)$  was the electrical drive signal applied to the phase modulator  $E_{in}(t)$  is the light source coming from the laser or the light wave coming from the laser and this would be the output of the phase modulator. Now you can very easily see that when  $U_1(t)=v\pi$  then I can replace this  $U_1(t)/v\pi$  and now you can see where easily that the numerator should be  $v\pi$  so this will cancel with each other and what is  $e^{j\pi}$  this is nothing but  $\cos\pi+j\sin\pi$   $\sin\pi$  is 0  $\cos\pi$  is -1 so this is nothing but -1.

So what you now have is that the output electric field is negative of the input electric field, there is no problem here sometimes I get a question as to how can the electric field be negative with respect to the output, I mean output electric field could be negative with respect to the input would it mean that power also is less than 0, which is wrong because the power associated with this output electric field is obtained by taking the magnitude square of the electric field and because this  $-$ sign would not really matter when you take the magnitude of the electric field.

The output power would be exactly equal to this input power, so there is no problem in power relationship electric fields can be negative but power would always be a positive quantity, okay so this is something very important to keep in mind many times we see that oh, power can be I mean some students have this thing that power can also go negative you know in this situations



but it cannot happen that way power is not negative, power is always positive because of this magnitude square thing.

Alright, having discussed our digressed a little bit let us get back to the track, so what we have observe is that when  $U_1(t)=v\pi$  then the output electric field is minus input electric field which means that if this is the reference voltage that you have given to the input of the phase modulator, the output of the phase modulator would be an exact opposite of that, right it would be  $180^\circ$  out of phase with the input. Now you look at the two red wave forms, one at the top and one at the bottom these two are now  $180^\circ$  out of phase.

Now you go back and think about the Young's double slit experiment that if I have two waves which are out of phase by  $180^\circ$  that is if they are out of phase then what would be the output and that you would see on the screen it would be a 0, because two waves which are in same in everywhere else except their phase is  $180^\circ$  out of phase with respect to each other will interfere will destructively interfere to give you a 0, right. So when you take this sign wave and add to that a sign wave which is  $180^\circ$  out of phase with the sign wave all you get is a 0 here, so what you get at the output is 0, and that is precisely what we have shown over here saying that the output electric field must be 0 at this particular characteristic.

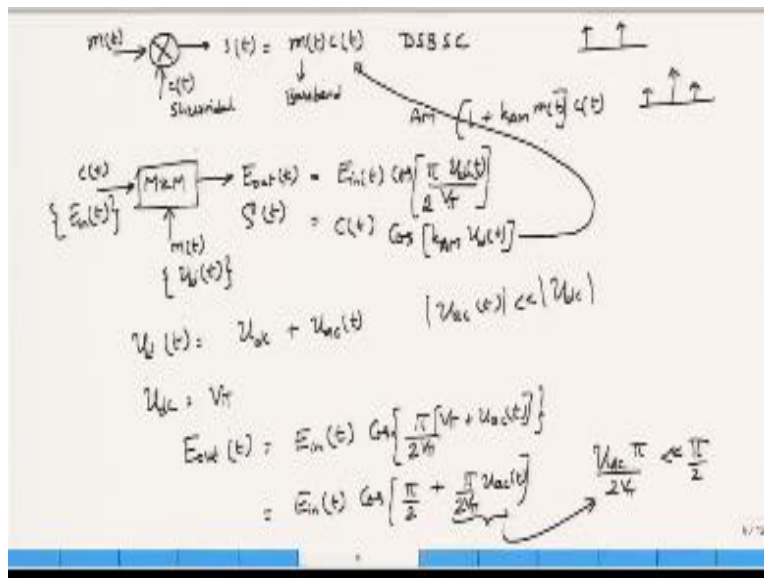
So it is hopefully understood that when the difference voltage  $U_2(t)$  is going through 0 then there is constructive interference here and when  $U_2(t)$  is going through  $v\pi$  there is destructive interference and the modulator produces nothing or the output of the modulator is 0, so in between these two extremes is the cosine characteristic, okay. As I said when you look at the power characteristic the characteristic of the power being power always being positive will mean that while the field is going negative the power is actually going positive.

And for this loss less modulator that we have assume this would be a periodic function, okay so this would be a periodic function the power is periodic with minus with the period of  $2v\pi$ , because this is over one period for the power. Whereas the electric field would be periodic over the same this one, but this would be this  $2v\pi$ , okay. So we have now discussed two modulators one is the phase modulator and the other is the Mack-Zehnder modulator let us try to put them

into use to see how we can modulate the phase of the light signal and to realize the phase modulation or amplitude modulation.

Since it is amplitude modulation that we are normally familiar with, let us actually begin with the slightly complicated example of modulation that is Mack-Zehnder modulation, okay.

(Refer Slide Time: 15:18)



Recall that for a simple amplitude modulator all you require in the functional terms is that I have the message signal  $m(t)$  and I have the carrier signal  $c(t)$  which would produce a signal  $s(t)$  which is sum  $m(t).c(t)$  this  $c(t)$  of course will be a sinusoidal carrier as we discussed in the last module and  $m(t)$  would be a message signal or the base band signal, so this is the base band signal. We are assuming that at this moment  $m(t)$  represents in analog wave form and this type of a modulator is called as a double side band suppressed carrier modulator, okay.

There could be other, there are in fact other ways of amplitude modulation one which is actually the original amplitude modulation will have some BIOS values it will have some  $1+k_{AM} m(t)$  where  $k_{AM}$  is some constant, this whole thing is getting multiplied by  $c(t)$ .

So in, if you look at the spectrum of this for sinusoidal modulation you will find not only the carrier term but you will also find the upper and lower side band, so the carrier is also transmitted here. Whereas for the double side band suppressed carrier system you do not have any carrier in the center, okay. So however both are amplitude modulators themselves, okay. Now can we realize this functional relationship with the Mack-Zehnder modulator, let us look at what the Mack-Zehnder modulator functional relationship is by putting a symbol for the Mack-Zehnder modulator, okay so this is my Mack-Zehnder modulator to which I am supplying light so let us call this as  $c(t)$  and I should of course also supply  $m(t)$ .

But I know that this relationship is not exactly this multiplication, right I know that the output of the Mack-Zehnder modulator if you take this  $c(t)$  and remember that is nothing but the electric field at the input and this  $m(t)$  as the difference signal  $U_d(t)$  write in our terms for the MZM, the output of this one will be called  $E_{out}(t)$  is actually given by  $E_{in}(t) \cdot \cos \pi U_d(t) / v\pi$ , sorry there is a  $2v\pi$ . So this is the characteristic that we have right and well instant of  $E_{out}(t)$  let us write this as  $s(t)$ .

So what you see here is that  $s(t)$  is actually given by  $c(t)$  this is for the Mack-Zehnder modulator I am actually trying to compare the conventional amplitude modulated discussion that we find in communication systems to the Mack-Zehnder modulator functional relationship that we have just derived, okay or that we have been discussing. This max ender modulator if you want to map  $m(t)$   $u_d(t)$  and  $c(t)$  sinusoidal carrier to the light input electric field  $e_{in}(t)$  and map assorted to  $e_{out}(t)$  this is the relationship that you will get  $C(t) \times \cos$  of some constant that we do not have to worry.

We can call that constant as  $ka$  and times  $u_d(t)$  okay clearly this is not exactly the same as this relationship  $m(t) \times c(t)$  because of the presence of this cosign term here right so this is not going to give you an exact double side bands of breast carrier is there a way out is there a way in which we can approximate this non linear characteristic with that of this characteristic that we have okay. Is there ea way in which I can kind of eliminate because by placing some restriction on  $u_d$  of  $t$  and  $km$  if I am pick this  $km$  and impose certain criteria on  $u_d(t)$ .

Is there a way for me to make this one almost equal to this particular characteristic let us see how we can do that well we remember what this  $K_{am}$  is  $k_m$  is nothing but  $\pi/2b\pi$  in this particular mapping that we have done and  $u_d(t)$  this is the other thing that we can play around we cannot of course play around with  $c(t)$  you require the carrier for modulation to happen let us suppose that I can write  $u_d(t)$  as consisting of some dc component plus some ac component okay.

So the dc component will be constant and the ac component will be another it is a small signal component okay ac component is very small the magnitude of ac component or the peak value of the ac signal is very small compare to the value  $U_{dc}$  okay, so let us also put down in terms of the magnitude itself so this has to be very small compare to  $U_{dc}$  okay, now if I take  $U_{dc}$  to be equal to  $V\pi$  okay if I substitute this  $V\pi$  condition in to this expression for the output electric fields I get e out of t of the max ender modulator is equal to  $E_n(t)$  times  $\cos \pi/2b\pi U_{dc}$  which is  $b\pi + U_{ac}(t)$ .

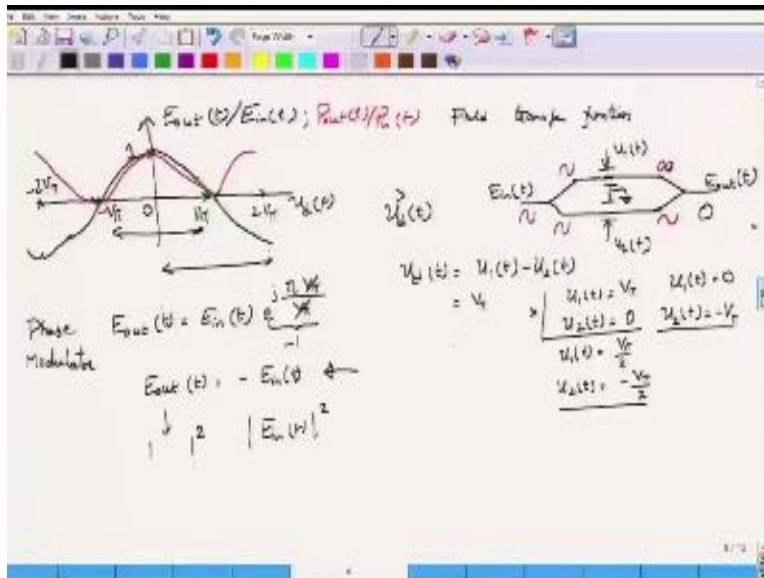
If you expound what is there in the bracket in side what you see is in of t times  $\cos b\pi$  will cancel from numerator and denominator what is get is  $\cos \pi/2 + \pi/2b\pi U_{ac}(t)$  so  $U_{ac}(t)$  is very small component compare  $U_{dc}$  at the same time while that is getting multiplied by  $\pi/2b\pi$  if I check take the peak value of  $sc(t)$  and demoted as  $U_{ac}$  this peak value multiplied by  $\pi/2b\pi$  will be peak value of this voltage correct?

That would be the peak value of this voltage if we choose this to be very small compare to one there is if you choose this component to be very small quantity compares to  $\pi/2$  okay. so that is the idea so we choose this one to be a very small quantity compare  $\pi/2$ , okay now let us expand this term in the bracket or let us expand this cosign term by recalling the trigonometric identity that  $\cos(a+b) = \cos a \cos b - \sin a \sin b$  okay.

So if I apply this trigonometric formula what I get for the output electric field is  $e(t) = E_n(t)$  times  $\cos \pi/2 \cos$  of let us call this  $\pi/2b\pi$  and whatever the  $A_{cp}$  value of some  $ka$  so I have  $\cos k_a U_{ac}(T) - \sin \pi/2$  this  $E_n(t)$  is common factor for everything  $\sin \pi/2 \times \sin(ka) U_{ac}(t)$  now I know that  $\cos \pi/2$  is 0 so this entire term will be equal to 0 I know that  $\sin \pi/2 + 1$  so what I get for the output electric field is  $-E_n(t)$ .

And we are assuming that  $k_a \times U_{ac}(t)$  is a small number right it is small number  $\sin\theta$  is approximately  $\theta$  itself so I can eliminate this  $\sin$  from this expression and I write  $k_a \times U_{ac}(t)$  okay the output is very small.

(Refer Slide Time: 23:02)



Because we are actually operating at the 0 point of the max ender modulator the output would be very small as you go around here what you would like would be to try and move the operating somewhere else so that is can still see a linear relationship that we will check later but at this point since we have chosen this in equal to  $b\pi$  as the input voltage swings back in fourth right as the input voltage changes over here.

The corresponding output change will happen over this transfer characteristic and this output is small because you are operating near the 0 point of the max ender modulator.

(Refer Slide Time: 23:39)

The image shows a whiteboard with handwritten mathematical derivations. At the top, the trigonometric identity  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$  is written. Below this, the output signal  $E_{out}(t)$  is derived from the input signal  $E_{in}(t)$  using a Taylor series expansion of a cosine function. The expansion is shown as  $E_{out}(t) = E_{in}(t) \left[ \cos \frac{\pi}{2} \cos k_a U_{ac}(t) - \frac{\sin \frac{\pi}{2} \sin k_a U_{ac}(t)}{2} \right]$ . This is simplified to  $E_{out}(t) = -E_{in}(t) k_a U_{ac}(t)$ , which is further written as  $E_{out}(t) = (-k_a) E_{in}(t) U_{ac}(t)$ . The final result is  $E_{out}(t) \propto \frac{c(t)}{m(t)}$ , where  $U_{ac}(t)$  is identified as an analog signal. To the right of the main derivation, the expression  $(E_{out}(t))^2 = (k_a)^4$  is written.

But what we have managed is to obtain this relationship we have obtain  $-k_a$  which is some constant does not matter but more importantly if you look at the remaining part of the expression you have  $E_{in}(t)$  which was equivalent  $C(t)$  for us and  $U_{ac}(t)$  which was equivalent of  $m(t)$  so as long as you max ender modulator is  $\pi(t)$  at be  $\pi$  and your applying ac signal ac drive voltage which is very small compare to the dc voltage that you can apply you can approximate the relationship of e out and e in by  $C(t) m(t)$  type of a relationship.

So that cosign nonlinear characteristic has been removed but at the expense that the output power has drastically reduce because what is the output power the output power is magnitude square of this term right and this magnitude square you will see there is a magnitude of  $k_a^2$  involves as  $k_a$  itself is quite small  $k_a^2$  is even smaller than  $k_a$  so the output power is very small but the good news is that you have been able to obtain amplitude modulation.

So if your  $m(t)$  are equivalently if your  $U_{ac}(t)$  happens to be analog signal what you have obtain is an analog modulation okay this is one way of operating the max ender modulator in order to perform amplitude modulation or amplitude modulation of a analog signal.

**Acknowledgement**

**Ministry of Human Resource & Development**

**Prof. Satyaki Roy**

**Co-ordinator, NPTEL IIT Kanpur**

**NPTEL Team**

**Sanjay Pal**

**Ashish Singh**

**Badal Pradhan**

**Tapobrata Das**

**Ram Chandra**

**Dilip Tripathi**

**Manoj Shrivastava**

**Padam Shukla**

**Sanjay Mishra**

**Shubham Rawat**

**Shikha Gupta**

**K. K. Mishra**

**Aradhana Singh**

**Sweta**

**Ashutosh Gairola**

**Dilip Katiyar**

**Sharwan**

**Hari Ram**

**Bhadra Rao**

**Puneet Kumar Bajpai**

**Lalty Dutta**

**Ajay Kanaujia**

**Shivendra Kumar Tiwari**

**an IIT Kanpur Production**

**©copyright reserved**