

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

**Course Title
Optical Communications**

**Week – VIII
Module-IV
Light sources-II**

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome to the module on optical communications. In this module we will continue the discussion of spontaneous and stimulated emissions, and then talk about when we can get laser oscillations okay.

(Refer Slide Time: 00:31)

Handwritten notes on a whiteboard:

$$\frac{R_{\text{spont}}}{R_{\text{stim}}} = e^{\frac{h\nu}{kT}} - 1 \rightarrow 10^{29}$$

$T = 300\text{K}$

$$k = 1.38 \times 10^{-23}$$
$$f = \frac{h\nu}{2\pi} \rightarrow \frac{6.626 \times 10^{-34}}{2\pi}$$

$700\text{nm} \rightarrow 10^{15} \text{ rad/s}$

$N_2 > N_1 \rightarrow \text{Population inversion}$

A small diagram on the right shows a rectangular block with a vertical axis labeled z and a horizontal axis labeled x . A vertical line is drawn inside the block, and a horizontal line is drawn across the top of the block.

So to do that we will first start by looking at in a matter in active material what is the spontaneous emission right, how much exactly is the spontaneous emission dominating over the

stimulated emission. So let us do that $R_{\text{spontaneous}}$ to $R_{\text{stimulated}}$ we had in the previous module shown that this would be equal to $e^{h/\omega/kT-1}$. So if you fix temperature h/ω is anyway fixed K is Boltzmann constant that is also fixed, what is left is the frequency ω .

So as you change the frequency ω you might end up the situation where h/ω will be larger than kt , if that happened, if h/ω is larger and if it is much larger than kt then this quantity on the right hand side will be larger than 1, which means that spontaneous emission will be larger than stimulated emission okay. So let us put some numbers so that this is, you know into perspective so $t=300k$ is good enough for a room temperature value.

And then put the small k which is Boltzmann constant is 1.38×10^{-23} and I also know that h is $h/2\pi$ h itself is 6.626×10^{-34} okay I am not writing the units, you can just refer to the units elsewhere and divided by 2π would be divide this value by 2π and let us assume that the radiation we are interested is actually radiating at 700 nanometer okay, we want to know what is the spontaneous emission at frequency corresponding to wavelength of 700 nanometer.

This 700 nanometer would correspond to a frequency of roughly 10^{15} rad/sec. So for getting all this $6.626/2\pi \cdot 1.38$ and everything, if you substitute these numbers into that expression, you know use a nice calculator which has scientific notation and calculate this ratio turns out for 700 nanometer wavelength, this ratio turns out to be very close to 10^{29} okay. You can see that in an ordinary matter, the spontaneous emission is so dominant over the stimulated emission that you essentially get no light at all, there is nothing to basically stimulate and generate a laser oscillation.

So if you simply take such as active material could be a helium, neon or could be a ruby, or could be another material, could be carbon dioxide right. And then simply connect up something and then keep hoping that it is going to lase, you know it is going to oscillate you would never find that it is oscillating, simply because the spontaneous emission in this material is so dominant over the stimulated emission.

So if you do not want the, this one to dominate over the stimulated emission, if you want the stimulated emission to dominate you want to invert this particular relationship, in the sense that the stimulated emission must become larger okay, and stimulated emission can become larger once n_2 becomes larger than n_1 okay. When n_2 becomes larger than n_1 , then one can actually have stimulated emission dominating this spontaneous emission. This phenomenon where n_2 is larger than n_1 if you recall is known as population inversion okay.

You can see that this is called as population inversion okay, strictly speaking population inversion does not happen in an two level lasers okay. You actually need to have a three level laser, first atoms have to be somehow brought into the third level, from third level they have to go to the second level and population inversion is created between these two levels, level one and level two okay.

So it is created between level one and level two, but this extra level three is needed in order to mediate or make this population inversion happen okay. But once the population inversion has happened you can consider only these two levels and then talk about a two level system okay. So we will not worry about this third level and everything at this point, we will simply assume that population inversion has happened okay.

The question is just population inversion sufficient for us to generate oscillations? The answer is no, because stimulated emission might be happening, but then someone has to keep sending in those input photons right. So maybe by luck you had one photon coming in and you had a nice stimulated emission which will last for some time but if you do not feed that stimulated emission back into the system it would not sustain itself.

So for sustaining the oscillations not just you know oscillations for a very brief period of time you want the oscillations to sustain only then you are actually making a source correct so to make this oscillations sustain or to sustain this oscillations you have to provide a feedback mechanism.

So any laser you know most typical lasers actually have three ingredients one ingredient is a pumping mechanism which is necessary to create population inversion of course you need a material in which can actually enable that population inversion I can take a wood and the try to create a population inversion okay some materials are easy they can actually be able to create population inversions okay.

And the third is the feedback mechanism so any laser actually as these three requirements and we will look at what happens when we put down all these three elements to form a laser at this point it might also be worthwhile to just give you the definition of a laser.

(Refer Slide Time: 06:12)

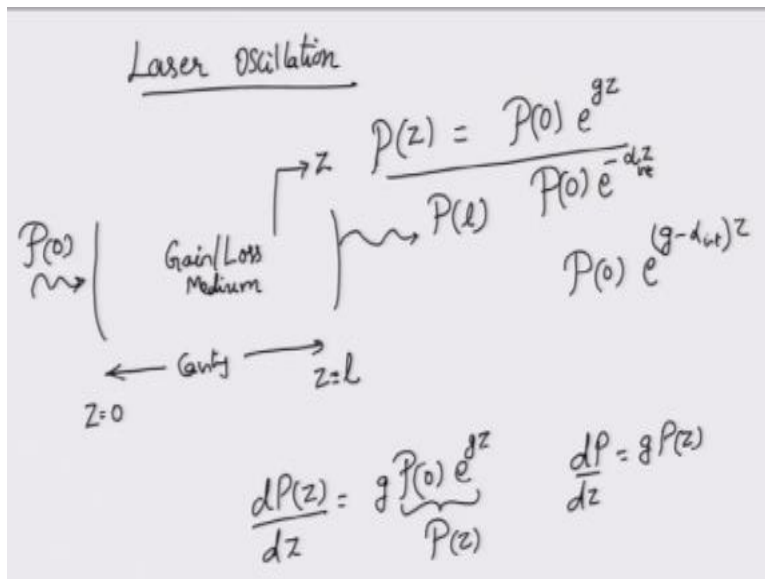
$\frac{R_{spont}}{R_{stim}} = e^{\frac{h\nu}{kT}} - 1 \rightarrow 10^{29}$
 $T = 300k$
 $k = 1.38 \times 10^{-23}$
 $h = \frac{h}{2\pi} \rightarrow \frac{6.626 \times 10^{-34}}{2\pi}$
 $700nm \rightarrow 10^{15} \text{ rad/s}$
 $N_2 > N_1 \rightarrow \text{Population inversion}$
LASER: Light Amplification by Stimulation Emission of Radiation
 WNW laser UN un

Or you know the acronym we can expand laser is actually an acronym okay although for the last 50 years are so we have been talking about laser as just a noun okay it actually originally meant as a acronym this is called as or this acronym is light amplification okay which means that there as to be some light to began with and that some light to began with comes from a spontaneous emission okay.

Somehow by thermal agitation you have caused a spontaneous emission photon to drop down but then you want to sustain that photon you want to create additional photons which are matching to that particular initial photon okay so you want to amplify the light by not by spontaneous emission by stimulated emission of radiation okay so this is the expansion of laser but for all our purposes we can laser light this okay notice the difference if it is an abbreviation you normally do not write it in this fashion right.

If it is a WWW you want to write WWW or if it is another one like say united nations you want to write UN you won't find as writing like this or if you won't found writing like this but lasers have become such commonly used word that you can write laser in this way at the begin of the sentence or in the middle of the sentence you can simply write this as laser okay, it is no longer consider to be the acronym but it is just consider to be the noun which encapsulates a broad range of phenomenon in which light is being generated okay. So the three ingredients for the laser oscillations to occur and sustain are a feed back.

(Refer Slide Time: 08:12)



Mechanism feed back mechanism in optics is actually in the form of a cavity the cavity keeps pumping things back and then you have a pump could be a flash lamp in some of the lasers or it

could be a electrically pumped signals so you have either an optical pump or an electrically pump the idea of pump is to basically push the atoms from N_1 to N_2 to create a population inversion okay.

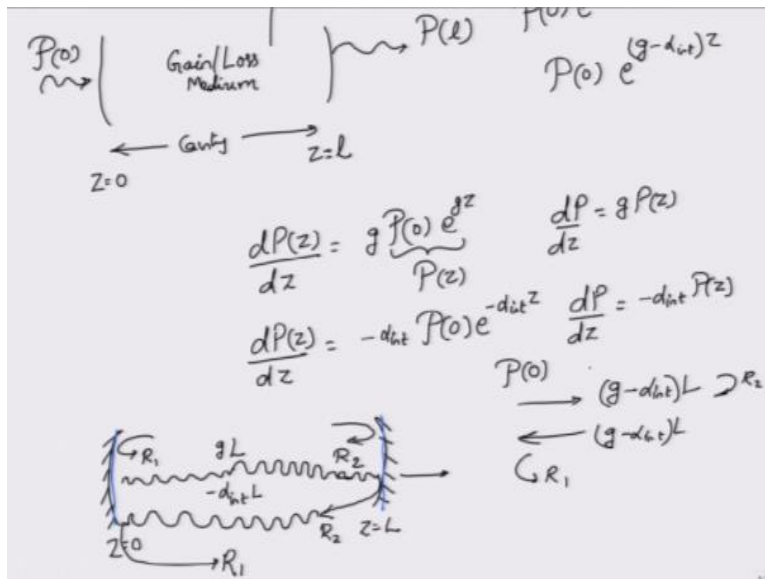
And then you want the material itself so you need an active material which can actually sustain this population inversion so these are essentially the three main ingredients of a laser so we will look at the laser oscillation in order to look at the laser oscillation let us assume a cavity and then dump our gain medium inside this cavity or put the gain medium inside the cavity let us also assume but the cavity as a length L okay which we are going to measure along the Z axis now if you this since this a, gain medium that is this medium as to provide gain right.

If I start with some optical power at the input of the cavity so at $Z=0$ you have the power $P = P(0)$ and then at any point along the length of the gain medium the power there would be $P(z)$ and this power would be equal to the power at the beginning times e^{gz} where g stands for the gain of the medium assume that it is independent of the frequency it is independent of anything else it is kind of constant at this point in place of gain if you had loss what would be the power at this point the power would be $P(0) e^{-z\alpha}$ okay.

Since this α is internal to the cavity we can write this as α_{int} just to indicate that this is an internal loss okay so you have a gain and you have a loss the effective power of course would be $P(0)e^{(g - \alpha_{int})Z}$ clearly if g is larger than internal losses then this will be a larger number than $p(0)$ and you would have then net amplification, in case $g < \alpha_{int}$ this term will be less than 1 so you will get a net attenuation or a net loss in the system right at $z = l$ these numbers in place of z you can change this into l okay, look at this equation again.

I can differentiate that equation so I can say $dP(z)/dz$ and since $p(0)$ is a constant that goes like that g is a factor which comes out and then you have e^{gz} but this is nothing but $p(z)$ itself so you get a nice differential equation $dp/dz = gP(z)$ okay which describes the gain equation okay.

(Refer Slide Time: 10:58)



So how the power is actually changing in this gain medium okay, now if of course you can also derive another equation for the loss which is dP/dz will be equal to in case you are only looking at the loss term then α_{int} will come out and then you still have $p(0)e^{-\alpha_{int}(z)}$ so another equation that you are going to get will be $dP/dz = -\alpha_{int}$ times $P(z)$ okay. Now let us look at what is called as a sustaining condition okay.

We want the oscillations to sustain let us assume the cavity so I am going to redraw the cavity over here is actually 1 kept at $z = 0$ and this one is kept at $z = 1$ and these cavity is essentially a mirror okay so this cavity is essentially a mirror and one can actually look at mirrors with a straight line edges or one can have a you know curved mirrors and the kind of modes that are generated will be slightly different so we will simply assume for simplicity now that these are straight line mirrors.

And they have been kept at $z = 0$ and $z = 1$ we also need to specify the reflectivity of this mirrors so if radiation comes in from the left hits the mirror then a part of their light is reflected back. These are not perfect mirrors okay if they were perfect mirrors then I would not have been able to tap any energy out of them so they are imperfect but their reflectivity's are quite high so this

reflectivity's R_2 similarly this reflectivity is R_1 let us say for some reason the power you know in the form of the optical wave is travelling in this way so may be it starts at the origin over here and travels all the way to the end, okay.

What would be the length over which it has travelled? A length of l what would be the gain it would have experienced gL , what would be the loss? $-\alpha L$ right at this point there is you know a reflection and R_2 times input actually keeps you know is actually reflected back, some power $1 - R_2$ is actually escaping outside but let us not worry about that one, the reflected power is R_2 which will again you know travel all the way upto the input, right.

At this point what will happen there would be further reflection, right. R_1 times this power would be reflected, so one way you have a propagation of L and the effective argument in the exponent would be $g - \alpha$ times L and the other way you have another propagation through L and you get $g - \alpha$ times L here in one way you are getting a reflection R_2 here you are getting a reflection R_1 , okay. You imagine that the initial power is $P(0)$.

Which is the one which has been incident just at the output of the first mirror so that is the power there is $p(0)$ so $p(0)$ times e to the power.

(Refer Slide Time: 14:08)

$$\frac{dP(z)}{dz} = -\alpha_{int} P(z) e^{-\alpha_{int} z} \quad \frac{dI}{dz} = -\alpha_{int} I$$

$P(0) \xrightarrow{(g - \alpha_{int})L} R_2$
 $\xleftarrow{(g - \alpha_{int})L} R_1$

$P(0) e^{(g - \alpha_{int})L} R_2 e^{(g - \alpha_{int})L} R_1$
 Self consistency $P(0) R_1 R_2 e^{2(g - \alpha_{int})L} = P(0)$
 $R_1 R_2 e^{2(g - \alpha_{int})L} = 1$

So $P(0)$ times $e^{g - \alpha_{int} L}$ times R_2 would be the power that is reflected which propagates again as $e^{g - \alpha_{int} L}$ and then reflects once more to give to R_1 okay, this would be the power that is so here you actually have an input power which is $p(0)$ and then after all this round trip propagation and this double reflection you get the power which is $p(0)$ times this particular quantity, of course I can combine this and this into a single exponent.

And what I get is $p(0) R_1 R_2 e^{2(g - \alpha_{int}) L}$, now sustaining condition or self consistency simply means that this must be equal to $P(0)$ if it is not equal to $P(0)$ if it is less than $P(0)$ then after about you know 3, 4 more round trips it will simply die down because R_1 and R_2 are never greater than 1, okay and if this number is less than 1.

(Refer Slide Time: 15:16)

$$\frac{dP(z)}{dz} = -\alpha P(z) e^{-\alpha z} \quad \frac{dI}{dz} = -\alpha I$$

$$P(0) \rightarrow (g-\alpha)L \rightarrow R_2$$

$$\leftarrow (g-\alpha)L$$

$$\leftarrow R_1$$

$$P(0) e^{(g-\alpha)L} R_2 e^{(g-\alpha)L} R_1$$

Self consistency $P(0) R_1 R_2 e^{2(g-\alpha)L} = P(0)$

$$R_1 R_2 e^{2(g-\alpha)L} = 1$$

Then you will not be able to get any gain so it simply keeps dropping, dropping, dropping eventually it goes off right eventually it goes to 0 so for oscillations to be sustained it is necessary that this power you know must be equal to the power $P(0)$, from this you can cancel $P(0)$ and then arrive at a relation which is $R_1 R_2 e^{2(g-\alpha)L}$ must be equal to 1, okay this is the self consistency requirement and you are need to really satisfy this particular condition.

(Refer Slide Time: 15:53)

The image shows a whiteboard with handwritten mathematical equations. At the top left, there is a small diagram of a light ray reflecting off a mirror labeled R_1 . The main text consists of the following equations:

$$P(0) e^{(g - \alpha_{int})L} R_2 e^{(g - \alpha_{int})L} R_1$$

Self Sustaining $P(0) R_1 R_2 e^{2(g - \alpha_{int})L} = P(0)$

$$R_1 R_2 e^{2(g - \alpha_{int})L} = 1$$
$$g > \alpha_{int}$$

Of course, here you might also observe that you cannot let g be less than α_{int} , right then it will not be satisfied up there, so you want g to be greater than α_{int} . in other words the gain of the medium has to be larger than any intrinsic losses, so one of those losses is coming from the mirrors because mirrors are only reflecting R_2 and R_1 some position of the light is actually escaping right.

So that can be you know losses but those losses are somewhat you know because of the mirror losses α_{int} is the medium losses, medium itself has some loss and that is α_{int} and this one is the loss which is the loss due to the mirror, okay.

(Refer Slide Time: 16:38)

$$P(0) e^{(g-\alpha_{int})L} R_2 e^{(g-\alpha_{int})L} R_1 = P(0)$$

Self-sustaining $P(0) R_1 R_2 e^{2(g-\alpha_{int})L} = P(0)$

$$R_1 R_2 e^{2(g-\alpha_{int})L} = 1 \quad e^{2(g-\alpha_{int})L} = \frac{1}{R_1 R_2}$$

$$g > \alpha_{int} \quad \alpha \quad (g-\alpha_{int})L = \frac{1}{2} \ln\left(\frac{1}{R_1 R_2}\right)$$

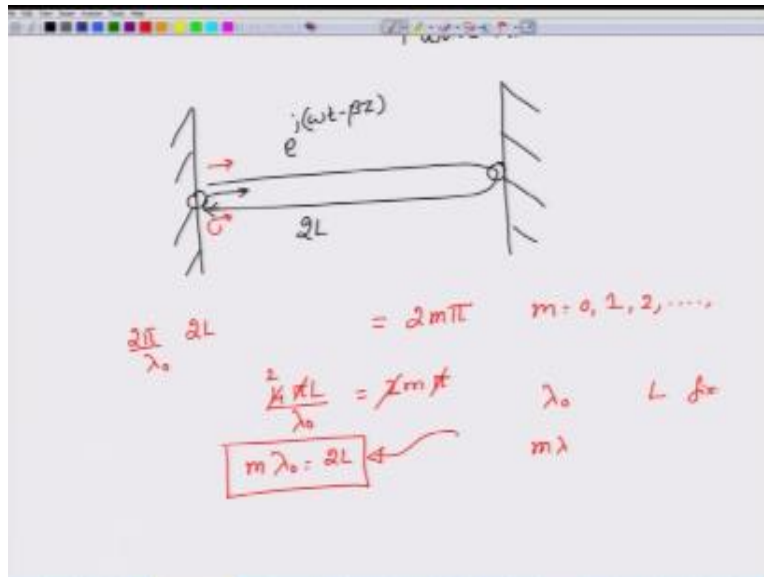
$$g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \quad g >$$

Mirror loss

So you can rearrange this equation to right in a slightly different form, so you can write $g = \alpha_{int}$ after taking out this exponential so take log on both sides so first divide this one by $R_1 R_2$ and then take the log so if you divide, if you do that one you get $e^{2(g-\alpha_{int})L} = 1/R_1 R_2$ and then if I take the log natural log on both sides you get $2 \log$ that log and exponential cancel with each other so you get $(2g-\alpha_{int})L = 1/R_1 R_2$ and then you can rearrange g now, so g can be written as internal loss plus $1/2L \log$ of $1/R_1 R_2$.

So you can see that the gain is getting balanced by the loss, so this is the intrinsic loss and this is the mirror loss, okay. The mirror loss is a function of $R_1 R_2$ as well as the length of the cavity L , intrinsic loss is just the material loss the gain medium loss whatever that is there. For oscillations to be sustained g must be greater than the total losses, okay. So only those particular modes which actually satisfy this condition will be sustained. Now this brings us to another question as to what exactly are these modes.

(Refer Slide Time: 18:05)



Well, if you take cavity right, and you assume that light is you know going back and forth between the two light will be sustained only when the total phase change would be an integral multiple of 2π , that is if you assume that there is a monochromatic wave which can be written as $e^{j(\omega t - \beta z)}$ when the wave propagates through a distance a total distance of $2L$ it undergoes a phase change, it will also undergo a certain phase change up on reflection here and upon reflection here.

But once after undergoing the phase changes the wave comes you know starts at goes at the right point, I mean at the input port you input point you will again have to apply the self consistency result in the sense that the total phase must be an integral multiple of 2π , okay only then this initial wave will be adding up to the new wave and therefore this process kind of gets repeated, okay or consistently repeated so that the wave is actually sustained.

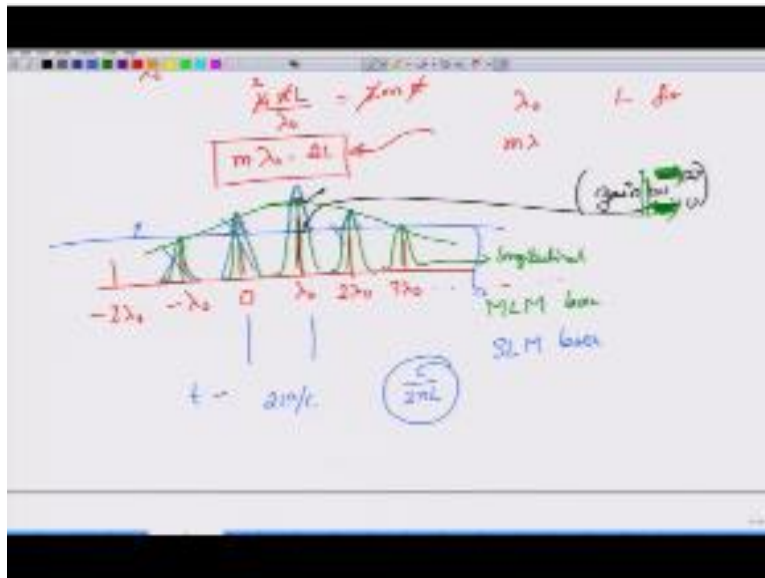
If the phase is different the sometimes the phase is positive sometimes the phase is negative on an average you will be seeing only 0 because on an average this would simply washout all the result for washed out, so you need a constructive interference and for constructive interference you need a total phase change of 2π , so what you need is the phase change to be integral multiple

of 2π , okay and what is the total phase change phase change is given over a distance of $2L$ so β is the propagation constant which can be written in terms of λ so you have $2\pi/\lambda_0$, λ_0 being the operating frequency of the radiation here.

So $2\pi/\lambda_0 \cdot 2L$ must be equal to $2m\pi$, m can be 0 although because of the propagation the first number would be 1 that would be interesting 2 and so on okay. This further actually can be rewritten as $4\pi L/\lambda_0 = 2m\pi$, π on both sides cancel 2 on both sides will cancel here and what you see is λ_0 times m must be $=2L$ this is interesting so what you see is that if you start with λ_0 if you fix the value of l so if you fixed the value of L then the only possible modes that are going to be present or the once which are satisfying this particular condition. Okay.

So the only one switch actually satisfy or the wave lengths that are going to exist or the once which satisfy this particular equation okay.

(Refer Slide Time: 20:49)



So given to mirrors which for my cavity which are kept at a distance of two l and a radiation whose wave length λ is there inside the only possible modes are going to be this integral multiple so you are going to get λ_0 you are going to get $2\lambda_0$ you are going to get $3\lambda_0$ and so on of course

you can also get $0\lambda_0$ so on but those frequency is can be you know these are the lower side frequency these are the upper side frequencies.

But the point is that you actually get a large number of discrete modes which are all integral multiples of these particular modes. Okay so you get 0 you get a $-\lambda_0$ you get $-2\lambda_0$ and so on for a length of $2L$ okay now there is one additional that happens over here we said that this cavity contains a gain medium right so this cavity contain a gain and this gain medium actually consist of those two levels of energy which we talked about 1 and 2.

So you would expect that this gain which is given by the frequency ω right I mean this gain medium which actually has a frequency ω or h/ω is the energy difference between the levels one and two would if it falls on one of these modes which are allowed by the cavity right so if this frequency somehow let us say falls on this λ_0 only that particular λ_0 will be constructively interference and supported all the other should in principle devout.

Because there is no power in other component right so the energy difference is such that this Ω falls only on this λ_0 and only this particular mode is picked up unfortunately that is not what happens in practice in practice you would see that this energy levels are not really single ones but they are actually a bands of energies okay so you actually have a band of energies and therefore there is actually a sprit in terms of Ω from the lowest point here to the highest point the lowest point here to the lowest point to the second level.

So there is actually a spirit of frequencies over which the you know the photons can be stimulated and observed in terms of the frequencies of the cavity what is means if the you actually have a wide spectrum of the frequencies and if all these modes which satisfy this condition I mean this all modes are actually sustained okay so there will actually a mode here there will be a mode here and there is a mode here mode here and a mode here.

These modes are called as longitudinal modes of the cavity okay and such a also which supports multiple longitudinal modes is called as MLM laser and MLM laser is multiple longitudinal mode which supports all these modes and the condition that g has to be greater than $\alpha x +$

whatever the losses means that the total gain loss curve must exceed a certain threshold okay and unfortunately in this particular example which I have shown there exists at least three modes because, this mode is exceeding the threshold, this fundamental mode exceeding the threshold, and there is one more mode which is exceeding the threshold, okay.

What is the spacing between these modes? We know that the length here is $T - L$, and you know that time is $2l/c$, where "c" is the speed of propagation, or in case if you looking at a medium which has the certain refractive index, then this would be $2ln/c$ or $2nl/c$, right. $1/\text{time}$ is frequency, so this is $c/2n(l)$, this is called as the modes of a cavity, okay.

So this cavity have all these modes, see which are all spaced at $c/2n(l)$, okay the mode spacing is $c/2n(l)$, okay these three modes are passing the gain and threshold, so this R going to exist our this particular laser systems, and they constitute what is called as the MLM laser.

However most sophisticated laser that are used for optical communications, do not have so many multiple longitudinal modes, there are special mechanisms that are made to suppress this additional modes, one of those mechanism system to actually incorporate resonant cavity, or put a filter inside, which will then select only one particular mode here, okay.

So such a action of that one results what is called as a SLM laser, SLM stands for single longitudinal mode, okay you can kind of suppress all this additional longitudinal modes, and then only the single longitudinal mode, okay so this completes our understanding of resonators and the associated laser oscillations, just to read iterate it is necessary that you need to have a cavity.

But the moment you have the cavity R1 and R2 as the reflectivity's, there are going to be some mirror losses plus the gain medium has a entrance classes, to overcome that gain you know, the losses one has to have gain, and gain comes from stimulated emission, okay gain comes from stimulated emission, but to cost stimulated emission you need a pump, so the pump causes population inversion which that then causes gain and then the gain has to be greater than the threshold, sorry! Gain has to be greater than the loss.

And the gain minus you knows! The threshold point at it which exactly happens, this called as the threshold, okay so this a way in a general laser could operate, we will then in the next class in the next module, we will specialize ourselves to semiconductor lasers, okay the discussion will be quite brief because we don't want to get in to much of details of semiconductor lasers, so we will get in to semiconductor lasers in as much detail as required, and then quickly after that move on to considering optical communication systems again , thank you very much

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL, IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production

©copyright reserved