Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – VIII
Module-II
Noise in photodiodes-II

by
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Hello and welcome to the moke module on optical communications. In this module we will complete the discussion of noise in photodiodes and then discuss the signal to noise ratio aspects of a photo detector circuit okay. We will not look at the actual circuit that is reserved for sometime later, what we will do instead is to assume that there is just a load resistor of some valve RL and the current that is generated by the photodiode actually flows through this RL and then generates a voltage or a current depending on whatever you are looking at.

And that is what we are going to use as the benchmark okay. So we will see what happens when this large, when the load resistor is large and when the load resistor is small, what kind of noise limits are signal to noise ratio okay. We have discussed several intrinsic noise processes in a photodiode right, but when you include a load resistor RL.

So this is the load resistor RL, there is actually a noise associated with a resistor which is kept at room temperature okay. This noise is called as thermal noise and this is purely because of the motion of electrons inside this resistive material as a result of thermally generated energy. So any matter kept at a energy or temperature T greater than 0 will experience a certain amount of energy and as a result of which there will be a vibration or there would be some juggling, jiggling of the electron hole pairs constituting current or equivalently a voltage resulting in addition.

I mean in addition to the actual voltage or the current that is in the resistor there is also a noise because of this irregular motion of the electrons. So the current is not exactly a deterministic process, but it becomes stochastic or statistical because of this thermal noise. If you look at thermal noise you are mostly interested in knowing what is the variance of the thermal noise right.

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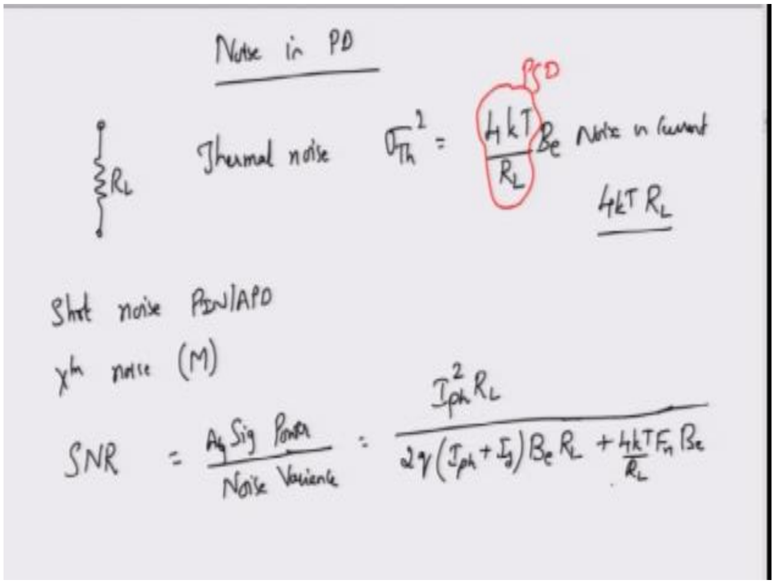

Noise in PD

R_L Thermal noise $\sigma_{Th}^2 = \frac{4kT}{R_L} B_e$ Noise in current

PSD

$4kT R_L$

Shot noise PIN/APD

x^{ln} noise (M)

$$SNR = \frac{Avg\ Sig\ Power}{Noise\ Variance} = \frac{I_{ph}^2 R_L}{2q(I_{ph}+I_d)B_e R_L + \frac{4kT}{R_L}F_n B_e}$$

So what would be the range over which I can expect the current to be in right, and this thermal noise variance surprisingly is independent of how much current you give to the load resistor RL okay or to any resistor R. It is in fact simply proportional to the thermal energy that it receives which is Kt right, you know which is proportional to KT and then divided by RL, this is divided by RL because you are looking at noise in current okay.

You are looking at current noise variance, but if you are someone who is looking or you are comfortable with looking at the voltage noise variance it would be 4KTxRL because current square into RL is voltage, so that can be related to the power V^2/RL and therefore you get voltage noise variance as 4kTxRL but because photo detectors mainly generate current we will look at the current noise variants okay the expressions will involve divide by R_L quantity okay and you can see that it does not depend how much current you are actually providing through the load resister it does not depend on any of that parameter it in fact does not even depend on the precede shape of the resister okay.

It all that shape and every things captured in this RL and it simply depends on what is the value of the resister and what temperature this is working in okay in addition to this thermal noise we have seen short noise in both PIN as well ABD's although with APD's you also get a multiplication noise or multiplicative noise mainly because of this multiplication factor M okay now we defined signal to noise ration which is a very important term in all communication systems because you want communication systems to have very high signal to noise ratio.

You want much of the signals that you have received to actually be the signals that you are looking at and not this fluctuations which you do not have any control or you do not have any use in obtaining the information right so you want the signal to be as large as possible noise to be as small as possible the way we defined signal to ratio is that we defined this as the average signal power okay to the noise variants which can be thought of as the average power that is dissipated in a one ohm resister okay.

If I have a photo current I_{ph} okay what would be the power that is noise power that is generated average noise power that is generated in a load resister R_L well I_{ph}^2 x R_L would be my average power that is generated right assuming that this I_{ph} is in the RMS value or something so you get RMS I_{ph}^2times R_L will give you the average signal power right this would be the signal power what would be the noise variants well you have short noise and you have thermal noise right so you add up the variants there but what should I add there should I add it over a infinite band width no I am going to add it over the bandwidth of the detector itself right.

So we have assumed that the detectors have bandwidth of B_e the low pass band width of B_e and therefore the two sided bandwidth is about two times B_e right so for that the amount noise power that is added will be multiplied by the power spectral density here in the thermal noise you multiply this one by B_e this is simply because you are looking at a bandwidth of two B_e in fact

And I had written on the variants this $4KT/ R_L$ is actually the power spectral density so this is power spectral density, okay. The power spectral density is constant and wide and it is independent of current $4kT/R_L$ but if you consider at two sided bandwidth of 2BE ten this would be the thermal noise variance, okay. And because this noise process is actually nothing to with the current the variance is actually not 2Be but rather only Be.

So this is actually the noise thermal noise variance that you are looking for, okay. So you can now write down what is the noise variance which is the sum of short noise as well as a thermal noise, so let me write down those terms so you have $2q\ T_p\ h + I_d$ assuming that dark current is there if you do not have dark current or you can neglect dark current you remove I_d from the equations right.

So you have $I_pH + I_d$ this is for the short noise times the bandwidth Be that you have and all this noise is in the load resistor R_l plus you have the thermal noise variance which is $4kT/R_L$ x $F_{n\ Ve}$ where F_n is called as the noise figure of the resistor it is the degradation in the signal to noise ration that would be experienced so it is not just $4kT/R_L$ but there is a multiplication of this quantity by an additional noise factor called Fn, okay.

In fact Fn is what is called as the excess noise figure and that is something that represents the degradation of the signal to noise ratio at the input and the output, remember output signal to noise ratio is always lesser than the input signal to noise ratio and the amount by which this is lesser is proportional to this noise figure or noise factor depending on how you actually call the ratios up there, okay.

So this is what you have $4kT/R_L$ x Fn this would be your noise variance and this is your signal to noise ratio, now signal to noise ratio is given by this expression but you might be tempted to see if there is any range over which one variance dominates over the other variance, right. So if I try and make R_L to be very large then the second term kind of starts to go down because it is inversely proportional to $R_{L.}$

While the first term kind of goes to go up, right. In that region you reign what is called as the short noise limit the opposite is true if your photo current is small right and your R_L is moderate to small then the thermal noise component will dominate over the short noise, so you get two limits in practice you get a thermal noise limit and you get a short noise limit, okay. Most Pin photo diodes are operated in the short noise limit however APD's actually have a very high internal short noise because remember.

Whatever the current electron whole pairs are generated because of the impact ionization and the avalanche process multiple photo carriers are generated, so the mean photo current increases and the internal short noise variance of an APD increases, okay so your APD circuits are mostly operated in the thermal noise limit. So we will see both noise limits and see what is the expression for the signal to noise ratio.

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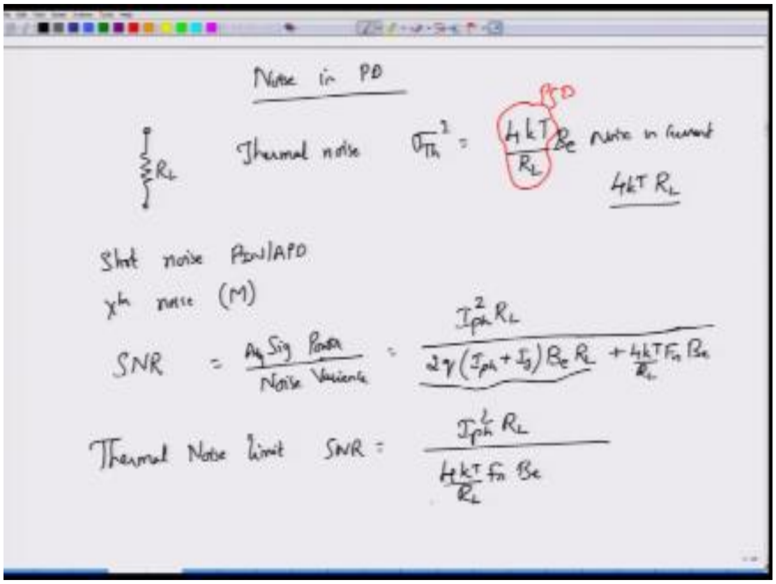

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$$\text{Thermal Noise Limit} \quad SNR = \frac{I_{ph}^2 R_L}{\frac{4kT}{R_L} F_n B_e}$$

So first consider the thermal noise limit, okay which happens when your RL is quite small so the first term the short noise term can be neglected with respect to the second term, right. So this can happen when the photo current Iph is quite small, okay. So in the thermal noise limit we assume that the short noise variance is small compared to the thermal noise and the SNR becomes Iph^2 RL/4kT/RL FnBe and you can see here that the thermal noise limit.

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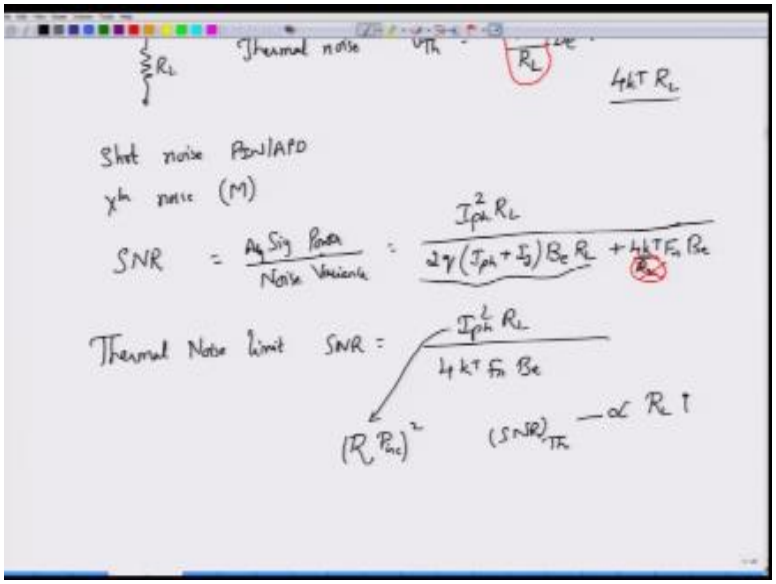

I multiply it by RL in the denominator and the numerator but I forget to remove this RL from this expression, okay so it is 4kT/RL FnBe but there is no RL originally in the short noise variance, correct short noise variance is 2qIph+Id.Be so when I multiply RL by numerator and denominator I should have remove this I did not remove it so now I am going to remove it, and then go back to the expression for thermal noise limit there is, so this RL is not here so let me remove this one from the consideration, okay.

So this is your thermal noise limit the signal to noise ratio, how is the photo current generated photo current is generated because of the responsively of the photo diode times the optical power that is incident, so Iph^2 is basically R into incident optical power square and what you observe here is that the signal to noise ratio when you are looking at the thermal noise limit, okay is directly proportional to RL in fact you can increase the signal to noise ratio by increasing the value of Rl, do not be so surprised because increasing RL only reduces the thermal noise variance, right.

Because it is inversely proportional to RL therefore thermal noise variance reduces, however as long as even this reduce thermal noise variance is larger than the short noise variance you are still in the thermal noise limit, do not increase RL too much otherwise you will go to the short noise limit, right because RL increases and the short current noise and it decreases completely. So this thermal noise in the thermal noise limit you can actually use a larger register RL and increase the signal to noise ratio only of two up to a certain limit of course, okay. let us go to the short noise limit, and in the short noise limit what happens is that.

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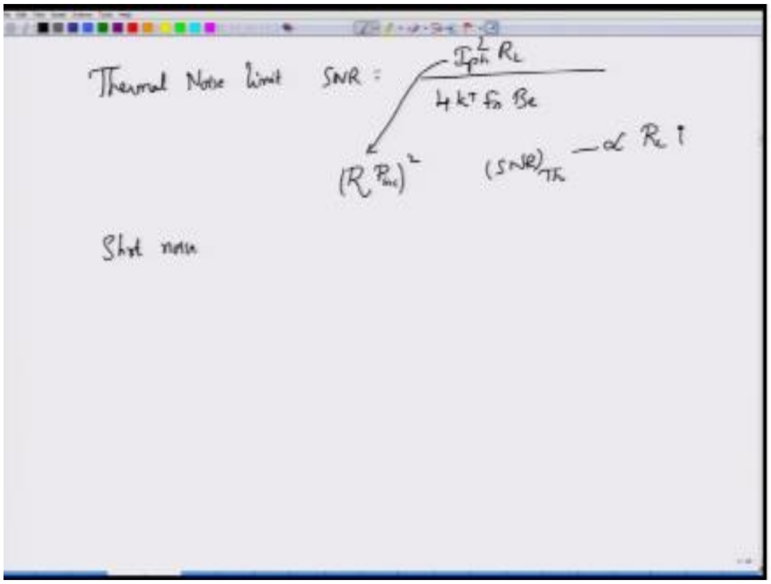

Thermal Noise Limit SNR $= \frac{I_{ph}^2 R_L}{4kT F_n B_e}$

$(R P_{in})^2$

$(SNR)_{Th} \propto R_L \uparrow$

Shot noise

The thermal noise can be almost neglected the load register RL is quite high, okay. How do you obtain a large RL physically you can take a big register and put it there, okay but in practice you implement what is called as trans impedance amplifier using an op amp, an op amp have a very, very high input impedance, okay. So because of that reason you are PIN photo detector circuits are normally short noise limited they are not usually thermal noise limited, okay. So in the short noise limit what happens is that.

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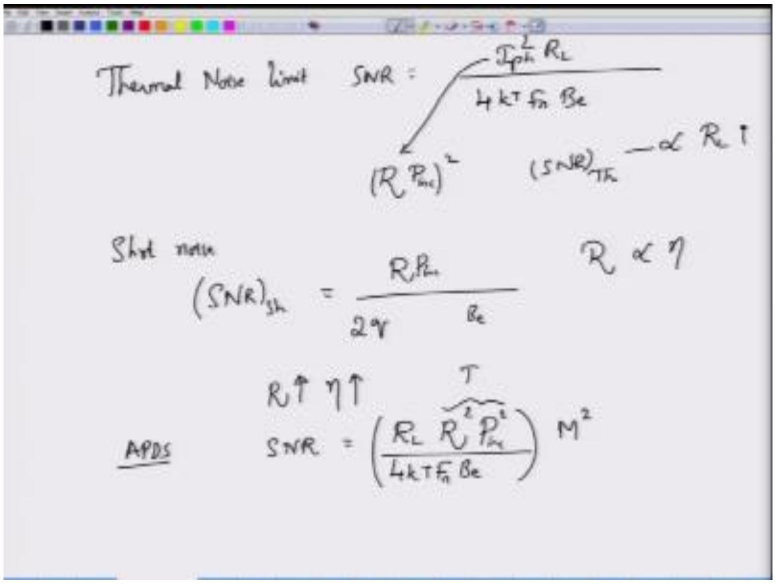

Thermal Noise limit $SNR = \frac{I_{ph}^2 R_L}{4kT F_n B_e}$

$(R P_{inc})^2$

$(SNR)_{Th} \propto R_L \uparrow$

Shot noise

$(SNR)_{sh} = \frac{R P_{in}}{2q B_e}$

$R \propto \eta$

$R \uparrow \ \eta \uparrow$

APDS $SNR = \left(\frac{R_L \overbrace{R^2 P_{in}^2}^{T}}{4kT F_n B_e} \right) M^2$

The signal to noise ratio in the short noise limit is given by the photo current which is R.Pin incident which is the optical power incident time RL/2qIphRL.Be, right. So here you can see that RL in a numerator cancels with RL in the denominator. So that is gone right and iph in the denominator cancels with one of the iph in the numerator because iph is R x pn right so this also cancels out here and the square terms goes away and then what you get is an nice short expression in which you can see that the signal noise ratio is actually proportional to responsivity.

But if you remember responsivity itself was proportional to quantum efficiency, right so it is η q/hc something son it is directly proportional to quantum efficiency. So the short noise signal to noise ratio can be increased by increasing the responsivity or equivalently increasing η and it does not really care what is the load register that you have kept as long as that load register is quite large.

So that thermal noise limit is all gone so this is the short noise limit for a pin structure however for apd is the situation is slightly more complicated because for apd is the signal to noise ratio in the short noise limit is actually dependent on Rl it is given by as I said apd is actually operate in the thermal noise limit so it is proportional to Rl but you actually if you put in the multiplicative terms and write down the short noise variants terms there you will see that this is given by Rl R^2 4Kt the excess noise factor fn times Be.

This entire thing gets multiplied by m^2 so this r x p incident is nothing but the photo current iph so this is iph^2 but what is se here is that there is a multiplication factor m^2 which is impacting over here so if you want to increase snr you can increase snr by operating the apd with a larger multiplicative gain, however larger multiplicative gain also have some effect on the excess noise factor F right.

Because we have seen that f versus m curves for a apd depend also of course on the additional factor k but in case you look at it there are actually linearly increasing so far larger value of k so it is actually a optimum value of the multiplication factor m which make the snr highest for a given optical input power you can maximize this multiplicative gain okay which you can obtain by going back to the short noise variants and then differentiating that with respect to m and setting the result in expression to 0 you will end up with a cubic polynomial equation okay which gives you the value of optimum value of the multiplicative term m.

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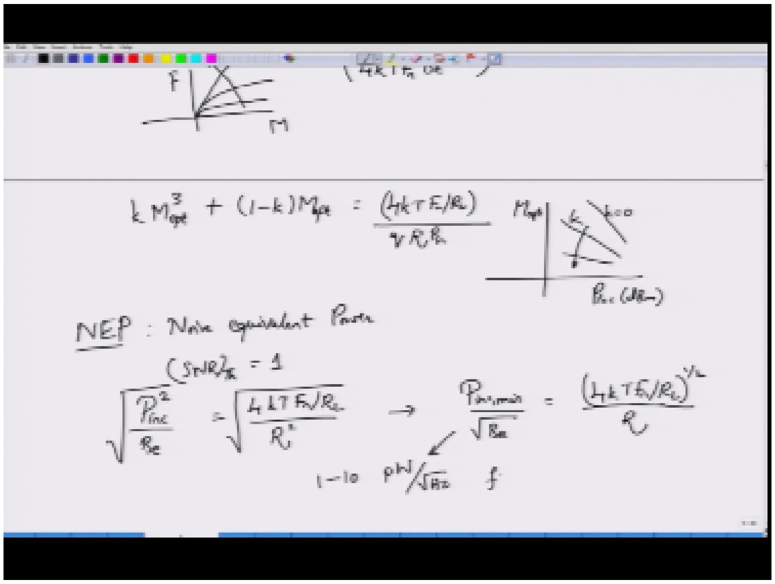

$$k M_{opt}^3 + (1-k) M_{opt} = \frac{(4kTF_n/R_L)}{q R_0 R}$$

NEP : Noise equivalent Power

$(SNR)_{th} = 1$

$$\sqrt{\frac{P_{inc}^2}{B_e}} = \sqrt{\frac{4kTF_n/R_L}{R^2}} \rightarrow \frac{P_{inc,min}}{\sqrt{B_e}} = \frac{(4kTF_n/R_L)^{1/2}}{R}$$

1–10 pW/$\sqrt{Hz}$ f

So you get this term by going to the equation for the apd noise variants and then differentiating that with respect to m and setting it to 0 I will just leave it an exercise for you to do that it is a straight forward exercise the resulting expression a little bit tds but you get here this is the thermal noise variants correct for kt fn/rl thermal noise variants divided by qR pin right I am of course neglecting the dark current in all this expressions and you can actually see that if you plot the incident optical power okay and m in terms of the optimum value of m for different optical power see which are measured in dbm you will actually see that the optimum value of m decreases as incidents optical power increases okay.

So this happens for k = 0 and these a e the values for increasing value of k remember that k is the ionization parameter dependent constant k okay so this is for the apd there is one additional factor by which we are additional term by which w quantify the photodiodes and this additional quantity is called as the noise equivalent power noise equivalent power (NEP),noise equivalent power is the situation where the noise variance actually is equal to optical input power, or you can as you start reducing the optical input power you will come t a situation where the signal to noise ratio becomes exactly equal to 1, at which point signal power is equal to noise power, this actually no improvement in to the, or there is no way in which you can distinguish signal and noise out there.

Because they both are equal to each other, and the minimum optical power at which the SNR goes to 1 is called as a noise equivalent power okay, so this noise equivalent power can be obtained by setting the SNR, in the thermal noise limit equal to 1, right? So if you do that what you get? you have the incident optical power P^2inc/Be=4KTFn/R_l / R^2 where R being the responsibility, right so you can take this R to the numerator to the left hand side, and you get Iph^2,right.

And that’s what I have just re written the equation, and then what we do is? We take the square root on both sides, so if I take the square root I get the minimum optical power, okay per square root of band width, okay this should be equal to 4KTFn/ Rl /R$^{1/2}$, this fellow which s on the left hand side, which represents the minimum optical power, per square root band width, is measured in terms of, typically in terms if pea co watt per square root Hz.

The typical values are anywhere between 1to10, all those some very good photo diodes have reach the range of fw/√hz okay.

If you know what is the total bandwidth of your circuit, then take the square root of the bandwidth of the circuit multiply it by the NEP that tells you what is the minimum optical power that is necessary in order to make signal noise ratio equal to 1, and then you can choose the optical power that you are going to input the detector to be, lets sat 10 times NEP 20 times NEP, and so on, okay.

So if I have a bandwidth of 1GHz then I take the square root of that 1 GHz bandwidth, multiplied by this 1 pea co what / √hz, , term so I get the minimum optical power that is required to make Snr=1, but of course you never operate, your circuits with Snr=1, you operate Snr of sat 10,100, and then you can scale up the required optical power.

So that s the essential use of NEP associated with NEP is one more factor called directivity, so if directivity simply one time 1/NEP, and it simply tells you that if this is the optical power weather this optical power will be detected or not, so it tells you how the noise is greater than signal.

So if you get a very small NEP then the directivity actual be very high , okay so this completes our discussion on noise in photo diodes and in the next module we are going to discuss the optical sources and then we will come back to combine optical sources, modulators and the WDM components we have discussed. And then put in the receiver at the end and then analyze what will happen to the systems, okay, thank you very much.

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